The Crowding-in Effect of Public Information on Private Information Acquisition

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Abstract

The dissemination of public information regarding an asset's fundamental value can encourage acquisition of private information by informed traders, leading to a crowding-in effect. Competing with the crowding-out effect analyzed in prior research, the crowding-in effect shapes the demand for private information in a hump-shaped curve against public information quality. I examine how a for-profit information seller strategically provides information, exploiting this hump-shaped demand curve, and offer theoretical support for the coexistence of free and paid information. The model yields distinctive insights into the equilibrium information structure and market quality when the crowding-in effect drives public information dissemination.

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This paper is based on Chapter 1 of Aoyagi's doctoral dissertation submitted to UC Berkeley in 2021. I am especially grateful to Thierry Foucault (the editor) and an anonymous referee for their detailed comments and suggestions, which substantially improved the paper. I also appreciate the constructive feedback from Nicolae Gârleanu, Takahiro Hattori, Terry Hendershott, Kei Kawakami, David Sraer, Yuki Sato, Christine Parlour, Yan Xiong, and the seminar participants at the University of Tokyo, Berkeley Haas, the TMU QFin Seminar, and HKUST.

I. Introduction

In financial markets, information is a valuable commodity, and investors are willing to pay for it to gain an edge in their investment decisions. However, not all information comes at a cost, and the same entities, such as independent financial analysts, often provide both paid and free information simultaneously.[1] Financial news websites often offer free yet fundamental information to the public, such as blog posts and market analyses. Another example is social media platforms, such as Reddit and 𝕏 (Twitter), where individual analysts may share their opinions and insights with the public for free. Notably, analysts who write for these websites and posts may also provide more precise signals to paying clients. The proliferation of the internet and social media in recent years has pronounced the coexistence of paid and free information.

The finance literature extensively examines information asymmetry, with significant research delving into the characteristics of paid private information (e.g., Admati and Pfleiderer (1986), (1988), (1990)). However, studies on free fundamental information, offered by for-profit independent analysts, are relatively scarce, leaving gaps in our understanding of its nature. Theoretical studies on economics of freebies extend to digital products and information, attributing the dissemination of free information to marketing and reputational motivations (Heiman, McWilliams, Shen, and Zilberman (2001), Li, Jain, and Kannan (2019)). Nonetheless, empirical evidence reveals mixed impacts of free products on sellers' medium-term profits, raising questions about the motives behind free information distribution (Scott (1976), Bawa and Shoemaker (2004), Aral and Dhillon (2021), Lee, Zhang, and Wedel (2021)). Furthermore,

[1]For instance, Morningstar and Value Line are leading providers of investment research. They offer a range of free services like stock and fund analysis reports on their websites and newsletters, alongside premium subscriptions featuring more comprehensive research and analysis.

ongoing discussions in the finance literature on the *crowding-out* effect complicate matters, suggesting that free public information may dilute the advantage of private information and diminish investors' demand for it (e.g., Verrecchia (1982), Goldstein and Yang (2017)).

These discussions pose a perplexing question: why do independent analysts, who aim to sell information for profit, also offer it for free, potentially reducing demand for paid information? Moreover, the lack of understanding regarding this motivation leads to further questions, such as the comparative quality of free versus paid information and how it varies with market conditions. Additionally, it remains unclear in which markets and for which assets analysts are more inclined to distribute free information. Investigating its overall impact on market quality is also essential.

The primary contribution of this paper is to demonstrate a *crowding-in* effect, whereby the provision of free public information stimulates the acquisition of private information. This effect stands in direct contrast to the existing crowding-out effect and elucidates why for-profit analysts disseminate fundamental information both with and without charge. Furthermore, this effect can be explained within the framework of a standard Kyle (1985) model without relying on arbitrary assumptions about signal correlation or trade timing, making it an inherent aspect of the standard market structure.

To begin, I analyze the optimal information-acquisition problem by an informed trader. The trader's *ex-ante* expected profit stems from her informational advantage over market makers, multiplied by trading intensity (i.e., the inverse price impact). The informed trader adjusts the quality (precision) of her private signal by balancing the following tradeoff: On one hand, a high-quality private signal boosts expected profits by conferring an informational advantage. On the other hand, it magnifies the price impact of her order flow and makes it challenging to exploit the informational advantage.

In examining the impact of free public information on the demand for private information, I consider a publicly available noisy signal regarding the asset's fundamental value, which is conditionally independent of the trader's private signal. The public signal conveys fundamental information to market makers and diminishes the informational advantage of the informed trader. Consequently, it reduces the marginal benefit of enhancing the quality of the private signal, thereby discouraging the demand for private information. This adverse effect is commonly referred to as the crowding-out effect in the theoretical literature.

However, in my model, the crowding-in effect also comes into play. As the free public signal diverts market makers' attention away from order-flow information, the price impact of order flow diminishes. In turn, the informed trader can engage in more intensive trading based on her private signal without affecting the price. Consequently, the public signal encourages the demand for private information. As a result of the competition between the crowding-in and crowding-out effects, the demand for private information (i.e., the optimal quality) follows a hump-shaped curve in response to the quality of free public information.

To unravel the implications for endogenous public information, I introduce a for-profit analyst, inspired by the framework in Admati and Pfleiderer (1986). The analyst charges a positive price for private information, exclusively conveyed to the informed trader, while public information is freely disseminated to all traders, including market makers. Due to the hump-shaped response of the demand for private information, a unique quality of public signal emerges that maximizes the analyst's profits, offering a theoretical rationale for the public information provision.

The crowding-in effect represents a novel contribution to our understanding of the interplay between public and private information. Existing theories have concluded that public information unequivocally discourages traders from acquiring and trading on private information through the crowding-out effect. However, empirical studies generally do not support the predicted substitution relation between public and private information (Francis, Schipper, and Vincent (2002), Frankel, Kothari, and Weber (2006)). The ambiguous reaction proposed in my model offers a theoretical rationale for this empirical observation, suggesting that whether the public signal enhances the quality and use of private information by traders depends entirely on market conditions, such as fundamental volatility and noise-trader risk.

Moreover, the outlined mechanism for information dissemination suggests that analysts are more inclined to offer free public information, and its quality improves, when the crowding-in effect outweighs the crowding-out effect. This comparison yields unique testable predictions.

Firstly, the findings suggest that markets characterized by high *ex-ante* uncertainty are more likely to witness the presence of high-quality free information. This is attributed to a strong crowding-in effect; given that the effect manifests as a reduction in the price impact, the informed trader benefits more when she has a larger *ex-ante* informational advantage. This result is consistent with empirical findings, indicating that analysts inject more informative information into the market when uncertainty is high (e.g., Frankel, Kothari, and Weber (2006), Amiram, Landsman, Owens, and Stubben (2018)).

Secondly, the model predicts that paid information endogenously becomes more precise than free information, with the best free signal being at most half as precise as paid information, even at the limit. This upper bound is determined by the level of information asymmetry that the for-profit analyst aims to maintain among traders to maximize her profits. Moreover, while public information may enhance the quality of private information, and despite the persistence of information frictions, its direct impact on market quality outweighs the changes in private information quality. Hence, it results in a deeper market, distinguishing itself from information sharing in other contexts, such as small short sellers engaging in rumor-mongering (e.g., Ljungqvist and Qian (2016); see the literature review).

Thirdly, the provision of free information, based on the crowding-in effect, is explained within a static framework, influencing the market through equilibrium price adjustments. Consequently, changes in the price and sale of paid information manifest within a relatively short time window. This presents a crucial testable implication in contrast to explanations rooted in marketing and reputational motivations, as these arguments assert that for-profit entities distribute free products to influence medium-term or long-term outcomes (Bawa and Shoemaker (2004)).[2]

Lastly, the baseline model is extended in several directions to analyze the impact of other market conditions. For instance, limited competition among informed traders intensifies the crowding-in effect, as each trader becomes more concerned about their price impact and benefits from its reduction triggered by the public signal. Furthermore, the effect becomes even more significant if information acquisition is not observable to market makers. When the quality of private information improves due to the crowding-in effect, and if that improvement is observable as in the baseline model, market makers become increasingly reliant on order flow information, thereby diminishing the crowding-in effect. Unobservable information acquisition eliminates this channel and slows down the decay of the crowding-in effect. In reality, the quality of privately held information is not readily observable. The model suggests that such opacity in information acquisition strengthens the crowding-in effect and facilitates the dissemination of freely available information.

[2] This argument relies on the notion that it takes time for the supply side to impact consumers' behavior and belief (e.g., Heiman et al. (2001), Li et al. (2019)).

A. Related Literature

Effect of public information. The crowding-out effect of public information on private information production has been analyzed by Verrecchia (1982), Diamond (1985), Kim and Verrecchia (1994), Gao and Liang (2013), and Colombo, Femminis, and Pavan (2014) within the context of information regulations, as summarized by Goldstein and Yang (2017).[3] Most of these studies establish this effect in either a perfectly competitive environment, following Grossman and Stiglitz (1980), or in a monopolistic environment where traders choose between being fully informed or remaining uninformed.

The crowding-in effect in my model emerges from changes in the price impact and is overlooked in the literature within a perfectly competitive environment, where traders are assumed to be price takers and unaware of the price impact. Furthermore, the effect arises in information acquisition concerning the intensive margin, and models with binary information acquisition cannot capture this effect.

Several studies have explored the positive influence of public information on private information acquisition by traders, incorporating various assumptions about information and trading structures. Firstly, Bertomeu, Beyer, and Dye (2011) and Cheynel and Levine (2020) examine an environment with a "mosaic" information structure, where a more precise private signal enables traders to better interpret and process public information. Secondly, McNichols and

[3]Empirical studies present mixed findings regarding the influence of public information. Bushee, Matsumoto, and Miller (2004), Chiyachantana, Jiang, Taechapiroontong, and Wood (2004), Eleswarapu, Thompson, and Venkataraman (2004), Gintschel and Markov (2004), and Chen and Lu (2019) find evidence consistent with the crowding-out effect, while Krinsky and Lee (1996), Coller and Yohn (1997), Straser (2002), and Sidhu, Smith, Whaley, and Willis (2008) report contrary findings.

Trueman (1994) derive the effect in a model involving multiple rounds of trading with the dissemination of public information occurring midway through the process. Thirdly, Han, Tang, and Yang (2016) analyze endogenous liquidity traders, showing that a more precise public signal attracts a larger set of liquidity traders, thereby encouraging potential informed traders to seek and trade on private information. In contrast, the crowding-in effect in my model emerges because the public signal acts as a substitute for order-flow information for market makers and reduces the price impact.[4]

Information sales and disclosure. A substantial body of research, initiated by the works of Admati and Pfleiderer (1986), (1988), and (1990), has explored the dissemination of various types of information under different circumstances.[5] In the context of distributing free fundamental information, Van Bommel (2003), Ljungqvist and Qian (2016), Liu (2017), and Schmidt (2020) analyze private information sharing by investors, such as small short-sellers and mutual fund managers.[6] Constrained by limited trading capacity or a short investment horizon, an investor may disclose privately held news to induce other traders to trade alongside, thereby influencing the price in a favorable direction.

[4]In the context of high-frequency trading, Aoyagi (2020) demonstrates the crowding-in effect of exogenous speed regulations on speed acquisition based on a similar mechanism.

[5]Fishman and Hagerty (1995), Cespa (2008), Garcia and Sangiorgi (2011), and Easley, O'Hara, and Yang (2016) examine the sale of fundamental information at positive prices, while Cheynel and Levine (2012) considers the sale of non-fundamental information. However, these studies rule out the dissemination of free fundamental information as a means to enhance analysts' profits.

[6]In contrast to the one-way information sharing in these studies, several papers, including those by Benveniste, Marcus, and Wilhelm (1992), Foucault and Lescourret (2003), Stein (2008), and Goldstein, Xiong, and Yang (2021), have investigated the mutual exchange of private information among investors driven by the complementarity of information structures.

In contrast, the crowding-in effect in this paper yields distinctive predictions. For instance, even an information seller without investment positions disseminates a fundamental signal without charge. Additionally, the purpose of the free signal is to mitigate the price impact of order flow, leading to predictions opposite to the above studies regarding price reactions. Moreover, unlike the findings in the aforementioned studies, trading on free information is not profitable in my model, as it is intended to be fully reflected in the price by market makers to generate the crowding-in effect.

The mechanism of the crowding-in effect is more closely related to the analyses by Admati and Pfleiderer (1986) and Pasquariello and Wang (2023).[7] A for-profit analyst in Admati and Pfleiderer (1986) sells private signals by introducing personalized noise or by limiting the number of customers, with the goal of constraining information revelation by the price (i.e., its informativeness) to preserve the value of sold signals. In my model, the analyst achieves a similar objective by providing fundamental information to the public and diverting market makers' attention away from order flow information. Contrary to the strategies in Admati and Pfleiderer (1986), however, free information in my model is linked to a reduction in the price impact, and the informed trader exploits private information more intensively, leading to increased equilibrium price informativeness. In a similar vein, Pasquariello and Wang (2023) argue that a trader may disclose a signal to influence market makers' belief updating toward her short-term endowment. Although it weakens the price impact in the short run, the value of long-run private

[7]Banerjee, Marinovic, and Smith (2022) discover a similar effect on corporate disclosure: public information reduces the reliance of informed traders on their private signals and diminishes price informativeness, prompting corporate managers to disclose fundamental information. In contrast, the substitution effect in my model targets different players, i.e., it influences market makers' pricing and encourages more active information acquisition and sales by traders and analysts.

information about the asset's fundamentals declines, opposing to the prediction based on the crowding-in effect.

