Strategic Mutual Fund Tournaments

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ABSTRACT

We characterize optimal mutual fund risk-taking strategies in competitive multi-period tournaments among multiple players. With multiple competitors, every player begins by taking maximum risk. In the final period, all players continue to take maximum risk except the leading player, who employs a "lock-in" strategy that depends on the magnitude of the lead. Our theory predicts the leader should strategically lock in advantage by reducing risk-taking if and only if the lead is great enough, rather than an increase in risk-taking by the trailers to try to catch up. Empirical evidence from style-adjusted mutual fund tournaments provides strong and robust support.

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I. Introduction

An economic tournament is said to exist whenever there is a contest among economic agents and the outcome results in a single winner whose prize is greater than that of others. In financial economics, interest in tournaments has often centered around the example of mutual funds, where new investments tend to flow to the mutual fund with the highest relative performance within a calendar year. As a consequence of the empirical observation that fund flows are convex with respect to performance rankings (Chevalier & Ellison, 1997; Sirri & Tufano, 1998), the seminal paper by Brown, Harlow, and Starks (1996) argues that mutual fund managers who are trailing have an incentive to increase risk in the second half of the year in an attempt to catch up. Consistent with this hypothesis, they empirically find that trailing funds increase risk.

One limitation of the subsequent literature is its incomplete characterization of tournament behavior in this setting. Much of this literature follows the original work (Brown, Harlow, & Starks, 1996) and asserts that trailing funds would want to increase risk. However, the literature has not considered the strategic reactions of the leading funds. The implicit assumption is that leaders would simply stand pat. However, if trailing funds increase risk, leading funds might also strategically change their risk-taking behavior. In addition, the magnitude of the leads and the number of competitors might also influence the strategy of the leaders.

We reexamine tournament models with N players engaging in strategic behavior, where competitors can change behavior after an initial stage. Our goal is to understand optimal strategy in tournament settings better by using only a minor deviation from the basic model setup and introducing more than two competitors ($N > 2$). Contrary to the assumed behavior in the literature, we find that all strategic behavior should be done by the leader. In a two-period model, when $N > 2$, all funds start by taking maximum risks in equilibrium. In the final period, we obtain a *lock-in equilibrium* where, the interim leader attempts to lock in her lead by taking minimal risk, but only if the lead is large enough. With a smaller lead, the leader optimally does *not* reduce risk. It is only in the special case with $N = 2$, when the decision to lock in does *not* depend on the magnitude of the lead. The derivation of this multi-player equilibrium and the empirical implication that the leader's lock-in decision depends on the size of the lead are new to the literature.

The benefit of locking in decreases when the leader faces more than one rival. We show that whether or not the leader reduces risk depends on the size of the lead relative to the distribution of the *maximum* of other players' performances. With multiple rivals, the distribution of the maximum is skewed because there is a greater chance that one rival gets a lucky break. As N increases, the leader requires a larger lead to reduce risk. Folding back to the initial period, all players compete aggressively by taking maximal risk because the interim leader gains this valuable option to lock in. Although our model has simplifying assumptions for tractability, we explore various extensions and find that this lock-in equilibrium is remarkably robust.

Our main testable implications are (1) only the leader reacts strategically in the interim and (2) her decision to reduce risk depends on the magnitude of the lead. We use mutual fund tournament data to test our theory, but our results apply more broadly to other multi-player tournament settings where competitors can react to interim rankings. We test our theory using a large sample of mutual funds from 1999-2022 in a panel of annual style-adjusted tournaments. We first show that funds above the 80th percentile of style-adjusted performance experience markedly larger inflows, and as a result, we define these funds to be the leading group and concentrate our analysis on them. While prior studies usually use monthly data, ours is one of the first large-scale studies of mutual fund tournaments to use daily frequency data. Higher frequency data allows us to better estimate the amount of risk taken by mutual fund managers within short windows of time and conduct a more powerful empirical analysis. Our empirical investigation could also potentially explain the differences between Brown, Harlow, and Starks' (1996) results for different time periods, as well as the difference between their results and Busse's (2001) who finds no evidence of increased risk-taking.

Consistent with our theory, we find strong evidence that funds in the leading group in interim-year (end-of-June) performance reduce risk relative to funds in the middle by annualized volatility of 0.421%. Surprisingly, we also find that funds in the trailing group reduce risk by an annualized volatility of 0.337%. The finding that trailers also reduce risk is not explained by our model nor by traditional explanations but could be explained by managerial career concerns where underperforming managers risk termination. Risk reduction by leaders relative to trailers is also consistent with Brown, Harlow, and Starks (1996) and others, who find that leading funds take less risk than trailing funds in the latter part of the year. Their interpretation is that trailers increase risk relative to leaders, whereas ours is the reverse.

We can differentiate between the two explanations because our theory implies that funds with the largest leads ought to reduce risk the most. We define a fund's lead as the difference between its interim performance and the performance of the 80^{th} percentile fund. A fund that outperforms this 80^{th} percentile fund by 10% at the interim date reduces volatility by 6.31%. Since there is less time for trailers to catch up later in the year, our theory predicts that funds in the top group with similar-sized leads ought to lock in more aggressively. Indeed, when the interim date is later (e.g., November rather than July), the volatility reduction for the top group is greater.

The study of tournaments in economics has a long and storied history. Seminal works, such as Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983), investigate the impact of dichotomous reward structures on individual effort and competitive behavior, rather than on risk-taking. Typical questions deal with whether the contest can be optimally designed to elicit effort or to efficiently separate types in an adverse selection context (Bhattacharya & Guasch, 1988). The major contribution of the theoretical literature in agency tournaments has been the optimal design of reward incentives (Akerlof & Holden, 2012). Empirical work has concentrated mainly in the context of labor markets, such as in O'Keefe, Viscusi, and Zeckhauser (1984) and Boganno (2001), or competitive sporting events, such as with golf in Ehrenberg and Boganno (1990) and Brown (2011) or weightlifting in Genakos and Pagliero (2012). Within financial economics, Kale, Reis, and Venkateswaran (2009) examine how competition for promotion to CEO elicits effort, while Kini and Williams (2012) and Coles, Li, and Wang (2017) examine the link between CEO compensation and firm risk.

Most theoretical models of tournament behavior involve just two competitors and focus on the costly choice of effort rather than the choice of risk. Lazear and Rosen (1981) find that when players choose effort, results of two-player tournaments generalize to multi-player tournaments and are unaffected by risk. Nalebuff and Stiglitz (1983) also allow for multiple players but focus on the choice of effort. Stein (2002) and Brown (2011) focus on the choice of costly effort where one player is a "superstar", whose presence reduces others' incentives to exert effort. When it comes to the choice of risk, Bronars (1987) is the first to note that in a two-player setting where competitors can react mid-tournament, the leader has an incentive to reduce risk while the follower has an incentive to increase it. Hvide (2002) and Coles, Li, and Wang (2020) examine the simultaneous choice of costless risk-taking and costly effort and find that leaders take a low-risk strategy, and the trailer takes a high-risk strategy while all competitors exert the same amount of effort. Overall, however, theoretical work has not addressed the effects of multiple competitors on risk-taking.

Meanwhile, theoretical work examining tournament behavior among mutual funds has focused instead on the choice of risk rather than on effort but continues to remain in a two-player setting. For instance, Taylor (2003) analyzes a game with two funds and two binary strategies (either a risky or riskless alternative) and finds results opposite to the standard tournament theory where higher-performing funds take more systematic risk than lower-performing funds – a *reverse tournament* effect. Goriaev, Palomino, and Prat (2003) also examine the dynamics of risk-taking in a model with two funds and find that the magnitude of the lead matters when changing risk-taking is costly. We show that in a costless environment, it is the presence of a *third* fund that makes the size of the lead relevant. Lóránth & Sciubba (2006) consider a two-fund tournament in which a third fund can enter at an interim date by paying a fee. In their equilibrium, the leading fund reduces risk, and they interpret this result as funds initially maximizing risk to minimize the threat of a new entrant. In contrast, our paper demonstrates that it is not the threat of a new entrant that drives the equilibrium but the very existence of a third fund.

Other theoretical works investigate the implications of tournaments on other aspects of competitor behavior. Bhattacharya and Pfleiderer (1985) considers the nonlinearity in reward schemes created by tournaments while Stoughton (1993) explores the resulting moral hazard issues. Optimal contract design in a multi-period game in which performance in one period would result in implications for fund flows in subsequent periods has also been considered (Heinkel & Stoughton, 1994). Basak and Makarov (2014) considers two-player tournaments with transparent informational environment in which managers have a relative performance component to their utility functions and find a multiplicity of equilibria in which there can be either returns chasing behavior or contrarian strategies.[1]

Empirically, the idea that new mutual fund flows chase past returns and create convex payoffs and tournament behavior has been well established. For instance, there are substantially greater fund flows into those mutual funds that are classified in the highest Morningstar five-star category relative to funds in the second-highest category (Reuter & Zitzewitz, 2021). Also, mutual funds that garner media attention by becoming the top performers in their investment category attract significantly greater fund flow (Kaniel & Parham, 2017). Kempf and Ruenzi (2008) documents that convex payoffs and tournament behavior also exist among mutual fund managers within the same mutual fund family. The empirical study of mutual funds is ideal for testing risk-taking in tournament models because performance and risk are readily observable. Some recent empirical papers find that tournament effects appear in some years and reverse tournament effects in others (Qiu, 2003). One contribution of our paper is to show that this kind of dichotomy can be consistent with tournament behavior since we predict that behavior should depend on the magnitude of the lead, not just whether a fund is leading.

The rest of the paper is organized as follows. We develop our model in Section 2, where we review the two-player model. The main part of the theory appears in Section 3, where we study a model in which a third player is introduced. Section 4 examines the robustness of our predictions

[1] A related literature also investigates the impact of mutual fund manager behavior in the presence of compensation contracts with benchmarking to an index. The tournament setting is different because managers are evaluated with respect to active competitors who might react strategically, rather than with respect to a passive benchmark that does not react at all. Basak, Pavlova and Shapiro (2007) find outperforming funds decrease tracking error risk relative to a benchmark, but not when the manager is very far ahead of the benchmark. Chen and Pennachi (2009) find that although trailing funds increase tracking error, they do not necessarily increase total level risk. Cuoco and Kaniel (2011) include an option payoff component in the compensation and find that risk-averse managers sufficiently above the benchmark will also lock in gains.

in more general settings. Empirical results are presented in Section 5, and conclusions are presented in Section 6. Proofs and model extensions are collected in the Appendix.

II. Base-Case Model Setup

Consider a tournament between $N \geq 2$ competitors. A three player setting ($N = 3$) is general enough to illustrate our main results. The tournament is divided into two periods. Initially there is a performance formation period when interim rankings among the mutual fund managers are established. We refer to the fund with the highest performance in the first period as the 'leader' and all other funds as the 'trailers'. There is an endgame where managers compete for a 'winner take all' prize given at the end of the second period, based on the final rankings among all funds.

In this setting, there is only a single winner and there is no difference in payoffs among any of the trailing funds. In reality, mutual fund managers are typically compensated by a proportionate fee schedule based on assets under management at the end of the year, as well as an implicit compensation through an expected increase in fund flows based on relative performance. To keep our model parsimonious and to focus specifically on the tournament aspect, we do not consider the linear part of the compensation schedule and focus only on the competition to be the winner. We also do not consider differential fund flows or termination outcomes for managers of the trailing funds. Therefore, the tournament payoff structure we investigate is an approximation of the well-known `convexity' of fund flows. Evidence (Kim, 2019) shows that the convexity in fund flows as a function of end of year rankings is the most prominent.

We specify fund return distributions for each period, $t \in \{1,2\}$, as $R_{it} = \sigma_{it}\epsilon_{it}$, where manager i selects a risk level $\sigma_{it} \in [0, \bar{\sigma}]$ bounded above by $\bar{\sigma}$, and ϵ_{it} is zero mean, unit variance, independent normally distributed random. This specification is general enough to allow for performance benchmarking, in which case ϵ_{it} is benchmark-adjusted idiosyncratic shocks. Although some of our proofs rely on the normality assumption, this can be generalized to some extent. Sufficient for our analysis is that return distributions are symmetric, continuous and unimodal, which is the same assumption Hvide (2002) uses.

