Debt Maturity and Investor Heterogeneity*

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Abstract

This paper studies how investor heterogeneity impacts equilibrium debt maturity. The optimal issuance strategy combines both long- and short-term debt. Long-term debt contains default risk, but hedges against intermediate downturns. Short-term debt provides repayment commitment but must be rolled and becomes risky during downturns. Issuing multiple debt maturities spreads the cost of these risky claims to investors most willing to hold risk at different points in time. Our model implies that debt ownership concentration can impact firm value and investment decisions.

Keywords: heterogeneous agents, debt financing, general equilibrium, investment, cost of capital, debt dilution

JEL Codes: D21, G11, G12, G31, G32, E22

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I. Introduction

This paper studies optimal debt maturity when investors have heterogeneous beliefs as in Geanakoplos (2003). This class of general equilibrium models with incomplete markets has been used to explain various phenomenon such as endogenous leverage cycles (Geanakoplos (2009)); asset price movements (Simsek (2013) and He and Xiong (2012)); the effects of financial innovation on prices, investment, and global capital flows (Fostel and Geanakoplos (2012), Fostel and Geanakoplos (2016); Fostel, Geanakoplos, and Phelan (2020); and the distribution of agent wealth (Cao (2018)). The insights of this literature have not been applied to corporate debt markets despite significant dispersion in investor and analyst forecasts for corporate earnings. For example, Diether, Malloy, and Scherbina (2002) find that higher analyst forecast dispersion is associated with lower equity returns, and forecast dispersion is a proxy for differences in opinion about a stock. In addition, De Franco, Vasvari, and Wittenberg-Moerman (2009) find significant analyst dispersion for buy versus sell recommendations in corporate debt markets. Following these empirical observations, our approach is to model investor heterogeneity through belief differences in the expectation of firm cash flows.[1]

We isolate the role of investor heterogeneity on firms' optimal debt maturity choice facing a standard trade-off between long- and short-term debt. Firms raise debt to finance long-term investment projects. Long-term debt contracts mature when investment proceeds are realized while short-term debt contracts mature before investment proceeds materialize. The optimal debt maturity decision balances the different costs and benefits of long- versus short-term debt.

[1]In addition, multiple investors are generally required to meet the debt financing needs of large corporate borrowers (Dass and Massa (2014), and Caglio, Darst, and Parolin (2019).

Long-term debt costs derive from default risk. The benefit is that firms can hedge against the cost of rolling over debt during bad times. By contrast, short-term debt must be rolled over, and is particularly costly during bad times when repayment likelihoods fall. The benefit of short-term debt is that the withdrawal threat renders it safe *ex ante* conditional on rollover *ex post*. Intermediate default is costly for firms because it prevents them from realizing potential profits, hence short-term debt has near-term repayment commitment. The main innovation is that debt is issued not to a representative investor, but to capacity constrained investors with heterogeneous beliefs over different states.

We derive three results. The main result is that the optimal maturity choice combines long- and short-term debt rather than relying on one debt maturity profile. The intuition is that firms minimize financing costs by splitting claims on cash-flows into different debt maturities to cater to heterogeneous investors' demand for risky securities.[2] Optimistic investors place a higher probability on debt repayment than pessimists and require less compensation to hold risk at each point in time. Thus, for a given level of investment, debt issuance costs are minimized by issuing risky debt claims to optimists *across time* rather than issuing to both optimists and pessimists *at a given point in time*. This mechanism gives rise to the optimal maturity profile that includes both long and short-term debt.

The novel insight is that investor heterogeneity endogenously generates different marginal buyers for debt securities at each point in time. Hence, a debt financing strategy of all short-term claims is not equivalent to one using all long-term claims. For a given level of investment, issuing debt via a single maturity requires a greater portion of capital to be sourced from pessimists

[2]Our catering result for debt maturity compliments Allen and Gale (1991) for the issuance of debt and equity claims, and Fostel and Geanakoplos (2012) for the tranching of cash-flows through financial innovation.

compared to issuing through multiple maturities. By extension, the marginal cost of capital is lower when a high marginal cost claim priced by a relative pessimist in one period is substituted to a different maturity claim whose price is determined by a more optimistic marginal buyer in a different period. Equilibrium in the debt market is achieved when substitution across claims of different maturities results in equivalent expected marginal costs of issuing debt.

The second result is that using a combination of short- and long-term debt is robust to allowing for debt dilution whereby future short-term debt dilutes the value of existing long-term claims. Specifically, the baseline model assumes that different claims are secured with separate collateral or equivalently through covenants that prevent dilution of long-term debt holders. We relax this assumption in an extension and show that a combination of both short- and long-term debt is optimal in both cases. The reason is that the firm's maturity choice becomes a margin of adjustment through which the impact of debt dilution on equilibrium prices is undone. In particular, each investor is willing to pay a lower price for long-term claims that they rationally anticipate will be diluted. Facing a steeper marginal cost of capital curve for long-term debt, the optimal response is to reduce the amount of long-term debt and substitute into more short-term debt. The substitution between the different debt claims allows a more optimistic investor who by definition is less concerned with dilution to price long-term debt in equilibrium and effectively un-do the initial price effect of dilution. Thus, equilibrium with multiple debt maturities in heterogeneous investor economies is robust to the impact of debt dilution.

The third and final result shows that in the representative agent economy with debt dilution, equilibrium is never a combination of both long- and short-term debt, and must be either all long term or all short term. Why do long- and short-term debt never coexist in the representative investor economy with dilution? The reason is a combination of equal seniority of claims and a wedge between the expected pricing of the claims stemming from the firm operating under limited liability and potentially a different repayment expectation than the representative investor. Equal seniority implies that the recovery values of long- and short-term debt are the same. Thus, the state-contingent payout streams of a sequence of short-term claims and the long-term claim are identical and are priced equivalently by the representative investor. However, limited liability on the firm side implies that firms do not internalize the impact of default on their issuance paths, even when firms and the representative investor agree on state probabilities. The pricing difference between the firm and the representative investor is magnified if there is any disagreement between the firm and the investor. Hence, the common price that the investor is willing to pay for the different maturities never equates the marginal costs from the firm's perspective, and a combination of long- and short-term debt is never an equilibrium outcome.[3]

What determines equilibrium in the representative investor economy with dilution? The optimal debt issuance strategy depends on the *relative optimism between the firm and the representative investor*. In particular, the firm issues all long (short) term debt when it is the relative pessimist (optimist). The reason is that long-term debt insulates the firm from intermediate debt repricing, a form of insurance against future shocks. This insurance option is priced by a relatively optimistic investor, which makes the insurance inexpensive from the firm's perspective. Alternatively, when the firm is the relative optimist it does not value the insurance option as much when it is priced by a relatively pessimistic investor. Furthermore, an optimistic firm does not view the down-state at time 1 as a likely event against which to insure.

Equilibrium in the representative investor economy with dilution when investors and firms

[3]More formally, the combination of equal seniority and dilution introduces a non-convexity into the economy.

share a common prior turns out to be a form of the maturity rat race studied in Brunnermeier and Oehmke (2013). In particular, any candidate allocation with both long- and short-term claims without dilution unravels to only short-term funding once you allow for dilution. The reason is the following: with dilution, the recovery values of long- and short-term debt are the same. Without dilution, long-term recovery values are higher than short-term claims. Hence, with dilution, long-term claimants "subsidize" short-term claimants' recovery rates because fewer claims need to be issued in bad states to rollover debt. From the firm's perspective, higher recovery rates on short-term claims incentivize additional short-term debt issuance. The recovery rate on short-term debt with dilution converges *in the limit* to the recovery rate of short-term debt without dilution. Thus, firms have the incentive to keep issuing short-term debt until short-term funding is the only type of funding issued.[4]

Lastly, we isolate the role of investor heterogeneity by first showing that the model nests as a special case the representative investor economy where the investor can either have the same common belief as firms or a different belief. If the unique equilibrium in either representative agent case entails a combination of long- and short-term debt, then investor heterogeneity would not be the driving force. Instead, we show that: 1) Modigliani-Miller holds under the common belief setting without dilution, so maturity is irrelevant. However, with dilution, the wedge between the expected pricing of claims implies that the marginal cost of the two securities are never equivalent across the firm and investor, which is a necessary condition for both types of securities to co-exist. 2) When the representative investor has a different belief from firms, she

[4]Notice that our unraveling result is a subset of the one derived in Brunnermeier and Oehmke (2013). Unraveling in our setting occurs only if the fraction of long-term debt in the liability structure is less than 1. In their model, unraveling occurs even if the share of all long-term debt is 1.

will always price one of the two securities more favorably than what firms expect and the equilibrium depends on who is more optimistic, the firm or the investor. Hence, the economy features either all long- or all short-term debt. In sum, conditional on Modigliani-Miller failing where debt maturity matters, equilibrium with a representative investor never combines both long- and short-term debt.

Taken together, the model makes several predictions regarding the impact of investor heterogeneity and debt ownership on firm debt liability choices and firm value. First, firms whose debt is held by a diverse (heterogeneous) set of investors will tend to issue a combination of debt maturities. By contrast, firms whose debt is concentrated among similar (homogeneous) or a certain type of investor will tend to rely on either long- or short-term debt. Second, financing costs are lower and investment opportunities are higher when firms use a combination of debt maturities compared to relying on a single maturity. Therefore, more dispersed debt ownership is associated with lower costs and more investment compared to concentrated debt ownership. Third, because debt maturity does not impact firm value when investors are homogeneous, debt maturity should not impact firm investment decisions when debt ownership is concentrated among similar investor types or held by a single investor. Finally, the model predicts that relative optimism between firms and investors impacts debt maturity choice. More optimistic (pessimistic) firms will issue more short-term (long-term) debt.

The organization of the paper is as follows: The related literature is below. Section II introduces the model, agents, the different debt contracts considered. Section III presents examples to highlight the main mechanism. Section IV derives the general solution to the model with the main analytical results. Section V discusses the empirical relevance of the model's predictions. Section VI includes a discussion of robustness to alternative assumptions, and Section VII concludes. All proofs that are not obvious from the text are included in the Appendix.

Related literature

Our work is related to the incomplete markets and heterogeneous agent framework of Allen and Gale (1991). They argue that firms cater to investor needs by splitting claims on cash flows into debt and equity securities. We endogenize investment and introduce a role for maturity to affect firm value. Many elements of our framework are similar to Bisin, Clementi, and Gottardi (2022) who show how to integrate production into general equilibrium analysis with incomplete markets. They analyze the efficiency properties of equilibrium. Our emphasis is on the role that debt maturity plays in the optimal behavior of the firm.

Debt maturity plays a non-trivial role in traditional corporate finance models with private information (Flannery (1986), Diamond (1991), Diamond (1993)). We show that a multi-period debt issuance strategy is optimal *without* intermediate liquidation along the equilibrium path, which is the focus of Diamond (1991).[5] Moreover, these papers do not consider investor heterogeneity. Debt maturity plays a role in optimal contracting models of Hart and Moore (1994), Hart and Moore (1995), Hart and Moore (1998), but repayment paths are either the fastest or slowest, never a combination of the two. Zwiebel (1996) shows that multiple repayment paths

[5]Liquidity risk is a necessary condition to generate an equilibrium with both long- and short-term claims in Diamond (1991). Proposition 2 in his paper shows that all short-term financing is used when short-term claims are always honored in a non-terminal state.

are possible but only for the *most risky firms* for whom debt is a possible financing source. Chong, Oehmke, and Zhong (2019) study debt maturity based on unobserved cash-flow risk.[6]

More broadly, the paper relates to debt maturity models that focus on lack of commitment (Brunnermeier and Oehmke (2013), and He and Milbradt (2016)) and debt overhang (Diamond and He (2014)). Our unraveling result with a representative investor and debt dilution shows that the rat race incentives in Brunnermeier and Oehmke (2013) extend beyond models with multiple creditors and the inability to commit to an aggregate maturity structure. We show that the underlying force generating maturity shortening in a single investor model with full commitment is debt-dilution, which is the common force with their model.

II. Model

A. Time and uncertainty

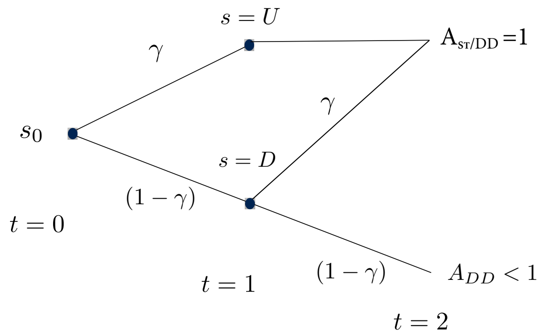
The model is a three-period production economy with incomplete asset markets. Time is denoted $t = \{0, 1, 2\}$. Uncertainty is given by a tree of state events $s \in S$ with root s_0, intermediate states $s \in S$ that take values $\{U, D\}$, and a set of terminal nodes denoted $S_T = \{UU, UD, DU, DD\} \subset S.$ Let state realization U be up or a "good" state and D be down or a "bad" state. There is a single durable consumption good available in the economy at $t = 0$, which is the numeraire.

The only uncertainty in the model is an aggregate shock that affects output at $t = 2$. The parameter A_{s_T} captures the effect of the shock to production. The expected value of the shock is

[6] See also Bolton and Scharfstein (1990), Bolton and Scharfstein (1996) for additional models on the optimality of debt in the presence of agency frictions.

FIGURE 1

Economy State Tree



conditional on the information revealed at $t = 1$. We assume for simplicity that good news at $t = 1$ resolves uncertainty at $t = 2$ and there is no shock: $A_{UU} = A_{UD} = 1$. Bad news at $t = 1$ raises uncertainty at $t = 2$ about the ability of the firm to repay debts, akin to "scary bad news" in Geanakoplos (2009). Specifically, there is no shock at terminal node $s = DU$, but there is a shock at terminal node $s = DD$, $A_{DD} < A_{DU} = 1$.[7] Figure 1 depicts the economy's state tree.

B. Debt contracts

There are two types of debt contracts in the economy. Short-term debt matures after one period and long-term debt matures after two periods. All debt contracts are non-contingent and pay zero-coupons. For simplicity, we normalize the repayment value of each contract to 1.