II. Model

Consider a one-shot trading model inspired by Kyle (1985) with three types of participants: an informed trader, competitive market makers, and a noise trader. The *ex-post* liquidation value of the asset is denoted as v and follows a normal distribution with mean $p_0 = 0$ and variance $\Sigma_0 > 0$, i.e., $v \sim \mathcal{N}(0, \Sigma_0)$. To examine the key mechanisms, this section introduces two minimal extensions: (i) the informed trader's selection of the optimal precision of her private signal at $t = 0$, and (ii) the noisy revelation of material information before the trading stage, manifested as a public signal. At $t = 1$, a trade occurs following the original Kyle model. In Section IV and Appendix B of the Supplementary Material, I discuss my modeling assumptions and assess the robustness of the main result when these assumptions are relaxed.

Based on prior research (e.g., Goldstein and Yang (2017)), the public signal is represented as

$$s_{pub} = v + e_{pub}, \tag{1}$$

where $e_{pub} \sim \mathcal{N}(0, \sigma_{pub}^2)$, and I denote $\tau_{pub} \equiv \sigma_{pub}^{-2}$ as the precision or quality of the public signal. Since it is public, all traders, including market makers, have access to s_{pub}. In what follows, I suppose that s_{pub} is distributed to traders free of charge, while the analysis below verifies that s_{pub}

is free as an equilibrium outcome.[8] Also, Section III delves into the endogenous distribution of s_{pub} by a for-profit analyst who controls τ_{pub} to maximize her profits.

Informed trader. At $t = 0$, before the public signal s_{pub} is revealed,[9] the informed trader obtains the private signal:

$$s = v + e, \tag{2}$$

where $e \sim \mathcal{N}(0, \sigma_e^2)$. The precision (quality) of the private signal is denoted as $\tau_e \equiv \sigma_e^{-2}$. In this stage, τ_e is a choice variable for the informed trader, and she must pay the cost $Q(\tau_e)$ with $Q'(\cdot) > 0$ and $Q''(\cdot) \geq 0$ to obtain the private signal with quality τ_e. In this section, Q and τ_{pub} are exogenously given, while Section III establishes them as equilibrium variables.

The trading stage is standard. Representing the asset price determined by market makers as p, the expected trading profit of the informed trader, given the signal realization, is

$$V(x, s, s_{pub}) = \mathbb{E}[(v - p)x | s, s_{pub}], \tag{3}$$

where x denotes the informed trader's trading quantity.

[8] The paper focuses on the impact of the additional piece of information, s_{pub}, which is public and available to all traders. It rules out the possibility that personalized signals are privately and exclusively provided to market makers, potentially at positive fees. This assumption is made to maintain a competitive market-making sector, although the fundamental mechanism for the crowding-in effect does not hinge on it.

[9] The informed trader acquires private information before the public signal is revealed. See Appendix B.4 of the Supplementary Material for the robustness of main results to the alternative timing assumption.

Noise trader and market makers. The noise trader's behavior is characterized by the random order flow, which is independent of other random variables. Specifically, she places a market order with quantity $u \sim \mathcal{N}(0, \sigma_u^2)$ in the trading stage.

Finally, market makers set the competitive price based on the available information, i.e., the aggregate order flow, $y = x + u$, and the public signal, s_{pub}. The competition between market makers leads to the semi-strong efficient price.

$$p = \mathbb{E}[v|s_{pub}, y]. \tag{4}$$

Equilibrium. The equilibrium concept of the model is the subgame perfect equilibrium. The first stage involves the informed trader's information acquisition (τ_e), and the second stage is the trading game. I assume that all random variables, (v, e, e_{pub}, u), are independent of each other. Figure 1 illustrates the timing of events.

[Insert Figure 1 approximately here]

Definition 1. *The equilibrium of the model is defined by the set of variables* (τ_e, x, p) *such that the following three conditions hold:*

(i) *For any alternate trading strategies,* $\bar{x}$*, and for any* (s, s_{pub})*, the informed trader does not have profitable deviation, i.e.,*

$$\mathbb{E}[(v-p)x|s, s_{pub}] \geq \mathbb{E}[(v-p)\bar{x}|s, s_{pub}].$$

(ii) *The equilibrium price,* p*, satisfies the efficiency condition in equation (4).*

(iii) *The precision of the private signal, τ_e, maximizes the ex-ante expected trading profit of the informed trader, i.e., $\mathbb{E}[V(x, s, s_{pub})]$ with V given in equation (3).*

A. Equilibrium in the Trading Stage

The informed trader forms the expectation using two signals, s and s_{pub}. Employing the standard filtering argument, her updated expectation is represented as

$$\hat{v} \equiv \mathbb{E}[v|s, s_{pub}] = \frac{\tau_e}{\Sigma_0^{-1} + \tau_e + \tau_{pub}} s + \frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_e + \tau_{pub}} s_{pub}. \tag{5}$$

Similarly, market makers' quote before observing order flow is computed as the following expectation based on the public signal.

$$p_{pub} \equiv \mathbb{E}[v|s_{pub}] = \frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_{pub}} s_{pub}. \tag{6}$$

I focus on the linear equilibrium, in which the informed trader places the following order:

$$x = \beta(\hat{v} - p_{pub}). \tag{7}$$

Namely, she exploits her informational advantage over market makers, represented by $\hat{v} - p_{pub}$, with intensity β.

Conversely, market makers set the trade-execution price by updating their belief conditional on the aggregate order flow, $y = x + u$:

$$p = p_{pub} + \lambda \left(y - \mathbb{E}[y|s_{pub}]\right), \tag{8}$$

where λ represents the price impact measure, and $\mathbb{E}[y|s_{pub}] = 0$ due to the law of iterated expectations.

Solution. Consider the informed trader's optimal trading strategy given the price in equation (8) and realized signals:[10]

$$x(s, s_{pub}) \equiv \arg\max_x V(x, s, s_{pub}) = \arg\max_x (\hat{v} - p_{pub} - \lambda x)x. \tag{9}$$

The FOC leads to

$$x(s, s_{pub}) = \frac{\hat{v} - p_{pub}}{2\lambda}, \tag{10}$$

suggesting that $\beta = \frac{1}{2\lambda}$. Next, consider the market efficiency condition given the linear trading strategy of the informed trader in equation (7). Once again, the standard filtering problem leads to the linear pricing rule in equation (8) with

$$\lambda = \frac{\beta \tau_e}{\beta^2 \tau_e + (\Sigma_0^{-1} + \tau_{pub}) \left(\Sigma_0^{-1} + \tau_e + \tau_{pub}\right) \sigma_u^2}. \tag{11}$$

Proposition 1. *There exits a unique linear equilibrium in the trading stage, in which the trading strategy of the informed trader and the asset price are given by equations (7) and (8) with the*

[10] By the law of iterated expectation, it holds that $\mathbb{E}[p_{pub}|s, s_{pub}] = \mathbb{E}[\mathbb{E}[v|s_{pub}]|s, s_{pub}] = p_{pub}$.

following coefficients.

$$\beta = \sigma_u \sqrt{\frac{\left(\Sigma_0^{-1} + \tau_{pub}\right)\left(\Sigma_0^{-1} + \tau_e + \tau_{pub}\right)}{\tau_e}}, \tag{12}$$

$$\lambda = \frac{1}{2\sigma_u} \sqrt{\frac{\tau_e}{\left(\Sigma_0^{-1} + \tau_{pub}\right)\left(\Sigma_0^{-1} + \tau_e + \tau_{pub}\right)}}. \tag{13}$$

Proof. Solving $\beta = \frac{1}{2\lambda}$ and equation (11) yields the result. □

The price impact, λ, increases with τ_e and decreases with τ_{pub}, as they result in significant information asymmetry between the informed trader and market makers, reducing market liquidity (Kyle (1985), Glosten and Milgrom (1985)). Additionally, these changes make the aggregate order flow relatively more informative than the public signal, leading market makers to rely more on order flow to learn v.[11]

B. Ex-ante Expected Profit

Employing Proposition 1, I derive the *ex-ante* expected profit of the informed trader, denoted as $V(\tau_e, \tau_{pub}) \equiv \mathbb{E}[V(s, s_{pub})]$, where the expectation is taken with respect to the realization of signals. By applying the optimal trading strategy, it holds that

$$V(\tau_e, \tau_{pub}) = \frac{\beta}{2} Var(\hat{v} - p_{pub}). \tag{14}$$

[11]The equilibrium converges to the original one-period Kyle (1985) model when $\sigma_{pub} \to \infty$ and $\sigma_e \to 0$, representing a model with a perfectly informed trader and no public signal.

This expression reveals that the expected profit stems from two key factors: the informational advantage of the informed trader, $Var(\hat{v} - p_{pub})$, and the intensity of trading based on this advantage, $\beta/2$. Further computations lead to the subsequent formula.

Lemma 1. *The ex-ante expected profit of the informed trader, before the cost of the signal, is given by*

$$V(\tau_e, \tau_{pub}) = \frac{\sigma_u b_{pub}}{2} \sqrt{\frac{\tau_e}{1 + b_{pub}\tau_e}}, \tag{15}$$

where $b_{pub} \equiv Var(v|s_{pub}) = \frac{\Sigma_0}{1+\tau_{pub}\Sigma_0}$.

Proof. See Appendix A.1. □

The expected profit is increasing and concave in the precision of the private signal, τ_e, indicating the diminishing marginal impact of τ_e. On one hand, a more precise private signal furnishes the informed trader with a larger informational advantage and higher expected profits.[12] On the other hand, it exacerbates the asymmetric information problem, prompting market makers to impose a larger price impact. The informed trader responds to this change by trading less intensively (i.e., β decreases), resulting in a decline in the expected profit. This negative impact of τ_e on V can be thought of as the *endogenous* marginal cost of information acquisition.

Equation (15) suggests that the first positive impact of τ_e dominates the second negative impact. This dominance holds because the illiquid market arises as a consequence of a better informed trader, indicating that the endogenous cost channel is an indirect effect of τ_e and cannot outweigh its positive direct impact.

[12] The informational advantage is explicitly computed as $Var(\hat{v} - p_{pub}) = \frac{\Sigma_0^2\tau_e}{1+\Sigma_0\tau_e+\Sigma_0\tau_{pub}} \frac{1}{1+\Sigma_0\tau_{pub}}$, which increases with τ_e.

Moreover, the public signal influences the profit function solely through the conditional variance of the fundamental value, denoted as $b_{pub} = Var(v|s_{pub})$. This arises from the fact that s_{pub} is available to all traders, and only the residual uncertainty of v after observing s_{pub} matters to the informed trader's profit.

C. Information Acquisition and Impact of Public Signal

The information acquisition problem of the informed trader at $t = 0$ is described as

$$\max_{\tau_e \geq 0} V(\tau_e, \tau_{pub}) - Q(\tau_e), \tag{16}$$

where $V(\tau_e, \tau_{pub})$ is given by equation (15).

Proposition 2. *Given Q, the optimal information acquisition is characterized by the unique solution to the following FOC.*

$$Q'(\tau_e) = \frac{\partial V(\tau_e, \tau_{pub})}{\partial \tau_e} = \frac{\sigma_u b_{pub}}{4\tau_e^2(\tau_e^{-1} + b_{pub})^{\frac{3}{2}}}. \tag{17}$$

Proof. Since the RHS of equation (17) monotonically decreases with τ_e and $Q'' \geq 0$, the SOC is satisfied. As the RHS converges to ∞ and 0 at $\tau_e = 0$ and $\tau_e \to \infty$, respectively, equation (17) has a unique positive solution. □

The optimal quality of private information balances the marginal benefit of obtaining high-quality private information and its exogenous marginal cost. For later use, the RHS of equation (17) is also interpreted as the informed trader's willingness to pay for the signal with quality τ_e. Given Q, it forms the downward-sloping demand function due to the diminishing impact of τ_e on the expected profit.

To comprehend the impact of the public signal on information acquisition, examine the partial derivative of the willingness to pay in equation (17) with respect to τ_{pub}.

$$\frac{\partial^2 V(\tau_e^*, \tau_{pub})}{\partial \tau_{pub} \partial \tau_e} = Q'(\tau_e)\Sigma_0 \frac{\frac{1}{2}\tau_e b_{pub} - 1}{\tau_e b_{pub}(\tau_e^{-1} + b_{pub})}. \tag{18}$$

Equation (18) can be both positive and negative, demonstrating that the impact of τ_{pub} on optimal information acquisition is ambiguous. This ambiguity stems from two competing effects.

Crowding-out effect. The motivation behind acquiring a higher-quality private signal is to amplify the informational advantage, i.e., $\frac{\partial Var(\hat{v}-p_{pub})}{\partial \tau_e} > 0$. However, as the public signal becomes more precise, this positive impact diminishes, as confirmed by $\frac{\partial}{\partial \tau_{pub}}\left(\frac{\partial Var(\hat{v}-p_{pub})}{\partial \tau_e}\right) < 0$, thereby making it less valuable to increase τ_e. This phenomenon is referred to as the *crowding-out* effect of public information on private information acquisition (Verrecchia (1982), Diamond (1985), Kim and Verrecchia (1994), and others).