In our basic specification, all managers have equal abilities and relative performances are due entirely to their risk choices and the resolution of uncertainty. Since our specification is based on idiosyncratic returns, expected excess return in each period is zero and there no risk premium. We also assume that there exists a costless risk-taking technology that is available to all managers. This is realistic because methods of adding risk, such as holding futures contracts, are available to all managers and require lower capital than investing in equities themselves.[2] In the Appendix, we discuss extensions of our base-case setup that account for additional features.

Denote the first period performance of each fund as $a_i = \sigma_{i1}\epsilon_{i1}$. The total return, without adjusting for compounding, is given by $R_i = a_i + \sigma_{i2}\epsilon_{i2}$. The tournament structure is a Nash equilibrium embodied in the following payoff for manager i, who chooses her risk level while taking other managers' choices as given:

$$U_i\big(R_i|\sigma_{)i(}\big) = \begin{cases} 1 \text{ if } a_i + \sigma_{i2}\epsilon_{i2} \geq \text{Max } a_j + \sigma_{j2}\epsilon_{j2}; j \neq i \\ 0 \text{ if } a_i + \sigma_{i2}\epsilon_{i2} < \text{Max } a_j + \sigma_{j2}\epsilon_{j2}; j \neq i \end{cases} \tag{1}$$

where $\sigma_{)i(}$ denotes the vector of risk choices for managers other than i. Each manager maximizes her expected utility, $E(U_i)$:

$$\text{Max}_{\sigma_i} E(U_i) = \text{Prob}(R_i \geq \max_{j\neq i}\{R_j\}). \tag{2}$$

[2] For instance, the May 6, 2010 `flash crash' is alleged to have been caused by one mutual fund selling a large number of futures contracts (SEC, 2010).

II.A Two-Player Tournaments with No Risk Limits

To build intuition, we begin with a simple, albeit unrealistic, case where $N = 2$ and there is no limit to risk-taking, $\bar{\sigma} = \infty$. Consider the final period risk choice of the fund managers. Let manager 1 have an interim lead over manager 2, i.e., $a_1 > a_2$. Suppose that manager 1 is currently choosing finite risk so that $\sigma_1 < \infty$ (we neglect the time subscript). If manager 2 also chooses finite risk, her probability of winning would be less than half. By selecting $\sigma_2 = \infty$, manager 2 can maximize her probability of winning at one-half, and manager 1's win probability is unaffected by her risk choice. Hence all equilibria feature $\sigma_2 = \infty$ but σ_1 can be anything: $\sigma_1 \leq \infty$. The usual interpretation of this effect in the literature is that the trailing manager takes on as much risk as possible to maximize the probability of catching up.

Now consider fund manager actions in the performance formation period. Our result above implies that the interim lead does not matter because both managers are equally likely to win. Therefore, there risk taking in period 1 is irrelevant, and any level of risk in the first period can be a Nash equilibrium. Therefore, allowing for infinite risk leads to no prediction on respective risk-taking in the performance formation period.

II.B Two-Player Tournaments with Finite Risk Limits

Since taking infinite risk is unrealistic and has little testable content, we bound the maximum risk each manager can take to a finite $\bar{\sigma} < \infty$. Consider first the final period. Again, suppose that manager 1 leads with $a_1 > a_2$. Now, manager 1's expected total benchmark-adjusted return exceeds that of manager 2's: $E[R_1] > E[R_2]$. Because σ_2 must be finite, by choosing $\sigma_1 = 0$, manager 1 wins with probability greater than one-half. Adding symmetrically distributed risk reduces this probability regardless of manager 2's actions. Conversely player 2 always increases her probability of winning by adding risk. There is now a unique Nash equilibrium where $\sigma_1 =$

0, which we call “locking in” and $\sigma_2 = \bar{\sigma}$. Manager 1 wins with probability greater than one-half and manager 2 wins with probability less than one-half.

We formally prove this second period equilibrium in Appendix I as our first Proposition:

Proposition 1. *In a two-player tournament where manager returns are symmetrically distributed, continuous, and unimodal, the leader always locks in and the follower always maximizes risk taking.*

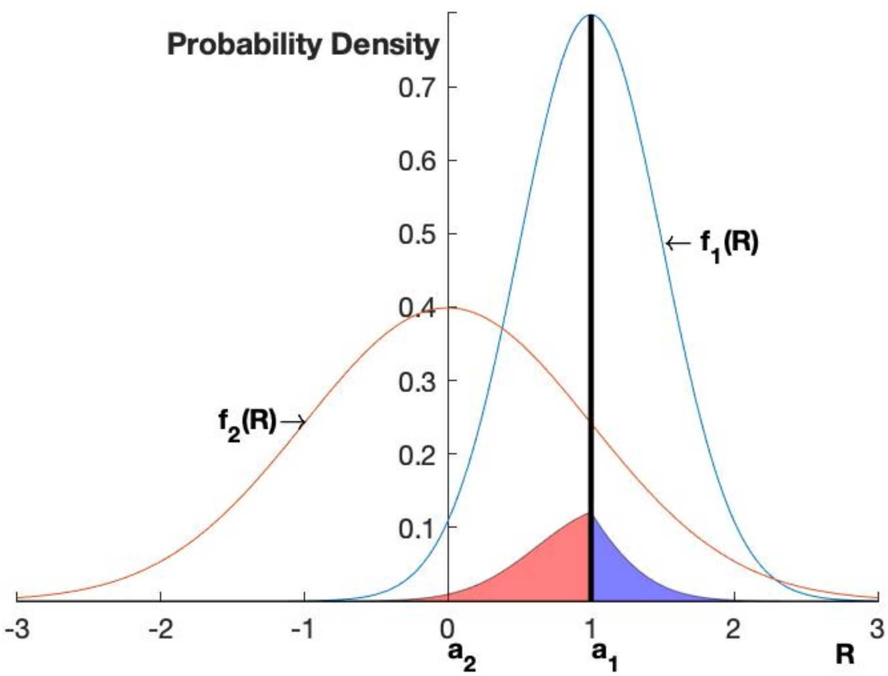

Figure 1: By locking in, the leading player 1 wins with the area below the trailing player 2’s probability density function to the left of a_1, which is the expected final period return of player 1. By increasing risk, player 1 gains the probability of winning in the blue shaded area above a_1, but loses in the red shaded area below a_1. The former, however, has lower probability mass than the latter. Hence player 1 should never increase risk above zero when player 2 has a finite risk limit.

For the case with normal distributions, Figure 1 illustrates the equilibrium described in Proposition 1. With $\bar{\sigma} < \infty$, there is an advantage to be the interim leader. If each fund begins the tournament without a head start, then no matter what initial risk strategy is employed, each fund will be in the lead with probability one-half at the interim stage. This means that in the first period, once again, taking any amount of risk is a Nash equilibrium. Bounding the risk limit leads to a sharper prediction than with unbounded risk limits, but only regarding the second period behavior. In the second period, the leading (trailing) manager takes zero (maximal) risk, but anything is still possible during in the initial period. We might expect leading funds to reduce risk and trailing funds to increase risk in the second period, but we cannot make any definitive statement about risk-taking behavior in the first period. A definitive prediction about first-period risk-taking requires considering richer cases, such as cases with more than just two funds.

III. Multiplayer Tournaments

We now consider a tournament with three managers. This one change to the model produces sharper results with and more interesting testable predictions. It will be apparent that what matters for our results is that each competitor faces multiple rivals.

We begin by characterizing the objective function for the final period of the tournament. For fund i to win the tournament, funds, j and k must have lower realized returns. Let $F_i(R_i)$ be the cumulative distribution function (cdf) for fund $i's$ return, given chosen risk level, σ_i. The probability that fund i wins with return R_i is $F_j(R_i)F_k(R_i)$, which is the cdf of the maximum of the returns of the other two funds, evaluated at fund $i's$ realized return, R_i. This expression is the cdf of the *maximum order statistic* of the other two funds. The event that manager i wins is equivalent to the event that manager i's return exceeds that of the maximum order statistic of the other two fund returns. Hence the objective function can be written as

$$E(U_i) = \int_{-\infty}^{\infty} F_j(R_i)F_k(R_i)f_i(R_i)dR_i, \qquad (3)$$

where $f_i(R_i)$ is the probability density function (pdf) of fund i's return. For illustrative purposes, consider the case where $f_i(R_i)$ is normally distributed with $f_i(R_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\frac{-(R_i-a_i)^2}{2\sigma_i^2}}$ as the pdf. To analyze the impact of changes in the level of risk, one can differentiate equation (3) with respect to σ_i. Simplifying yields:

$$\frac{dE(U_i)}{d\sigma_i} = \int_{-\infty}^{\infty} \left(-\frac{1}{\sigma_i} + \frac{(R_i - a_i)^2}{\sigma_i^3}\right) F_j(R_i) F_k(R_i) f_i(R_i) dR_i. \tag{4}$$

This expression reveals that there are two resulting effects when a fund manager increases the risk level. The first is negative and occurs when returns are near their initial (expected) values and the level of risk is low. The second is positive and occurs when returns deviate from their initial (expected) values. Which effect dominates depends on the relative interim position of the fund and the potential for risk increases. We now analyze the various cases.

III.A Multi-Player Tournaments with Finite Risk Limits

Since the infinite risk limit case is uninteresting, we begin by considering a finite risk limit, $\bar{\sigma} < \infty$. Proposition 2 provides the essential properties we require for the distribution of the maximum order statistic in the case where fund returns are normally distributed. The proof of the following Proposition applies some properties of maximum order statistics derived by Nadarajah and Kotz (2008) and is presented in Appendix I.

Proposition 2. *The distribution of the maximum order statistic for two independent normal random variables is unimodal and positively skewed.*

Given these characteristics regarding the distribution of fund manager returns, we begin by considering the second-period best response strategies of the trailing managers.

Proposition 3. *Suppose all managers' risk choices are restricted to a finite risk limit,* $\sigma_i \leq \bar{\sigma} < \infty$, $\forall i$. *Then during the second period of the tournament, trailing managers always maximize risk-taking.*

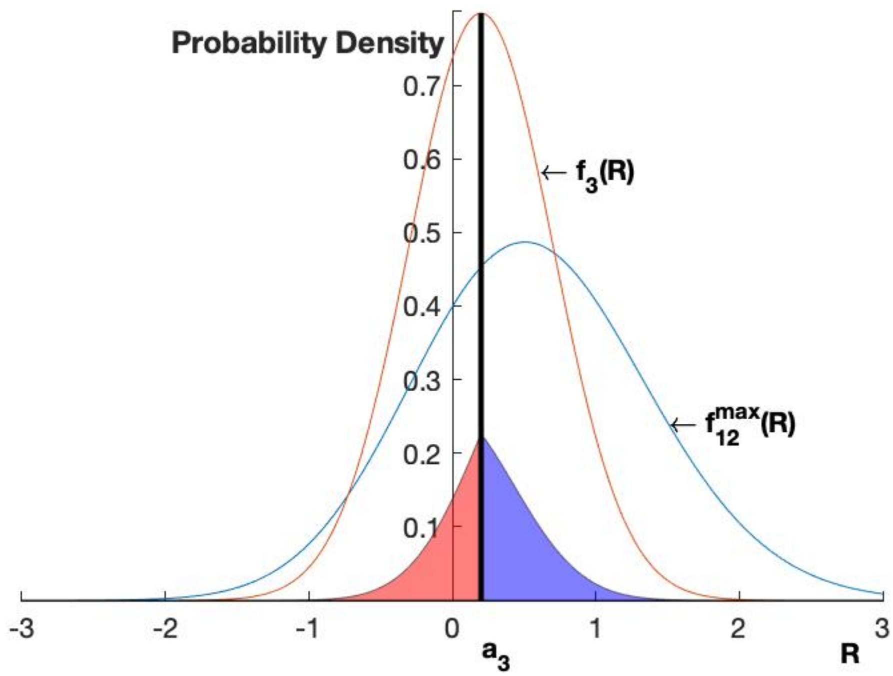

Figure 2: This figure depicts the situation of player 3 whose mean return distribution lies below the mode of the maximal order statistic distribution between players 1 and 2. If player 3 increases risk, her distribution slightly the area of the shaded region above a_3 increases by more than the shaded region below if risk is symmetric. This argument can be extended to all possible values of σ_3.