[7] Note that this uncertainty structure is the same as the simplification made in the continuous time version of Diamond and He (2014). They assume in example 2 of their paper that asset volatility is state contingent. Specifically, $\sigma_H = \epsilon > 0 = \sigma_L$ where σ_i is asset volatility conditional on state i. Clearly uncertainly is resolved when $\sigma_L = 0$.

Let the quantity of debt issued at any state and time be q_{s/S_T}. The quantity of long-term debt issued at $t = 0$ is denoted q^ℓ and the market price denoted by p^ℓ. Let q^ς denote the quantity of short-term debt issued at $t = 0$ and q_s^ς, $s = \{U, D\}$ denote short-term debt issued at $t = 1$. The prices of short-term debt at $t = 0, 1$ are respectively $p^\varsigma, p_U^\varsigma,$ and p_D^ς. Following much of the literature, we assume equal seniority between short- and long-term debt.

C. Agents

We first describe the firm's objective followed by the investors' problem.

1. Firm

There are a large number of identical price taking firms, which allows us to focus on a representative firm. A manager (equity claimant) operates the firm with access to a two-period decreasing returns to scale production technology. The production function is denoted by $f(I; \alpha, A_s) = A_s I^\alpha,\ \alpha < 1$, where I is the amount of capital the manager raises at time 0 and puts into production. We follow Diamond (1991) and assume the firm has no cash endowment, does not generate cash flow at $t = 1$, and that new promises issued at $t = 1$ do not increase the initial investment I.[8]

The firm maximizes expected profits, choosing investment and the maturity of the debt

[8] Alternatively, one could assume that there is an extreme form of limited commitment at the interim date in which no cash flows can be verified at a reasonable cost so debt repayments cannot come from cash flow. Under this alternative, cash flow is independent of how the project is financed. The debt maturity mix will affect the investment cost that generates the cash flow, and management would still issue the types of debt securities to minimize these cost as in our model.

contracts it issues subject to limited liability.[9] Let ρ denote the portion of debt that is raised long-term, $\rho = \frac{p^\ell q^\ell}{I}$, and let γ denote the probability of good news. Firms maximize over γ, which is the true state probability of their production process. Formally, the firm maximizes the following problem:

$$\text{(1)} \qquad \begin{cases} \max\limits_{I,\rho} \prod = & \Sigma_s \overline{\gamma}_s \max\left(A_s I^\alpha - q^\ell - q_s^\varsigma, 0\right) \\ \text{s.t.} & I = p^\ell q^\ell + p^\varsigma q^\varsigma \\ & 0 \leq \rho = \frac{p^\ell q^\ell}{I} \leq 1 \end{cases}$$

where $\overline{\gamma}_s$ is the product of state probabilities along each path from $t = 0 \to 2$.

At $t = 1$ the firm decides whether to roll over expiring short-term claims. If so, the firm repays short-term debt holders by raising $p_s^\varsigma q_s^\varsigma = q^\varsigma$, $s = \{U, D\}$. For simplicity, we assume the firm can always repay debts conditional on good news at $t = 1$ and $p_U^\varsigma = 1$. Bad news raises uncertainty about repayment and $p_D^\varsigma < 1$ if the firm defaults at $t = 2$.

The price of short-term debt issued at time 0 depends on if there is default at $t = 1$. If there is no default, short-term debt is initially risk-free and $p^\varsigma = 1$. If there is default, then $p^\varsigma < 1$. Shareholders receive no payments in default while creditors liquidate its assets and obtain δI,

[9] We restrict the analysis to debt and abstract away from equity without loss of generality for maturity structure. Incorporating equity would not change the maturity results. The reason is the same as Allen and Gale (1991). The firm will split the claims it issues into debt and equity. The most optimistic investors will purchase equity, the next most optimistic will purchase long-term debt at time 0 and risky short-term debt at time 1. Hence, allowing for equity issuance would simply push down the set of investors that purchase risky debt in each period.

where $\delta < 1$.[10] We make the following assumptions on parameters to keep the problem interesting and highlight the importance of supply considerations in our mechanism.

Assumption 1 Parameter restrictions governing default

1. To ensure there is fundamental credit risk in the economy, let $A_{DD} < \alpha$.

2. To prevent liquidation at $t = 1$, let $\delta < \overline{\delta}$, where $\overline{\delta}$ is defined at the end of Lemma 1 in Appendix A.

As will become clear, the first condition ensures that no debt is mechanically risk free. The second condition implies that intermediate liquidation needs to be costly so that it is not optimal to issue risky short-term debt claims at time 0. One could allow for liquidity risk at time 1, but it would obfuscate the mechanism driving optimal maturity structure relative to existing demand-side theories.[11]

Lemma 1 *(Short-term rollover). Given assumption 1, if a funding strategy with short-term debt exists, it is unconditionally rolled over $t = 1$, and $p^{\varsigma} = 1$.*

The proof of Lemma 1 also demonstrates that a risky short-term funding strategy, while always feasible, is not always optimal relative to a long-term funding strategy. Hence, the parameter restrictions are not necessary, but we maintain them to distinguish the mechanism.

Using Lemma 1, we can simplify the firm's problem. Specifically, using the roll over

[10]The fractional recovery is a stand-in for any number of reasons why liquidation is costly: for example, bankruptcy costs or inefficient second-best use of assets, etc.

[11]See Diamond (1991) Proposition 2 on the necessity of liquidity risk in models with private information.

condition for short-term debt, $q^{\varsigma} = p_s^{\varsigma} q_s^{\varsigma},\ s = \{U, D\}$, the firm becomes:

$$\max_{I,\rho} \prod = \gamma \left(I^{\alpha} - \frac{\rho I}{p^{\ell}} - \frac{(1-\rho)\, I}{p^{\varsigma}} \right) + (1-\gamma)\, \gamma \left(I^{\alpha} - \frac{\rho I}{p^{\ell}} - \frac{(1-\rho)\, I}{p_D^{\varsigma}} \right)$$

where the constraints $\rho = \frac{p^{\ell} q^{\ell}}{I}$ and $q^{\varsigma} = p_D^{\varsigma} q_D^{\varsigma}$ are substituted to write the problem in terms of choice variables I and ρ.

If an interior maximum for ρ exists, the first-order necessary conditions with respect to I and ρ, respectively, are

$$\alpha I^{\alpha-1} \left[1 - (1-\gamma)^2\right] = \frac{\rho \left[1 - (1-\gamma)^2\right]}{p^{\ell}} + (1-\rho) \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right] \tag{2}$$

$$\frac{\left[1 - (1-\gamma)^2\right]}{p^{\ell}} = \gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}. \tag{3}$$

Equation (2) is the first order condition *w.r.t* investment, I. It says that the marginal product of capital in states where the firm makes profits–which occurs with probability $1 - (1-\gamma)^2$–must equal the maturity-weighted expected marginal cost of debt. The first term on the right is the expected marginal cost of long-term debt, $\frac{\left[1-(1-\gamma)^2\right]}{p^{\ell}}$. This is the probability that the firm repays claims and retains equity, $1 - (1-\gamma)^2$, divided by the bond price. The marginal cost is weighted by the fraction of long-term debt in the liability structure, ρ. The second term on the right is the marginal cost of a sequence of short-term bonds, $\frac{1}{p_U^{\varsigma}} \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right]$. With probability γ, the sequence of claims is risk-free ($p_U^{\varsigma} = 1$). With probability $\gamma\,(1-\gamma)$, the firm pays a higher short-term borrowing cost, $p_D^{\varsigma} < p_U^{\varsigma} = p^{\varsigma} = 1$, per bond to roll over existing claims. The fraction of short-term debt in the liability structure is $(1-\rho)$.

Equation (3) is the first order condition *w.r.t* to maturity, ρ. It says that the expected marginal costs of long- and short-term bonds must be equal in any equilibrium where both long- and short-term claims are issued in non-zero quantities. In other words, for a given level of investment, the firm issues a liability structure that minimizes its expected marginal cost. The expected marginal costs of long- and short-term claims must be equal in any interior optimum; otherwise, the firm will issue the lower of the two. It will be useful to combine (2) and (3) as a single necessary condition in terms of either maturity:

$$\alpha I^{\alpha-1} = \frac{1}{p^{\ell}}, \tag{4}$$

$$\alpha I^{\alpha-1}\left(1-(1-\gamma)^2\right) = \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right]. \tag{5}$$

The above equations are also the first order conditions for either corner solution obtainable by substituting $\rho = 0$ or 1 into the firm's maximization problem. This shows that for any investment amount, I, the firm's problem boils down to choosing the issuance strategy with the lowest marginal cost.

We now turn to the investors' problem to derive debt prices which determine the optimal funding strategy.

2. Investors

There exists at $t = 0$ a continuum of uniformly distributed investors with unit mass, $h \in H \sim U[0,1]$, each of whom is endowed with the durable consumption good in all non-terminal states. Investors are risk-neutral, expected utility maximizers that consume at $t = 2$, and do not discount the future. We assume investors have different priors.[12] The uniform

[12]We discuss below the importance of different priors for the generality of our results.

distribution is not critical to the optimal issuance strategy we derive. It simply allows one to rank investors according to the likelihood each places on the subsequent state being good, denoted by h. In fact, any continuous and well-behaved distribution in which investor types are monotonic will suffice.[13] Assuming alternative distributions on beliefs would quantitatively affect how debt is priced, but will not change the outcome that different investors may price debt in different periods, depending on how much long- versus short-term debt is issued. What is important is that investors value debt differently–through some form of heterogeneity–and no single investor can finance the economy's financing needs.[14]

Investors also have access to a risk-less storage technology and form portfolios consisting of cash and debt securities issued by firms. Investor preferences are given by:

(6) $$U^h\left(x_{UU}, x_{UD}, x_{DU}, x_{DD}\right) = h^2 x_{UU} + h\left(1-h\right) x_{UD} + \left(1-h\right) h x_{DU} + \left(1-h\right)^2 x_{DD}.$$

Given debt prices, $\left(p^{\ell}, p^{\varsigma}, p^{\varsigma}_U, p^{\varsigma}_D\right)$, each investor, $h \in H$, chooses cash holdings, $\left\{x^h, x^h_D, x^h_U\right\}$, debt holdings, $\left\{q^{\ell,h}, q^{\varsigma,h}, q^{\varsigma,h}_U, q^{\varsigma,h}_D\right\}$, and final period consumption decisions, $\left\{x^h_s\right\}$, $s \in S_T$, to

[13]The reason, as will become clear, is that it will always be beneficial to substitute a marginal claim from one period to another if there are no risky claims issued in the other period. In essence, as long as there is an atomistic investor in some period that wants to hold a risky claim, she will price that claim more favorably than increasingly less willing investors in another period.

[14]Heterogeneity would be immaterial if a single investor could finance the entire capital raise and the investor with the highest marginal valuation for firm assets would fund all investment, which is equivalent to assuming a representative agent.

maximize utility given by (6) subject to the budget set defined by:

$$
\begin{aligned}
B^h\left(p^\ell, p^\varsigma, p_U^\varsigma, p_D^\varsigma\right) \quad = \quad & \Big\{\left(x, x_D, x_U, q^\ell, q^\varsigma, q_U^\varsigma, q_D^\varsigma, x_s\right)_{h\in H} \\
& x^h + p^\ell q^{\ell,h} + p^\varsigma q^{\varsigma,h} = 1, \\
& x_U^h + p_U^\varsigma q_U^\varsigma = 1 + q_U^\varsigma d_U\left(q^\varsigma\right) + x^h \\
& x_D^h + p_D^\varsigma q_D^\varsigma = 1 + q_D^\varsigma d_D\left(q^\varsigma\right) + x^h \\
& x_s^h = x^h + x_{U,D}^{h_1} + q^\ell d_s\left(q^\ell\right) + q_s^\varsigma d_s\left(q_s^\varsigma\right), s \in S_T \Big\}. \qquad (7)
\end{aligned}
$$

Each investor uses their initial cash endowment to purchase either type of debt security at $t = 0$. The endowment received at $t = 1$ and cash carried forward are used to purchase short-term debt at $t = 1$ or held for final consumption.[15] Investors carry all unused cash forward for consumption.

Given that all debt repayments are normalized to \$1, optimists purchase the debt security with the highest yield, *i.e.,* long-term debt at $t = 0$ and risky short-term debt at $t = 1$.[16] Relative pessimists will hold safe short-term debt at $t = 0$ and remain in cash at $t = 1$. We now solve for the debt delivery or repayment functions, $d_s(\cdot)$, that determine the expected repayment values and subsequent debt pricing.

[15]Note that the firm is using the proceeds from time 1 short-term debt issuance to repay its initial time 0 short-term liabilities.

[16]Given the equal seniority assumption, a basic no-arbitrage argument shows that long-term bonds in the secondary market conditional on $s = D$ must be priced equivalently to short-term bonds issued on the primary market. Hence, long-term bonds will not change hands at $t = 1$ given that optimists already possess them at $t = 0$.

3. Interpretation of Investor Beliefs

The assumption of different priors generates heterogeneity among investors' marginal utility of consumption.[17] For any modeling strategy, firm investment choices will be identical in different economies when equilibrium debt prices are the same. Thus, one can always construct an alternative formulation of preferences and endowments in which the market clearing prices are the same as those obtained in our formulation.[18] For example, one could assume investors differ in a measure of risk aversion; have different endowments across states, which produces different marginal utilities across states; or have different degrees of "patience" and derive the same conclusions as our model. The differences between investor beliefs may arise due to differences in expectations about firm betas, firm cash flows to macroeconomic shocks, or simply differences in expectations about macroeconomic events. Recent survey evidence in Giglio, Maggiori, Stroebel, and Utkus (2021a), Giglio, Maggiori, Stroebel, and Utkus (2021b) shows that disagreement among investors widens during times of stress and persists through events and short-term cycles. Moreover, their findings suggest that investor disagreement leads to differences in portfolio allocations and trading patterns.

[17]The assumption of different priors is not *per se* necessary, though it makes the modeling simpler. For example, agents may have different posteriors even with common priors if one drops the "common knowledge assumption." In this case, agents cannot disagree in the limit and must converge after a finite number of iterations (see Geanakoplos and Polemarchakis (1981) . Therefore, our model would be one of the short- or medium-run. That said, Fryer, Harms, and Jackson (2019) provide a microfoundation for persistent belief differences that arises from the way agents interpret signals and store the information.