Crowding-in effect. On the contrary, equation (18) reveals a positive reaction of the optimal τ_e to τ_{pub}, referred to as the *crowding-in* effect of public information. When the public signal becomes more precise, the market makers' pricing behavior becomes more dependent on it, making the price impact of the order flow less responsive to information acquisition. This allows the informed trader to trade more intensively based on her informational advantage, making it more valuable to increase τ_e. This phenomenon is captured by $\frac{\partial}{\partial \tau_{pub}}\left|\frac{\partial \beta}{\partial \tau_e}\right| < 0$ and is interpreted as a reduction in the endogenous marginal cost of increasing τ_e. Consequently, it becomes optimal for the informed trader to increase the precision of her private signal even in the presence of a more precise public signal.

Due to competition between the crowding-in and crowding-out effects, the optimal information acquisition exhibits an ambiguous reaction to the public signal. To formalize this argument, define $\bar{b}_{pub}$ as a unique solution to

$$Q'\left(\frac{2}{b_{pub}}\right) = \frac{\sigma_u}{4}\left(\frac{b_{pub}}{6}\right)^{\frac{3}{2}}, \tag{19}$$

and introduce the following condition:

$$Q'\left(\frac{2}{\Sigma_0}\right) < \frac{\sigma_u}{4}\left(\frac{\Sigma_0}{6}\right)^{\frac{3}{2}}. \tag{20}$$

Proposition 3. *If condition (20) holds, the optimal quality of the private signal follows a hump-shaped curve in relation to the quality of the public signal. The unique tipping point is given by*

$$\bar{\tau}_{pub} = \frac{1}{\bar{b}_{pub}} - \frac{1}{\Sigma_0}. \tag{21}$$

Otherwise, the optimal quality of private information is monotonically decreasing in τ_{pub}.

Proof. See Appendix A.2. □

[Insert Figure 2 approximately here]

Figure 2 depicts a numerical example of Proposition 3. The crowding-in effect of public information dominates (resp. is outweighed by) the crowding-out effect when τ_{pub} is small (resp. large), resulting in a single-peaked curve of the optimal τ_e against τ_{pub}.

To grasp the intuition behind the hump-shaped curve, consider a scenario where the quality of the public signal is very low (e.g., $\tau_{pub} = 0$) and gradually increases. This triggers both the crowding-in and crowding-out effects, but their impacts on the optimal information acquisition differ significantly.

On one hand, owing to the imprecise public signal, the informed trader possesses a substantial informational advantage. Consequently, changes in trading intensity (β) have a sizable impact on expected trading profit. Thus, the crowding-in effect, stemming from changes in trading intensity, is greatly magnified in a small-τ_{pub} region.

On the other hand, a small τ_{pub} prompts market makers to heavily rely on the order flow, resulting in a significant price impact and weak trading intensity. Since trading intensity is already weak, the expected trading profit does not react much even if the informational advantage deteriorates due to the crowding-out effect. Consequently, the crowding-out effect has a limited impact on information acquisition in a small-τ_{pub} region. Since the opposite argument holds in a large-τ_{pub} region, the optimal τ_e draws a hump-shaped curve, as Proposition 3 suggests.

From equations (19) and (21), the tipping point, $\bar{\tau}_{pub}$, is (weakly) decreasing in a upward shift in Q', as it amplifies the crowding-out effect of τ_{pub} by making a high-quality private signal more expensive exogenously. Conversely, the noise-trader risk, σ_u, and the initial fundamental uncertainty, Σ_0, positively influence $\bar{\tau}_{pub}$. This is because these parameters provide a larger informational advantage to the informed trader and strengthen the crowding-in effect.

D. Market Quality

This subsection constructs the measures of market quality by incorporating the equilibrium information acquisition by the informed trader.

Price efficiency. Firstly, I derive the variance of v conditional on the price information. Since $p = p_{pub} + \lambda y$, observing the price information leads to $Var(v|p) = \frac{\Sigma_0}{1+\Sigma_0 \eta}$, where $\eta \equiv \tau_{pub} + \frac{\tau_e}{2+b_{pub}\tau_e}$. Then, the price informativeness is defined by the signal-to-noise ratio of the price.

$$\Sigma \equiv \frac{Var(v)}{Var(v|p)} = 1 + \Sigma_0 \eta. \tag{22}$$

Σ measures the amount of value uncertainty that is resolved by observing the equilibrium price. The higher the value of Σ, the more efficient the price is.

Incorporating the equilibrium information acquisition in equation (17), the following result holds.

Proposition 4. *The price informativeness is increasing in the precision of the public signal.*

Proof. See Appendix A.3. □

Firstly, τ_{pub} directly improves the price informativeness by prompting market makers' learning of v. Secondly, a higher τ_{pub} triggers the crowding-in and the crowding-out effects on information acquisition by the informed trader (Proposition 3), making the order flow more or less informative. When the crowding-in effect is dominant, both of these channels improve the price efficiency. Even when the crowding-out effect is dominant, it is partially offset by the crowding-in effect and cannot be strong to overturn the direct positive effect of public information.

Price impact. Consider the price impact, λ, as the measure of trading cost and market illiquidity. Proposition 1 indicates that τ_{pub} reduces λ because it alleviates the asymmetric information problem. However, it also affects the precision of the private signal: the informed trader becomes more or less informed, generating a non-trivial impact on λ. Once again, the result below shows that the indirect effect cannot dominate the direct effect of τ_{pub} due to the crowding-in versus crowding-out effects.

Proposition 5. *The market liquidity, measured by λ^{-1}, is increasing in the precision of the public signal.*

Proof. See Appendix A.3. □

Overall, Propositions 4 and 5 provide an optimistic prediction regarding the impact of public information. Both the market liquidity and the price efficiency improve when a public signal provides precise information about the asset's fundamentals. Also, these results justify the use of τ_{pub} as the metric of market quality when I endogenize this variable in Section III, as the market quality measures are increasing in τ_{pub} after incorporating changes in τ_e.

E. Discussion and Policy Implication

The notable crowding-in effect observed in this study arises from two distinctive aspects of the model. Firstly, it considers the strategic motive of the informed trader, who incorporates the equilibrium price impact of her behavior. The equilibrium price becomes more responsive to order flow when the informed trader acquires more private information, thereby imposing an implicit cost on her information acquisition. The presence of a public signal, in turn, weakens this response of the price impact by making market makers rely less on the order flow. It leads to the crowding-in effect, as the informed trader can acquire information more aggressively without affecting the price. This aspect is absent in perfectly competitive models with price-taking informed traders (e.g., those based on Grossman and Stiglitz (1980)).

To assess the importance of this assumption, Section IV presents two natural extensions of the baseline model: one with $N \geq 1$ informed traders, where N controls the degree of competition among them and their awareness of the price impact, and the other with unobservable information acquisition, in which the informed trader cannot commit to her choice of τ_e nor control market makers' beliefs about it.

Secondly, this model focuses on the intensive margin of information acquisition, where the informed trader is not perfectly informed and adjusts the precision of her signal. This differs from models based on Kyle (1985), where a private signal perfectly reveals v, and information acquisition is a binary choice between being fully informed and uninformed.[13] In such models, the crowding-in effect does not arise, and public information consistently diminishes private information production by reducing the expected profit level.

Previous studies on crowding-in effects rely on specific assumptions about information structures (Cheynel and Levine (2020)) or about the timing of trades (McNichols and Trueman

[13] In reality, information acquisition involves both the extensive and intensive margins. For instance, as suggested by IEX, modern high-frequency traders need to invest in sophisticated communication technologies, incurring fixed investment costs. Additionally, they subscribe to various information services, such as direct data feeds and colocation of information servers, where subscription fees depend on the quality of technologies.

(1994)). Conversely, Proposition 3 demonstrates that it is an inherent feature of the basic model of strategic information acquisition, provided both the aforementioned aspects are considered.

Furthermore, Proposition 3 has implications for potential consequences of information regulations aiming to increase the amount of public information in financial markets, such as fair-disclosure regulation and changes in accounting standards or disclosure enforcement. My results suggest that such regulations may promote private information production. This finding goes counter to the existing models that focus on the crowding-out effect, which raise doubts about the effectiveness of regulation by unintentionally discouraging private information production among potential informed traders. Empirically, the literature argues that the effect of public information is usually ambiguous and difficult to capture in the data (Leuz and Wysocki (2016)). The crowding-in effect in this paper proposes the additional channel through which those policies may enhance market quality by partially offsetting their unfavorable impact on private information production.

III. Endogenous Public Signal

This section explores the crowding-in effect to understand why a for-profit information seller may choose to disseminate public information for free. It also delves into the equilibrium characteristics of such information and the market conditions that encourage free information distribution.

To maintain focus on the behavior of a specialized information seller and for tractability, this analysis assumes that the analyst does not trade on her own account using the information she uncovers.[14] This could be justified by risk aversion (Admati and Pfleiderer (1986)) or specialization (Golec (1992)), e.g., an independent analyst possesses information-gathering abilities but lacks capital to invest, while traders have capital and market access but lack technologies to discover primary information.

A. Analyst

A for-profit analyst provides both a private signal and a public signal, characterized by qualities τ_e and τ_{pub}, respectively. At the first stage, she determines τ_{pub} prior to the trading game, and subsequently, τ_e is determined by the demand and supply in the market for private information, as elaborated below.[15]

Cost of producing information. To act as an information provider, the analyst makes an investment to gather information regarding the asset's fundamentals. With this in mind, the following information-production cost is imposed.

$$C(T) = \frac{c}{2}T^2, \tag{23}$$

with $c > 0$ and $T \equiv \max\{\tau_e, \tau_{pub}\}$. Intuitively, the cost associated with uncovering a noisy signal about v increases and becomes convex as its precision improves, reflecting the increasing difficulty of obtaining more precise information. Moreover, according to the definition of T, the

[14]I also abstract away from incentive problems faced by the analyst and mechanism design arguments by assuming that information is communicated truthfully.

[15]See Appendix B.2 of the Supplementary Material for the robustness of main results when multiple information sellers sell signals to a single trader.

analyst incurs the cost only once to acquire the primary source of information. Once she obtains a signal with precision $\tilde{\tau}$ by paying the cost, she can provide a signal with lower quality, such as $\hat{\tau} \leq \tilde{\tau}$, without incurring additional costs. This scenario mirrors real-world information supply, where substantive information is produced by making investments in information technologies (e.g., hiring skilled economists, purchasing monitoring equipment, and acquiring raw data), while replicating and disseminating a garbled version of the original information entails almost zero marginal costs (Romer (1990), Veldkamp (2006)).

Market for Private Information. In the market for private information, I assume that the trader pays (the analyst receives) $Q(\tau_e) = q\tau_e$ for the private signal with quality τ_e, where q denotes the unit price of information quality.[16]

Regarding the market for the private signal, I focus on the competitive Walrasian equilibrium, in which the informed trader and the analyst propose the demand and the supply of information given the unit price q, and the market clears. As the bargaining literature has established (Yildiz (2003), Dávila and Eeckhout (2008), Penta (2011)),[17] this equilibrium is achieved as a result of a two-agent alternating-offer process even if agents have market power and are not price-takers. In this process, the analyst offers price q, and the trader either demands her optimal signal quality at the proposed price, or rejects it. In the former case, the demanded trade is realized, and the market ends. If she rejects the offer, the bargaining proceeds to the next round,

[16] To motivate this payment, τ_e can be interpreted as the *number* of private signals sold at unit price q, where each signal is denoted as $s_l = v + e_l$ with iid errors $e_l \sim \mathcal{N}(0, 1)$. Acquiring τ_e units of such signals is informationally equivalent to observing $s \equiv \frac{1}{\tau_e}\sum_{l=1}^{\tau_e} s_l = v + \frac{1}{\tau_e}\sum_{l=1}^{\tau_e} e_i$, so that rewriting $e \equiv \frac{1}{\tau_e}\sum e_i \sim \mathcal{N}(0, \tau_e^{-1})$ maintains consistency.

[17] Dávila and Eeckhout (2008) show the convergence result when agents offer a price and a maximum amount to be exchanged, while Penta (2011) extends the convergence result to games with an arbitrary number of agents.

where the trader offers an alternative price, and the analyst either supplies her optimal signal or rejects the offer. This alternating-offer process goes on until they reach an agreement.

1. Private Information

By providing information with qualities (τ_e, τ_{pub}), the analyst earns the following profit:

$$\pi_A(\tau_e, \tau_{pub}) = q\tau_e - C(T). \tag{24}$$

Due to the cost structure in equation (23), two possible cases arise based on $\tau_e \gtrless \tau_{pub}$. However, setting $\tau_e < \tau_{pub}$ cannot be optimal, as the marginal benefit (before the cost) of increasing τ_e is always positive ($q > 0$), whereas the benefit from increasing τ_{pub} turns negative due to the crowding-out effect. Therefore, the focus is on the equilibrium where $\tau_e \geq \tau_{pub}$ in the subsequent analysis.

From the FOC of equation (24) with respect to τ_e, the marginal cost of information production establishes the following upward-sloping supply curve.

$$q = c\tau_e. \tag{25}$$

Conversely, the informed trader's demand for information is characterized by the FOC given in equation (17):

$$q = \frac{\sigma_u b_{pub}}{4\tau_e^2(\tau_e^{-1} + b_{pub})^{\frac{3}{2}}}. \tag{17'}$$

Due to the monotonicity of these functions, the equilibrium is uniquely determined by the market-clearing price.