The intuition behind Proposition 3 is to suppose that manager 3 takes additional mean-preserving risk as depicted in Figure 2. This increases the probability of winning if the area of the additional partial distribution gained to the right (shaded in blue) is greater than the area of the partial distribution lost to the left (shaded in red). With symmetrically distributed returns, this is always the case if the manager's expected return is less than the mode of the maximum of the other managers' returns. If competing funds' returns are symmetrically distributed, then, for either trailing manager, the mode of the maximum is always greater than the expected return and the distribution of the maximum is positively skewed.

Proposition 3 implies that the two trailing managers take as much risk as possible. We now show that the leader might not choose to increase risk to the maximum. If the trailers are limited to finite risk, manager 1's lead still might not be safe enough for her to reduce risk. Consider what happens when manager 1 has only a slight lead over managers 2 and 3.

Proposition 4. *Suppose that the initial return for fund manager 1,* a_1*, is less than the mode of the maximal order statistic density between managers 2 and 3. Then, increasing risk increases player 1's probability of winning. Hence the optimal response is to set* $\sigma_1 = \overline{\sigma}$.

Our proof in Appendix I reveals that when $N > 2$, it is not the symmetry of competitor returns that drives the results, but the extent of the leader's lead relative to the mode of the maximal order statistic of the competitions' returns. The intuition for this result lies in the skewed distribution of the maximum order statistic. This result implies the following corollary.

Corollary 1. *If manager 1s' lead is sufficiently small and lies below the mode of the maximum order statistic density of managers 2 and 3, then the unique Nash equilibrium in the single period tournament is for each manager to choose maximal variance,* $\sigma_1 = \sigma_2 = \sigma_3 = \overline{\sigma}$.

This result with three managers is different from that with two managers, where the leader always sits on a lead, no matter how small. With $N = 3$, if the lead is not greater than the mode of the maximal order statistic, the leader maximizes risk. With two other managers rather than just one to worry about, it is more likely that a trailing competitor will achieve a high outcome from a high-risk strategy. As a result, the leader must also continue to take maximal risk.

However, if manager 1 has a large lead, then there is an asymmetric equilibrium in which the leader minimizes risk.[3] This is depicted in Figure 3 and described in the following Proposition.

Proposition 5. *Suppose that the leading manager return,* a_1*, lies above the mode and the median of the maximal order statistic density of competing fund returns. Then the optimal strategy is always to lock in. If* a_1 *is below the median but greater than the mode, there exists some critical value,* $\widehat{\mathrm{a}_1}$*, above which the optimal strategy is to lock in but to take maximal risk otherwise. The critical value depends on the magnitude of maximal risk,* $\overline{\sigma}$.

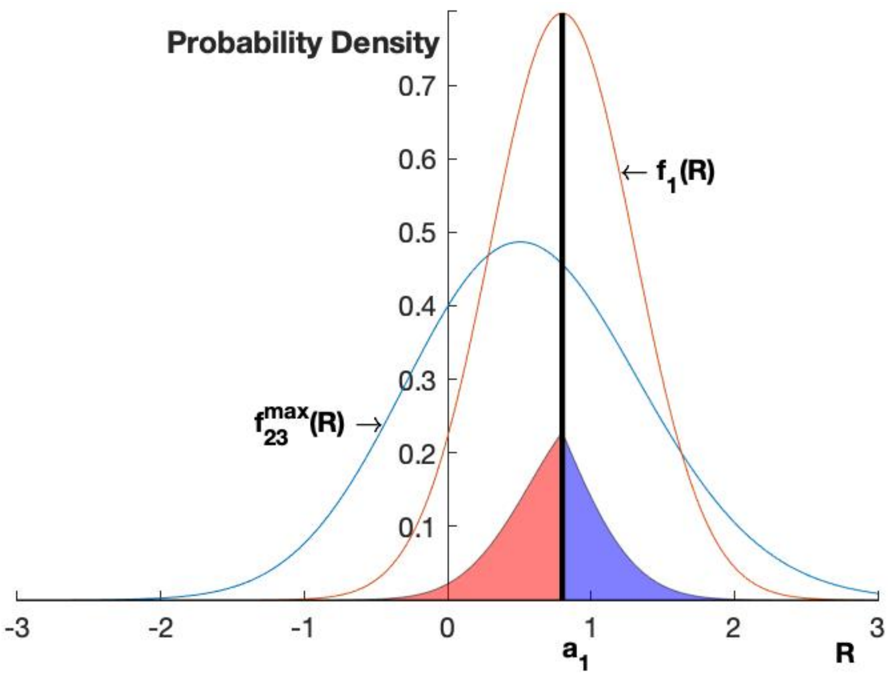

Figure 3: The leader finds it counterproductive to add some amount of risk. The density, $f_{23}^{\max}(R)$, is the maximum order statistic of the two trailing funds. With zero risk, the probability that fund 1 wins is the area to the left of a_1. With risk-taking, the density, $f_1(R)$, is that of the leader causes a higher probability in red of losing versus a lower probability in blue of winning.

The following corollary characterizes the equilibrium in the final period.

[3] In particular, the lead has to be greater than the median of the maximal order statistic, rather than the mode, to guarantee that locking in is the dominant strategy. To see why being above the mode is not sufficient, consider the situation where the lead is greater than the mode but less than the median. If the leader locks in, her probability of winning is less than half, by the definition of the median. If she maximizes risk-taking, her probability of winning approaches one half as her risk limit becomes infinite.

Corollary 2. *If manager 1's returns to date,* a_1, *exceed the mode of the maximal order statistic density between managers 2 and 3, then there exists a unique Nash equilibrium in which either: (1) manager 1 chooses zero risk and managers 2 and 3 choose maximum risk; or (2) all managers choose the highest permissible levels of risk. The determination of which equilibrium prevails depends on the level of* a_1 *and the magnitude of the risk limit.*

We have therefore identified when the leader sits optimally on a lead in the final period. It involves the joint determination of the relative interim position of the leader *vis-a-vis* the likely values of the maximal order statistic of the other trailing funds. When the lead is slim the leader optimally maximizes risk. However, with a sufficient lead and finite risk-taking, the leader optimally minimizes risk.[4]

This result shows how the multi-player case differs from the two-player case. With two players, the distribution of the maximum order statistic is just the distribution of a single competitor's return. Hence, the leader takes zero risk and the trailer maximizes risk because the mode and the median of the maximum are equal to each other when returns are unimodal and symmetrically distributed. With $N \geq 2$, the distribution of the maximum does not equal that of any single fund return. For tournaments with N>3, this intuition regarding the distribution of the maximum of the competing funds carries forward, and generally implies that the lead of the leader must be greater to observe the lock-in strategy.

[4] We have numerically analyzed the case when a_1 is above the mode but below the median of the maximum order statistic and found there exists a cut-off point where the leader switches from maximizing risk to locking in with no interior optimal level of risk-taking.

III.B Value of the Lock-in Option

The possibility to lock in a lead after the initial period has implications for first period risk taking. We have demonstrated that if the interim leader has a sufficient lead, she will minimize risk; otherwise, it will be optimal to increase risk to the maximum. Because a leader only reduces risk when her lead is safe, we can define the value of this option to be the difference between the probability of winning by reducing risk to zero and the probability of winning by taking maximum level of risk. The probability that both the middle and bottom players lose if manager 1 takes no risk is $F_2(a_1)F_3(a_1)$. On the other hand, if manager 1 takes maximum risk, the probability of winning is $\int_{-\infty}^{\infty} F_2(R)F_3(R)f_1(R)dR$, where $f_1$denotes the pdf of the leader with mean a_1 and standard deviation $\overline{\sigma}$. This means that the option value of locking in is

$$\text{Lock-in Option Value} = \max\left(F_2(a_1)F_3(a_1) - \int_{-\infty}^{\infty} F_2(R)F_3(R)f_1(R)dR, 0\right). \tag{5}$$

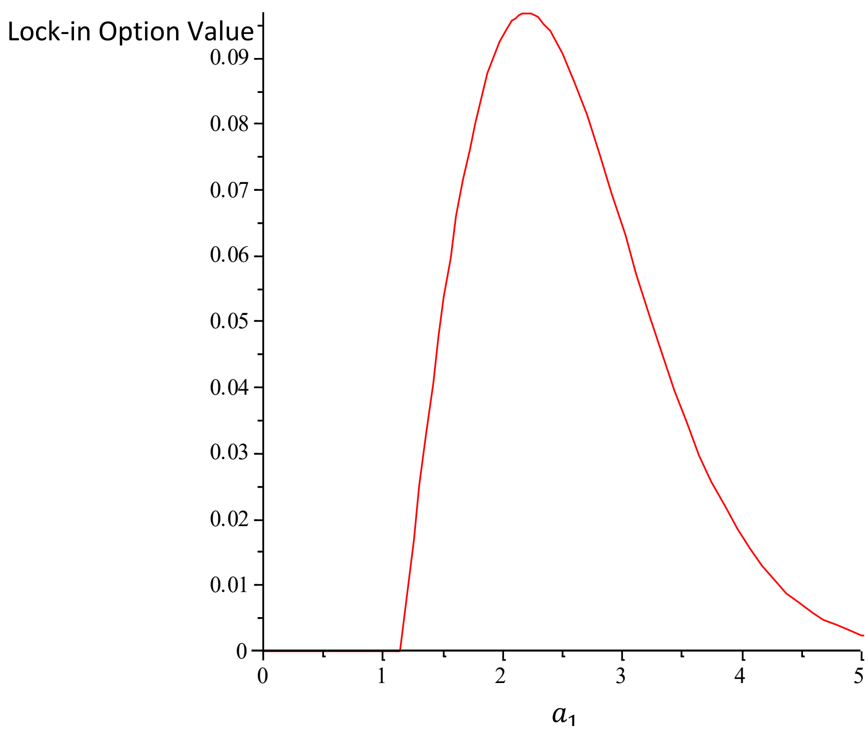

Figure 4: The value of the lock-in option in terms of increased probability of winning for fixed realization of the bottom two funds. This simulation is calibrated using the parameters $a_3 = 0$, $a_2 = 1$, and $\overline{\sigma} = \sigma_2 = \sigma_3 = 1$. The value a_1 represents the first periods outcome of the highest-ranking fund.

Figure 4 illustrates the value of this lock-in option under normal distributions. As illustrated, unless the lead is sufficiently large, the value of the lock-in option is zero. However, the lock-in option is 'in the money' once the lead is big enough. The value of the lock-in option has a maximum which is where the leader gains greatest advantage over taking risks in the second period. As is indicated by Figure 4, this relative advantage is maximized at an interior point. If the top player's lead is huge, there is a positive but small advantage of locking in a lead, because the leader is likely to win regardless of her actions.

III.C Dynamics of Optimal Risk Taking in Multi-Player Tournaments

We now discuss the effects of introducing multiple ($N > 2$) funds and the lock-in option on the dynamics of risk-taking behavior in tournaments. With only two players, any amount of initial risk taken by either player would be a Nash equilibrium. Therefore, tournaments among two players with equal abilities yield no predictions regarding optimal risk-taking during the initial period. As a result, we could observe either the leader reducing risk or keeping risk constant while the trailer keeps risk constant or increases risk. Considering $N > 2$ players now yields a sharper prediction for the initial risk-taking and the dynamics of risk-taking for all players.

Note that the optimal strategies of all players in the multiperiod game are equivalent to playing a reduced form single period game without an interim reaction period. Consider the following single-period baseline tournament. The ex-ante identical players select a single level of risk up to a finite maximum level, $\overline{\sigma}_T = \overline{\sigma}_0 + \overline{\sigma}$, where $\overline{\sigma}_0$ is the risk limit during the initial period of the multiperiod game. Here, there is no possibility to depart from this maximum risk strategy at an interim date. Since performance in the multiperiod game is independent across sub-periods, this baseline game is a single-period tournament which we have already analyzed with a finite risk limit. Applying Proposition 4, the optimal strategy is $\sigma_i = \bar{\sigma}_T, \forall i = \{1, \dots 3\}$.