[18]Back (2010) provides a formal proof of the asset pricing equivalence between heterogeneous and homogeneous agents with state-dependent utility models.

D. Debt repayment

The key friction in our model is that agents cannot be coerced to repay debts. As in Fostel and Geanakoplos (2016), the pledgeable value of the firm serves as the payment enforcement mechanism. Specifically, creditors have the right to seize firm assets up to the value of the promise but nothing more. "Collateral" in our economy will be the firm itself, and can be thought of as the physical assets it produces from its investment decision. To isolate the role that investor heterogeneity plays in the model, we first solve a version of the model where we prevent short-term debt from diluting the value of existing long-term debt. Section IV considers a version where short-term debt dilutes the value of long-term debt when both types of claims have equal seniority in bankruptcy.

To prevent dilution, we assume that the firm pledges separate assets as collateral for long- versus short-term debt.[19] Long-term debt holders receive ρ portion of firm assets financed with long-term debt, and the remaining $(1-\rho)$ portion of firm assets are financed with short-term debt. Accordingly, debt delivers either the full \$1 promise or there is default and creditors receive their pro-rata share of firm output:

$$\begin{cases} d_{DD}\left(q^{\ell}\right) = & \frac{\rho A_{DD} I^{\alpha}}{q^{\ell}}, \text{ long-term recovery} \\ d_{DD}\left(q^{s}_{D}\right) = & \frac{(1-\rho) A_{DD} I^{\alpha}}{q^{s}_{D}}, \text{ short-term recovery} \end{cases} .$$

[19] For example, the firm can pledge separate machines, or can use real-estate for one type of debt and accounts receivable for another, etc. An alternative interpretation is that long-term debt contains negative pledge clauses that prevent firms from raising additional capital that may jeopardize the firm's ability to repay.

One can solve out for the endogenous values, $I^{\alpha}, \rho, q^{j}, j = \ell, \varsigma$, using the definition of $\rho = \frac{p^{\ell} q^{\ell}}{I}$ and the F.O.C (4) for $I^{\alpha-1}$ to obtain

$$\begin{cases} d_{DD}\left(q^{\ell}\right) = & \frac{A_{DD}}{\alpha} \\ d_{DD}\left(q^{\varsigma}_{D}\right) = & \frac{A_{DD}}{\alpha}\left(\frac{p^{\varsigma}_{D}\gamma+\gamma(1-\gamma)}{\gamma+\gamma(1-\gamma)}\right) \end{cases} . \tag{8}$$

Equation (8) makes clear that the value of long-term claims is tied to firm fundamentals and independent of short-term debt. Intuitively, large (adverse) technology shocks, low A_{DD}, leave fewer assets available for investors to recover. In addition, investor recovery is higher for more productive firms, low α. Furthermore, note that $\frac{p^{\varsigma}_{D}\gamma+\gamma(1-\gamma)}{\gamma+\gamma(1-\gamma)} < 1$, which means that recovery value of short-term claims issued when uncertainty rises at time 1 is less than the recovery of long-term claims. The reason is that the price to issue risky debt at $s = D$ rises and additional claims must be issued to avoid default.

Given (8), investors price claims based on their repayment expectation and perfect competition implies that marginal claimants break even in expectation. Formally, the marginal long-term claimant at $t = 0$ determines:

$$1 - (1 - h_0)^2 + (1 - h_0)^2 \, d_{DD}\left(q^{\ell}\right) = p^{\ell}. \tag{9}$$

Likewise, the marginal short-term claimant at $t = 1$ determines risky short-term debt prices:

(10) $$h_1 + (1 - h_1)\, d_{DD}\left(q^{\varsigma}_D\right) = p^{\varsigma}_D.$$

Market clearing for risky debt at time 0 and 1 determines the endogenous marginal claimants in equations (9) and (10) above. Specifically, total investor demand for long-term securities at time 0 is given by the total endowment of capital among investors that purchase long-term debt, $(1 - h_0)$. The supply of long-term bonds firms issue is price times quantity, $p_\ell \times q_\ell$, and market clearing implies:

(11) $$\frac{1 - h_0}{p^\ell} = q^\ell, \text{ Long-term debt market clearing.}$$

The short-term debt market must clear at both $t = 0, 1$. Assumption 1 ensures all short-term debt issued at time 0 is rolled over at time 1 and risk-free with $p^{\varsigma}_0 = 1$. Hence, investors $\underline{h} < h < h_0$ use their endowment to purchase q^{ς} short-term debt at time 0. Together with the measure of investors in the long-term debt market, $(1 - h_0)$, the measure of total capital invested in the firm at time 0 is $(1 - \underline{h})$. At time 1, firms issue $q^{\varsigma}_U = q^{\varsigma}$ in the good state and $q^{\varsigma}_D = \frac{q^{\varsigma}}{p^{\varsigma}_D}$ in the bad state to repay time 0 short-term claimants. Therefore, conditional on $s = U$, all short-term debt is risk free. However, conditional on $s = D$, the face value of short-term debt due at time 2 must rise to clear the market. There will be a marginal buyer, h_1 indifferent to holding risky short-term debt and cash. To understand the amount of available supply of capital at time 1, note that the total measure of investors at time 0 from whom firms raise capital, $1 - \underline{h}$, have only their new endowment at time 1. The total capital available to the remaining measure of investors, $\underline{h}$, is the sum of their two endowments or $2\underline{h}$ in total. Thus, the total market capital available at time 1 is $1 + \underline{h}$. However, the demand for risky short-term debt at time 1 will come exclusively from the time 0 investors that already own debt, $1 - \underline{h}$; none of the additional capital from the measure of investors $\underline{h}$ is needed. The reason is that firms will not raise more at time 1, from the measure $1 - h_1$, than what is owed to the measure of short-term creditors $h_0 - \underline{h}$. Hence, the marginal risky short-term debt holder at time 1, h_1, will never be more pessimistic than $\underline{h}$ and $h_1 > \underline{h}$. Therefore, market clearing for risky short-term debt market at time 1 is given by

$$\frac{1 - h_1}{p_D^{\varsigma}} = q_D^{\varsigma}, \text{ Short-term debt market clearing.} \tag{12}$$

We can now formally state the definition of a competitive equilibrium.

Definition 1 *A competitive equilibrium is, given a price vector* $\left\{p^{\ell}, p_t^{\varsigma}\right\}_{(t=0,1)}$*, investors choose portfolios of cash and debt holdings,* $\left\{x_0, q^{\ell}, q_t^{\varsigma}\right\}_{(t=0,1)}$*, that maximize* (6) *subject to* (7)*, firms choose a liability structure and investment outlay,* $\{\rho, I\}$*, that maximize the firm program in* (1)*, and debt markets clear at* $t = 0, 1$ *through* (9) *and* (10).

To sum up, the model consists of seven equations: (2), (3), (8)-(12) that solve for the seven endogenous variables: $\left\{I, p^{\ell}, q^{\ell}, p_D^{\varsigma}, q_D^{\varsigma}, h_0, h_1\right\}$.[20] Before characterizing equilibrium, we present a series of numerical examples to highlight the role investor heterogeneity plays in determining debt maturity.

[20]Technically, $\underline{h}$ is an eighth endogenous variable which is the marginal buyer at time 0 holding risk-less short-term debt. The equation to pin it down is $I = (1 - \underline{h})$–the total supply of loans used to raise funds equals the total endowment of all agents that purchase debt.

TABLE I

Representative Investor Equilibrium

	p_D^{ς}	p^{ℓ}	q_D^{ς}	q^{ℓ}	MC	I	ρ	V	Π	$d\left(q_D^{\varsigma}\right)$	$d\left(q^{\ell}\right)$
Maturity mix	.8685	.9663	.1588	.1428	1.034	.276	.50	.357	.065	.562	.625
Long-term	-	.9663	-	.2850	1.034	.276	1	.357	.065	-	.625
Short-term	.8685	-	.3177	-	1.034	.276	0	.357	.065	.562	-

III. Examples of investor beliefs and debt maturity choice

We use the following parameters throughout the examples below and include the results in Table I: $A_{DD} = .5$, $\alpha = 0.8$, $\gamma = 0.7$.

Example 1 *(Representative investor and debt maturity).*

The special case of the general model is the representative investor, which is obtained by substituting $h_t = \gamma$. This case considers a global believe common to all agents, which is the classic starting point of most models.

Long-term only–$\rho = 1$: To compare outcomes across economies, define the value of the firm output as $V_{\gamma}^{\ell} = I^{\alpha}$ and expected profits as $\Pi_{\gamma}^{\ell} = \left(1 - [1-\gamma]^2\right)\left(V_{\gamma}^{\ell} - q^{\ell}\right)$, where the superscript ℓ denotes the long-term debt regime and the subscript γ denotes the representative investor's γ. Long-term equilibrium debt prices from (9) are
$1 - (1-\gamma)^2 1 + (1-\gamma)^2 d_{DD}\left(q^{\ell}\right) = p^{\ell}$, and the debt recovery function is defined as in (8).

Short-term only– $\rho = 0$: Setting $h_1 = \gamma$ in (10), risky short-term debt pricing at $t = 1$ becomes $\gamma(1) + (1-\gamma)\, d_{DD}\left(q_D^{\varsigma}\right) = p_D^{\varsigma}$, and the short-term debt recovery rates are also given by (8).

Interior maturity mix–$0 < \rho < 1$: From the firm's perspective, equations (4) and (5) show that the interior allocation can be boiled down to either corner solution. Hence, all that matters is the relative pricing across the different funding regimes. Since the price of long- and short-term claims must be equivalent for a representative investor, issuing any combination of the two is no different than issuing at either corner. Table I shows this equivalence for a random $\rho = 0.5$. Hence, the Modigliani-Miller (M-M) Theorem holds for a global belief.

Example 2 *(Debt maturity with heterogeneous investors).*

Now consider the general heterogeneous investor case described in Sub-section 2 where $h_t \neq \gamma$. This example shows that heterogeneity breaks the M-M Theorem because the expected marginal cost of a sequence of short-term claims need not be equivalent to a long-term claim.

Long-term only–Relative to Example 1, the only change we make is that risky long-term debt prices are determined by (9). Table II contains the equilibrium values for the same parameters as Example 1.

Notice the value of firm output and profits are higher under the heterogeneous investor regime: $\Pi_h^\ell = .0665 > \Pi_\gamma^\ell = .0650$ and $V_h^\ell = .3629 > V_\gamma^\ell = .3570$. This is because the marginal investor's prior is higher in equilibrium than the common belief, $h = .7182 > \gamma = .70$, which results in higher debt prices, expands firm budget sets, and leads to more investment and output.[21]

Short-term only–Relative to example 1, the only change we make is that risky short-term debt prices are determined by (10). The solution to this economy is given in Table II.

The reason why the short- and long-term funding regimes are no longer equivalent is because the same investor will never price risky short- and long-term debt equivalently. In particular, the market-clearing conditions, (11) and (12), equate the supply of risky debt with

[21]Clearly, one could generate an outcome for which the homogeneous equilibrium would dominate the heterogeneous long-term candidate solution by setting $\gamma > h$.

TABLE II

Optimal debt maturity in a Heterogeneous Investor Equilibrium

	p^s_D	p^ℓ	MC	I	ρ	h_0	h_1	V	Π	$d(\cdot)$
Maturity mix	.9495	.9879	1.012	.3083	.5752	.8227	.8690	.3900	.0710	.6159
Long-term	-	.9702	1.030	.2817	1	.7182	-	.3629	.0665	.625
Short-term	.8786	-	1.032	.2800	0	-	.7199	.3612	.0657	.5286

demand at each point in time. Maturity is irrelevant only when all debt is priced through a common expectation: $h_0 = h_1$. However, this is not feasible because there is more uncertainty at $s = D$ than at $s = 0$.

Example 3 *(Optimal debt maturity with heterogeneous investors).*

Our final example shows that using a combination of long- and short-term debt is optimal in the heterogeneous investor example. Table II contains the solution to the model with an interior maturity choice and both corner solutions. All risky debt prices are higher in the interior solution than in either corner solution because both marginal buyers, $h_0 = .8227$ and $h_1 = .8690$, are more optimistic than their counterparts in the long- and short-term only candidates. Consequently, firm budget sets expand, allowing for more investment and production. By substituting a portion of the risky debt issuance from one period into two distinct periods, the firm is able to place its risky claims to investors in both periods that are more willing to hold risk. This is the essence of the substitution benefit that dispersed debt maturities have in incomplete market economies with heterogeneous investors that is not present in homogeneous investor economies.

IV. Analytical results

The examples in the previous section show that investor heterogeneity allows debt maturity to impact firms' debt-maturity decisions. This section formalizes those results. We first show why debt maturity structure is irrelevant (and thus M-M holds) in the representative investor economy but is relevant in the heterogeneous investor economy. The second result shows that issuing a combination of long- and short-term debt is always optimal in the heterogeneous agent economy. As a corollary, short-term debt is always rolled over. There is no liquidity risk, in contrast to Diamond (1991). This helps to highlight our new result is driven by investor heterogeneity. Finally, we show that allowing for debt dilution is *de-facto* irrelevant for both the representative and heterogeneous investor economies.

Proposition 1 *(Debt Maturity Irrelevance for Representative Investors).*

Consider an economy where a representative investor prices all risky claims with the same state probability as the firm, γ*. Let an equilibrium allocation be given by* $\mathcal{E}^{\varsigma} = (I^*, q^{\varsigma *}, q^{\varsigma *}_{s})$ *for a given price vector* $(p^{\varsigma}, p^{\varsigma}_{s})$ *where short-term debt is the only financing source. Similarly, define an equilibrium allocation in a long-term debt economy by* $\mathcal{E}^{\ell} = \left(I^*, q^{\ell *}\right)$*, for a given long-term price vector,* $\left(p^{\ell}\right)$*. Lastly, for the mixed-maturity economy let the equilibrium allocation be given by* $\mathcal{E}^{\mathcal{I}} = \left(I^*, q^{\ell *}, q^{\varsigma *}, q^{\varsigma *}_{s}\right)$ *for the corresponding price vector,* $\left(p^{\ell}, p^{\varsigma}, p^{\varsigma}_{s},\right)$*. The three economies are equivalent,* $I^*|_{\mathcal{E}^{\varsigma}} = I^*|_{\mathcal{E}^{\ell}} = I^*|_{\mathcal{E}}\ \forall \rho$*.*

The argument proceeds by noting that equivalent operating scale, I^*, implies equivalent first-order conditions in (4) and (5). Then direct substitution of the representative investor debt pricing and debt delivery functions shows that the marginal costs implied by the candidate corner solutions are always equivalent. Hence, the long- and short-term debt regimes are the same.