Lemma 2. *Given τ_{pub}, the market for the private information yields the following equilibrium quality and price of the private signal.*

$$\tau_e^* = \frac{\sqrt{1 + 4b_{pub}\left(\frac{\sigma_u b_{pub}}{4c}\right)^{\frac{2}{3}}} - 1}{2b_{pub}}, \tag{26}$$

and $q^* = c\tau_e^*$.

Proof. See Appendix A.4. □

Note that these equilibrium variables are influenced by the analyst's choice of τ_{pub} at the first stage, thereby motivating the dissemination of public information.

2. Public Information

Considering τ_e^* in equation (26) as a function of τ_{pub} and adopting the optimal supply in equation (25), the analyst's objective function during the stage of public-information dissemination is summarized by $\pi_A = \frac{c}{2}\tau_e^{*2}$. As per Proposition 3, competition between the crowding-in and crowding-out effects shapes the profit into a single-peaked curve concerning τ_{pub}.[18] Thus, the optimal τ_{pub} in the first stage is determined by $\frac{d\tau_e^*}{d\tau_{pub}} = 0$. As shown in equation

[18]The remainder of this subsection focuses on the case where condition (20) holds, while Proposition 6 and discussions thereafter incorporate the possibility that the demand function becomes monotonically decreasing in τ_{pub}.

(18), this condition is equivalent to

$$\tau_e^* \frac{\Sigma_0}{1 + \Sigma_0 \tau_{pub}} - 2 = 0. \tag{27}$$

Initially, an increase in the quality of the public signal benefits the analyst by enhancing the trader's willingness to pay for the private signal through the crowding-in effect. This underpins the core rationale for disseminating free public information. However, this benefit begins to diminish as τ_{pub} becomes sufficiently large, amplifying the crowding-out effect. Condition (27) signifies that the analyst establishes the strategy such that these effects cancel out each other.

B. Equilibrium Information Structure

Solving conditions (26) and (27) leads to the following information structure in the equilibrium.

Proposition 6. *The equilibrium qualities of the private and the public signals are given by*

$$\tau_e^* = \frac{1}{3} \left(\frac{3\sigma_u}{2c} \right)^{\frac{2}{5}} \tag{28}$$

and

$$\tau_{pub}^* = \left[\frac{1}{6} \left(\frac{3\sigma_u}{2c} \right)^{\frac{2}{5}} - \frac{1}{\Sigma_0} \right]^+, \tag{29}$$

where $z^+ \equiv \max\{z, 0\}$.

Proof. See Appendix A.4. □

Once again, the analyst may distribute fundamental information to the public without charge to exploit the crowding-in effect. This public information diverts market makers' attention away from order-flow information and enables the informed trader to leverage her informational advantage, thereby enhancing the value of private information. The characteristics of the equilibrium in Proposition 6 will be analyzed in more detail in the following subsection.

It is worth noting that the fundamental mechanism underlying the distribution of free information can be linked to the discussion in Admati and Pfleiderer (1986), where an information seller with no trading positions introduces personalized noise to sold signals or distributes them to selected investors. Generally, more precise private information leads to more aggressive use by the trader and faster information revelation, ultimately diminishing its informational value. The information-provision strategies outlined in Admati and Pfleiderer (1986) impede information aggregation by the price, thereby slowing down the decay of information value. In my model, the analyst can achieve a similar objective by directly providing information to market makers and diverting their attention away from order flow information. As the analysis below demonstrates, the free public signal yields distinct predictions regarding the equilibrium information structure.

Why the public signal is free. Although the analyses so far have assumed that the public signal is distributed without charge, Proposition 6 and the competitive market-making sector verify that s_{pub} being free is both necessary and sufficient for the equilibrium. On one hand, Proposition 6 shows that the analyst optimally chooses to provide public information ($\tau^*_{pub} > 0$) even if it is free because of the crowding-in effect, demonstrating its sufficiency for the equilibrium. On the other hand, competitive market makers require the public signal to be free, as their profits from trading (with the cost of the public signal, if any, already sunk) are competed away.[19] Hence, they intend to learn s_{pub} only if it is free, suggesting its necessity for the equilibrium.

C. Comparative Statics and Testable Implications

Proposition 6 provides several testable predictions about information quality and the market characteristics facilitating the coexistence of free and paid signals.

Corollary 1. *(i) τ_e^* increases with noise-trader risk, σ_u, decreases with the marginal information-production cost of the analyst, c, and is independent of fundamental uncertainty, Σ_0.*

(ii) The equilibrium unit price of the private signal, q^*, increases with σ_u and c but is independent of Σ_0.

(iii) The analyst disseminates free public information if and only if the following condition holds.

$$\frac{1}{6}\left(\frac{3\sigma_u}{2c}\right)^{\frac{2}{3}} > \frac{1}{\Sigma_0}. \tag{30}$$

In this case, τ_{pub}^* increases with σ_u and Σ_0, while decreases with c.

Proof. The results directly follow from equations (28) and (29). □

The impact of σ_u and c on the quality and the price of the private signal is clear. A greater noise-trader risk augments the value and demand for private information, thereby increasing both

[19] In this paper, "public" refers to information that is potentially accessible to all market participants. Although a fee may be charged to access it, s_{pub} is not exclusive to any single player, thereby maintaining the competitive nature of the market-making sector.

its quality and price in the equilibrium. Conversely, a higher information-production cost prompts an upward shift in the supply schedule of the private signal, resulting in contrasting responses in the price and quality. The irrelevance of Σ_0 to τ_e^* will be discussed below.

Regarding the quality of the public signal, σ_u and Σ_0 reinforce the crowding-in effect, resulting in high-quality public information. Conversely, an increase in c has the opposite effect, as the analyst becomes more hesitant to produce information and drives up the equilibrium signal price from the supply side, intensifying the crowding-out effect.

Based on this insight, the analyst disseminates free public information, alongside paid private information, in markets exhibiting a strong crowding-in effect, as indicated by condition (30). Therefore, the model suggests that markets characterized by high uncertainty, stemming from active noise trading and volatile asset fundamentals, are more likely to witness the distribution of free public information by for-profit analysts.

Information quality gap. How precise is the free public information compared to the paid private information? This question is crucial because private signals are generally unobservable. Yet, understanding the relationship between signal qualities allows us to infer the quality of private information from publicly available information.

Proposition 6 yields two interesting observations. Firstly, σ_u and c affect τ_{pub}^* only indirectly through τ_e^*, and their impacts on τ_{pub}^* are disproportionally weaker than their impacts on τ_e^*. Secondly, Σ_0 is irrelevant to τ_e^*, but it increases τ_{pub}^*. These responses contribute to the information quality gap, defined as $\tau_e^* - \tau_{pub}^*$.

To grasp the intuition, recall that the informed trader controls τ_e considering the level of asymmetric information, defined by how much uncertainty market makers face relative to the informed trader before observing order flow information:

$$I \equiv \frac{Var(v|s_{pub})}{Var(v|s, s_{pub})} = 1 + \tau_e b_{pub}. \tag{31}$$

Note that $I > 1$ implies information asymmetry, with a larger I indicating a more severe friction. Hence, the residual uncertainty, $b_{pub} = Var(v|s_{pub}) = (\Sigma_0^{-1} + \tau_{pub})^{-1}$, is crucial to the informed trader, rather than the value uncertainty, Σ_0, itself (see equation (15)). Moreover, equation (31) can be rearranged as

$$\tau_{pub} = \frac{\tau_e}{I - 1} - \frac{1}{\Sigma_0}, \tag{32}$$

which delineates the relationship between private and public information qualities given I. The public signal must be disproportionately less precise than the private signal to achieve information friction I, with this disparity widening as I increases.

In the equilibrium, equation (29) establishes the relationship (32) as

$$\tau_{pub}^* = \frac{\tau_e^*}{2} - \frac{1}{\Sigma_0}, \tag{33}$$

indicating that information asymmetry remains constant at $I^* = 3$. The analyst aims to fully exploit the crowding-in effect and achieves this objective by maintaining this target level of asymmetric information among traders.

Since the analyst's objective is to achieve I^*, she manages τ_{pub} and fully adjusts to changes in fundamental uncertainty, Σ_0. For instance, heightened volatility in the asset's value exacerbates *ex-ante* information asymmetry. In response, the analyst disseminates a more precise public signal to offset the effect of Σ_0 on I^*, leaving τ_e^* unaffected. This irrelevance of fundamental uncertainty to private information in equilibrium, while technically derived from the envelope condition of equation (27), stems from endogenous public information and is a unique outcome of this model.

The following proposition summarizes the implications of equation (33).

Proposition 7. *(i) The private signal is always more precise than the public signal, $\tau_e^* > \tau_{pub}^*$.*

(ii) When σ_u increases or c decreases, τ_e^* increases more than τ_{pub}^*. At the limit of $\sigma_u/c \to \infty$, the quality gap becomes infinitely large.

(iii) When Σ_0 increases, the quality gap shrinks. At the limit of $\Sigma_0 \to \infty$, the public signal is half as precise as the private signal.

Proof. The result directly follows from equation (33) and Corollary 1. □

The first statement aligns with the observation in real markets: analysts provide more comprehensive and high-quality information to paid customers compared to their free blog posts and social media feeds. Importantly, this gap arises without imposing exogenous restrictions in information-production costs, as the analyst aims to leave information asymmetry between traders by setting $\tau_e^* > \tau_{pub}^*$.

Furthermore, when Σ_0 is high, the level of *ex-ante* asymmetric information is already severe before the realization of s_{pub}. Therefore, given τ_e^*, the analyst aims to disseminate more precise public information to achieve $I^* = 3$. The last statement in Proposition 7 is a consequence of equation (32) at the limit:

$$\lim_{\Sigma_0 \to \infty} \tau_{pub} = \frac{\tau_e}{I-1}.$$

Thus, the degree of asymmetric information sets the upper bound on τ_{pub} given τ_e. Since

$I^* = 3$ in the equilibrium, even the best free public signal is only half as precise as the paid private signal.

Overall, the model delineates the characteristics of paid and free information in equilibrium when the dissemination of the latter is driven by its crowding-in effect. They are differently influenced by market conditions, leading to fluctuations in the information quality gap. However, changes in the quality gap cannot serve as a direct indicator of fluctuations in information frictions (I) because the analyst adjusts the qualities to achieve a constant level of I^*.

IV. Extension and Robustness Check

This section relaxes the assumptions in the baseline model to assess the robustness of the key results and to derive additional implications. The details and solutions of the extended models are provided in Appendix B of the Supplementary Material.

A. Multiple Informed Traders

One of the primary drivers of the crowding-in effect is the strategic choice of information quality by the informed trader, considering its impact on the behavior of market makers. This phenomenon relies on the monopolistic nature of the trader and may become more prominent when competition among traders is limited. The following extension introduces multiple informed traders and investigates this prediction. It also provides further insights into the type of market participants to whom analysts provide information, either for a positive fee or free of charge.

1. Environment

Consider an extension of the baseline model with $N \geq 1$ informed traders indexed by $i = 1, 2, \cdots, N$. Prior to the trading session, trader i acquires a private signal $s_i = v + e_i$ with $e_i \sim \mathcal{N}(0, \tau_i^{-1})$, where $\{e_i\}_{i=1}^N$ are mutually independent. As in the baseline model, the analyst earns the following profit by supplying private signals with qualities $\{\tau_i\}_{i=1}^N$ at prices $\{q_i\}_{i=1}^N$, along with the public signal of quality τ_{pub}:

$$\pi_A = \sum_{i=1}^{N} q_i \tau_i - C(T), \tag{34}$$

where $T \equiv \max\{\tau_1, \cdots, \tau_N, \tau_{pub}\}$. In this extension, the analyst also adjusts N before entering the market for private information, which can be seen as controlling the customer base or establishing business relationships with investors to sell information at a positive price. Note that liquidity takers without private information have no influence on the equilibrium because they have no informational advantages relative to market makers and opt out of the market at the trading stage. The remaining parts of the model are the same as those in the baseline setting, and I focus on the symmetric equilibrium (i.e., $\tau_i = \tau_j = \tau_e$ for all $i, j = 1, \cdots, N$).[20]

2. Crowding-in Effect and Equilibrium Information Structure

Firstly, consider the market for private information given τ_{pub} and N.

Proposition 8. *Both the individual demand and the equilibrium quality of private information*

[20] As Appendix B.1 of the Supplementary Material formally attests, the cost structure renders the quality of supplied signals identical in the equilibrium, thereby supporting the symmetric equilibrium analyzed below.

take a hump-shaped curve in relation to τ_{pub}. The tipping point for the individual demand is

$$\bar{\tau}^{D}_{pub} = \left[\left(\frac{\sigma_u}{2c} G(N) \right)^{\frac{2}{5}} - \frac{1}{\Sigma_0} \right]^{+}, \tag{35}$$

and that for the equilibrium information quality is

$$\bar{\tau}_{pub} = \left[\left(\frac{\sigma_u}{2c} N G(N) \right)^{\frac{2}{5}} - \frac{1}{\Sigma_0} \right]^{+}, \tag{36}$$

where $G(N)$ is given by equation (B.24) in Appendix B.1 of the Supplementary Material.

Proof. See Appendix B.1 of the Supplementary Material. □

[Insert Figure 3 approximately here]

Proposition 8 attests that the crowding-in effect persists, forming the equilibrium quality of private information into a hump-shaped curve with respect to τ_{pub}.