The multiperiod tournament is equivalent to taking the above baseline tournament and appending a sub-tournament in which players compete to win the lock-in option. Incorporating interim information and strategic reaction is equivalent to granting the interim leader with the lock-in option to reduce risk over the continuation period. The benefit of being the interim leader is that only the leader earns the lock-in option. The question is whether this additional option gives anyone an incentive to deviate from the baseline equilibrium of taking maximum risk throughout. Since only the interim leader earns the lock-in option, this is also a single period tournament in the same form as before, where players start out the tournament with equal initial positions. We have already seen above that maximum risk-taking is the unique equilibrium of such a tournament.

Therefore, the multiperiod tournament is the combination of the baseline tournament with this additional sub-tournament for the lock-in option. Both tournaments are single period, and both tournaments involve maximum initial risk-taking as the unique equilibrium. The decision regarding risk-taking is whether to reduce risk or to maintain the maximum level of risk at the interim period. The only time it might pay to reduce risk is as a reaction to interim information. Taking less than maximal risk initially is sub-optimal. In Proposition 6, we summarize the implication of this result for first-period risk-taking.

Proposition 6. *Suppose funds have equal fund managerial abilities and identical starting positions. Let the upper bound on first period risk,* $\overline{\sigma}_0$*, be finite. Then there exists a unique symmetric equilibrium in which each manager selects an identical level of risk* $\sigma_1 = \sigma_2 = \sigma_3 = \overline{\sigma}_0$ *in the first period.*

By putting together our analysis of the first-period equilibrium and the final-period equilibrium, we have the following corollary summarizing the equilibrium in a multi-period case.

Corollary 3. *In a tournament with multiple players (N>2) where managers have equal abilities and can choose finite maximum risk, there exists a unique Nash equilibrium in which every manager begins the tournament choosing maximal risk. In the final period, all managers continue to take maximum risk except the leading player who may reduce risk-taking to zero if the lead manager's returns to date exceeds the mode of the maximal order statistic density among the other managers.*

This corollary implies our main testable empirical prediction: it should be the case that it is the leading manager, rather than the trailing managers, who strategically alter risk-taking behavior, but if and only if the magnitude of the lead is great enough.

IV. Generalizations of the Tournament Model

Before we analyze if empirically observed behavior is consistent with our theoretical predictions, we first consider how various generalizations of our model that brings it closer to implementable empirical design might affect our predictions.

In our model, we analyzed a three-fund tournament with a single winner. One question is: what happens if there are more than three competitors? Moreover, our empirical analysis considers the case where funds compete to be among the top group, rather than to be the very top fund. Hence, it is natural to consider how our predictions change if there were more than three competitors and if funds competed simply to be among the top *group* of funds.

It turns out in these cases, our model's qualitative results would go through. However, with more rivals, the leader must have an even larger lead to be safe. This follows because adding additional competitors unambiguously increases the mode of the maximal order statistic distribution among the remaining N-1 funds. But our proof remains largely unchanged because maximal order statistic is always unimodal and skewed. Furthermore, if our tournament pay-off went to being in the top group of funds, rather than just to the single top fund, then our results remain unchanged once again. Here, rather than comparing a fund's return to the distribution of the maximal order statistic, one would need to compare a fund's return to the distribution of the corresponding central order statistic, which is also unimodal and skewed when the underlying distribution is normally distributed. For example, if the payoff went evenly to all funds in the top 20%-tile ranking, the relevant distribution is the upper 20%-tile central order statistic, which is unimodal and skewed if the underlying returns are normally distributed. Given these distributional properties, our proof would follow as before.

We can also consider the case where fund managers can choose to take systematic risk or idiosyncratic risk. We provide more complete analysis of the case where fund managers may take non-negative systematic risk in Appendix II. From the point of view of the trailing funds, taking systematic risk rather than idiosyncratic risk has no advantage. If anything, if a trailing fund were to take systematic risk, it would provide an additional way in which the competing leading fund can lock in her lead by taking on identical correlated systematic risk. Hence, in equilibrium, when fund managers take risk, they take only idiosyncratic risk to maximize the chance of realizing an outcome different from those of other competitors. Therefore, it remains true that the leading fund will lock in her lead if the lead is sufficiently large.

A potential criticism of our model, and of the literature on mutual fund tournaments in general, is that given the common knowledge about abilities in our core model, there is no reason why new assets should flow to the highest performers in the first place. If investors were aware that managers were all unskilled, there should not be a tournament pay-off to begin with. In general, the mutual fund tournament literature assumes that pay-off are exogenously determined. If we generalize our model slightly so that funds have differential ability, then on average, the funds with highest performance will have more able managers, and new assets flow towards high performing funds because it signals ability. Generating a convex fund flow relation with optimizing investors is beyond the scope of this paper. However, it can be shown that if managers have different abilities, investors will optimally infer that high returns are associated with more skilled managers. In Appendix III, we illustrate this in a simplified version of the environment of Berk and Green (2004) and Jiang, Starks and Sun (2021).

V. Empirical Study

We now examine whether the behavior of mutual fund managers is consistent with our predictions. Specifically, we investigate whether leaders rather than trailing funds change their risk-taking by examining whether the magnitude of the lead affects their response.

V.A Data Description and General Mutual Fund Characteristics

Our data comes from the CRSP Mutual Fund Database. We use data on equity mutual funds starting in 1999 when fund performance data first became available at the daily frequency for an entire calendar year. While most prior research on tournament effects in mutual funds use monthly data, this would not allow us to reliably investigate changes in risk-taking behavior across time within a calendar year because of the lack of data points. This is particularly important when we focus on risk-taking changes during the last couple of months of a calendar year. Using daily data also allows us to more accurately control for factor risk exposures and better distinguish idiosyncratic risk from total risk-taking. For each mutual fund, we collect daily fund returns net of fees and expenses. We also collect each fund's Lipper style category. We then merge our data with the MFLINKS database (Wermers, 2000) to eliminate redundant mutual fund share classes. We end our sample in 2022, which provides us with 24 years of data.[5] Our approach is to compare how mutual funds change their risk-taking behavior in the later months relative to earlier months of a calendar year. This is akin to using fixed effects for each mutual fund and accounts for fund characteristics that do not vary within the year.

Table 1 shows the distribution of mutual funds used in our study. We limit our sample to funds in one of twelve Lipper equity fund categories, which are the intersections of four size categories (Large-Cap, Mid-Cap, Small-Cap, and Multi-Cap) and three value categories (Value, Core, and Growth).[6] Overall, we have 58,419 fund-year observations over 24 separate years in 12 fund style categories. Each category is well-represented and ranges from an average of 79.8 funds per year (Mid-Cap Value) to an average of 332.8 funds per year (Large-Cap Core). For our analysis, we consider each tournament spanning a single calendar year ending in December within each of the 12 Lipper classifications. As mentioned earlier, we believe it is most realistic to define the end of a calendar year as the conclusion of the tournament and the beginning of the calendar year as the start (Kim, 2019). Thus, in our sample, we have 288 distinct tournaments.

We assume that the tournament winners are based on cumulative raw returns because recent literature has found that fund flows respond more to unadjusted performance (or CAPM-adjusted performance) than to multi-factor adjusted performance (Berk & van Binsbergen, 2016; Barber, Huang, & Odean, 2016). Moreover, because we rank fund returns within a style category, comparisons using raw returns are effectively comparisons using benchmark-adjusted returns.

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[5] Our sample does include the financial crisis period of 2008, but our results are robust to removing this period.
[6] This categorization is reasonable since some practitioner publications (e.g., *Barron's*) rank funds based on Lipper rather than Morningstar categories.

Finally, unlike in our model, where there is a single winner, empirically, funds flow toward a group of top performers. Hence, we examine in our empirical study whether a fund ends up among a group of top performers (defined in various ways), rather than the absolute top fund.

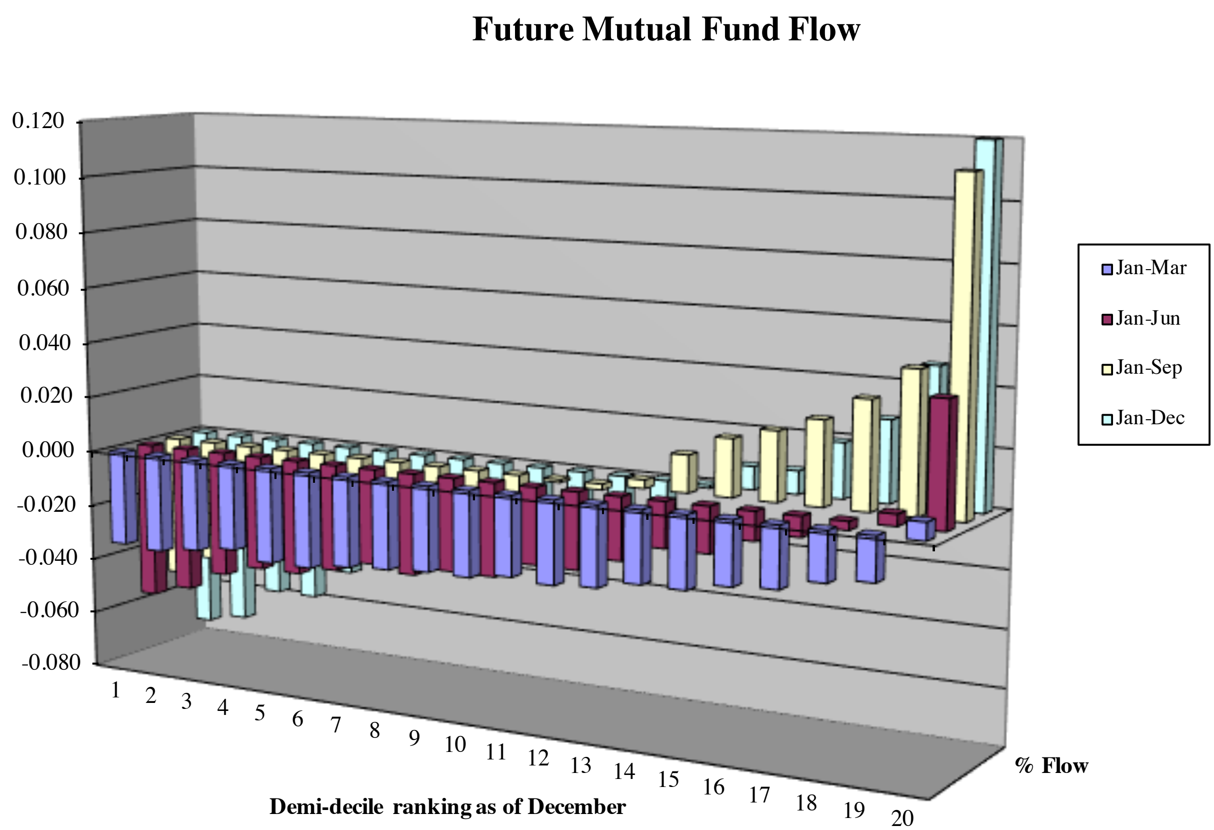


Figure 5: The relation between returns in year t and fund flows over different portions of year $t+1$. Mutual funds are ranked into 20 demi-decile groups within each style and year according to their annual returns in year t. Subsequent fund flows are represented on the vertical axis as the average percentage increases in the size of the funds in each demi-decile not accounted for by returns.

We first establish that empirically, there exists a tournament pay-off in our sample. The most visible way in which tournament incentives arise is the relation between fund flows and past performance. Figure 5 illustrates the empirical relation between returns in year t and fund flows over subsequent periods. Funds are divided into 20 demi-decile groups within each style and year by year-t annual returns. Within each demi-decile, we compute following period fund flows starting in January of next year. Moreover, there may be additional benefits to producing superior performance, such as promotion of fund managers within a fund family and spill-over effects to other funds of having a star manager in a fund family.

Next, for empirical work, we define which funds are leaders and which are trailers. Consistent with a tournament effect and prior literature, the relation between past performance and future fund flows is convex. For example, there is nearly 12% growth in fund size over the subsequent year for the top five percent performing funds. The increase in fund flow is most pronounced for the top 20% funds. Based on this observation, we classify that a fund is a leader if its performance is in the top fifth of funds within a category. This definition is the same one used by Morningstar for their five-star (top) funds, albeit using their categories rather than Lipper's. Similarly, we define trailers as funds in the bottom 20^{th} percentile in terms of performance.