Moreover, without dilution, the recovery values of long- and short-term debt are the same in the interior maturity regime as the respective corner solutions. Thus, any combination of long- and short-term debt, $0 < \rho < 1$, equates the marginal cost of each debt maturity with its marginal cost in the corner solution. Thus, debt maturity is irrelevant. The details are provided in Appendix A.

We now show that maturity is generally relevant in the heterogeneous investor economy. To clarify that the mechanism is driven by investor heterogeneity rather than liquidity risk, we first characterize when there is no liquidity risk and all short-term debt is endogenously safe. Short-term debt is "safe" at $t = 0$ if and only if it is unconditionally rolled over at $t = 1$. The rollover condition states that profits must be greater than or equal to zero after repaying both long- and short-term debts:

$$I^{\alpha} \geq q^{\ell} + q^{\varsigma}_{D}. \tag{13}$$

We focus on the down-state without loss of generality because the firm is always better off conditional on $s = U$ than $s = D$.[22] The price of short-term debt at $t = 0$ must be $p^{\varsigma} = 1$ if the firm is not liquidated.

Lemma 2 *(Short-term Debt Rollover). For a given quantity vector, Q, short-term debt at $t = 0$ is safe if and only if*

$$\alpha \frac{1 - \rho}{(1 - \alpha\rho)} \leq \frac{p^{\varsigma}_{D}}{p^{\ell}} < 1. \tag{14}$$

[22]Note that state probabilities, γ, do not factor in this decision because the firm only retains profits when it fully repays all debts, both of which occur with probability $1 - (1 - \gamma)^2$.

Lemma 2 says that there must be a balance between risky debt prices in order for both maturities to exist with safe short-term debt issued at $t = 0$. If all risky debt prices were equivalent, then firms would choose only short-term debt because it can be financed at the risk free rate conditional on $s = U$. Alternatively, if long-term debt prices are significantly higher than short-term debt prices, which would obtain as $\rho \to 0$, then the refinancing cost of the short-term component will be too high and the benefit of insulating against higher roll-over costs will dominate.

We now examine the conditions under which (14) holds. Let ρ approach 1 and debt maturity tilt more towards long-term debt. Substitution into long-term debt does two things: 1) by market clearing, long-term debt prices must fall as more pessimistic marginal investors finance the investment; and 2) it reduces the amount of short-term debt that needs to be rolled over conditional on $s = D$, leading to higher short-term debt prices. Thus, the price ratio is increasing in ρ and approaches 1, and the rollover condition always holds. Intuitively, defaulting on a small portion of short-term debt becomes more costly as firm profits and production rest more heavily on long-term debt. This is a standard disciplining argument where a little short-term debt keeps the firm from defaulting on all of its claims.

Alternatively, let ρ approach 0 and maturity tilt more towards short-term debt. Substitution into short-term debt causes the price ratio to fall. The relationship thus requires that the ratio of risky debt prices remain greater than a measure of firm production, α. Consider the two limiting cases.

Case 1: $\alpha \to 0$

Short-term debt is always rolled over as $\alpha \to 0$ for any positive price ratio. Intuitively, α

measures the return to a unit of capital input.[23] Higher marginal returns to production mean that firms generate large revenues from small capital raises. Therefore, firms can repay all claims even if they are all short-term and require rollover.

Case 2: $\alpha \to 1$

Production is linear when $\alpha = 1$. An interior optimum requires that both risky debt prices be equal to 1, hence risk free. However, debt is never risk free $\forall\, A_{DD} < 1$. Therefore, an interior optimum cannot exist when $\alpha = 1$.

We can define an upper bound, $\bar{\alpha} < 1$, for any candidate interior choice, $0 < \rho < 1$, by combining cases 1 and 2 for $\rho \to 0$ and the fact that Lemma 2 holds for $\rho \to 1$. Otherwise, Lemma 2 holds for all parameters in the model. We now state the main proposition of the paper.

Proposition 2 *(Debt Maturity Relevance for Heterogeneous Investors).*

Let investors have heterogeneous beliefs given by $h \in [0, 1]$. *Define a vector of debt quantities for the candidate interior solution with* $0 < \rho < 1$ *as*

$Q := \left(q^{\ell}, q_0^{\varsigma}, q_s^{\varsigma}\right) \in R^3_{++},\ s = \{U, D\}$.

1. *In general, debt maturity is always relevant:* $I^*|_{\mathcal{E}^{\mathcal{I}}} \neq I^*|_{\mathcal{E}^{\varsigma}} \neq I^*|_{\mathcal{E}^{\ell}},\ \forall \rho \in [0, 1]$.

2. *For* $\alpha < \bar{\alpha}$, *the optimal debt maturity strategy is characterized by* $Q := \left(q^{\ell}, q_0^{\varsigma}, q_s^{\varsigma}\right) \in R^3_{++},\ s = \{U, D\}$. *Moreover, if* $\nexists\, \bar{\alpha} < 1$ *that violates Lemma 2, then the optimal debt maturity strategy is always* $Q := \left(q^{\ell}, q_0^{\varsigma}, q_s^{\varsigma}\right) \in R^3_{++},\ s = \{U, D\}$.

Proposition 2 states that firms' choice of debt maturity structure always matters in the heterogeneous agent economy. Compared to a representative investor economy, the marginal

[23] Firm marginal productivity rises as α falls because $0 < I < 1$.

investors pricing risky claims are no longer the same, and their beliefs are no longer given by γ. The equivalence between the three different maturity structures in Proposition 1 requires that $h_0 = h_1 = \gamma$, which is no longer a necessary outcome with heterogeneous investors.

The second part of the proposition says that it is generally optimal to issue both long- and short-term claims to heterogeneous investors when short-term claims are rolled over. The intuition is the following: Moving from all long-term to an interior issuance strategy has two benefits. First, the first marginal short-term claim issued at $s = D$ is effectively priced risk free by optimist $h = 1$. This marginal short-term claim is always cheaper than the marginal long-term claim it replaces, which is a consequence of the fact that $h_1 \approx 1$ when $q_D^s = \epsilon \approx 0$. Moreover, the difference in pricing the last marginal long-term claim at $t = 0$ and the first marginal claim at $t = 1$ is why any continuous distribution of types, for which prices are monotonic in types, gives rise to the interior optimum. Hence, the uniform distribution is not special, rather it is especially easy to see the economics. Second, there is more competition among long-term investors for fewer long-term claims at $t = 0$, leading to higher long-term debt prices. Thus, the firm is substituting high marginal cost long-term debt issued at $t = 0$ for low marginal cost short-term debt rolled over at $t = 1$ until, in equilibrium, expected marginal costs are equivalent.

The mechanism works in both directions. Consider moving from issuing all short-term to an interior allocation. In doing so, the firm replaces a high marginal cost short-term claim at $t = 1$ with a low-cost long-term claim financed by an optimist at $t = 0$. Hence, substituting replaces anticipated rollover costs with nearly risk free long-term debt until, in equilibrium, the expected marginal costs are equivalent.

In sum, for a given investment opportunity, firms minimize financing cost through a combination of long- and short-term claims. Interior debt issuance strategies ease firm budget sets and allow for more investment and production than either all long- or all short-term issuance strategies.[24]

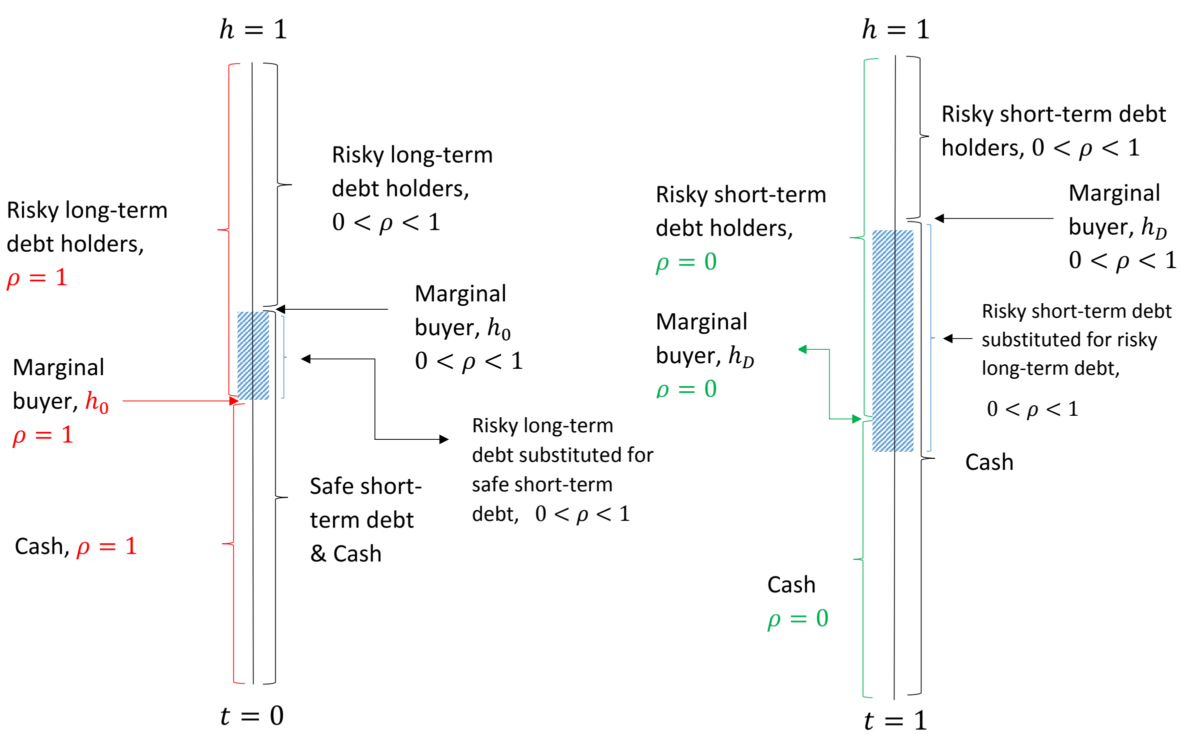
Figure 2 is a graphical representation of the costs and benefits of substituting from either corner solution to an interior solution. Using a combination of debt maturities concentrates fewer total long-term claims to investors most willing to hold risk at $t = 0$ than a maturity with no short-term debt. Concurrently, the debt issuance needed to ensure short-term debt is rolled over at $t = 1$ is also more concentrated to investors with higher willingness to hold risk than if the firm only issued short-term debt.

Proposition 2 establishes that with incomplete markets and default, heterogeneity between investors is sufficient for multiple debt maturities to trade. It turns out, in the current model, that heterogeneity between investors is also necessary. Consider a generalization of the representative investor economy where the investor's belief is different from firms'. As before, firm beliefs are given by γ, and the investor's belief is given by $\omega\gamma,\ 0 < \omega < \frac{1}{\gamma}$. This belief specification captures

[24]Note that multiplicity is not an issue in our model. The reason is that both short-term debt prices and quantities are free to adjust at time 1, and there is sufficient investor capital to fund the firm (there is always a mass of investors at time 1, h_1, holding cash in equilibrium). In models with funding runs, the supply of claims is assumed to be fixed at the roll-over stage. In other words, if an agent issues q claims at $1, it cannot raise more than q claims to rollover. This prevents agents from offering additional claims at higher returns. In our model, the firm can issue more claims at time 1 if prices fall. All agents who wish to hold risk at time 1, $h > h_1$, stand to gain in expectation at lower prices (higher implied interest rates). Therefore, if any individual investor, h, threatens to withdraw and not roll over at time 1, the firm can offer a higher return to the next most pessimistic investor, $h_1 - \epsilon$, to raise the needed capital by simply offering a higher expected return. The higher equilibrium return on risky claims at time 1 then benefits all remaining investors that agree to rollover. Thus, any individual investor deviation to withdraw *increases* the payout to rollover rather than decreasing it, which is what one needs for multiplicity.

FIGURE 2

Marginal buyer regimes



the global belief case considered in Proposition 1, where $\omega = 1$, as a special case. More generally, $\omega < (>)\, 1$ indicates that the representative investor is less (more) optimistic than the firm. We show in the appendix that firms will never choose a combination of long- and short-term debt. The reason is that the investor will never price long- and short- equivalently. Essentially, short-term recovery rates are a function of how much short-term debt firms issue, which depends on the firm's belief. However, the investor, with a different belief over how likely repayment is, prices short-term debt. This difference drives a wedge between the price of short-term debt relative to long-term debt whose value in recovery is determined by the fundamental, $\frac{A_{DD}}{\alpha}$, over which there is no disagreement. Hence, the price of the two claims are never equal, which is necessary for them to both co-exist.

Proposition 3 *Consider a representative investor whose belief over the probability of repayment differs from firms, and there is fundamental default in the economy given by* $\frac{A_{DD}}{\alpha} < 1$. *The optimal debt issuance strategy is* $q^{\ell} = 0$ *if* $\omega < 1$ *and* $q^{\varsigma} = 0$ *if* $\omega > 1$.

The equilibrium maturity structure is either all short- or all long-term debt depending on whether the firm is a relative optimist compared to the investor. If the firm is a relative optimist, then the price of locking in long-term debt at time 0 is too low (implied interest rates are too high) compared to the expected refinancing cost of short-term debt tomorrow because the firm does not view the down-state as a likely outcome. By contrast, when the firm is a relative pessimist, it views the down-state tomorrow as more likely. Hence, the cost of locking long-term debt today is relatively attractive.