To address the impact of competition, numerical experiments are conducted.[21] Figure 3 reports the response of the individual demand to τ_{pub} (Panel A) and the reaction of the tipping point $\bar{\tau}^{D}_{pub}$ to changes in N (Panel B). The result suggests that the crowding-in effect remains robust in this environment, albeit weakening as the number of traders increases. Therefore, a market with limited competition, involving a small N, fosters a strong crowding-in effect. It is consistent with the aforementioned prediction: as informed traders become less competitive, each of them becomes more concerned about the impact of her information acquisition on the

[21] As Appendix B.1 of the Supplementary Material attests, whether N strengthens the crowding-in effect depends only on N itself, as other parameters enter the analysis as a coefficient of the demand function. Numerical experiments with large values of N (e.g., $N = 10^5$) exhibit a monotonically decreasing tipping point in relation to N, as in Panel B of Figure 3, suggesting the robustness of this relation.

equilibrium price, thereby amplifying the main channel of the crowding-in effect of public information.

Secondly, considering the choice of τ_{pub} and N by the analyst at the first stage, I obtain the following information structure in the equilibrium.[22]

Proposition 9. *(i) There exists a unique N^* that maximizes the analyst's profit.*

(ii) With N^*, the equilibrium qualities of private and public signals are given by

$$\tau_e^* = \left[\left(\frac{\sigma_u}{c} \right)^2 (1 - m(N^*)) m(N^*)^3 N^* R(N^*)^2 \right]^{\frac{1}{5}}, \tag{37}$$

and

$$\tau_{pub}^* = \left(\frac{\tau_e^*}{2} \frac{1 - m(N^*)}{m(N^*)} - \frac{1}{\Sigma_0} \right)^+,$$

where $m(N)$ and $R(N)$ are characterized in Appendix B.1 of the Supplementary Material.

Proof. See Appendix B.1 of the Supplementary Material. □

The first point indicates that the analyst optimally limits the sales of information ($N^* < \infty$). The intuition can be understood by a simple price-quantity tradeoff: as established in the literature (e.g., Kyle (1989)), fierce competition among informed liquidity takers diminishes individual expected returns and discourages their information acquisition.[23] Although expanding the customer base directly increases the analyst's revenue, the profit margin shrinks, and this tradeoff determines N^*.

[22]For tractability, this analysis ignores the integer restriction of N and takes it as a continuous variable.

[23]See inequality (B.19) in Appendix B.1 of the Supplementary Material.

Additionally, the second point establishes the dissemination of public information in this extension. As in the baseline model, the hump-shaped reaction of τ_e to τ_{pub} motivates the analyst to provide free information to exploit the crowding-in effect through the market makers' belief updating.

3. Target Audience and Prices of Information

The above results also aid in analyzing which types of market participants the analyst charges a positive or zero price for information. Propositions 8 and 9 indicate that information provided exclusively to liquidity takers is always positively priced ($q^* > 0$), as competition among informed traders crowds out the analyst's profit margin.

Conversely, when fundamental information is publicly distributed for free, it facilitates use by competitive market makers and triggers the crowding-in effect. Since this enhances the analyst's profit, she is willing to provide information without charge, supporting the public dissemination of information in the equilibrium.

Overall, this analysis clarifies how information generates crowding-in and -out effects depending on the type of market participants using it. It sheds light on the intended audience and effects of positively priced and publicly disseminated information provided by the for-profit analyst.

B. Secret Information Acquisition

Another crucial assumption is that market makers observe the informed trader's information acquisition (τ_e). In reality, however, it may not be readily observable. The following extension investigates the impact of such an opacity on the equilibrium.

To address this situation, assume that τ_e is not observable to market makers and, following Xiong and Yang (2023), they form a belief about the precision of private information to set the price impact, denoted as $\tilde{\tau}_e$ and $\tilde{\lambda}$, respectively. Appendix B.3 of the Supplementary Material provides the model's solution and proof for the following results.

In the trading stage, the actual signal quality τ_e and market makers' belief $\tilde{\tau}_e$ lead to the following *ex-ante* expected profit of the informed trader.

$$\tilde{V}(\tau_e, \tilde{\tau}_e, \tau_{pub}) = \frac{\sigma_u}{2}\sqrt{\frac{1 + b_{pub}\tilde{\tau}_e}{\tilde{\tau}_e}}\frac{b_{pub}\tau_e}{1 + b_{pub}\tau_e}. \tag{38}$$

On one hand, $\tilde{V}$ decreases with $\tilde{\tau}_e$ because market makers set the price impact $\tilde{\lambda}$ based on this belief. On the other hand, τ_e increases $\tilde{V}$ because, given the trading intensity, a higher-quality private signal results in a larger informational advantage.

The optimal information acquisition is determined by the FOC of $\tilde{V}$ with respect to τ_e while holding $\tilde{\tau}_e$ fixed, as the informed trader cannot influence market makers' belief due to the lack of a commitment device:

$$q = \frac{\partial \tilde{V}}{\partial \tau_e} = \frac{\sigma_u}{2}\sqrt{\frac{1 + b_{pub}\tilde{\tau}_e}{\tilde{\tau}_e}}\frac{b_{pub}}{(1 + b_{pub}\tau_e)^2}. \tag{39}$$

Focusing on the perfect Bayesian equilibrium, in which market makers' belief is consistent ($\tilde{\tau}_e = \tau_e$), and by incorporating the supply-side of information, the following results hold:

Proposition 10. *(i) Given τ_{pub}, the equilibrium quality of private information takes a* *hump-shaped reaction to τ_{pub}. The tipping point is given by*

$$\bar{\tau}_{pub} = \left[\frac{1}{6} \left(\frac{3\sigma_u}{c} \right)^{\frac{2}{5}} - \frac{1}{\Sigma_0} \right]^+ . \tag{40}$$

(ii) In the equilibrium with endogenous τ_{pub}, qualities of the private and the public signals are given by

$$\tau_e^* = \frac{1}{3} \left(\frac{3\sigma_u}{c} \right)^{\frac{2}{5}} \tag{41}$$

and $\tau_{pub}^* = \bar{\tau}_{pub}$.

Proof. See Appendix B.3 of the Supplementary Material. □

Compared to the baseline model (equation (17)), the opacity in information acquisition has the following implications.

Proposition 11. *When private information acquisition is unobservable to market makers, the crowding-in effect of the public signal becomes stronger, and the equilibrium qualities of private and public information become higher than the case with observable information acquisition.*

Proof. Comparing Proposition 10 with Propositions 3 and 6 leads to the result. □

Recall that the crowding-in effect arises because the public signal substitutes for order flow in market makers' belief updating. When information acquisition is observable, increases in τ_e due to the crowding-in effect diminish the substitution effect, as market makers reweigh order flow. Hence, the crowding-in effect weakens as τ_e increases. However, this channel is absent when information acquisition is unobservable: even if τ_e increases, market makers' belief updating is unaffected, preserving the magnitude of the crowding-in effect. Since the crowding-in effect is amplified due to the opacity of information acquisition, free public information is more likely to be provided, and its quality tends to be higher compared to the case with observable information acquisition.

V. Conclusion

This paper demonstrates that the dissemination of public information regarding an asset's fundamental value can incentivize acquisition of private information by an informed trader, resulting in a crowding-in effect. Public information diverts market makers' attention away from order flow information and reduces the price impact of informed order flow. Consequently, the informed trader can trade more intensively on her private information without affecting the price and is more inclined to invest in acquiring private information.

By introducing this crowding-in effect, which competes against the crowding-out effect studied in existing literature, the model establishes a theoretical foundation for the coexistence of free and paid information provided by a for-profit information seller, such as independent financial analysts. While the analyst aims to maximize profits from selling private information, she also has an incentive to distribute public information without charge, as it can enhance the demand for private information due to the crowding-in effect.

The model characterizes the equilibrium information structure and provides unique insights into the quality of paid and free information, indicating that market uncertainty and the informed trader's awareness of her price impact are crucial determinants of the equilibrium quality of free and paid information.

A. Proof

A.1. Proof of Lemma 1

With the optimal trading strategy, $x(s, s_{pub}) = \frac{\hat{v}-p_{pub}}{2\lambda}$, the *ex-ante* expected profit becomes

$$V(\tau_e, \tau_{pub}) = \mathbb{E}[(\hat{v} - p_{pub} - \lambda x(s, s_{pub}))x(s, s_{pub})] = \frac{\mathbb{E}[(\hat{v} - p_{pub})^2]}{2\lambda}.$$

The difference between beliefs of the informed trader and market makers is

$$\hat{v} - p_{pub} = \frac{\tau_e}{\Sigma_0^{-1} + \tau_e + \tau_{pub}} \left(s - \frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_{pub}} s_{pub} \right),$$

meaning that

$$\mathbb{E}\left[(\hat{v} - p_{pub})^2\right] = \frac{\tau_e}{(\Sigma_0^{-1} + \tau_e + \tau_{pub})(\Sigma_0^{-1} + \tau_{pub})}.$$

Applying λ in Proposition 1 to V leads to equation (15).

A.2. Proof of Proposition 3

With exogenous Q, the demand for information is characterized by the solution to

$$0 = H(\tau_e, \tau_{pub}) \equiv Q'(\tau_e) - \frac{\sigma_u b_{pub}}{4\tau_e^{\frac{1}{2}}(1 + \tau_e b_{pub})^{\frac{3}{2}}}. \tag{A.42}$$

Denote this solution as $\hat{\tau}_e$. It holds that $\frac{\partial H}{\partial \tau_e} > 0$ and

$$\frac{\partial H}{\partial \tau_{pub}} = b_{pub}^2 \sigma_u \frac{1 - \frac{1}{2}\tau_e b_{pub}}{4\tau_e^{\frac{1}{2}}(1 + \tau_e b_{pub})^{\frac{5}{2}}}.$$

Therefore, the implicit function theorem implies that $\hat{\tau}_e$ is increasing in τ_{pub} if and only if $\tau_{pub} < \bar{\tau}_{pub}$ where $\bar{\tau}_{pub}$ is the solution to

$$\hat{\tau}_e \frac{\Sigma_0}{1 + \tau_{pub}\Sigma_0} = 2.$$

Since $\hat{\tau}_e$ is the solution to $H = 0$ and depends on τ_{pub}, the above condition is equivalent to $H(\frac{2}{b_{pub}}, \tau_{pub}) = 0$, i.e., $Q'(\frac{2}{b_{pub}}) = \frac{\sigma_u}{2(2/b_{pub})^{\frac{3}{2}}(3)^{\frac{3}{2}}}$. Considering $b_{pub} = \frac{\Sigma_0}{1+\tau_{pub}\Sigma_0} \in [0, \Sigma_0]$, the equation above has a unique solution, $\bar{b}_{pub}$, in this range if, and only if, condition (20) holds. Otherwise, $\bar{b}_{pub} > \Sigma_0$, and the corresponding $\tau_{pub} > 0$ does not exist. Given $\bar{b}_{pub}$, rearranging $\frac{\Sigma_0}{1+\tau_{pub}\Sigma_0} = \bar{b}_{pub}$ with respect to τ_{pub} leads to $\bar{\tau}_{pub}$ in Proposition 3.

A.3. Proof of Propositions 4 and 5

Based on equation (A.42), the implicit function theorem leads to

$$\text{(A.43)} \qquad \frac{d\hat{\tau}_e}{db_{pub}} = \frac{\sigma_u \frac{1-\frac{1}{2}\hat{\tau}_e b_{pub}}{4\hat{\tau}_e^{\frac{1}{2}}(1+\hat{\tau}_e b_{pub})^{\frac{5}{2}}}}{Q''(\hat{\tau}_e) + \frac{\sigma_u b_{pub}}{2}\frac{1+4\hat{\tau}_e b_{pub}}{4\hat{\tau}_e^{\frac{3}{2}}(1+\hat{\tau}_e b_{pub})^{\frac{5}{2}}}}.$$

Price informativeness. Consider the impact of b_{pub}. Since $\eta = 2\frac{\tau_e^{-1}+b_{pub}}{b_{pub}(2\tau_e^{-1}+b_{pub})} - \Sigma_0^{-1}$, I analyze the behavior of

$$S(\tau_e, b_{pub}) \equiv \frac{\tau_e^{-1} + b_{pub}}{b_{pub}(2\tau_e^{-1} + b_{pub})}$$

evaluated at $\hat{\tau}_e$:

$$
\begin{aligned}
\frac{dS(\hat{\tau}_e, b_{pub})}{db_{pub}} &= \frac{\partial S(\hat{\tau}_e, b_{pub})}{\partial b_{pub}} + \frac{d\hat{\tau}_e}{db_{pub}} \frac{\partial S(\hat{\tau}_e, b_{pub})}{\partial \tau_e} \\
&\propto -\left[2\frac{1}{\hat{\tau}_e}\left(\frac{1}{\hat{\tau}_e} + b_{pub}\right) + b_{pub}^2\right] + \frac{b_{pub}^2}{\hat{\tau}_e^2}\frac{d\hat{\tau}_e}{db_{pub}} \\
&\equiv \Omega(b_{pub}).
\end{aligned} \tag{A.44}
$$

Firstly, if $1 - \frac{1}{2}\hat{\tau}_e b_{pub} < 0$, then the above equation directly implies $\Omega < 0$. In contrast, if $1 - \frac{1}{2}\hat{\tau}_e b_{pub} > 0$, then $Q'' \geq 0$ implies that

$$
\frac{d\hat{\tau}_e}{db_{pub}} < \frac{1}{b_{pub}} \frac{1 - \frac{1}{2}\tau_e b_{pub}}{\frac{1}{2}\tau_e^{-1} + 2b_{pub}}.
$$

Hence,

$$
\begin{aligned}
\Omega &< -\left[2\frac{1}{\hat{\tau}_e}\left(\frac{1}{\hat{\tau}_e} + b_{pub}\right) + b_{pub}^2\right] + \frac{b_{pub}^2}{\hat{\tau}_e^2}\frac{1}{b_{pub}}\frac{1 - \frac{1}{2}\tau_e b_{pub}}{\frac{1}{2}\tau_e^{-1} + 2b_{pub}} \\
&\propto -\left(\frac{1}{2} + 2b_{pub}\hat{\tau}_e\right)\left[2\frac{1}{\hat{\tau}_e}\left(\frac{1}{\hat{\tau}_e} + b_{pub}\right) + b_{pub}^2\right] + b_{pub}\left(\frac{1}{\hat{\tau}_e} - \frac{1}{2}b_{pub}\right) \\
&< 0.
\end{aligned}
$$

Therefore, η decreases with b_{pub}, meaning that it increases with τ_{pub}.