The measure of risk-taking in our model is idiosyncratic volatility, which must be measured net of systematic risk. Following the standard approaches (Fama & French, 1993; Carhart, 1997), we consider up to four potential systematic risk factors: the market factor $(r_m - r_f)$, size factor (SMB), value factor (HML), and momentum factor (UMD). Mutual fund returns are calculated in excess of the risk-free rate, as residuals of CAPM (market factor only), as residuals of a 3-factor model (market, SMB, and HML), and as residuals of a 4-factor model (3-factors plus UMD). Within each fund-year, we compute the residuals by regressing daily mutual fund returns on daily factor returns each month. We measure risk-taking using the annualized standard deviation of these residuals over different periods within a calendar year.

Table 2 reports the summary statistics of mutual fund risk-taking measures for each of the four metrics used. Risk-taking measures reported here are measured over the entire calendar year. The average (median) total volatility of mutual fund returns in our sample is 20.0% (17.0%). Total return volatility at the 5th percentile is 9.8% and at the 95th percentile it is 39.0%, suggesting considerable heterogeneity in risk-taking. As we adjust for systematic risk, average idiosyncratic volatility falls. Nevertheless, idiosyncratic volatility relative to a 4-factor model is 1.7% at the 5th percentile but a much larger 9.9% at the 95th, indicating that heterogeneity in risk-taking remains substantial even after accounting for risk-factors.

V.B Second Half Tournament Effect

The main objective of our investigation is to see how these risk-taking measures change within each tournament year for the funds in the leading group relative to others. Following prior literature beginning with Brown, Harlow and Starks (1996), we first compare risk-taking over the last half of the year to the first half of the year. We measure changes in risk-taking behavior in two ways. One is the ratio of annualized return volatility over the second half of the year divided by the volatility over the first half (VOL_RATIO). This measure is commonly used in the literature, but because VOL_RATIO is hard to interpret when the denominator is small, we prefer to use VOL_DIFF, the difference between annualized return volatility in the second half and the first half of the calendar year. Prior researchers have also used this measure (Koski & Pontiff, 1999), but we also use VOL_RATIO as a robustness check.

According to our theory, a fund's relative interim performance ranking should affect how funds alter their risk-taking. We employ two types of ranking measures. Within each category and each year, we rank funds by cumulative percentage returns over the first half of the calendar year (January to June). This six-month period is a proxy for the period 1 in our model. The first ranking type is a leading indicator set to one if performance is in the top 20%, a middle indicator if performance is in one of the three middle quintiles, and a trailing indicator if performance is in the bottom 20%. We omit the middle indicator so the coefficients on the other indicators can be interpreted as changes in fund risk-taking for a group relative to the changes in the middle group.

Since only some of the leading funds have sufficiently large leads, we also employ a second type of measure of lead. This measure interacts the leading and trailing group indicators with the absolute difference between a fund's cumulative past return and that of the fund at the top or bottom quintile breakpoints (distance). For leading funds, this measures how far ahead they are from the breakpoint. For the funds in the trailing group, it measures how far behind the fund is from the breakpoint. The median fund in the leading group is ahead of the 80th percentile fund by a mid-year return of 1.53% in the middle of the year, while the 95th percentile fund is ahead by 3.35%. Among the trailing group, the median fund is behind the 20th percentile fund by 1.52% in the middle of the year, while the 95th percentile fund is behind by 3.19%.

Table 3 shows results from our base-case specification, where we treat the initial period to be January to June of each calendar year and assess the changes in risk-taking over July to December. Our basic regression specification is a pooled-panel regression of changes in risk-taking on measures of where a fund stands relative to its peers in July. We include fixed effects for each fund category by year, which absorb common volatility changes within each fund category and calendar year. We also include a separate indicator (Crisis Period) for the 2008 Financial Crisis to distinguish its effects on overall volatility. Standard errors are clustered by fund to control for seasonal patterns experienced by funds across years.

Panel A of Table 3 revisits the basic tournament effect without considering any distance measures for leading or trailing funds. We make no adjustments for risk factors in the first column. The coefficients on the indicators can be interpreted as the average increase in percentage volatility by the funds in the leading group (-0.421) and the trailing group (-0.337) relative to the decrease in volatility by the funds in the middle group (-1.981). That is, a typical leader decreases volatility by a statistically significant 0.421% relative to what a typical mutual fund in the middle group does. However, a typical mutual fund in the trailing group decreases volatility by 0.337% as well. As with Busse (2001), we also find that it is difficult to find an asymmetry in risk reduction without accounting for the magnitude of the lead. The intercept term of -1.981% can be interpreted as the decrease in volatility by funds in the middle group. During a typical year, we find that funds reduced risk during the latter half of the year. The coefficient on the Crisis Period indicator implies an exceptional increase of 32.439% in total volatility for a typical fund during the 2008 Financial Crisis. We also include controls for factor risk exposures as we move across columns. The regression R^2 statistics fall as risk factors are added because the variation in idiosyncratic volatility also falls.

To illustrate the effects of the magnitude of the lead, Panel B of Table 3 shows results when we interact our indicator functions with our distance metric. This allows us to examine how much funds are leading or lagging change risk levels. To make our results easier to interpret, we de-mean our distance metric so that the indicator variables are unaffected. As with the earlier specification, the coefficients on the indicators can still be interpreted as the increase in percentage volatility by the average fund in that group. We are most interested in the coefficient on leader distance. Across all specifications, we find that funds with even greater leads reduce risk more. In the first column, without any factor risk adjustments, we find a statistically significant coefficient of -0.631. This implies that a fund that is ahead by 10% as of June, reduces volatility by $0.631 \times 10\% = 6.31\%$ more than other funds in the leading group. Since the cross-sectional standard deviation across all funds is 11.6%, this is a small but not insignificant change. We consider changes in risk-taking with factor risk adjustments in other columns. Overall, the results are similar, which suggests that when funds change risk, they are not doing so by changing only their factor risk exposures.

Table 3 also reveals that extreme trailers also reduce risk. As discussed above, Panel A shows that funds in the trailing group decrease risk relative to the middle group. Their decrease is less than that of the leading funds, which is still consistent with the findings of Brown, Harlow, and Starks (1996), who found that trailers take more risk than leading funds. Panel B of Table 3 includes the coefficient on trailer distance. Across all factor risk adjustments, the coefficient on the distance below the trailer cut-off is negative and statistically significant. Hence, trailers reduce risk more when they are trailing by more. The magnitude is less than for the leaders, but trailers reduce risk nonetheless. This is contrary to the predictions of our model, where trailers should continue to take maximum risk in a tournament without cost or effort. Clearly, our model is incomplete, and the evidence suggests there are additional mechanisms; perhaps it is costly for managers to exert effort or take risks, or there are disincentives against poor performance. We leave the consideration of additional mechanisms for future research.

One potential concern is that risk-taking may mean-revert for exogenous reasons and funds that take high risk in the initial period tend to exhibit lower risk in the later period. This would induce a sorting bias if high-risk funds appeared more often in the leading group than in the trailing group (Schwartz, 2012). However, once we account for systematic risk exposures and, therefore, also account for risk premia, there is no ex-ante reason why we would expect to see more risk-takers in one group than another. We further investigate this possibility by comparing the coefficients on the trailing and leading distances. Across all models in Tables 3, the coefficients for extreme trailers are noticeably smaller than for extreme leaders. This suggests asymmetric behavior across the leading group and trailing group: the magnitude of the lead matters much more for leaders then trailers. The asymmetry indicates that there are some deliberate choices being made in risk-taking and this choice differs more for extreme leaders than trailers.

As a robustness check, we also use VOL_RATIO as an alternative measure of changes in risk-taking. It is important to recognize that using ratios can skew results when the denominator is close to zero, but it is still important to demonstrate consistency of our results with prior literature, such as Brown, Harlow and Starks (1996). Table 4 reports these results where we use only indicator functions in Panel A and add our distance metric in Panel B. In both specifications, results are consistent with those in Tables 3. Based on the coefficients on the top/bottom 20% indicator functions, we generally find that both leading group and trailing funds decrease risk. The coefficients on the interaction term with distance show that risk reduction for the leading group is a function of the magnitude of the lead. Once again, we find that extreme leaders reduce the most risk, but less so for the extreme trailers.

V.C Changes in Mutual Fund Risk-Taking Behavior Across Time

By changing the length of the continuation period, we can investigate the effects of changing the level of maximal risk, $\overline{\sigma}$. Empirically, we interpret risk in terms of risk per unit of time, such as with annualized volatility. This way, the total amount of idiosyncratic volatility is proportional to the square root of period length. Hence, the shorter the second period, the lower $\overline{\sigma}$ becomes. This implies that the value of the lock-in option increases as the length of the second period falls. Intuitively, it is easier to hold on to a lead in the second period, and even a smaller lead in the second period is safer.

We now investigate how tournament behavior changes by measuring fund risk-taking and fund leads at different points in time. Rather than just comparing volatilities over January to June with those over July to December, we also compare the same January to June volatility with the volatility between months starting in July, August, September, October, November, and December to the end of the year. We use annualized volatility so that the risk measures are comparable across units of time. Funds are ranked according to cumulative returns ending the day before each starting month.

The results are presented in Table 5 with factor risk adjustments using different models and using different measures of leads. In Panel A, we regress volatilities of excess returns across time and on our distance measures as before. The coefficient on distance among the top 20%-tile group is consistently negative and statistically significant. Furthermore, the coefficient on being in the top 20%-tile group is also negative and these point estimates generally increase in magnitude over time. For instance, in Panel A for the December column, the point estimate of -0.821 for the top 20%-tile group indicates that a typical top 20%-tile fund reduces risk by 0.821% in terms of annualized volatility as compared to the risk-taking behavior of funds in the middle group. In contrast, this point estimate was about half when evaluated in July with a point estimate of -0.421. This indicates that as the tournament approaches the final evaluation point later in the calendar year, leading funds tend to lock in their leads more aggressively. Furthermore, the coefficient on being in the bottom 20%-tile group become statistically indistinguishable from zero later in the calendar year. This suggests that there is less evidence that trailing funds also tend to decrease their risk-taking later in the calendar year. This is particularly true in Panel B when we control for factor risk exposures.

We take this logic further in Figure 6 and illustrate the increasing marginal impact by plotting the cumulative effects for the leading group. For each starting month, rather using our distance measures, we estimate a series of panel regressions with nested indicator functions. We replace the interaction term with two indicator functions, one for being in the top 10%-tile and another being in the top 5%-tile. We then plot the indicator functions for top 20%-tile to show risk-changing by a typical top 20%-tile (but not in the top 10%-tile) fund. We then plot the sums of the indicator functions for the top 10%-tile and the top 20%-tile to show the risk-changing behavior by a typical top 10%-tile (but not in the top 5%-tile) fund. Finally, we plot the sums of all three indicator functions to illustrate the risk-changing behavior of funds in top 5%-tile.

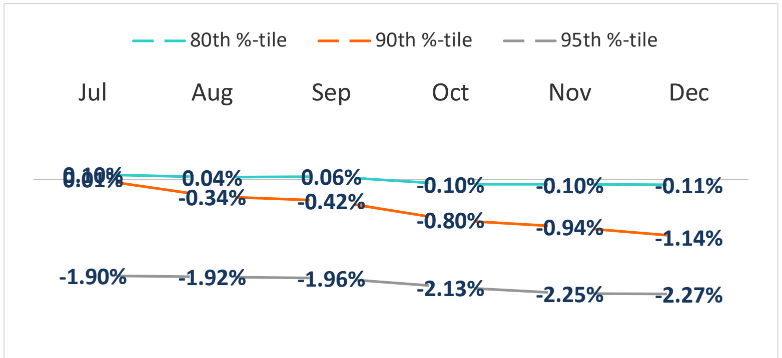
Panel A of Figure 6 shows the results when risk is measured using excess returns, and we find results that are consistent the predictions of our tournament model. At any point, funds with the largest leads, such as those in the top 5% tile, reduce risk more than funds with smaller leaders, such as those in the top 10%-tile. Moreover, as time goes on, the most leading funds reduce risk even more. Finally, the extent to which funds with larger leads (in the top 10%-tile and top 5%-tile) reduce risk increases further closer to the end of the calendar year.