Debt dilution: Heterogeneous Investor Economy

In this section we allow for short-term debt to dilute long-term debt and study its impact on the optimal maturity choice. For continuity, we focus first on the economy with heterogeneous investors and then representative investors. We will show that debt dilution is *de-facto* irrelevant, meaning that the optimal debt maturity structure that emerges in equilibrium remains unchanged with or without dilution. This is robust to representative and heterogeneous agents. Dilution is irrelevant when investors are heterogeneous because the firm undoes the effect of dilution through its maturity choice–the maturity mix is simply a margin of adjustment through which the firm maintains its efficient investment level. However, when facing a representative investor, dilution introduces a non-convexity into the firm's maximization problem that prevents both long- and short-term debt from co-existing; but by definition, dilution does not not impact the corner solutions. Hence, dilution impacts the *equilibrium characterization* in the representative investor framework, but does not impact the firm's efficient investment choice.

We first augment the model to allow short-term debt to dilute long-term debt by relaxing the assumption that the firm collateralizes the different debt maturities with distinct and separate assets. Instead, we assume the firm collateralizes all debt of equal seniority with the full value of the firm. Let variables with hats define objects in the economy with debt dilution. Conditional on rolling over short-term debt, both long- and short-term debt deliveries are equivalent on a per-claim basis, $\hat{d}_s\left(q^{\ell}\right) = \hat{d}_s\left(q^{\varsigma}_s\right)$, or generally $\hat{d}_s\left(\cdot\right), s \in S_T$. Hence, the debt recovery function for any possible maturity can be written as follows:

(15) $$\hat{d}_{DD}\left(\cdot\right) = \begin{cases} 1, & s \neq DD \\ \frac{A_{DD} I^{\alpha}}{q^{\varsigma}_D + q^{\ell}}, & s = DD \end{cases},$$

where setting either $q^{\varsigma}_{D} = 0$ for long-term only funding or $q^{\ell} = 0$ for short-term only funding. Then, using the same process described in the baseline economy, one can express debt recovery as a function of fundamental parameters and a dilution factor:

$$\hat{d}_{DD}\left(\cdot\right) = \frac{A_{DD}}{\alpha} \underbrace{\left(\frac{p^{\varsigma}_{D}}{\left(1-\rho\right) p^{\ell} + \rho p^{\varsigma}_{D}} \right)}_{\text{dilution factor}} < \frac{A_{DD}}{\alpha}. \tag{16}$$

The dilution factor represents the loss to long-term claimants due to rolling over short-term debt when prices fall at time 1; there is no dilution if short-term debt is not issued, $\rho = 1$, and as $\rho \to 0$, the recovery value converges to the short-term recovery value, $\hat{d}_{DD}\left(q^{\varsigma}_{D}\right) = \frac{A_{DD}}{\alpha}\left(\frac{p^{\varsigma}_{D}}{p^{\ell}}\right) = d_{DD}\left(q^{\varsigma}_{D}\right)$ in (8).[25] The dilution factor is decreasing in ρ and long-term debt is "maximally diluted" in the interior solution as $\rho \to 0$. Hence, the recovery value of long-term (short-term) claims monotonically increases (decreases) in ρ.

Proposition 4 *Consider any diluted recovery value,* $\hat{d}_{DD}\left(\cdot\right)$*, defined by* (16) *and corresponding* $d_{DD}\left(\cdot\right)$ *without dilution defined by* (8)*. In any interior debt financing strategy where* $Q := \left(q^{\ell}, q^{\varsigma}_{0}, q^{\varsigma}_{s}\right) \in R^{3}_{++}$, $s = \{U, D\}$*, the following hold:*

- $d_{DD}\left(q^{\ell}\right) > \hat{d}_{DD}\left(\cdot\right) > d_{DD}\left(q^{\varsigma}_{D}\right)$,
- $\frac{1-(1-h_0)^2+(1-h_0)^2\left[d_{DD}\left(q^{\ell}\right)\right]}{p^{\ell}} > \frac{1-(1-h_0)^2+(1-h_0)^2\left[\hat{d}_{DD}(\cdot)\right]}{p^{\ell}}$.
- $\frac{h_1+(1-h_1)\hat{d}_{DD}\left(q^{\varsigma}_{D}\right)}{p^{\varsigma}_{D}} > \frac{h_1+(1-h_1)d_{DD}\left(q^{\varsigma}_{D}\right)}{p^{\varsigma}_{D}}$

[25] From (3), $\frac{p^{\varsigma}_{D}}{p^{\ell}} = \frac{p^{\varsigma}_{D}\gamma+\gamma(1-\gamma)}{\gamma+\gamma(1-\gamma)}$.

Proposition 4 formalizes the relationship between recovery values in economies with and without debt dilution. Relative to an economy without debt dilution, the expected recovery value of long-term claims are lower and the expected recovery value of short-term claims are higher, and investors price claims accordingly. An immediate implication of Proposition 4 is that any interior candidate equilibrium with dilution entails less long-term debt.

Corollary 1 *Let ρ be the equilibrium portion of long-term debt in a non-dilution economy where recovery is determined by* (8) *and $\hat{\rho}$ be the equilibrium portion of long-term debt in a dilution economy where recovery is determined by* (16). *Then, $\hat{\rho} < \rho$ and $\hat{q}^{\varsigma}_D > q^{\varsigma}_D$.*

A consequence of corollary 1–that maturity choice is a margin of adjustment for firms to price movements–and the fact that total investment is a linear combination of long- and short-term debt issuance, is that debt dilution does not impact firm value (investment and profits).

Proposition 5 *(Investor Heterogeneity and Debt Dilution Equivalence). Let $\hat{I}$ be the investment allocation from the solution to program* (1) *with diluted debt deliveries given by* (16) *and corresponding interior maturity choice $\hat{Q} := \left(q^{\ell}, q^{\varsigma}_0, q^{\varsigma}_s\right) \in R^3_{++}$, $s = \{U, D\}$, $\forall q \in \hat{Q} > 0$ and risky debt prices, $\left(\hat{p}^{\ell}, \hat{p}^{\varsigma}_s\right)$. Let I^* be the investment allocation from interior maturity choice $Q^* := \left(\hat{q}^{\ell}, \hat{q}^{\varsigma}_0, \hat{q}^{\varsigma}_s\right) \in R^3_{++}$, $s = \{U, D\}$, $\forall q^* > 0$ with risky debt prices $\left(p^{\ell^*}, p^{\varsigma^*}_s\right)$ given non-diluted debt recovery values in* (8). *Then, $I^* = \hat{I}$.*

The intuition is the following: Debt secured by distinct assets does not create any inherent value over and above what can be achieved by securing a mix of securities. For example, securing long-term debt with exclusive assets prevents short-term debt from extracting value from those assets in default. This reduces the value of short term claims relative to a contract that allows short term claimants to dilute long-term debt holders. Consequently, firms adjust their maturity structure by issuing more long-term debt in response to the new prices investors are willing to pay. Market clearing implies that the marginal buyers of non-diluted long-term debt are less optimistic, while the marginal buyers of short-term debt are more optimistic than in the economy without debt-dilution. Hence, the equilibrium prices adjust so that they are unchanged across the two economies.

An immediate implication of Propositions 2 and 5 is that the interior maturity mix remains the optimal debt issuance strategy.

Corollary 2 *The optimal liability choice with debt dilution is interior:*

$\hat{Q} := \left(\hat{q}^{\ell}, \hat{q}_0^{\varsigma}, \hat{q}_s^{\varsigma}\right) \in R^3_{++}$, $s = \{U, D\}$, $\forall q \in \hat{Q} > 0$.

Debt dilution: Representative Investor Economy

Our next result shows that in representative investor economies, debt dilution has no impact on firm values (i.e. they are no better or worse-off). However, debt dilution does introduce a non-convexity into the firm's maximization problem that prevents the interior allocation from emerging as an equilibrium. In fact, any interior allocation without dilution converges to short-term only funding once dilution is permitted, a result reminiscent of the maturity rat-race in Brunnermeier and Oehmke (2013). However, investment and profitability are unaffected because both corner solutions remain viable equilibrium strategies.

Proposition 6 *(Debt Dilution under Representative Investors). Consider a global belief economy where the representative investor prices all risky claims with the same state probability as the firm, γ. Let allocations in an economy with short-term debt only, long-term debt only, and a mix of the two debt maturities be respectively given by $\hat{\mathcal{E}}^{\varsigma} = (I, q^{\varsigma}, q_s^{\varsigma})$, $\hat{\mathcal{E}}^{\ell} = \left(I, q^{\ell}\right)$, and*

$\hat{\mathcal{E}}^{\mathcal{I}} = \left(I, q^{\ell}, q^{\varsigma}, q^{\varsigma}_{s}\right)$. *In presence of debt dilution, equilibrium is characterized exclusively by either all short-term or all long-term debt. Moreover, maturity structure is de-facto irrelevant: as* $\hat{I}|_{\mathcal{E}^{\varsigma}} = \hat{I}|_{\mathcal{E}^{\ell}}$, $while \nexists \hat{I}|_{\hat{\mathcal{E}}^{\mathcal{I}}}$.

The intuition is the following: debt dilution is irrelevant in the respective corner solutions where only one type of security is issued. Thus, the equivalence between the two allocations follows from Proposition 1. In any interior solution, both debt securities must be priced according to the representative investor's outside option of holding cash, and those prices must imply the equivalent marginal funding costs for the firm. Moreover, because of equal seniority, the recovery value of both debt maturities is the same, which implies that the payout vectors of long- and a sequence of short-term claims are identical. However, given the representative investor's pricing of claims, the implied marginal costs of the two securities from the firm's perspective are equivalent only when the recovery values are different or when the two securities are risk free (see Appendix A for proof). The reason is that the firm operates under limited liability and only considers states in which they have an equity claim—the upstate—while the investor cares also about recovery in default in the down state. Hence, the two sides never price the two securities the same in expectation.

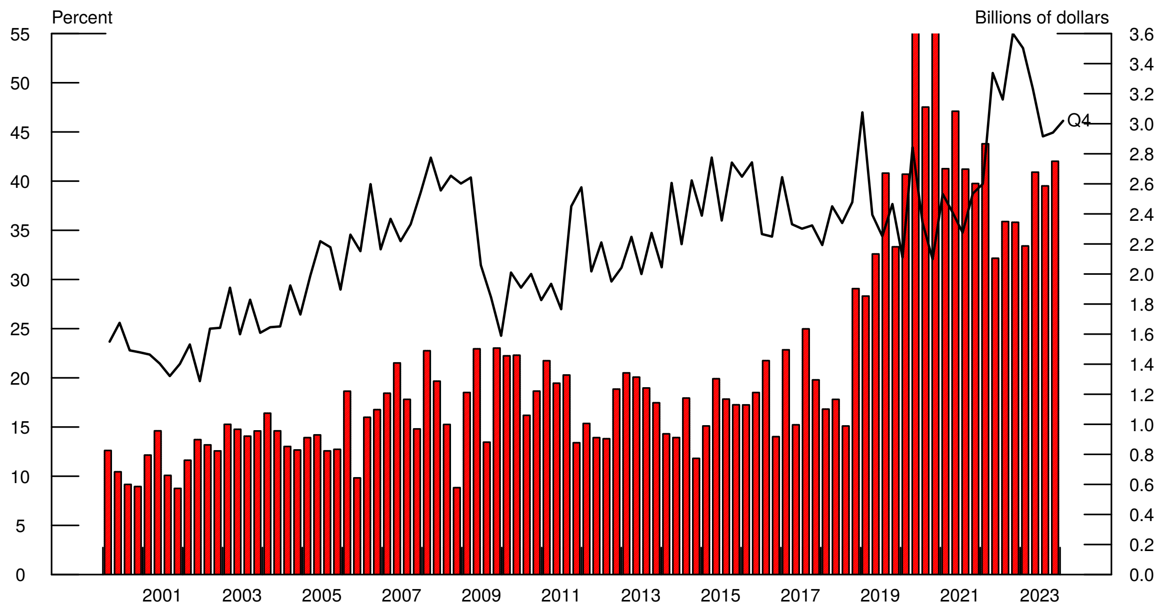
In fact, from Proposition 4, starting from any interior allocation where $0 < \rho < 1$ with separately secured claims, the recovery value must fall for long-term and rise for short-term claims in the presence of dilution. Hence, the firm prefers to issue even more short-term claims, lowering ρ. A lower ρ begets additional dilution of long-term claims, giving the firm additional incentive to issue short-term claims. Therefore, any interior allocation without dilution must unravel to all short-term with dilution.

Corollary 3 *In the representative investor economy with global belief γ, suppose that debt recovery values are given by (15), and conjecture an equilibrium where $0 < \hat{\rho} < 1$. The firm has the incentive to raise the proportion of short-term debt until $\hat{\rho} = 0$ and only short-term debt is issued.*

Corollary 3 is similar to the maturity rat race result for financial firms of Brunnermeier and Oehmke (2013). They show that when facing multiple creditors and lack of commitment to aggregate maturity structure, financial firms have the incentive to shorten the maturity structure to a single period. While debt dilution plays a prominent role in their analysis, the friction on which they focus is the lack of commitment to a maturity choice. Our complementary result shows that the "rat race" forces are obtainable with a representative investor and full ex-ante commitment to the aggregate maturity structure–maturity is adjusted at time 1 with short-term debt, but it is fully priced at issuance. Firms and investors have rational expectations at time 0 that additional short-term debt must be issued with positive probability. The maturity choice is determined *ex ante* and followed through *ex post*. Hence, the common thread in the two papers that drives maturity to the short end is debt dilution.[26] More importantly, Proposition 5 and Corollary 2 show that investor heterogeneity can play a critical role in preventing the rat race from occurring.

FIGURE 3

Percentage and Volume of Multiple Debt Maturity Issuances



The percentage line measures the fraction of all firms in a given quarter that issued more than one maturity debt instrument in that quarter. The bars report the total dollar value of the multiple maturity debt issuances in the quarter. Notes: Own calculations based on Mergent-FISD data.

V. Empirical predictions and implications

The model generates several new predictions on the impact of investor heterogeneity on debt maturity and firm outcomes and is consistent with several existing empirical studies, which we discuss below.[27]

Prediction 1a. Investor heterogeneity leads to co-issuance of long- and short-term debt.