Liquidity. The price impact measure is rewritten as $\lambda = \frac{1}{2\sigma_u}\sqrt{\frac{\tau_e b_{pub}^2}{1+\tau_e b_{pub}}}$, and I analyze the reaction of $\frac{\tau_e b_{pub}^2}{1+\tau_e b_{pub}}$ to changes in b_{pub}. It holds that

$$\begin{aligned}\frac{d}{db_{pub}}\left(\frac{\hat{\tau}_e b_{pub}^2}{1+\hat{\tau}_e b_{pub}}\right) &\propto \hat{\tau}_e(2+\hat{\tau}_e b_{pub}) + b_{pub}\frac{d\hat{\tau}_e}{db_{pub}} \\ &\equiv \omega(b_{pub})\end{aligned}$$

If $1-\frac{1}{2}\hat{\tau}_e b_{pub} > 0$, then the above equation implies $\omega > 0$. In contrast, if $1-\frac{1}{2}\hat{\tau}_e b_{pub} < 0$, then it holds that

$$\frac{d\hat{\tau}_e}{db_{pub}} > \frac{1}{b_{pub}}\frac{1-\frac{1}{2}\tau_e b_{pub}}{\frac{1}{2}\tau_e^{-1}+2b_{pub}}.$$

By applying this inequality to ω,

$$\begin{aligned}\omega &> \hat{\tau}_e(2+\hat{\tau}_e b_{pub}) + \frac{1-\frac{1}{2}\tau_e b_{pub}}{\frac{1}{2}\tau_e^{-1}+2b_{pub}} \\ &\propto 2(1+\hat{\tau}_e b_{pub})^2 \\ &> 0.\end{aligned}$$

Hence, λ is increasing in b_{pub} and decreasing in τ_{pub}.

A.4. Proof of Proposition 6

Denote the inverse demand function of the informed trader in equation (17) as $q_D(\tau_e)$ and the supply schedule of the analyst in equation (25) as $q_S(\tau_e)$. A unique solution exists for the market clearing condition because $q_D(0) = \infty > q_S(0)$ and $q_D(\infty) = 0 < q_S(\infty)$. It holds that

$$0 = \bar{H}(\tau_e, \tau_{pub}) = c\tau_e - \frac{\sigma_u b_{pub}}{4\tau_e^{\frac{1}{2}}(1 + \tau_e b_{pub})^{\frac{3}{2}}}.$$

Rewriting this condition leads to the following quadratic equation:

$$0 = h(\tau_e) = \tau_e(1 + \tau_e b_{pub}) - \left(\frac{\sigma_u b_{pub}}{4c}\right)^{\frac{2}{3}}.$$

The equilibrium τ_e^* is a unique solution to $h(\tau_e) = 0$ and is given by equation (26). Moreover, together with equation (A.43), the implicit function theorem implies that the optimal τ_{pub} is given by the solution to $\frac{\tau_e \Sigma_0}{1+\tau_{pub}\Sigma_0} = 2$. Applying this condition to τ_e^* in equation (26) leads to the results in Proposition 6.

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FIGURE 1

Timeline of the Model

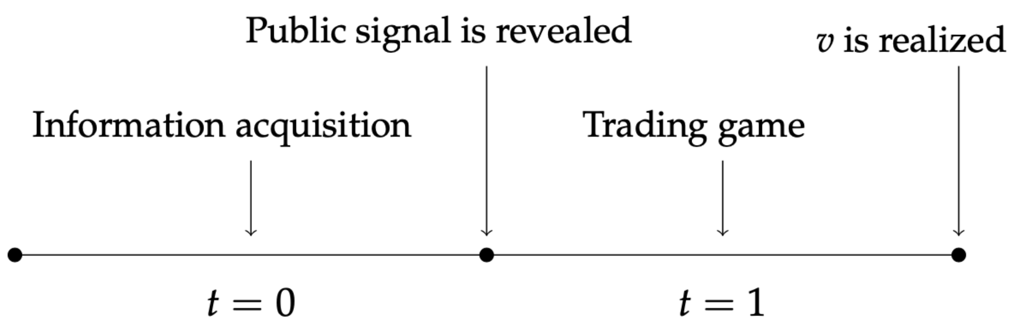


FIGURE 2

Optimal Information Acquisition

This figure plots the optimal quality of private information for the informed trader. The information cost is $Q(\tau_e) = q\tau_e$ with constant q, and parameters are $q_H = 0.25$, $q_L = 0.20$, $\sigma_u^2 = 25.0$, $\Sigma_0 = 10.0$.

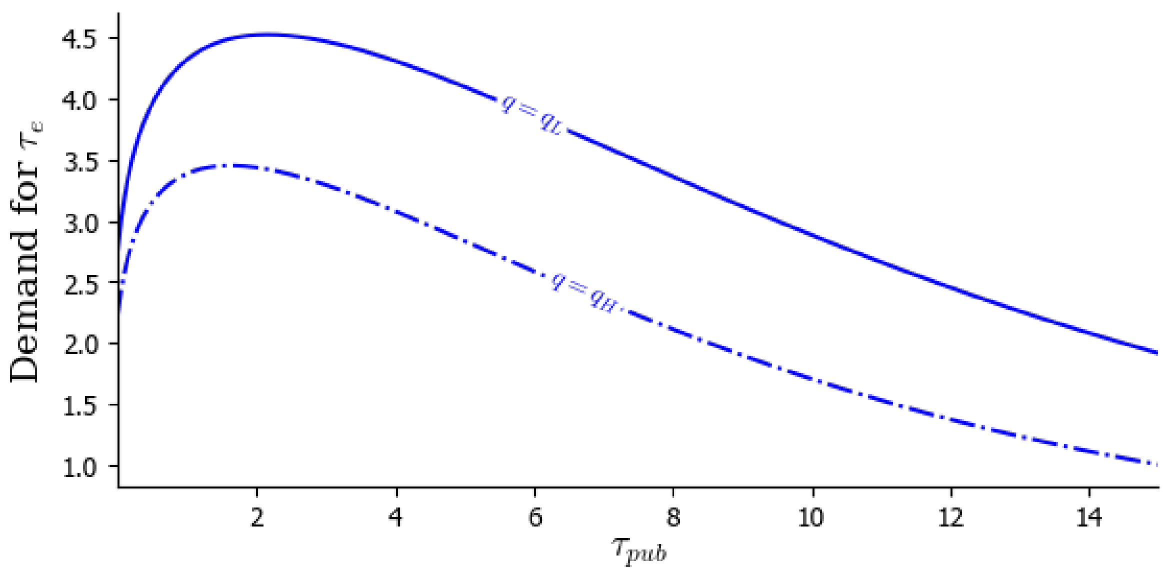
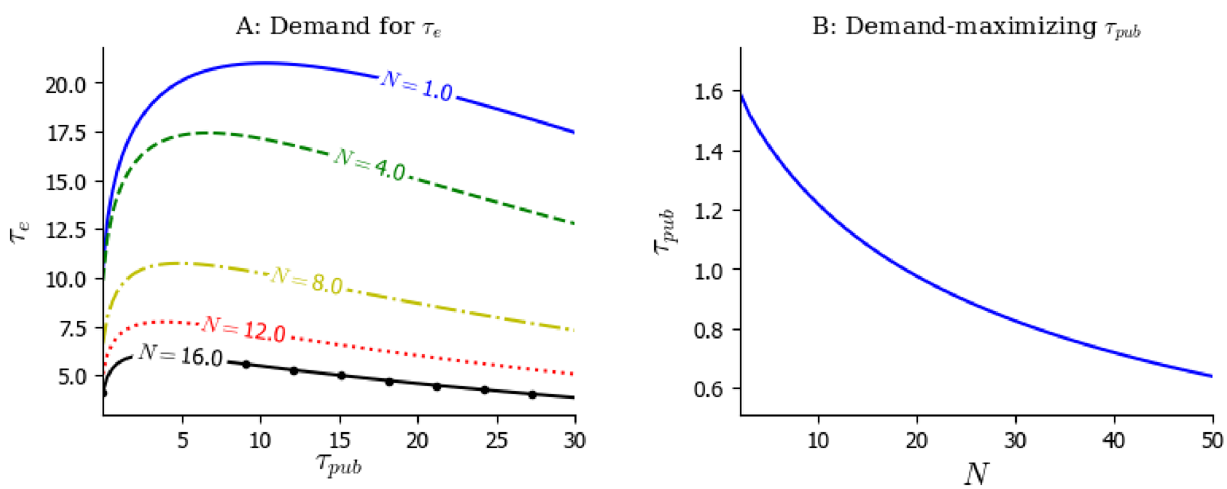


FIGURE 3

Individual Demand for the Private Signal

These figures are illustrated by using $\sigma_u^2 = 25, \Sigma_0 = 25, c = 0.01$. Panel A plots the reaction of the individual optimal demand for τ_e to changes in τ_{pub}. Panel B plots the reaction of the demand-maximizing τ_{pub}^D to changes in N.



The Supplementary Material for

"The Crowding-in Effect of Public Information on Private Information Acquisition"

Jun Aoyagi

B. Extensions

This appendix provides formal models and solutions to the extensions offered in Section IV.

B.1. Model with N Informed Traders

Trading strategy and execution price. There are $N \geq 1$ informed traders with index $i = 1, 2, \cdots, N$. In what follows, I consider generic informed trader i. The expected asset's value conditional on available information is

$$v_i \equiv \mathbb{E}[v|s_i, s_{pub}] = \frac{\tau_i}{\Sigma_0^{-1} + \tau_i + \tau_{pub}} s_i + \frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_i + \tau_{pub}} s_{pub}.$$

The trader i's trading behavior in the linear equilibrium is characterized by

(B.1) $$x_i = \hat{\beta}_i (v_i - p_{pub}).$$

For simplicity, I rewrite it as a function of available signals:

(B.2) $$x_i = \beta_i s_i + \gamma_i s_{pub},$$

where

$$\beta_i = \hat{\beta}_i \frac{\tau_i}{\Sigma_0^{-1} + \tau_i + \tau_{pub}},$$

and

$$\gamma_i = -\hat{\beta}_i \frac{\tau_i}{\Sigma_0^{-1} + \tau_i + \tau_{pub}} \frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_{pub}}.$$

Given that all informed traders follow the same linear trading rule, the aggregate order flow is

$$y = \sum_i x_i + u = Bv + \sum_i \beta_i e_i + G s_{pub} + u,$$

with $B \equiv \sum_i \beta_i$ and $G \equiv \sum_i \gamma_i$. Therefore, the filtering problem below pins down the market makers' pricing strategy.

$$p = \mathbb{E}[v|s_{pub}, y] = \phi s_{pub} + \lambda y,$$

where

$$\lambda = \frac{B\left(\sum_i \beta_i^2 \tau_i^{-1} + \sigma_u^2\right)^{-1}}{\Sigma_0^{-1} + \tau_{pub} + B^2\left(\sum_i \beta_i^2 \tau_i^{-1} + \sigma_u^2\right)^{-1}}, \tag{B.3}$$

and

$$\phi = \frac{\tau_{pub}(\sum \beta_i^2 \tau_i^{-1} + \sigma_u^2) - BG}{(\Sigma_0^{-1} + \tau_{pub})(\sum \beta_i^2 \tau_i^{-1} + \sigma_u^2) + B^2}. \tag{B.4}$$

Trader i decides on her optimal trading behavior by solving the following problem given that other informed traders take the strategy in equation (B.1).

$$\max_{x_i} \mathbb{E}[(v-p)x_i|s_i, s_{pub}] = \mathbb{E}[(v - \phi s_{pub} - \lambda \sum_j x_j)x_i|s_i, s_{pub}].$$

Since $\mathbb{E}[s_j|s_i, s_{pub}] = \mathbb{E}[v|s_i, s_{pub}] = v_i$ for all $j \neq i$, the FOC leads to

$$x_i = \frac{\hat{v}_i - \phi s_{pub} - \lambda \sum_{j\neq i} \mathbb{E}[x_j|s_i, s_{pub}]}{2\lambda}$$
$$= \frac{(1-\lambda B_{-i})\left(\frac{\tau_i}{\Sigma_0^{-1}+\tau_{pub}+\tau_i}s_i + \frac{\tau_{pub}}{\Sigma_0^{-1}+\tau_{pub}+\tau_i}s_{pub}\right) - (\phi + \lambda G_{-i})s_{pub}}{2\lambda},$$

where $B_{-i} = \sum_{j\neq i} B_j$ and $G_{-i} = \sum_{j\neq i} G_j$. Hence, the coefficients for the trading strategy are expressed as