We investigate risk-changing across time with adjustments for factor risk exposures in the remaining panels of Table 5 and Figure 6. The results are consistent whether we account for factor risk exposures using the CAPM, the 3-Factor Model or the 4-Factor Model. Overall, leading funds reduce risk while the funds with the largest leads reduce risk the most. With controls for factor risk, the effects are consistent, and risk-reduction increases with lead size and become stronger later in the year. The coefficient estimates on the intersection term in Panel B of Table 5 are all consistent and statistically significant. Looking at Figure 6, we see consistent risk reduction by the largest leaders later and the effects get stronger later in the year.

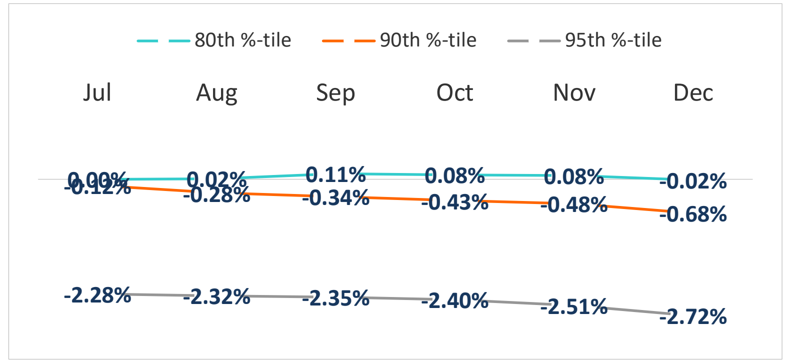
Figure 6: Leader Lock-in Effect across Time

This figure plots pooled-panel regression estimates of changes in mutual fund volatility on rankings of mutual fund performance. The dependent variable is VOL_DIFF, which is the annualized percentage volatility of daily mutual fund returns over starting month to December of each year minus volatility of mutual fund returns over January to June of the same year. We vary the starting month. Mutual fund returns are calculated in excess of the risk-free rate, residuals of CAPM, residuals of 3-Factor Model, and residuals of 4-Factor Model. We plot the sums of the point estimates associated with indicator functions that equal one if a mutual fund is within the top 20%, top 10%, and top 5% rankings among its peers in the same Lipper classification category according to its cumulative returns over January to the end of the month before. The sample is from 1999 to 2022 with 58,419 fund-year observations. All regressions include fixed-effects for each fund category by year.

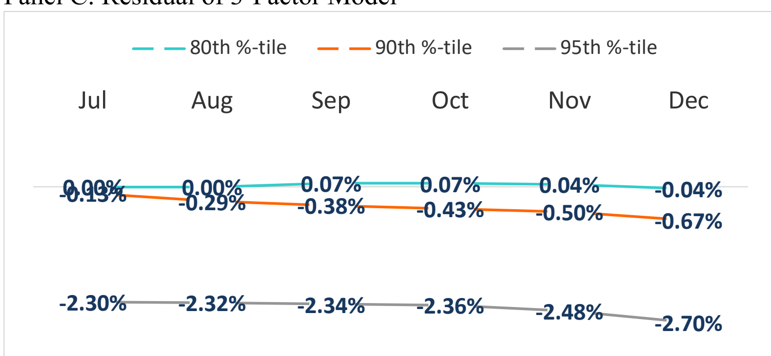
Panel A: Excess Return



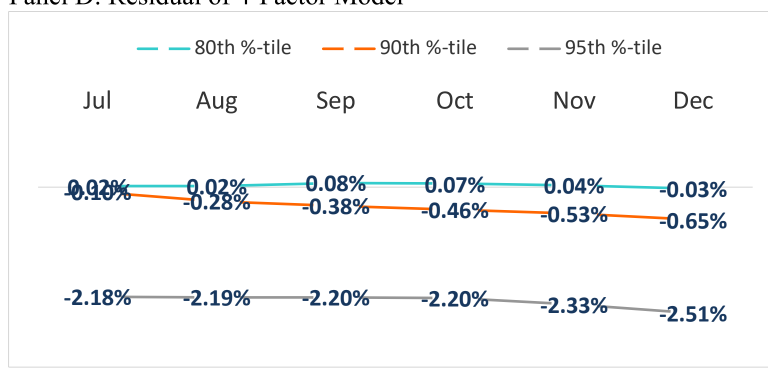
Panel B: Residual of CAPM



Panel C: Residual of 3-Factor Model



Panel D: Residual of 4-Factor Model



Overall, these empirical results support the predictions of our theoretical analysis of the tournament model. We find it is the leading funds, rather than the trailing funds, that respond by reducing risk. Moreover, funds with the largest leads reduce risk the most. Finally, as the time length to the end of the tournament becomes shorter near the end of the calendar year, leading funds reduce risk more aggressively. These results are robust to controlling for various factor risk exposures, using various measures of lead, various measures of risk-changes and are not due to a mechanical bias.

VI. Conclusion

Tournament behavior has important implications for mutual funds and the way in which they are managed. We have extended the literature both theoretically and empirically. Theoretically, we have shown that in a multi-player setting, risk-taking strategies interim tournament leaders can be very different depending on the magnitude of their leads relative other competitors, and the amount of time left to compete for the top prize. In the context of mutual funds, in the final period of the tournament, all fund managers should maximize the amount of risk that they take, except for those the fund with the highest returns to date. It is optimal for this manager to reduce risk if and only if the fund's lead is sufficiently, where a sufficient lead must be larger when the time until the end of the tournament is smaller. In contrast, prior literature has mostly assumed that trailing funds, rather than the leading fund, increase risk-taking.

Empirically, we analyze tournaments with heterogeneous characteristics, and the main predictions of our model are borne out. Our results are consistent with our theoretical predictions and we find that changes in risk are greatest among leaders rather than trailers and the degree of risk reduction depends on the magnitude of the lead. These results are robust to using different ways of controlling for systematic risk exposures, alternate measures of lead, and different measures of risk changes. Interestingly, our empirical results also show that trailing funds, particularly those trailing others by a larger margin, also reduce risk. This latter finding suggests that our model is incomplete, leading future work to consider other extensions.

Of course, managers are affected by various forms of explicit and implicit contract arrangements in addition to tournament incentives. Among these is the desire not to finish last in the tournament, i.e. a reverse tournament. Adding a reverse tournament to our core model would cause managers who are not in last place but are sufficiently unlikely to win to reduce risk later in the year. Another complication would occur when the manager's compensation contract is not linear, although adding a linear contract to a tournament would not affect tournament incentives. Furthermore, fund managers may possess ex-ante known degrees of ability or risk-taking may be costly, either of which can affect the optimal dynamic strategy of risk-taking. Finally, fund managers may need to exert costly effort to perform well, while simultaneously deciding how much risk to take. We leave these various extensions for future research.

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Appendix I: Proofs

Proof of Proposition 1: If the leader (manager 1) takes zero risk, her expected utility is

$$E(U_1) = Prob(R_1 > R_2) = \int_{-\infty}^{a_1} f_2\,(R)dR,$$

where $f_i(R)$ denotes the probability density function of manager i's return at the end of the second period. In contrast, if manager 1 takes any amount of risk, her expected utility is now

$$E(U_1) = Prob(R_1 > R_2) = \int_{-\infty}^{\infty} [1 - F_1(R)] f_2(R)\,dR,$$

where $F_1(R)$ is the cumulative distribution function of manager 1's return at the end of the second period. To understand this probability, note that we integrate across the possible outcomes of manager 2's return, $f_2(R)$, the probability that manager 1's return is above that of manager 2's return. When manager 1 takes zero risk, this is a step-function that equals one below a_1 and zero otherwise.

Therefore, the increase in manager 1's expected utility from risk-taking equals

$$\Delta E(U_1) = \int_{-\infty}^{\infty} [1 - F_1(R)] f_2\,(R)dR - \int_{-\infty}^{a_1} f_2\,(R)dR,$$

which can be rewritten as

$$\Delta E(U_1) = -\int_{-\infty}^{a_1} F_1(R) f_2\,(R)dR + \int_{a_1}^{\infty} [1 - F_1(R)] f_2\,(R)dR.$$

The first term corresponds the red shared area in Figure 1 of the main text, which is the probability that manager 2 catches up to manager 1 because manager 1 took risk and fell behind. This measures the expected loss due to risk-taking. The second term corresponds to the blue shaded area in Figure 1, which is the probability that manager 2 would have caught up to manager 1, but because manager 1 took risk that paid-off and she maintained her lead. This measures the expected gains due to risk-taking.

To see if expected utility gain is positive or negative, it is useful to redefine returns relative to manager 1's first period return a_1. That is, let $R' = R - a_1$. Then, the expected utility gain can be written as

$$\Delta E(U_1) = -\int_{-\infty}^{0} F_1(R') f_2\,(R' + a_1) dR' + \int_{0}^{\infty} [1 - F_1(R')] f_2\,(R' + a_1) dR'.$$

Since $f_1(x)$ is symmetric around a_1, $F_1(-x)$=$1 - F_1(x)$. Therefore, since a symmetric continuous unimodal distribution has the property that $f_2(a + x) \leq f_2(a - x) \forall x > 0$ whenever a is greater than the mode of the distribution, we have that

$$\Delta E(U_1) < 0.$$

Hence manager 1 would not choose to take any risk, since that would result in a lower expected utility.

Conversely, for trailing manager 2, where $a_2 < a_1$, if the leader locks in, manager 2's expected utility is simply

$$E(U_2) = Prob(R_2 > a_1) = \int_{a_1}^{\infty} f_2\,(R) dR.$$

Since f_2 is symmetric, continuous and unimodal, manager 2 maximizes this probability by taking as much risk as possible. More generally, suppose the leader takes any action, which may possibly be sub-optimal, with distribution $f_1(x)$ which is symmetric around a_1 and unimodal. Since $f_1(a + x) > f_1(a - x)\ \forall x > 0$ and manager 2 can only take actions with symmetric distributions, manager 2 is always better off taking a mean-preserving spread. □

Alternative Proof of Proposition 1: To help develop intuition for subsequent propositions, we can also integrate across the leader's (manager 1's) outcomes rather than integrate across the competition's outcomes. From this point of view, manager 1's expected utility if she takes zero risk is simply

$$E(U_1) = F_2(a_1),$$

where $F_2(R)$ is the cumulative distribution function of the competition's return at the end of the second period. If manager 1 chooses to take risk, then the expected utility is now

$$E(U_1) = \int_{-\infty}^{\infty} F_2(R) f_1\,(R) dR.$$

Since $F_2(a_1)$ is a constant and $\int_{-\infty}^{\infty} f_1(R)\, dR = 1$ by definition, the utility gain from taking risk is

$$\Delta E(U_1) = \int_{-\infty}^{\infty} [F_2(R) - F_2(a_1)] f_1\,(R) dR.$$

Since $f_1(x)$ is a symmetric distribution around a_1, it suffices to show that

$$[F_2(a_1 + x) - F_2(a_1)] \leq [F_2(a_1) - F_2(a_1 - x)]\ \forall (x > 0).$$

This is true when

$$f_2(a + x) \leq f_2(a - x)\ \forall (x > 0),$$

which is true for symmetric continuous unimodal distribution whenever a is greater than the mode of the distribution. □

Proof of Proposition 2: Suppose X_1 and X_2 are two independent normal random variables with means μ_1 and μ_2, identical variances σ_1=σ_2= σ, and zero correlation ρ=0. Denote $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{\left(-z^2/2\right)}$ and $\Phi(z)$ as the pdf and the cdf of a standard normal distribution, respectively. It is known, for example in Nadarajah and Kotz (2008), that the maximum of these two random variables, M=max(X_1, X_2), have the pdf

$$f(x) = \frac{1}{\sigma}\phi\left(\frac{x-\mu_1}{\sigma}\right)\Phi\left(\frac{x-\mu_2}{\sigma}\right) + \frac{1}{\sigma}\phi\left(\frac{x-\mu_2}{\sigma}\right)\Phi\left(\frac{x-\mu_1}{\sigma}\right).$$

Figure A depicts this pdf across varying means. Note that $\frac{\partial\phi(z)}{\partial z} = -z\,\phi(z)$ and $\frac{\partial\Phi(z)}{\partial z} = \phi(z)$. Therefore, if we differentiate f(x),

$$f'(x) = \frac{-1}{\sigma^2}\left(\frac{x-\mu_1}{\sigma}\right)\phi\left(\frac{x-\mu_1}{\sigma}\right)\Phi\left(\frac{x-\mu_2}{\sigma}\right) + \frac{2}{\sigma^2}\phi\left(\frac{x-\mu_1}{\sigma}\right)\phi\left(\frac{x-\mu_2}{\sigma}\right) + \frac{-1}{\sigma^2}\left(\frac{x-\mu_2}{\sigma}\right)\phi\left(\frac{x-\mu_2}{\sigma}\right)\Phi\left(\frac{x-\mu_1}{\sigma}\right).$$

To show that M = max(X_1, X_2) is unimodal, it suffices to show that the derivative of its pdf has a unique solution to $f'(x) = 0$.