The main result stated in Proposition 2 is that the presence of heterogeneous investors leads to a debt issuance that combines long- and short-term debt. Empirically, most large firms whose bonds are held by a combination of insurance companies, bond funds, and banks issue both long- and short-term debt when tapping capital markets. For example, Figure 3 shows that nearly 50% of all firms that issue corporate bonds issue more than one maturity in any given quarter, consistent with our model's prediction.

The model predicts that firms will issue either long or short term debt when debt is owned by a representative investor. One can interpret a representative investor in our model as either a large single investor, or a dispersed set of investors with similar portfolios allocations. Therefore,

[26]Our unraveling result is weaker than Brunnermeier and Oehmke (2013) because it occurs iff $\rho < 1$, while their result also holds for $\rho = 1$.

[27]More broadly, our framework shows that capital supply matters for equilibrium financing strategies. The survey results in Graham and Harvey (2001) and Servaes and Tufano (2006) support our model's implications. They find less support for traditional demand factors such as information asymmetries (Flannery (1986)) and debt overhang (Myers (1977)) as driving maturity choice. In addition, Custodio, Ferreira, and Laureano (2013) find that supply-side factors–*i.e., investors demand for debt*–have more explanatory power in explaining debt maturity than traditional firm demand factors.

the ownership structure of firms that do not issue multiple maturity debt should be more concentrated and less dispersed.

Prediction 1b. A combination of long- and short-term debt minimizes financing costs

Our model also predicts that a combination of debt maturities is the least costly financing option. Consistent with the prediction, Choi, Hackbarth, and Zechner (2018) show that corporations typically issue debt into, on average, more than three distinct maturity bins, and Norden, Roosenboom, and Wang (2016) show that borrowing costs are lower and leverage is positively associated with debt granularity *i.e.,* a mix of debt maturities rather than a single debt maturity. The converse is that more concentrated debt ownership is associated with higher financing costs. Manconi, Massa, and Zhang (2016) exploit an exogenous shock to bond holder concentration and find that greater bond ownership concentration is associated with greater credit spreads.

Prediction 1c. A combination of long- and short-term debt is associated with more investment opportunities

Using our model, we can interpret investment opportunities as growth options defined as the market-to-book value of firm assets. The market-to-book value of assets is simply total firm production divided by the amount of capital raised to produce, or the book value of its liabilities. Using the first order conditions (2) and (3), one can derive market-to-book in terms of either long-term or short-term bond prices since the expected costs across maturities must be the same in an interior maturity equilibrium. In terms of the long-term bond price, p^ℓ, we have

$$\text{market-to-book} = \frac{I^\alpha}{I} = I^{(\alpha-1)} = \frac{1}{\alpha p^\ell\left(\alpha, A_{DD}, \gamma\right)}. \tag{17}$$

Therefore, firms that utilize a combination of both long- and short-term debt with lower financing costs also have higher investment opportunities. This result is confirmed in Choi, Hackbarth, and Zechner (2021) who find that firms with more granular debt profiles tend to have better investment opportunities. By extension of Predictions 1a and 1b, investor heterogeneity drives the granularity of the debt profiles that are associated with better investment opportunities.

Prediction 2. Debt maturity has no impact on firm outcomes when debt ownership is concentrated

In the absence of investor heterogeneity, Propositions 1 and 6 predict debt maturity structure does not impact firm investment decisions or profitability when debt is held by a representative investor. Again interpreting a representative investor as either a large single investor or a dispersed set of investors with similar portfolios allocations, debt maturity should not impact investment and profitability when a firm's debt is concentrated among a single investor or a set of similar investors with common preferences. Thus, studies that assess the impact of debt maturity should control for the investor ownership structure of a firm's liabilities to accurately tease out the impact of maturity choice.

Prediction 3a. *The impact of covenants on debt maturity depends on debt ownership concentration.*

Our model shows that investor ownership interacts in subtle ways with debt covenants that prevent dilution on debt maturity and firm outcomes. For example, when debt ownership is dispersed among heterogeneous investors, Corollary 1 shows that the measure of long-term debt as a fraction of total debt, ρ, increases when long-term debt is secured against dilution. This result suggests covenants substitute for short-term debt, consistent with the findings of Billet et al. (2007). The authors find that firms with more protective covenants in their public bond indentures tend to have less short-term debt in their capital structure. By contrast, when debt ownership is concentrated among similar investors, covenants lead to either long- or short-term debt profiles.

Prediction 3b. *Debt covenants, debt maturity, and growth options are jointly determined.*

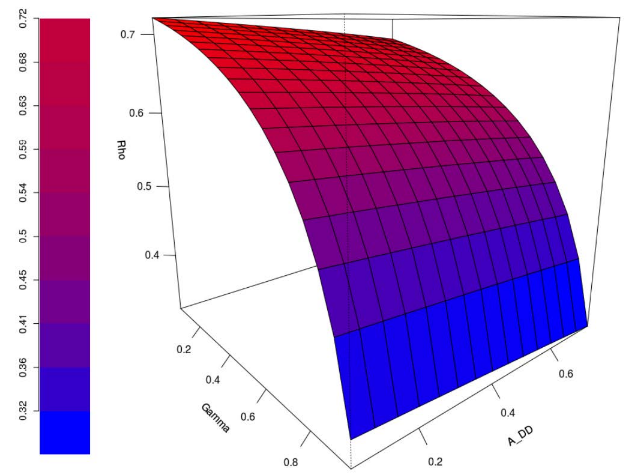
Equation (17) above shows that the value of growth options is endogenous to pricing, which is jointly determined by maturity choice and investor heterogeneity and ownership. Therefore, growth options are not exogenous processes to firm capital structure. Empirically, researchers first studied the impact of growth options on debt maturity using single equation models (*e.g.* Barclay and Smith (1995), Guedes and Opler (1996), Barclay, Marx, and Smith (2003). Johnson (2003) and Billett, King, and Mauer (2007) extend these studies to simultaneous equation models of maturity and leverage and maturity, leverage, and covenants. However, in all these studies, growth options are included as exogenous right hand side variables. Our model suggests this approach is also miss-specified and studies should model growth options jointly rather than treat them as exogenous covariates.

Prediction 4. *More firm optimism implies more short-term debt.*

There are two parameters in the model related to firm optimism that govern the relative amount of short-term debt in a firm's debt structure. The first is γ, the likelihood that firm places on the good state. Figure 4 plots equilibrium values of ρ for the range of γ. In the model, firms issue more short-term debt when good news is more likely, higher γ. The reason is that the likelihood of rolling over short-term debt at the risk-free rate increases, which lowers expected

FIGURE 4

Comparative statics for ρ



Source: Similuation results for the optimal amount of long-term debt the liability structure as a function of γ and A_{DD}. The value of α–curvature parameter–used is 0.8

rollover costs relative to long-term financing. Consistent with this prediction, Landier and Thesmar (2009) find that firms with more optimistic managers tend to issue more short-term debt and less long-term debt than firms with less optimistic managers.

The second parameter is ω, which measures the relative optimism between firms and investors. $\omega < (>)1$ implies that the firm is a relative optimist (pessimist) compared to investors, and leads to more short-term (long-term) debt in equilibrium. Thus, the model predicts that differences *between* investor and firm sentiment drive the relative amount of long- versus short-term debt issued in equilibrium.

VI. Discussion and extensions

This section provides a discussion about extensions and the robustness of the main results to alternative modeling assumptions.

The models makes the simplifying assumption that states are independent and identically distributed. Suppose alternatively that good news is more likely to follow good news than bad news. This assumption alleviates the concern that bad news is effectively "not as bad." The relative expected cost between long- and short-term debt would change, but the effect would be a *quantitative* adjustment in the interior value ρ and not affect the equilibrium financing regime. In particular, the state-contingent costs of rolling over short-term debt change due to different repayment probabilities, but as long as the cost of rolling over short-term debt rises in one intermediate state relative to the ex ante long-term issuance cost, issuing both long- and short-term claims will remain optimal. Hence, any structure in which repayment volatility increases in at least one succeeding state will satisfy the condition in Lemma 2.

One could also allow for uncertainty conditional on $s = U$ with $A_{UD} \neq A_{DU}$, which would be consistent with the formulation in He and Xiong (2012). This too would only affect the interior value of ρ. The additional uncertainty lowers both long-term prices at $t = 0$ and short-term debt prices conditional on $s = U$ at time 1. However, intermediate states would still be characterized by more uncertainty than the initial state; in fact, there would be even more uncertainty. Therefore, the rollover costs of short-term debt will remain higher than the ex ante costs of issuing long-term debt and satisfy Lemma 2.

While the model of the economy has been intentionally kept simple to highlight the main results, the structure can be derived from principles that are more general. For example, Fostel and Geanakoplos (2010) show that when choosing from a menu of projects, agents have the incentive to produce projects with more volatile payouts conditional on bad news. The reason is simple: uncertainty following bad news is not informative, which implies that price declines in bad intermediate periods are relatively small. Alternatively, if uncertainty were completely resolved after bad news, then prices in bad intermediate periods would fall much further and reflect the certain bad outcome in the final period. With the collateral constraint imposed by the repayment enforcement frictions in these models, lower intermediate prices limit *ex ante* how much agents can borrow.

Short-term debt issued at time 0 is *endogenously* risk-free. This is not an innocuous outcome, though there are many ways one can argue that short-term debt is informationally insensitive up to some signal (Dang, Gorton, and Holmstrom (2015)). In the model, firms will never issue both long- and short-term debt if there is liquidation along the equilibrium path at time 1 and all firm assets are used to repay creditors. The argument is the following: risky short-term debt at time 0 must be priced higher than long-term debt because long-term debt allows equity to be retained in 3 of 4 terminal states, while intermediate liquidation implies equity retention in only 2 terminal states. Long-term debt does not insulate the firm from liquidation conditional on bad news at time 1. Hence, there is no benefit. The equilibrium maturity choice will depend on the probability of good news, determined by γ. Short-term (long-term) debt will dominate long-term (short-term) debt when good (bad) news is likely. This is consistent with Diamond (1991) if γ is interpreted as a credit rating. This alternative also highlights the fact that firms generally not subject to rollover risk can issue both long and short-term debt, which is consistent with the data.

Finally, we do not allow investors to purchase bonds with leverage as in the collateral equilibrium models developed by Fostel and Geanakoplos. This restriction is not necessary for the result that issuing multiple types of securities is optimal. The reason is that, due to investor heterogeneity, it will remain optimal to issue multiple types of debt securities to cater to investor needs, as in Allen and Gale (1991). The firm will not want to close off the security market to time 1 investors by issuing only long-term debt at time 0, even though allowing for leverage at time 0 will lower long-term spreads and raise prices. Issuing more long-term debt at time 0 implies that less short-term debt needs to be rolled over at time 1, which also raises the prices of short-term debt. The more tilted the issuance becomes to one maturity, the greater is the substitution benefit of the other maturity.

VII. Conclusion

This papers studies the role of investor heterogeneity for corporate debt maturity structure. We find that firms facing heterogeneous investors will tend to issue a combination of long- and short-term debt to minimize the cost of a given debt issuance. Firms use debt maturity to spread financing cost to investors most willing to pay at any point in time. By contrast, debt maturity is irrelevant when facing a representative investor that prices all claims in expectation, equivalently. We show that these results are robust to allowing for debt dilution where future short-term debt can dilute the value of existing long-term claims. The model provides new predictions for the impact of dispersed versus concentrated investor ownership on debt maturity and investment opportunities, and is consistent with several existing empirical findings regarding the relationship between the use of multiple debt maturities and debt pricing, investment opportunities, and the impact of firm optimism on corporate debt-maturity choices.

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A. Appendix Omitted Proofs

Lemma 1: We will derive an upper bound on δ that will equate a debt strategy of issuing all long-term debt and all risky-short-term debt. All δ less than the upper bound will necessarily lower risky short-term debt prices without altering the all long-term candidate solution, which implies that the all long-term solution always dominates an all risky short-term solution. We can then focus the rest of the paper on candidate solutions for which all risky short-term debt is dominated by the all long-term candidate solution.

Let V_d be the firm value function when issuing only risky short-term debt at $t = 0$ that defaults at $s = D$ at time 1. Then $V_d = \gamma\left(I^{\alpha} - q_0^{\varsigma}\right) + (1-\gamma)\times 0$ as the firm only retains equity at $s = U$ and defaults at $s = D$. If the firm raises all capital short-term, then $I = q_0^{\varsigma} p_0^{\varsigma}$. The first order condition for a maximum is

$$\alpha I^{\alpha-1} = \frac{1}{p_0^{\varsigma}}. \tag{18}$$

Investors are also perfectly competitive price takers. Because of the connectedness of the set of agents and monotonicity of utility in h, there will be a marginal buyer at time 0, h_0, who's expectations price risky short-term debt. All agents $h > h_0$ will purchase short-term debt and all other agents remain in cash. The bond pricing condition is

$$h_0 + (1-h_0)\,\delta I = p_0. \tag{19}$$

Rearranging, we get $p_0 = \frac{h_0}{1-(1-h_0)\delta}$, which bounds risky short-term bond prices between h_0 and 1 as $\delta \overset{\lim}{\to} (0,1)$. Investor recovery in default, δI, is an increasing function of δ for any given I. Thus, the lower is δ, the lower the price any investor is willing to pay for risky short-term debt. The lower bond price raises the marginal cost of capital, which lowers investment and profits. Finally, market clearing will determine a candidate solution to the risky-short term debt problem:

$$(1-h_0) = I \tag{20}$$

As $\delta \overset{\lim}{\to} 0$, $p_0 \overset{\lim}{\to} h_0$, and $I \overset{\lim}{\to} (\alpha h_0)^{\frac{1}{1-\alpha}}$ from the first order condition. Plugging the limiting I into the market clearing condition, we obtain a polynomial that solves for the fixed point:

$$h_0 + (\alpha h_0)^{\frac{1}{1-\alpha}} - 1 = 0. \tag{21}$$

Clearly there is a unique $h_0 > 0, \forall \alpha \in (0,1)$ that solves (21) as the left hand side is monotonically increasing in h_0 ranging from 0 to $1 + \alpha^{\frac{1}{1-\alpha}} > 1$.