(B.5) $$\beta_i = \frac{1-\lambda B_{-i}}{2\lambda}\left(\frac{\tau_i}{\Sigma_0^{-1}+\tau_{pub}+\tau_i}\right),$$

(B.6) $$\gamma_i = \frac{(1-\lambda B_{-i})\left(\frac{\tau_{pub}}{\Sigma_0^{-1}+\tau_{pub}+\tau_i}\right) - (\phi+\lambda G_{-i})}{2\lambda}.$$

Introducing notations $\kappa_i \equiv \frac{\tau_i}{\Sigma_0^{-1}+\tau_{pub}+\tau_i}$ and $m_i \equiv \frac{\kappa_i}{2-\kappa_i} = \frac{\tau_i}{2(\Sigma_0^{-1}+\tau_{pub})+\tau_i}$, equation (B.5) is rewritten as

$$\beta_i = \frac{1-\lambda B_{-i}}{2\lambda}\kappa_i = \frac{1-\lambda(B-\beta_i)}{2\lambda}\kappa_i,$$

meaning that

(B.7) $$\lambda\beta_i = (1-\lambda B)\frac{\kappa_i}{2-\kappa_i} = (1-\lambda B)m_i.$$

By summing up the both sides of equation (B.7) for all $i = 1, 2, \cdots, N$, and denoting $M = \sum_i m_i$, it holds that

(B.8) $$\lambda B = \frac{M}{1+M}.$$

Also, this result simplifies equation (B.7):

$$\beta_i = \frac{1}{\lambda(1+M)} m_i = \frac{B}{M} m_i.$$

Moreover, it holds that

$$\sum_i \beta_i^2 \tau_i^{-1} = \frac{\sum_i m_i^2 \tau_i^{-1}}{\lambda^2 (1+M)^2},$$

so that equation (B.3) becomes

$$B = \sqrt{\frac{M(\Sigma_0^{-1} + \tau_{pub})\sigma_u^2}{1 - (\Sigma_0^{-1} + \tau_{pub}) \frac{1}{M} \sum_i m_i^2 \tau_i^{-1}}} = \sqrt{\frac{\sigma_u^2}{\frac{1}{M(\Sigma_0^{-1} + \tau_{pub})} - \sum_i \left(\frac{m_i}{M}\right)^2 \tau_i^{-1}}}.$$

The above equation specifies B as a function of $\boldsymbol{\tau} = (\tau_i)_{i=1}^N$ and τ_{pub}.

Also, using $\frac{\tau_i}{\Sigma_0^{-1} + \tau_{pub} + \tau_i} \frac{1 - \lambda B_{-i}}{2\lambda} = \beta_i$, equation (B.6) is rewritten as

$$\gamma_i = 2\tau_{pub} \frac{B}{M} \frac{m_i}{\tau_i} - \frac{\phi}{\lambda} - G. \tag{B.9}$$

Denote $l_i \equiv \frac{\tau_{pub}}{2(\Sigma_0^{-1} + \tau_{pub}) + \tau_i}$ and $L = \sum_i l_i$. Aggregating the above equation for i leads to

$$G = \frac{1}{1+N} \left(2\frac{B}{M} L - N \frac{\phi}{\lambda} \right). \tag{B.10}$$

To simplify the above equation, compute that

$$\phi = \frac{\frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_{pub}} \frac{1}{M} B - G}{\frac{1+M}{M} B}.$$

From equation (B.10), we obtain

$$G = \left(2L - N \frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_{pub}} \right) \frac{B}{M}.$$

Then, individual γ_i is derived from (B.9):

$$\gamma_i = \left(2l_i - \frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_{pub}}\right)\frac{B}{M} = -\frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_{pub}}\frac{B}{M}m_i.$$

In summary, the equilibrium is characterized by the proposition below.

Proposition 1. *In the trading stage, (i) trader i places a market order of the form $x_i^* = \beta_i s_i + \gamma_i s_{pub}$ with*

$$\beta_i = \frac{B}{M}m_i, \tag{B.11}$$

and

$$\gamma_i = -\frac{\tau_{pub}}{\Sigma_0^{-1} + \tau_{pub}}\beta_i, \tag{B.12}$$

where

$$B = \sqrt{\frac{\sigma_u^2}{\frac{1}{M(\Sigma_0^{-1}+\tau_{pub})} - \sum_i \left(\frac{m_i}{M}\right)^2 \tau_i^{-1}}},$$

$$m_i = \frac{\tau_i}{2(\Sigma_0^{-1} + \tau_{pub}) + \tau_i},$$

and $M = \sum_i m_i$. Note that this is equivalent to the following formula that expresses the trading behavior as a function of the belief gap:

$$x_i = \hat{\beta}_i(v_i - p_{pub}) \tag{B.13}$$

with

$$\hat{\beta}_i = \frac{B}{M}\frac{\Sigma_0^{-1} + \tau_i + \tau_{pub}}{2(\Sigma_0^{-1} + \tau_{pub}) + \tau_i}.$$

(ii) The market makers execute the order flow at price $p^* = \mathbb{E}[v|s_{pub}, y] = \phi s_{pub} + \lambda y$ with the

coefficients being

$$\phi = \frac{1}{1+M}\left(\frac{(1+N)\tau_{pub}}{\Sigma_0^{-1}+\tau_{pub}} - 2L\right), \tag{B.14}$$

and

$$\lambda = \frac{1}{B}\frac{M}{1+M}, \tag{B.15}$$

where $L = \sum_i \frac{\tau_{pub}}{2(\Sigma_0^{-1}+\tau_{pub})+\tau_i}$.

Note that this proposition characterizes the trading behavior and the price as functions of $\boldsymbol{\tau}$ and τ_{pub}.

Expected trading profits and information acquisition. Trader i computes the expected return by assuming that all her rivals take the same information quality and behavior, i.e., $\tau_j = \tau_k = \tau_{-i}$ for all $j, k \neq i$. Then, the optimized profit is

$$\begin{aligned} V(s_i, s_{pub}; \tau_{-i}) &= \mathbb{E}[(v-p)x_i^*|s_i, s_{pub}] \\ &= \lambda x_i^{*2}(s_i, s_{pub}). \end{aligned}$$

Taking the expectation with respect to s_i and s_{pub}, we obtain the *ex-ante* expected profit.

Lemma 1. *Given* $\boldsymbol{\tau} = (\tau_i)_{i=1}^N$, *the ex-ante expected profit of informed trader* i *is given by*

$$V_i(\boldsymbol{\tau}, \tau_{pub}) = \mathbb{E}[\lambda x_i^{*2}(s_i, s_{pub})] = \sqrt{\frac{\sigma_u^2}{2}\frac{1}{\Sigma_0^{-1}+\tau_{pub}}}\mathcal{V}_i(\boldsymbol{\tau}), \tag{B.16}$$

where

$$\mathcal{V}_i(\boldsymbol{\tau}) \equiv \frac{m_i(1+m_i)}{1+\sum_i m_i}\sqrt{\frac{1}{\sum_i m_i(1+m_i)}},$$

and

$$m_i = \frac{\tau_i}{2(\Sigma_0^{-1}+\tau_{pub})+\tau_i}.$$

Using the *ex-ante* profit function (B.16), the optimal information acquisition of trader i given q_i and τ_{-i} is characterized by the following FOC:

$$q_i = \frac{\partial V_i}{\partial \tau_i} = \sqrt{\frac{\sigma_u^2}{2} \frac{1}{\Sigma_0^{-1} + \tau_{pub}}} \frac{dm_i}{d\tau_i} R(\tau_i, \tau_{-i}), \tag{B.17}$$

where

$$R(\tau_i, \tau_{-i}) \equiv \frac{\partial \mathcal{V}_i}{\partial m_i} = \frac{[\sum_i m_i(1+m_i)]^{-\frac{3}{2}}}{(1+\sum_i m_i)^2} \left[(1+\sum_i m_i)(1+2m_i) - m_i(1+m_i)\right] \times$$
$$\left[\sum_i m_i(1+m_i)\right] - \frac{1}{2} m_i(1+m_i)(1+\sum_i m_i)(1+2m_i).$$

Conversely, deciding on the supply of private signals, the analyst solves the following problem:

$$\pi_A(\tau_{pub}, N) \equiv \max_{\{\tau_i\}_{i=1}^N} \sum_{i=1}^N q_i \tau_i - \frac{c}{2} T^2.$$

Since the marginal benefit of increasing τ_i is $q_i > 0$, setting $\tau_l > \max\{\tau_i\}_{i \neq l}$ for some l is suboptimal. Therefore, the supply is characterized by the identical quality, i.e., $\tau_i = \tau_e$ for all $i = 1, \cdots, N$, leading the supply function for signal s_i to be $\tau_i = \tau_e = \frac{1}{c} \sum_{j=1}^N q_j$, where the RHS is independent of i.

Incorporating equation (B.17) for all i and summing them up, the market clearing condition for the private information is summarized by the following equation:

$$\frac{c}{N} \tau_e = \sigma_u \frac{(1 - m(\tau_e))^2}{[2(\Sigma_0^{-1} + \tau_{pub})]^{\frac{3}{2}}} R(m(\tau_e), N), \tag{B.18}$$

where I denote $m(\tau_e) = \frac{\tau_e}{2(\Sigma_0^{-1} + \tau_{pub}) + \tau_e}$ and $R(\tau_e, \tau_e) = R(m(\tau_e), N)$. Note that the RHS of equation (B.18) represents the individual demand for private information in the equilibrium. It holds that

$$\frac{\partial R(m, N)}{\partial N} = -\frac{H(m, N)}{N^{\frac{5}{2}} (1 + N m(\tau))^3 \sqrt{m(\tau)(1 + m(\tau))}},$$

where, for $N \geq 2$,

$$\text{(B.19)} \qquad H = m^3N^2(3N-5) + \frac{N}{2}m^2(3N^2 - \frac{7}{2}N - 5) + \frac{m}{2}(4N^2 - 3N - 3) + \frac{1}{2}(N - \frac{3}{2})$$
$$> 0.$$

Hence, the individual demand is monotonically decreasing in N for $N \geq 2$.

Note that condition (B.18) guarantees the symmetric equilibrium. It holds that

$$R(m, N) = \frac{(N - \frac{1}{2}) + m(N^2 + \frac{1}{2}N - 1) + 2N(N-1)m^2}{N^{\frac{3}{2}}(1 + Nm)^2[m(1+m)]^{\frac{1}{2}}}$$

with

$$\text{(B.20)} \qquad \frac{\partial R}{\partial m} = \frac{F(m, N)}{N^{\frac{3}{2}}(1 + Nm)^3[m(1+m)]^{\frac{3}{2}}},$$

where

$$\begin{aligned} F(m) = & -2N^2(N-1)m^4 - N^2m^3(3N-2) - \frac{3}{4}N^2m^2(2N+1) \\ & - m\frac{N}{2}(4N-1) - \frac{1}{4}(2N-1) \\ & <0. \end{aligned}$$

The last inequality comes from $N \geq 1$. Hence, it establishes that $\frac{\partial R}{\partial m} < 0$. Together with

$$\lim_{\tau_e \to 0} \sigma_u \frac{(1 - m(\tau_e))^2}{[2(\Sigma_0^{-1} + \tau_{pub})]^{\frac{3}{2}}} R(m(\tau_e), N) = \infty > 0,$$

and

$$\lim_{\tau_e \to \infty} \sigma_u \frac{(1 - m(\tau_e))^2}{[2(\Sigma_0^{-1} + \tau_{pub})]^{\frac{3}{2}}} R(m(\tau_e), N) = 0,$$

condition (B.18) has a unique solution, $\tau_e = \tau_e(\tau_{pub}, N)$.

Finally, the equilibrium profit of the analyst from supplying private signals (given τ_{pub} and N)

is expressed as

$$\pi_A = \frac{c}{2}\tau_e^2(\tau_{pub}, N).$$

Proof of Proposition 8. Denote the RHS of equation (B.18) as $\bar{R}(\tau)$ and consider the following partial derivatives:

$$\frac{\partial \bar{R}}{\partial \tau} = \frac{\sigma_u(1-m)}{[2(\Sigma_0^{-1}+\tau_{pub})]^{\frac{3}{2}}}\frac{dm}{d\tau}\left(\frac{\partial R}{\partial m}(1-m) - 2R(m)\right) < 0, \tag{B.21}$$

and

$$\frac{\partial \bar{R}}{\partial \tau_{pub}} = -\frac{\sigma_u(1-m)}{4(\Sigma_0^{-1}+\tau_{pub})^{\frac{3}{2}+1}}\frac{\Phi(m)}{N^{\frac{3}{2}}(1+Nm)^3[m(1+m)]^{\frac{3}{2}}},$$

where

$$\Phi(m) = N^{\frac{3}{2}}(1+Nm)^3[m(1+m)]^{\frac{3}{2}}\left[m(1-m)\frac{\partial R}{\partial m} - \left(2m - \frac{3}{2}\right)R(m)\right]. \tag{B.22}$$

Since the LHS of equation (B.18) is independent of τ_{pub}, the implicit function theorem indicates that τ_e is increasing in τ_{pub} if and only if $\Phi < 0$. By plugging equation (B.20) into equation (B.22), Φ is rewritten as $\Phi(m) = \sum_{l=0}^{5}\phi_k m^k$, where

$$\begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} N - \frac{1}{2} \\ N^2 + \frac{1}{2}N - \frac{3}{2} \\ 4N^2 - 7N + \frac{3}{2} \\ N^3 - \frac{11}{2}N^2 + \frac{3}{2}N + 2 \\ -2N(N^2+2N-3) \\ -2N^2(N-1) \end{pmatrix}. \tag{B.23}$$

Note that this polynomial is defined over $m \in [0, 1]$.