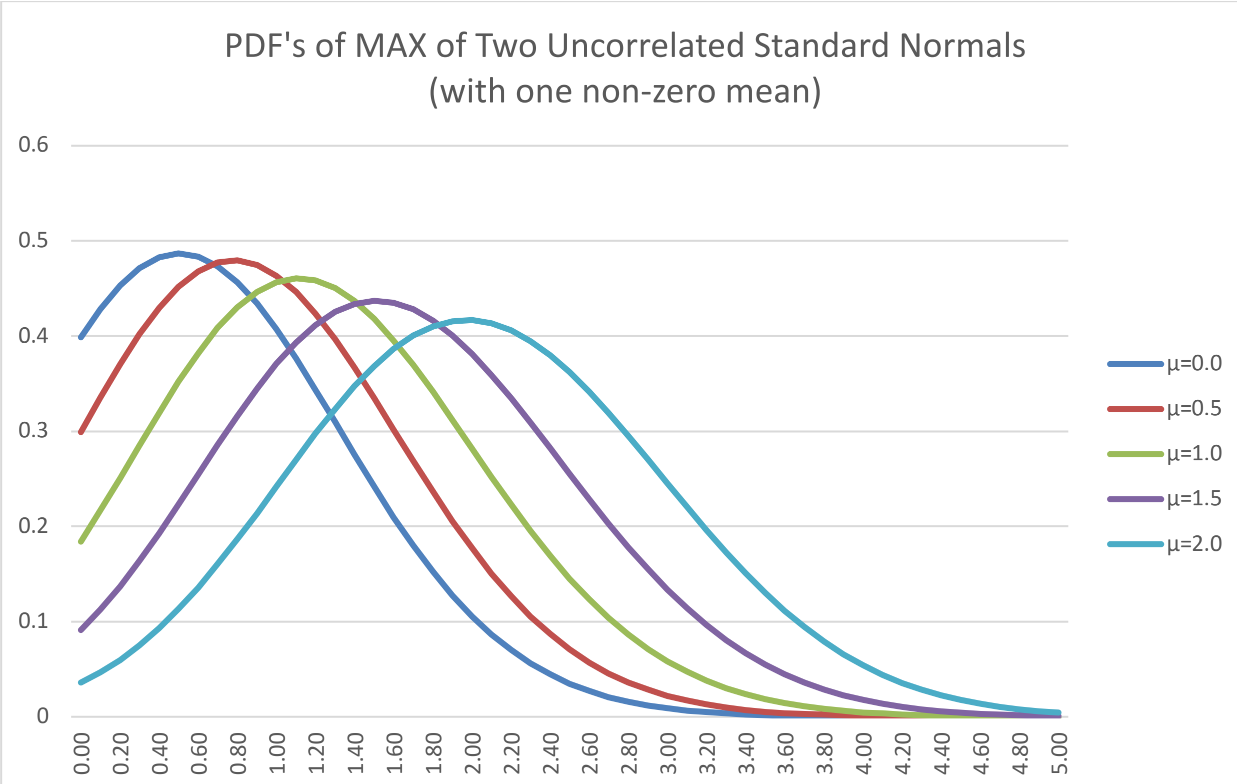

Figure A: This figure illustrates the probability density functions of $\mathbf{M}$=max($\mathbf{X_1}$, $\mathbf{X_2}$), where $\mathbf{X_1}$and $\mathbf{X_2}$ are two independent normal random variables with means $\boldsymbol{\mu_1} = \boldsymbol{\mu}$ and $\boldsymbol{\mu_2} = \mathbf{0}$, identical unit variances $\boldsymbol{\sigma_1}$=$\boldsymbol{\sigma_2}$= $\mathbf{1}$, and zero correlation $\boldsymbol{\rho}$=0. We vary one of the means, $\boldsymbol{\mu}$, through the range 0.0 to 2.0.

In the special case where the two distributions have identical means, μ_1=μ_2= μ, we have $f'(x) = \frac{2}{\sigma^2}\phi\left(\frac{x-\mu}{\sigma}\right)\left[\phi\left(\frac{x-\mu}{\sigma}\right) - \left(\frac{x-\mu}{\sigma}\right)\Phi\left(\frac{x-\mu}{\sigma}\right)\right]$. Since z and $\Phi(z)$ are both strictly increasing, $z\Phi(z)$ is also strictly increasing. Letting z=$\left(\frac{x-\mu}{\sigma}\right)$, we see that since $\phi(z) = z\Phi(z)$ has a unique solution,

$f'(x) = 0$ also has a unique solution. Therefore, M=max(X_1, X_2) is unimodal in this case.

In another special case where the two means are maximally separated, μ_2= $-\infty$, we see that the distribution of one of the two random variables becomes irrelevant. In this case, $f(x) = \frac{1}{\sigma}\phi\left(\frac{x-\mu_1}{\sigma}\right)$, which is a normal distribution and is also unimodal. We show numerically in Figure A that, in all intermediate cases, that the maximum order statistic remains unimodal.

To compute the moments of the maximum order statistic, we set $\sigma = 1$ without any loss of generality. It is also known (Nadarajah & Kotz, 2008) that M=max(X_1, X_2), have the moment generating function of $m(t) = m_1(t) + m_2(t)$, where

$$m_1(t) = \exp\left(t\mu_1 + \frac{t^2}{2}\right)\ \Phi\left(\frac{\mu_1-\mu_2+t}{\sqrt{2}}\right) \text{ and } m_2(t) = \exp\left(t\mu_2 + \frac{t^2}{2}\right)\ \Phi\left(\frac{\mu_2-\mu_1+t}{\sqrt{2}}\right).$$

By differentiating each term, we obtain

$$m_1'(t) = (\mu_1 + t)m_1(t) + \left(\frac{1}{\sqrt{2}}\right)\exp\left(t\mu_1 + \frac{t^2}{2}\right)\ \phi\left(\frac{\mu_1-\mu_2+t}{\sqrt{2}}\right) \text{ and}$$

$$m_2'(t) = (\mu_2 + t)m_2(t) + \left(\frac{1}{\sqrt{2}}\right)\exp\left(t\mu_2 + \frac{t^2}{2}\right)\ \phi\left(\frac{\mu_2-\mu_1+t}{\sqrt{2}}\right).$$

Note that the mean of M=max(X_1, X_2) is E[M]= $m'(0) = m_1'(0) + m_2'(0)$. Hence,

$$E[M] = \mu_1\ \Phi\left(\frac{\mu_1-\mu_2}{\sqrt{2}}\right) + \sqrt{2}\phi\left(\frac{\mu_1-\mu_2}{\sqrt{2}}\right) + \mu_2\ \Phi\left(\frac{\mu_2-\mu_1}{\sqrt{2}}\right).$$

For example, if both means are equal to zero, μ_1=μ_2= 0, and with unit variances, the expected maximum is $\frac{1}{\sqrt{\pi}}$. By differentiating once again, we obtain

$$m_1''(t) = m_1(t) + (\mu_1 + t)m_1'(t) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{3}{2}\mu_1 - \frac{1}{2}\mu_2 + \frac{3}{2}t\right)\exp\left(t\mu_1 + \frac{t^2}{2}\right)\ \phi\left(\frac{\mu_1-\mu_2+t}{\sqrt{2}}\right)$$

and

$$m_2''(t) = m_2(t) + (\mu_2 + t)m_2'(t) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{3}{2}\mu_2 - \frac{1}{2}\mu_1 + \frac{3}{2}t\right)\exp\left(t\mu_2 + \frac{t^2}{2}\right)\ \phi\left(\frac{\mu_2-\mu_1+t}{\sqrt{2}}\right).$$

Hence,

$$E[M^2] = m''(0) = (1+\mu_1^2)\Phi\left(\frac{\mu_1-\mu_2}{\sqrt{2}}\right) + \sqrt{2}(\mu_1+\mu_2)\phi\left(\frac{\mu_1-\mu_2}{\sqrt{2}}\right) + (1+\mu_2^2)\Phi\left(\frac{\mu_2-\mu_1}{\sqrt{2}}\right).$$

For example, with zero means and unit variances, $E[M^2]$=1 and variance of the maximum equals $E[M^2] - (E[M])^2 = \frac{\pi-1}{\pi}$. By differentiating a third time,

$$m_1^{(3)}(t) = 2m_1'(t) + (\mu_1 + t)m_1''(t) + \frac{1}{\sqrt{2}}\left[\frac{3}{2} + \left(\frac{3}{2}\mu_1 - \frac{1}{2}\mu_2 + \frac{3}{2}t\right)^2\right]\exp\left(t\mu_1 + \frac{t^2}{2}\right)\ \phi\left(\frac{\mu_1-\mu_2+t}{\sqrt{2}}\right)$$

and

$$m_2^{(3)}(t) = 2m_2'(t) + (\mu_2 + t)m_2''(t) + \frac{1}{\sqrt{2}}\left[\frac{3}{2} + \left(\frac{3}{2}\mu_2 - \frac{1}{2}\mu_1 + \frac{3}{2}t\right)^2\right]\exp\left(t\mu_2 + \frac{t^2}{2}\right)\,\phi\left(\frac{\mu_2-\mu_1+t}{\sqrt{2}}\right).$$

Hence,

$$E[M^3] = m^{(3)}(0) = 2m_1'(t) + 2m_2'(t) + \mu_1 m_1''(0) + \mu_2 m_2''(0)$$

$$+\frac{1}{\sqrt{2}}\left[3 + \left(\frac{3}{2}\mu_1 - \frac{1}{2}\mu_2\right)^2 + \left(\frac{3}{2}\mu_2 - \frac{1}{2}\mu_1\right)^2\right]\,\phi\left(\frac{\mu_1-\mu_2}{\sqrt{2}}\right),$$

which can be expanded out to

$$E[M^3] = (3\mu_1 + \mu_1^3)\Phi\left(\frac{\mu_1-\mu_2}{\sqrt{2}}\right) + (3\mu_2 + \mu_2^3)\Phi\left(\frac{\mu_2-\mu_1}{\sqrt{2}}\right)$$

$$+\frac{1}{\sqrt{2}}[7 + 6\mu_1^2 - 4\mu_1\mu_2 + 6\mu_2^2]\,\phi\left(\frac{\mu_1-\mu_2}{\sqrt{2}}\right).$$

To compute the skewness of M=max(X_1, X_2), consider the case where $\mu_1 > 0$ and $\mu_2 = 0$, without any loss of generality, which helps with the notations. Then letting $u = \frac{\mu_1}{\sqrt{2}}$, we have

$$E[M^3] = \Phi(u)\mu_1^3 + 3\sqrt{2}\phi(u)\mu_1^2 + 3\Phi(u)\mu_1 + \frac{7}{\sqrt{2}}\,\phi(u).$$

Since, $E[M] = \mu_1\,\Phi(u) + \sqrt{2}\phi(u)$,

$$\left[E[M]\right]^3 = [\Phi(u)]^3\mu_1^3 + 3\sqrt{2}\,[\Phi(u)]^2\phi(u)\mu_1^2 + 6\,\Phi(u)[\phi(u)]^2\mu_1 + \frac{4}{\sqrt{2}}[\phi(u)]^3.$$

By comparing each term of μ_1^n, and recognizing that $\mu_1 > 0$, $\Phi(u) < 1$, and $\phi(u) < \frac{1}{2\pi}$, we see that $E[M^3] > \left[E[M]\right]^3$, hence skewness of M=max(X_1, X_2), $E[M^3] - \left[E[M]\right]^3$, is positive. □