The candidate solution is the lowest price equilibrium determined by $\delta \overset{\lim}{\to} 0$, where

$\underline{p}_0^* = \underline{h}_0^*$. Now consider the two period long-term investment strategy with value function given by $V_\ell = 1 - (1-\gamma)^2 (I^\alpha - q_\ell)$. The firm raises $I = q_\ell p_\ell$, and the first order condition is

$$\alpha I^{\alpha-1} = \frac{1}{p_\ell}. \tag{22}$$

The optimality condition is the same form as the all short-term problem due do limited liability in default. The marginal investor equation pricing two-period risky long-term debt is

$$1 - (1-h_0)^2 + (1-h_0)^2 \frac{A_{DD}}{\alpha} = p_\ell. \tag{23}$$

The recovery in default term $\frac{A_{DD}}{\alpha}$ comes from plugging the first order condition for I and $I = q_\ell p_\ell$ into the recovery value function $\frac{A_{DD} I^\alpha}{q_\ell}$. Lastly, market clearing takes the same form as the short-term funding regime:

$$(1-h_0) = I. \tag{24}$$

The lowest price equilibrium in the long-term regime is when $A_{DD} = 0$ and there is no recovery in default, just as in the risky short-term regime. The lowest two-period long-term price is then given by $p_\ell = 1 - (1-h_0)^2 = h_0(2-h_0)$. Plugging this into the market clearing condition, we obtain a similar polynomial as the short-term regime

$$h_0 + (\alpha h_0 (2-h_0))^{\frac{1}{1-\alpha}} - 1 = 0. \tag{25}$$

Comparing (21) with (25), we see that the marginal buyer in the lowest long-term funding regime is always more optimistic than the marginal buyer in the lowest-price short-term funding regime. This reflects the fact that at time 0, any marginal buyer places far less weight on two periods of bad news leading to long-term default, than a single period of bad news leading to short-term default. The highest long-term bond prices will be given by $\frac{A_{DD}}{\alpha} = 1$, which implies that $p_\ell = 1$. Therefore, both short-term with liquidation and long-term have the same risk-free price when debts are always repaid, but the long-term regime has a higher lowest price equilibrium. Clearly, prices in the two regimes are monotonically increasing in the parameters δ and A_{DD} respectively. Thus, for any $0 < A_{DD} < 1$, there will be at most 1 value of δ where the two pricing function cross, if they cross at all. Otherwise, long-term always dominates short-term. Let $\delta = \hat{\delta}$ be the value when the two pricing functions cross. To find $\overline{\delta}$, we must compare the value function for the two regimes because the expectations of retaining equity are different due to the firm being profitable in 3 of 4 states in the long-term regime, but only 2 of 4 in the short-term only regime.

Using the respective first order conditions for I^* and the fact that $q^* = \frac{I^*}{p^*}$ in both regimes, we can write the equivalence condition for risky short-term and long-term funding $V_d = V_\ell$ as

$$(1 + (1-\gamma)) q_\ell^* = q_0^*. \tag{26}$$

This says that even if the firm raises the same amount of capital on the same terms in the two regimes in which $q_0 = q_\ell$, the long-term regime will dominate due to the fact that the firm retains equity in a superset of states with long-term funding. Risky short-term debt must be less expensive by the factor $(1 + (1 - \gamma))$ than long-term funding, giving us $\overline{\delta} = (1 + (1 - \gamma))\,\hat{\delta}$. Therefore, for any parameters (α, A_{DD}, γ) that determine the solution to the long-term funding regime, if there is a value of $\overline{\delta}$ high enough to raise risky short-term prices enough to incentivize issuing short-term debt with default at time 1, then $\forall \delta < \overline{\delta}$ long-term funding will dominate.

Proposition 1: The condition for equivalence is that the expected marginal cost of either source is the same. From the F.O.C, equivalence implies

(27)
$$\frac{1}{p^{\ell}} = \frac{1}{1 - (1-\gamma)^2}\left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right].$$

Equilibrium long-term debt pricing is given by $p^{\ell} = 1 - (1-\gamma)^2 + (1-\gamma)^2 \frac{A_{DD}}{\alpha}$, which can be re-written as

(28)
$$p^{\ell} = (\gamma + \gamma(1-\gamma))\left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha}.$$

Equilibrium short-term debt pricing is given by $p_D^{\varsigma} = \gamma + (1-\gamma)\frac{A_{DD}}{\alpha}\left[\frac{p_D^{\varsigma}\gamma + \gamma(1-\gamma)}{\gamma + \gamma(1-\gamma)}\right]$, which, after some algebra, can be re-written as

(29)
$$p_D^{\varsigma} = \frac{\gamma\left[(\gamma + \gamma(1-\gamma))\left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha}\right]}{\left[\gamma(1-\gamma)\left(1 - \frac{A_{DD}}{\alpha}\right) + \gamma\right]}.$$

Plugging (28) and (29) into (31), equivalence requires that

(30)
$$1 - (1-\gamma)^2 = \gamma\left[(\gamma + \gamma(1-\gamma))\left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha}\right] + \frac{\gamma(1-\gamma)\left[(\gamma + \gamma(1-\gamma))\left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha}\right]}{\frac{\gamma\left[(\gamma+\gamma(1-\gamma))\left(1-\frac{A_{DD}}{\alpha}\right)+\frac{A_{DD}}{\alpha}\right]}{\left[\gamma(1-\gamma)\left(1-\frac{A_{DD}}{\alpha}\right)+\gamma\right]}}.$$

Notice that two terms in the fraction in second term on the right hand side, γ and $\left[(\gamma + \gamma(1-\gamma))\left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha}\right]$, cancel out. After re-writing $1 - (1-\gamma)^2 = \gamma + \gamma(1-\gamma)$,

(31) becomes

$$
\begin{aligned}
\gamma+\gamma\left(1-\gamma\right) &= \gamma\left[\left(\gamma+\gamma\left(1-\gamma\right)\right)\left(1-\frac{A_{DD}}{\alpha}\right)+\frac{A_{DD}}{\alpha}\right]+\left(1-\gamma\right)\left[\gamma\left(1-\gamma\right)\left(1-\frac{A_{DD}}{\alpha}\right)+\gamma\right] \\
&= \gamma\left(1-\gamma\right)\left(1-\frac{A_{DD}}{\alpha}\right)+\gamma\frac{A_{DD}}{\alpha}+\gamma^{2}\left(1-\frac{A_{DD}}{\alpha}\right)+\gamma\left(1-\gamma\right) \\
1 &= \left(1-\frac{A_{DD}}{\alpha}\right)\left(\gamma+1-\gamma\right)+\frac{A_{DD}}{\alpha} \\
1 &= 1.
\end{aligned}
$$

Hence, the two regimes are always equivalent.
Lemma 2: Short-term debt can only be safe for a candidate interior solution if its price at time 0 is equal to 1. In other words, all short-term debt is unconditionally rolled over. We will find under what conditions a candidate interior optimization can be achieved and all debts rolled over assuming $p_0^{\varsigma}=1$. Plugging the first order conditions for an interior maximum are (2) and (3) with $p_0^{\varsigma}=1$ into the necessary rollover condition in (13) immediately gives (14). This is the necessary condition for an interior optimum with short-term debt rollover.

Proposition 2: We first show that the long and short-term economies are no longer equivalent, and then show why the interior solution always dominates either corner solution.

First, the long and short-term debt pricing equations in the heterogenous agent economy are given by (9) and (10). Rewrite these similar to Proposition 1 as

$$
\begin{aligned}
p^{\ell} &= \left(h_0+h_0\left(1-h_0\right)\right)\left(1-\frac{A_{DD}}{\alpha}\right)+\frac{A_{DD}}{\alpha}, \qquad \text{and} \\
p_D^{\varsigma} &= h_1+\left(1-h_1\right)\frac{A_{DD}}{\alpha}\left[\frac{p_D^{\varsigma}\gamma+\gamma\left(1-\gamma\right)}{\gamma+\gamma\left(1-\gamma\right)}\right] \\
&= \frac{h_1\left(\gamma+\gamma\left(1-\gamma\right)\right)+\left(1-h_1\right)\frac{A_{DD}}{\alpha}\gamma\left(1-\gamma\right)}{\gamma+\gamma\left(1-\gamma\right)-\left(1-h_1\right)\frac{A_{DD}}{\alpha}+\gamma} \\
&= \frac{\gamma\left[h_1+\left(1-\gamma\right)\left[h_1\left(1-\frac{A_{DD}}{\alpha}\right)+\frac{A_{DD}}{\alpha}\right]\right]}{\gamma\left[\left(1-\gamma\right)-\left(1-h_1\right)\frac{A_{DD}}{\alpha}\right]+\gamma}
\end{aligned}
$$

Plugging these into the equivalence condition, (31), we get

$$
\gamma+\gamma\left(1-\gamma\right)=\gamma\left[\left(h_0+h_0\left(1-h_0\right)\right)\left(1-\frac{A_{DD}}{\alpha}\right)+\frac{A_{DD}}{\alpha}\right]+\frac{\gamma\left(1-\gamma\right)\left[\left(h_0+h_0\left(1-h_0\right)\right)\left(1-\frac{A_{DD}}{\alpha}\right)+\frac{A_{DD}}{\alpha}\right]}{\frac{\gamma\left[\left(1-\gamma\right)\left[h_1+\left(1-h_1\right)\frac{A_{DD}}{\alpha}\right]+h_1\right]}{\gamma\left[\left(1-\gamma\right)-\left(1-h_1\right)\left(\frac{A_{DD}}{\alpha}\right)\right]+\gamma}}
$$

Inspection of the above equation shows that equivalence requires $h_0=h_1=\gamma$ in order to cancel out the first part of the fraction in the second term on the right. In particular, plug in $h_1=\gamma$ into $\gamma\left[\left(1-\gamma\right)\left[h_1+\left(1-h_1\right)\frac{A_{DD}}{\alpha}\right]+h_1\right]$ and rearrange to get $\gamma\left[\left(\gamma+\gamma\left(1-\gamma\right)\right)\left(1-\frac{A_{DD}}{\alpha}\right)+\frac{A_{DD}}{\alpha}\right]$, which cancels with the numerator iff $h_0=\gamma$. The leftover term from the second term on the right becomes $\left(1-\gamma\right)\left[\gamma\left(1-\gamma\right)\left(1-\frac{A_{DD}}{\alpha}\right)+\gamma\right]$. And with

$h_0 = \gamma$, the first term on the right becomes $\gamma\left[(\gamma + \gamma(1-\gamma))\left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha}\right]$ allowing one to arrive at equivalence to the left hand side as in the proof of proposition 1 above. However, the marginal buyer at time 0 will never have the same repayment expectation when uncertainty rises at 1. Therefore, $h_0 \neq h_1$ even if by coincidence either happens to equal γ.

The second part of the proof is by contradiction and shows that for any investment level, I, that can be financed by either all long- or all short-term debt, financing with both long- and short-term has a lower marginal cost. Suppose maturity is irrelevant, and the same investment plan, I, can be raised through all long or an interior, $0 < \rho \leq 1$. Maximization requires $MPK = MC$, which defines investment as a function of prices. Irrelevance implies $I^*\left(p^{*\ell}\right) = I^*\left(p^{*\ell}, p_0^{*\varsigma}, p_D^{*\varsigma}\right)$. By Lemma 2, if $q_D^{*\varsigma} > 0$, then the short-term component of the interior solution is rolled over at time 1 and $p_0^{*\varsigma} = 1$. Equivalence of marginal costs and efficient investment scale across the two funding strategies implies that the amounts of debt issued must also be the same. Let $\tilde{Q} = \tilde{q}_0^{\ell}$, and $p^{*\ell} = \tilde{p}_0^{\ell}$ be the long-term candidate equilibrium quantities and prices, and $\hat{Q} = \hat{q}_0^{\ell} + \hat{q}_0^{\varsigma}$, and $\hat{p}_0^{\ell}$, $\hat{p}_D^{\varsigma}$, $\hat{p}_0^{\varsigma}$ be the interior-candidate equilibrium quantities and prices. Equivalence implies that $\tilde{Q} = \hat{Q} \Rightarrow \tilde{q}_0^{\ell} > \hat{q}_0^{\ell}$, $\forall \hat{q}_0^{\varsigma} > 0$. Moreover, since the firm takes prices as given, it must be the case that $\tilde{p}_0^{\ell} > \hat{p}_0^{\ell}$. Market clearing in the long-term corner solution is given by $\left(1 - \tilde{h}_0\right) = \tilde{p}_0^{\ell}\tilde{q}_0^{\ell} = I^*$ and in the interior solution given by $\left(1 - \hat{h}_0\right) + \left(1 - \hat{h}_D\right) = \hat{p}_0^{\ell}\hat{q}_0^{\ell} + \hat{p}_D^{\varsigma}\hat{q}_D^{\varsigma} = I^*$. Equating the two market clearing conditions for the same I^* gives $\left(1 - \tilde{h}_0\right) = \left(1 - \hat{h}_0\right) + \left(1 - \hat{h}_D\right)$. This can only hold if $\hat{h}_D = 1$ meaning that $\hat{q}_D = 0$–no short-term debt is issued–or if $\tilde{h}_0 < \hat{h}_0$–the marginal long-term bond buyer in an interior solution is more optimistic than the marginal bond buyer in the corner solution. However, a more optimistic marginal buyer in the interior solution implies $\tilde{p}_0^{\ell} < \hat{p}_0^{\ell}$, which contradicts $\tilde{q}_0^{\ell} > \hat{q}_0^{\ell}$, $\forall \hat{q}_0^{\varsigma} > 0$. The same logic also shows that the firm can never be indifferent between a short-term corner solution and the interior.