Firstly, observe that

$$\Phi(1) = \frac{3}{4} + \frac{7}{4}N + \frac{1}{2}5N^2 - 7N^3 < 0,$$

where the last expression holds due to $N \geq 1$. Moreover, $\Phi(0) = \phi_0 > 0$. Therefore, there exists at least one $m^* \in (0, 1)$ such that $\Phi(m^*) = 0$.

Secondly, consider the first-order derivative of Φ with respect to m multiplied by and evaluated at $m = m^*$:

$$\begin{aligned} m^*\Phi'(m^*) &= 5\phi_5 m^{*5} + 4\phi_4 m^{*4} + 3\phi_3 m^{*3} + 2\phi_2 m^{*2} + \phi_1 m^* \\ &= 2\phi_5 m^{*5} + \phi_4 m^{*4} - \phi_2 m^{*2} - 2\phi_1 m^* - \phi_0 \\ &< 0, \end{aligned}$$

where the second line comes from $\Phi(m^*) = 0$, and the last inequality holds due to coefficients (B.23). This implies that m^* is a unique solution to $\Phi(m) = 0$, and $\Phi(m) > 0$ if and only if $m < m^*$. In other words, τ_e is increasing in τ_{pub} when $m > m^*$. Note also that m^* is characterized only by N.

Finally, based on equation (B.21) and $\frac{dm}{d\tau_e} > 0$, the individual demand (i.e., the RHS of equation (B.18)) is increasing in τ_{pub} if and only if

(B.24) $$\tau_{pub} < \bar{\tau}_{pub}^D \equiv \frac{1}{2}\left[\frac{\sigma_u}{c}\frac{(1-m^*)^3}{m^*}R(m^*, N)\right]^{\frac{2}{5}} - \frac{1}{\Sigma_0},$$

while the equilibrium τ_e is increasing in τ_{pub} if and only if

(B.25) $$\tau_{pub} < \bar{\tau}_{pub} \equiv \frac{1}{2}\left[\frac{\sigma_u}{c}\frac{(1-m^*)^3}{m^*}NR(m^*, N)\right]^{\frac{2}{5}} - \frac{1}{\Sigma_0}.$$

Rewriting $G(N) = \frac{(1-m^*)^3}{m^*}R(m^*, N)$ leads to the expressions in Proposition 8.

Proof of Proposition 9. At the first stage, the analyst controls τ_{pub} and N to solve

$$\max_{\tau_{pub},N} \pi_A = \max_{\tau_{pub}} \frac{c}{2}\tau_e^2(\tau_{pub}, N).$$

Firstly, given N, the objective function takes a hump-shaped response to τ_{pub}. From the argument above, the maximizer, denoted as τ_{pub}^*, is obtained by setting

$$m = \frac{\tau_e(\tau_{pub}, N)}{2(\Sigma_0^{-1} + \tau_{pub}) + \tau_e(\tau_{pub}, N)} = m^*(N).$$

Therefore, by plugging this condition into equation (B.18), the equilibrium is characterized by τ_e^* and τ_{pub}^* in Proposition 9, where I use notations $m = m^*$ for simplicity.

Also, consider the partial derivative of τ_e with respect to N. Based on the market-clearing condition in (B.18), the implicit function theorem implies that $\frac{\partial \tau_e}{\partial N}$ is directly proportional to

$$D \equiv R(m, N) + N\frac{\partial R(m, N)}{\partial N}.$$

Firstly, D can be expressed as the following polynomial of N, and $\partial \tau_e / \partial N = 0$ holds if $D = 0$:

$$\begin{aligned} D = &-N^3 m^2 \left(m + \frac{1}{2}\right) + N^2 m^2 \left(3m + \frac{17}{4}\right) \\ &+ \frac{1}{2}N(7m^2 + 3m + 1) + \frac{1}{4} + \frac{1}{2}m. \end{aligned}$$

Since $D(0) > 0$ and $\lim_{N\to\infty} D = -\infty < 0$, it is positive at $N = 0$ and negative at $N \to \infty$. Hence, there is at least one N^* such that $\partial \tau_e / \partial N = 0$.

Secondly, at this N^*, it holds that $\frac{\partial D}{\partial N}|_{N^*} < 0$. This implies that N^* is the unique solution to $D(N) = 0$, suggesting that $\partial \tau_e / \partial N > 0$ for $N < N^*$ and $\partial \tau_e / \partial N < 0$ for $N > N^*$. Therefore, the equilibrium $\tau_e(\tau_{pub}, N)$ takes a single-peaked curve with respect to N, and $N = N^*$ maximizes the analyst's profit.

B.2. Model with Multiple Information Sellers

The model presented in Section III can be expanded to accommodate multiple information sellers. As the simplest case, consider two analysts, indexed by $j = A, B$, serving as information sellers. I assume that they are endowed with heterogeneous information-production skills captured by the marginal cost parameters, $c_A < c_B$. Analyst j provides $s_j = v + e_j$ and $s_{pub,j} = v + e_{pub,j}$ with the noise components characterized by $e_j \sim \mathcal{N}(0, \tau_j^{-1})$ and $e_{pub,j} \sim \mathcal{N}(0, \tau_{pub,j}^{-1})$, respectively. All signals are conditionally independent of each other.

Consider the demand side of information. Following the sufficient statistics argument of Gaussian random variables, observing two signals (s_A, s_B) is informationally equivalent for the informed trader to observing a single signal, $s = v + e$, where

$$e \equiv \frac{\tau_A}{\tau_A + \tau_B} e_A + \frac{\tau_B}{\tau_A + \tau_B} e_B.$$

Hence, by defining $\tau_e \equiv \tau_A + \tau_B$, the informed trader's information-acquisition problem is almost identical to the baseline model:

$$(\tau_A^D, \tau_B^D) = \arg\max_{\tau_A, \tau_B} V(\tau_A + \tau_B, \tau_{pub}) - \sum_{j=A,B} q_j \tau_j, \tag{B.26}$$

where $V(\tau_e, \tau_{pub})$ is given by equation (15), and q_j denotes the unit price of information provided by analyst j. This objective function implies that two signals are perfect substitutes for the informed trader with respect to V.

On the supply side, assuming the same profit function for analysts as in Section III, analyst j sets her supply schedule as $q_j(\tau_j) = c_j \tau_j$. Hence, whether the equilibrium information supply involves multiple analysts depends on the structure of the information market. As the extension of the baseline model with the bilateral double auction, I consider two scenarios: one with a single auction with uniform pricing and the other with separated auctions with heterogeneous prices.

If two signals are sold at the uniform price, $q = q_A = q_B$, equation (B.26) implies that the bid for the private information is identical to the baseline model, specifying only the total demand, $q_D(\tau_e)$. The equilibrium is determined by the market-clearing condition, imposing $\tau_e^* = \tau_A^* + \tau_B^*$ and $q^* = c_A\tau_A^* = c_B\tau_B^*$ with τ_e^* given by the baseline model. Hence, analyst A supplies $\tau_A^* = \frac{c_B}{c_A+c_B}\tau_e^*$,while analyst B covers the remaining demand.

This scenario is akin to multiple analysts providing information to a platform, which aggregates the signal and distributes revenue to contributors. Hence, both analysts provide information to the market, with the one possessing a higher information-production skill producing more information than her rival. Note that this equilibrium also leads to the same result as the information structure in the baseline model, providing the additional specification about the share of analysts.

In contrast, if two signals with distinct quality levels are traded at separated bilateral auctions at different unit prices ($q_A \neq q_B$), the informed trader buys only from the analyst who supplies the signal at the lowest price, i.e., $\tau_e^* = \tau_A^*\mathbb{I}_{\{q_A \leq q_B\}} + \tau_B^*\mathbb{I}_{\{q_B < q_A\}}$. Therefore, the only equilibrium is characterized by analyst A, who has a higher information-production skill, supplying all private information at q_A^*, while analyst B remains inactive. This equilibrium results in the same argument as the analysis in the baseline model.

Overall, the equilibrium analysis proposed in the baseline model remains robust in the presence of multiple information sellers, although the specifics of the supply side may depend on assumptions regarding information-production costs and restrictions on the price structure of signals.

B.3. Secret Information Acquisition

Suppose that the choice of τ_e by the informed trader is not observable for market makers. Instead, they form a belief, $\tilde{\tau}_e$, and set the price

$$p = \mathbb{E}[v|s_{pub}, y] = p_{pub} + \tilde{\lambda}y,$$

where $\tilde{\lambda}$ is the price impact coefficient computed *based on the belief* $\tilde{\tau}_e$.

Given the market makers' belief, the informed trader solves

$$\max_x \mathbb{E}[(v-p)x|s, s_{pub}] = \mathbb{E}[(v - p_{pub} - \tilde{\lambda}x)x|s, s_{pub}],$$

leading to the FOC:

$$x = \frac{\hat{v} - p_{pub}}{2\tilde{\lambda}},$$

which implies $\beta = \frac{1}{2\tilde{\lambda}}$. In contrast, given the informed trader's trading strategy and belief $\tilde{\tau}_e$, market makers compute

$$\tilde{\lambda} = \frac{\beta\tilde{\tau}_e}{\beta^2\tilde{\tau}_e + \left(\Sigma_0^{-1} + \tau_{pub}\right)\left(\Sigma_0^{-1} + \tilde{\tau}_e + \tau_{pub}\right)\sigma_u^2}.$$

Solving the above coefficients and following analogous computations to the baseline model, the equilibrium is characterized by

$$\beta = \sqrt{\frac{\left(\Sigma_0^{-1} + \tau_{pub}\right)\left(\Sigma_0^{-1} + \tilde{\tau}_e + \tau_{pub}\right)\sigma_u^2}{\tilde{\tau}_e}}$$

and $\tilde{\lambda} = \frac{1}{2\beta}$. Accordingly, the ex-ante expected profit of the informed trader is given by (38) with the FOC with respect to τ_e (with $\tilde{\tau}_e$ being fixed) being (39). By imposing the consistency condition for market makers' belief, $\tau_e = \tilde{\tau}_e$, it reduces to

$$q = \frac{\sigma_u b_{pub}}{2\tau_e^2(\tau_e^{-1} + b_{pub})^{\frac{3}{2}}}. \tag{B.27}$$

Proof of Propositions 10 and 11. Since the demand function in equation (B.27) is directly proportional to that in the baseline model, the solution to the market-clearing condition takes hump-shaped curve in relation to τ_{pub}, where the tipping point is characterized by $\tau_e b_{pub} = 2$. Applying equation (B.27), this condition is identical to $\tau_{pub} = \bar{\tau}_{pub}$ in the baseline setting. Following the same argument as the baseline model, the analyst controls τ_{pub} to achieve this point, leading to τ_e^*

and τ_{pub}^* in equations (40) and (41). Also, since the coefficient for the demand function in equation (B.27) is twice that of the baseline model, the equilibrium τ_e^* is larger in this extension. This also implies a larger τ_{pub}^* than the baseline model from equation $\tau_e^* \frac{\Sigma_0}{\tau_{pub}^* + \Sigma_0} = 2$.

B.4. Arrival Timing of the Public Signal

Assume that the informed trader chooses the quality of her private information, τ_e, after observing the public signal, s_{pub}. All other assumptions are the same as the baseline model. The following lemma holds.

Lemma 2. *The informed trader's expected profit at the information-acquisition stage is the same as in the baseline model and is independent of the realized value of the public signal,* s_{pub}.

Proof. Note that the equilibrium in the trading stage (β and λ) and the optimized trading profit conditional on realized signals (V) are the same as in the baseline model. The objective function of the informed trader choosing the private information quality is then

$$\begin{aligned} V(\tau_e, s_{pub}) &\equiv \mathbb{E}[V(s, s_{pub})|s_{pub}] \\ &= \frac{\beta}{2}\mathbb{E}\left[(\hat{v} - p_{pub})^2|s_{pub}\right], \end{aligned} \tag{B.28}$$

where $x^* = \beta(\hat{v} - p_{pub})$, and β is the same as the benchmark model. It holds that

$$\begin{aligned} \mathbb{E}\left[(\hat{v} - p_{pub})^2|s_{pub}\right] &= \left(\frac{\tau_e}{\Sigma_0^{-1} + \tau_e + \tau_{pub}}\right)^2 Var(s|s_{pub}) \\ &= \frac{\tau_e}{(\Sigma_0^{-1} + \tau_e + \tau_{pub})(\Sigma_0^{-1} + \tau_{pub})}. \end{aligned}$$

By adopting β in equation (12), the expected profit at the information-acquisition stage is

$$\mathbb{E}[V|s_{pub}] = \frac{\sigma_u}{2}\sqrt{\frac{\tau_e}{(\Sigma_0^{-1} + \tau_e + \tau_{pub})(\Sigma_0^{-1} + \tau_{pub})}},$$

which is identical to equation (15). □

Therefore, the timing of the revelation of public information and acquisition of private information is irrelevant to the result of the baseline model.