Proposition 3: Let the representative investor's belief that $s = U$ be given by $\omega \times \gamma$, where γ is the firm's belief that $s = U$, and $0 \leq \omega < \frac{1}{\gamma}$. Following the proof of Proposition 1, the first order condition for the interior solution must satisfy

$$\frac{1}{p^{\ell}} = \frac{1}{1 - (1-\gamma)^2}\left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right]. \tag{31}$$

Using the investor's belief, the investor must break even when purchasing both long- and short-term debt. Hence the respective prices must satisfy,

$$p^{\ell} = 1 - (1-\omega\gamma)^2 + (1-\omega\gamma)^2\frac{A_{DD}}{\alpha}, \tag{32}$$

and

$$p_D^{\varsigma} = \omega\gamma + (1-\omega\gamma)\frac{A_{DD}}{\alpha}\left[\frac{p_D^{\varsigma}\gamma + \gamma(1-\gamma)}{\gamma + \gamma(1-\gamma)}\right]. \tag{33}$$

Substituting (32) and (33) into (31) and working through the algebra, one can easily show that (31) holds with equality iff $\frac{A_{DD}}{\alpha}=1$, which requires that both long- and short-term debt are risk-free. The equivalence breaks down because the recovery value of short-term debt given by the term $\frac{p_D^{\varsigma}\gamma+\gamma(1-\gamma)}{\gamma+\gamma(1-\gamma)}$ in (33) is a function of the firm's belief, γ, and how much debt is issued. But the pricing of that debt is determined by the investor's belief, $\omega\gamma$. Therefore, unlike the case with a global belief in which the short-term dilution factor is fairly priced leading to equivalence between both long- and short-term debt prices, there is disagreement between the firm and the investor over the relative price of short-term debt compared to long-term debt. This concludes the first statement of the proof.

One can then show for the general case with default, $\frac{A_{DD}}{\alpha}<1$, that $\omega>(<)\,1 \implies MC^{\ell}<(>)\,MC^{\varsigma}$ and the cheapest funding option is immediate depending on what one assumes about relative beliefs ω.

Corollary 1: From Proposition 4 and equation (8) we know that $d_{DD}\left(q^{\ell}\right)>d_{DD}\left(q_D^{\varsigma}\right)$ for a given $\left(I^{*},\rho^{*}\right)$. Suppose the firm does not alter its debt structure and ρ^{*} is unchanged. Then, long-term debt prices must rise to reflect greater recovery values, $\hat{p}^{\ell}>p^{\ell^{*}}$. But if long-term debt is now cheaper in equilibrium, then the maturity structure for a given $\left(I^{*},\rho^{*}\right)$ cannot be optimal and the firm must adjust. Thus ρ must rise. For the second statement that short-term issuance must fall, suppose not and that $\hat{q}_D^{\varsigma}=q^{*\varsigma}_D$. From Proposition 4, the marginal cost of short-term debt increases because recovery values fall. Hence the expected marginal cost of issuing $\hat{q}_D^{\varsigma}$ is higher than $q^{*\varsigma}_D$. Interior optimality requires that the expected marginal costs of the two debt maturities must be the same, $E\left[MC\right]=E\left[MC^{\ell}\right]=E\left[MC^{\varsigma}\right]$. Hence, leaving $q^{*\varsigma}_D$ unchanged results in $E\left[MP\right]<E\left[MC\right]$ and cannot be an equilibrium.

Proposition 6: From Proposition 1, the two corner solutions are identical allocations. Hence we only need to show that the interior allocation cannot exist as an equilibrium because the short- and long-term corner solutions, by definition, do not contain dilution.

Any interior candidate solution must satisfy the firm first order condition that the expected marginal cost of each type must necessarily be the same, given by equation (3). Consider any generic delivery function where equally senior claims repay the same per claim amount at $s=DD$, $\hat{d}_{DD}\left(\cdot\right)$. Substitute the recovery value into the bond pricing equations given by equations (9) and (10) using the representative agent, γ, as the marginal buyer for each type of debt instrument. With some simple algebra, the marginal pricing equivalence condition, (3), simplifies to: $\hat{d}_{DD}\left(\cdot\right)=1$. Hence the only way two securities with different repayment probabilities and identical repayment vectors can imply the same marginal cost is if they are in fact risk-free and the same security.

Corollary 3: Suppose the firm chooses an investment policy function $\hat{I}$ with some small amount of short-term debt, $\hat{q}^{\varsigma}=\epsilon>0$, and long-term debt, $\hat{q}_s^{\ell}-\epsilon$, such that the allocation $0<\hat{\rho}<1$ is equivalent to $\hat{\rho}=1$. When short term debt can dilute long term debt, the recovery values for each long and short term bond are equivalent. Thus under representative agents, $d_{DD}(q^{l})=d_{DD}(q_D^{\varsigma})$. Moreover, the investor must be indifferent to holding either long-term debt or a sequence of short-term debt and cash, and hence the return to the two types of debt portfolios must be equivalent to the outside option of holding cash:

$$\gamma+\frac{(1-\gamma)\gamma}{p_D^{\varsigma}}+\frac{(1-\gamma)^2 d_{DD}}{p_D^{\varsigma}}=\frac{1-(1-\gamma)^2+(1-\gamma)^2 d_{DD}}{p^{\ell}}=1.$$

Solving each expected return condition for $d_{DD}(\cdot)$ and setting them equal yields a pricing relationship: $p^{\ell} = p^{\varsigma}_D(1-\gamma)+\gamma$. For any candidate interior solution (with long and short term debt issuance) to unravel to the short end, it must be the case that the marginal cost of short-term debt is less than long-term debt. Thus the following condition (equation (3)) must hold: $\gamma + \frac{\gamma(1-\gamma)}{p^{\varsigma}_D} < \frac{[1-(1-\gamma)^2]}{p^l}$. Plugging in $p^{\ell}(p^{\varsigma}_D)$ from above and working through the algebra, this condition simplifies to $p^{\varsigma}_D < 1$, which always holds when there is default, $d_{DD}(\cdot) < 1$. Note that this is true for any amount of short-term debt, $\epsilon > 0$. Therefore, the firm should increase its short-term debt position $\forall \epsilon > 0$ and continue issuing short-term debt until $\rho = 0$ and only short-term debt is issued.

B. Equilibrium Conditions

A. Interior Maturity Choice

The ten endogenous variables are $\left(p^{\varsigma}_0, p^{\ell}, p^{\varsigma}_D, q^{\varsigma}_0, q^{\ell}, q^{\varsigma}_D, I, \rho, h_0, h_D\right)$. The system of equations,

$$
\begin{aligned}
p^{\varsigma}_0 &= 1 \text{ time 0 short-term debt price} \\
\alpha I^{\alpha-1} &= \frac{1}{p^{\ell}}, \text{ combined first order} \\
1-(1-h_0)^2 + (1-h_0)^2 d_{DD}(\cdot) &= p^{\ell}, \text{ long-term debt pricing} \\
h_D + (1-h_D)\, d_{DD}(\cdot) &= p^{\varsigma}_D, \text{ short-term debt pricing} \\
\frac{1-h_0}{p^{\ell}} &= q^{\ell}, \text{ long-term debt market clearing} \\
\frac{1-h_D}{p^{\varsigma}_D} &= q^{\varsigma}_D, \text{ short-term debt market clearing} \\
I &= p^{\ell} q^{\ell} + p^{\varsigma}_D q^{\varsigma}_D, \text{ firm funding condition} \\
q^{\varsigma}_0 &= p^{\varsigma}_s q^{\varsigma}_s \text{ short-term rollover condition} \\
\rho &= \frac{p^{\ell} q^{\ell}}{I}, \text{ long-term debt portion} \\
d_{DD}(\cdot) &= \frac{A_{DD}}{\alpha}\left(\frac{p^{\varsigma}_D}{(1-\rho)\, p^{\ell} + \rho p^{\varsigma}_D}\right) \text{ debt recovery value.}
\end{aligned}
$$

B. Long-term Equilibrium

The endogenous variables are in this economy: $\left(I, p^{\ell}, q^{\ell}, h_0\right)$, and four equations: (4), (9), (11), and $I = p^{\ell} q^{\ell}$.

C. Short-term Equilibrium

The the endogenous variables are: $(I, p_D^{\varsigma}, q_D^{\varsigma}, h_1)$. Equilibrium is found by simultaneously solving market clearing via (12), the pricing equation, (10), the debt delivery function, (8), and the firm's funding condition, $I = p_0^{\varsigma} q_0^{\varsigma}$.

C. Appendix

Here we show that changing the uncertainty structure of the economy does not materially alter the optimal choice to issue both long- and short-term debt. Instead of the structure given by Figure 1 where $\gamma = \gamma|_{s=D}$, let $\gamma_1 = \Pr(s = U) > \gamma_2 = \Pr(s = DU|_{s=D})$ so that the likelihood of receive a good state following a bad state is less that receiving an unconditional good state. Breaking the firm's problem given by (1) into its constituent pieces, we can write profits as

$$\max_{I,\rho} \prod = \left\{ \gamma_1 \left[I^{\alpha} - \rho \frac{I}{p^{\ell}} - (1-\rho) \frac{I}{1} \right] + (1-\gamma_1) \gamma_2 \left[I^{\alpha} - \rho \frac{I}{p^{\ell}} - (1-\rho) \frac{I}{p_D^{\varsigma}} \right] \right\}.$$

This profit expression simply states that conditional on good news at $t = 1$, both long- and short-term debt is repaid, and conditional on bad news at $t = 1$ long- and short-term debts are repaid only if good news arrives at $t = 2$. Notice that the only difference between this problem and the one presented in the main body of the paper is that $\gamma_2 < \gamma_1 = \gamma$. The first order conditions for a maximum simply become:

$$\begin{aligned} \frac{[\gamma_1 + \gamma_2 (1-\gamma_1)]}{p^{\ell}} &= \frac{1}{p_0^{\varsigma}} \left[\gamma_1 + \frac{\gamma_2 (1-\gamma_1)}{p_D^{\varsigma}} \right] \\ \alpha I^{\alpha-1} [\gamma_1 + \gamma_2 (1-\gamma_1)] &= \frac{\rho [\gamma_1 + \gamma_2 (1-\gamma_1)]}{p^{\ell}} + \frac{(1-\rho)}{p_0^{\varsigma}} \left[\gamma_1 + \frac{\gamma_2 (1-\gamma_1)}{p_D^{\varsigma}} \right]. \end{aligned}$$

Plugging into the other we obtain $\alpha I^{\alpha-1} = \frac{1}{p^{\ell}}$ which of course arises because in equilibrium the marginal cost of a long-term bond must equal the marginal cost of a short-term bond for $0 < \rho < 1$ allowing us to express the first order condition for a maximum as a function of either long- or short-term debt. Let $A \equiv \gamma + \gamma(1-\gamma)$ when $\gamma = \gamma|_{s=D}$ and $B \equiv \gamma_1 + \gamma_2 (1-\gamma_1)$ from the restated problem above and $A > B$. Then, $\forall (I, \rho) : \alpha I^{(\alpha-1)} A > \alpha I^{(\alpha-1)} B$. This implies that $\frac{1}{p^{\ell}}\big|_A > \frac{1}{p^{\ell}}\big|_B \Rightarrow p^{\ell}\big|_B > p^{\ell}\big|_A$ at the optimum. In other words, for a given ρ, the firm will only raise the same amount of capital across the two economies if long-term bond prices are higher in the economy with more uncertainty at $s = D$, which is a contradiction because the firm is less likely to repay debt at $s = DU$ with in the more uncertainty case. Alternatively, the firm can raise less long-term debt and more short-term debt in the economy with more uncertainty at $s = D$, leaving total I_0 unchanged and tilting ρ more toward short-term debt. This results in lower short-term bond prices and higher long-term bond prices. And by proposition 3, starting from a corner solution, it will always be less costly to balance long- and short-term borrowing costs against one another rather than issuing all long- or short-term debt. The only thing that will change is the relative maturity tilt.

The same logic applies if we were to allow for uncertainty at $s = U$ and default at $s = UD$. For this, assume that firm deliver at $s = UD$ is higher than $s = DD$, where generically

$d_{UD}(Q) = d_{DD}(Q) + \epsilon < 1$. This simply reflects the fact that the ultimate shock to collateral is worse in two consecutive bad states than in an up state followed by a down state. The firm's maximization problem can be split and written as follows:

$$\max_{I,\rho} \prod = \left\{ \gamma^2 \left[I^\alpha - \rho \frac{I}{p^\ell} - (1-\rho) \frac{I}{p_U^\varsigma} \right] + (1-\gamma)\gamma \left[I^\alpha - \rho \frac{I}{p^\ell} - (1-\rho) \frac{I}{p_D^\varsigma} \right] \right\}.$$

Only two things change in the problem. 1) Debts are no longer repaid conditional on $s = U$ so that now the first set of repayment states are given by γ^2 rather than γ. 2) $p_U^\varsigma \neq 1$ as it does with full repayment. Taking first order conditions for an interior maximum and plugging in, one can express the same marginal product equals marginal cost as $\alpha I^{\alpha-1} = \frac{1}{p^\ell}$. And by proposition 3, we know that for any given I_0 and a candidate corner solution, it is always be cheaper to fund a portion of the investment outlay by substituting into either long or short-term debt rather so that both debt maturities are utilized. *QED*.

D. Negative pledge covenant

Our treatment of protected long-term debt can be thought either as an explicit pledge or earmark, or the inclusion of a negative pledge covenant that explicitly spells out how long-term debt is secured from short-term debt dilution. The benefit of thinking about negative pledge covenants, as detailed below, is two fold: 1) negative pledges are among the most common covenants found in public debt indentures, 2) given their prominence, surprisingly little is known in the academic literature of their impact. We thus attempt to fill this void with the support of strong practical relevance.

Negative pledges are widely recognized in law and economics (see Bjerre (1999) , Wood (2007), Wood (2008)). The covenant stipulates that the firm cannot issue secured debt in the future without securing the current debt issue. For example, Billett et al. (2007) classify negative pledge covenants as "Secured Debt Restrictions" because they restrict the security of future debt issues. Table III in their paper shows that negative pledges are typically the 3rd or 4th most common covenant, behind cross default or acceleration, asset sale, and merger clauses. Negative pledges are more common than leverage, dividend, and share repurchase restrictions. Table **??** gives a general sense for the basic statistics on types of bonds that contain a negative pledge covenant. They are more prone in medium-to-long-term non-financial corporate indentures.

Source: Covenant data are from Mergent-FISD. Own calculations based on Mergent-FISD data.