Proof of Proposition 3: Consider any manager, i, with a_i less than the mode of the maximum of the other managers' returns. Figure 2 illustrates the situation of such manager selecting a risk level strictly less than $\bar{\sigma}$. Let the derivative of the pdf of maximum be $f_{max}'(x)$. Since a_i is below the mode, $f_{max}'(a_i) > 0$. Moreover, note that the distribution of the maximum order statistic, f_{max}, satisfies the property $f_{max}(a_i + y) > f_{max}\,(a_i - y)$ for all $y > 0$ whenever a_i is less than the mode of the maximum and the underlying distribution, f, is symmetric, continuous unimodal distribution. Intuitively, this is because the distribution of the maximum, f_{max}, takes the underlying distribution and shifts the mass to the right. Hence, manager i is better off taking a symmetric mean-preserving spread. Since the trailing managers can only take symmetric risk, this means that any increase in risk is beneficial. □

Proof of Proposition 4: The same argument used above for Proposition 3 applies here for the leading manager as well as for the trailing managers. Increasing risk always adds more probability mass for managers below the mode of the distribution of the maximum if the underlying distributions are symmetric, continuous and unimodal. □

Proof of Proposition 5: The case when the value a_1 lies above the mode of the density of the order statistic, f_{max}, is illustrated in Figure 3. In this case, increasing risk by a small amount starting from zero risk is counterproductive at first. This is because the shaded area to the left of a_1 is locally greater than the shaded area to the right. However, because of the positive skewness of the maximum order statistic density, this argument cannot be extended to the case of global optimality. This is because there are values, $y > 0$, sufficiently large such that for the density in the right tail, $f_{max}(a_1 + y) > f_{max}(a_1 - y)$. Therefore, for a large enough risk-taking, the marginal benefit of increasing risk can eventually become positive and remain positive.

Suppose a_1 is not only above the mode, but also lies above the median. Even at the upper risk bound, the probability of winning cannot exceed 0.5 as risk is increased. Therefore, if a_1 lies above the median of the maximum order statistic, the optimal strategy for the leader is to lock in. By locking in, the probability of winning is greater than 0.5 by the definition of the median, which is greater than any other amount of risk-taking.

If a_1 lies below the median of the maximum order statistic, as the manager increases risk-taking the probability of winning approaches 0.5 if there is no upper bound. Therefore, if the upper bound, $\bar{\sigma}$, is sufficiently high, then the global optimum will be to take maximal risk, $\bar{\sigma}$. If the risk limit is sufficiently low it is possible that even if the leading manager's first period return is below the median, she might not be able to increase the probability of winning to 0.5 and therefore will lock in. Therefore, there exists some critical value, $\widehat{a_1}$, such that if the leading manager's position is higher, she would lock in. Otherwise she takes maximum risk. The leading manager would never take an intermediate level of risk, $0 < \sigma < \bar{\sigma}$, because once the manager surpasses the initial cost of risk-taking, the marginal benefit to increasing risk is positive. □

Appendix II: Extension to Systematic and Idiosyncratic Risks

We can also adapt our model to the case where funds can take systematic or idiosyncratic risks up to a finite maximum level of total variance. Consider the case where returns in each period are

$$R_{it} = (1 - x_i)\sigma_{it}\epsilon_{it} + x_i\epsilon_m,$$

where the variance of ϵ_m, systematic risk, is equal to σ_m^2, and $x_i \in [0,1]$, represents the amount of non-negative systematic risk taken in the fund's portfolio. The idiosyncratic risk, ϵ_i, is uncorrelated with the idiosyncratic risk of another fund, ϵ_j, as well as being uncorrelated with the systematic risk, ϵ_m. The total variance of the portfolio is therefore $\mathrm{Var}(R_{it}) = (1 - x_{it})^2\sigma_{it}^2 + x_i^2\sigma_m^2 \leq \overline{\sigma^2}$, which is bounded above by a risk limit. For simplicity, let us further assume that the systematic risk is normalized so that $\sigma_m = \bar{\sigma}$. The strategic choices for each fund in each period are now both level of systematic risk, x_{it}, and the level of idiosyncratic risk, σ_{it}. The fund is allowed to set total risk equal to zero with $x_{it} = 0$ and $\sigma_{it} = 0$, or to choose maximal idiosyncratic risk with $x_{it} = 0$ and $\sigma_{it} = \bar{\sigma}$, or to choose maximal systematic risk with $x_{it} = 1$.[7] For tractability, we assume that systematic risk level is bounded below at zero, and furthermore, we remove risk premia from the analysis by assuming $E(\epsilon_{it}) = E(\epsilon_m) = 0$. This is equivalent to fund managers not being rewarded by investors for taking systematic risk. However, systematic risks are perfectly correlated across funds, and we focus our analysis on the impact of taking this correlated risk on the strategic equilibrium.

Analysis of the Lock-in Equilibrium

Suppose we consider the lock-in equilibrium as before where the leading fund has sufficient lead as to choose zero risk, while the two trailing funds choose maximal risk level. If these risks are idiosyncratic, the equilibrium is preserved. If the two trailing funds are taking maximal idiosyncratic risk, then clearly there is no reason for the top fund to switch to taking either idiosyncratic or systematic risk. This follows from our previous proof as increasing any risk increases the probability of losing. Since taking on systematic risk is uncorrelated with the risk

[7] The cases of interior levels of risk are off-equilibrium, as in the case with only idiosyncratic risk.

taken by other two players, it doesn't matter whether the off-equilibrium strategy is systematic or idiosyncratic.

Now consider whether it pays for the second place fund to switch to taking systematic risk. The second place fund knows that the last fund is taking idiosyncratic risk. So, the probability of beating the last fund is the same no matter whether the risk taking is systematic or idiosyncratic. The leading fund is locking in, so once again there is no gain in the probability of beating the leading fund whether the risk taken by the second place fund is systematic or idiosyncratic. Hence the equilibrium strategy of maximal idiosyncratic risk is a weak best response. The last fund also has similar behavior: if she switches to systematic risk, it still does not improve her chances against either of the other two funds which are taking idiosyncratic risks and zero risk, respectively. Hence, we have shown that the lock-in equilibrium with idiosyncratic risks is robust.

We can also see that there cannot be an equilibrium in which both trailing funds take maximal systematic risk while the leading fund locks in with zero risk. In this case, the bottom fund will want to switch to maximal idiosyncratic risk because taking on systematic risk that is perfectly correlated with the second fund will guarantee a loss. However, it can be the case that the bottom fund takes maximal idiosyncratic risk while the middle fund takes maximal systematic risk. Clearly both funds have no incentive to switch their types of risk-taking. The distribution of the maximum between these two funds is identical to that when they are both taking idiosyncratic risks. However, it is possible the top fund may want to switch to systematic risk now because by doing so it guarantees a victory against the second fund. With perfectly correlated systematic risk, mimicking the action of the trailing fund becomes the better response than simply going to zero risk. Therefore, the only way to support the lock-in form of equilibrium is when the lead of the top fund is very large and the third fund is sufficiently far below the interim performance of the second fund.

Analysis of the equilibrium where all funds take maximal idiosyncratic risks

Now we consider the second period symmetric equilibrium in which all funds take maximal idiosyncratic risks. Clearly this is also an equilibrium since it is a weak best response to not switch to systematic risks and since the leading fund's lead was insufficient to warrant a lock-in strategy. Therefore, this equilibrium is robust in the second period.

Clearly there cannot be an equilibrium in which either of the bottom two funds take systematic risk while the top fund takes idiosyncratic risk, since there is no benefit to taking on systematic risk when there are no rewards for taking systematic risk. If anything, if the leading fund takes on systematic risk, the leader beats any trailing fund that takes on systematic risk and does not decrease her probability of winning against the other. Hence the trailing funds take maximal idiosyncratic risk and the top fund can take either idiosyncratic risk or systematic risk.

The First Period Equilibrium

We have shown that both the lock-in equilibrium and the maximal idiosyncratic risk-taking equilibrium are robust to adding the possibility of systematic risk-taking in the second period. As before the lock-in option is valuable. Hence, if there is no disadvantage to portfolio revisions at the interim date, the best way for each fund to gain an advantage in the second period is maximum risk-taking in the first period. It is never an equilibrium for more than one fund to take systematic risk in the first period. Otherwise, one of the funds taking systematic risk would achieve a higher probability of taking a lead into the second by switching to idiosyncratic risk. The asymmetric equilibrium in which only one fund takes systematic risk is isomorphic in terms of risk dynamics to the one where all firms take idiosyncratic risk. Hence the overall predictions of risk changing in our model is robust to allowing for both non-negative systematic and idiosyncratic risks, with virtually identical empirical predictions.

Appendix III: Endogenous Pay-Performance Sensitivity

Here we demonstrate how expected managerial ability is increasing in returns using a simplified version of Jiang, Starks and Sun's (2021) model. The purpose of this extension is to show that fund flows in a learning model based on Bayesian updating can be consistent with the equilibrium behavior we have derived in our two period, three fund model.

Suppose that $R_t = \mu_i + \sigma_\epsilon \epsilon_t$ where, $\epsilon_t \sim N(0,1), t \in \{1,2\}$, for each fund manager i, and μ_i denotes each manager's unknown ability. Following the literature (Berk & Green, 2004; Jiang, Starks, & Sun, 2021), assume that managers do not know their own ability, and that their prior expectation of their own ability is that they have none, $\mu = 0$. We assume that the initial prior about manager ability is normally distributed with variance σ_0^2 . Managers and investors are symmetrically informed, and they simultaneously learn manager ability by observing returns conditional on equilibrium strategies.

First Period Case: Recall that equilibrium behavior is for each manager to take maximum risk. Suppose all managers add normally distributed risk with variance σ_ϵ^2. Then, the posterior distribution for μ, is a function of R_1: $E[\mu|R_1] = \frac{\sigma_0^2}{\sigma_0^2+\sigma_\epsilon^2} R_1 = \frac{\sigma_0^2}{\sigma_0^2+\sigma_\epsilon^2}(\mu_i + \epsilon_1)$ and $\text{var}[\mu|R_1] \equiv \sigma_\mu^2 = \left[\frac{1}{\sigma_0^2} + \frac{1}{\sigma_\epsilon^2}\right]^{-1}$. Hence, managerial ability is increasing in returns, and, in the usual way, end of period variance is identical across managers no matter what the first period return is.

Second Period Case: There are now two types of equilibria: the first type where all funds continue to take maximum risk, and the second lock-in equilibrium type where the leading fund in the first period takes zero risk. Consider the first type of equilibrium and define $\bar{R} = \left(\frac{1}{2}\right) R_1 + \left(\frac{1}{2}\right) R_2$ to be the average return over the two periods. Given this observation and the specification above, the posterior distribution of managerial ability is normally distributed with mean $E(\mu|\bar{R}) = \frac{\sigma_0^2}{\left(\frac{\sigma_\epsilon^2}{2}\right)+\sigma_0^2}\bar{R}$, and variance $\sigma_\mu^2 = \left[\frac{1}{\sigma_0^2} + \frac{2}{\sigma_\epsilon^2}\right]^{-1}$. Again, expected managerial ability is increasing in the average period returns of each manager with identical variance. Hence the posterior distribution of the top two-period fund manager will stochastically dominate the other two managers and investors will optimally invest everything in the top manager, leading to the tournament reward we have postulated in the specific form of our model.

Now consider the second type of equilibrium with the leading fund locks in. Since there is no additional uncertainty during the second period in the return of a leading fund that locks in, her expected ability will be the same as it was at the end of the first period: $E[\mu^1|R_1] = \frac{\sigma_0^2}{\sigma_0^2+\sigma_\epsilon^2} R_1$, where we use μ^1 to denote the ability of the leading fund. As before the posterior variance is $\sigma_\mu^2 = \left[\frac{1}{\sigma_0^2} + \frac{1}{\sigma_\epsilon^2}\right]^{-1}$.

There are now two cases to consider: (1) the leading manager from the first period finishes below another fund and does not finish as the top fund; and (2) the leading manager from the first period remains on top at the end of the second period. In case (1), the winning fund must have taken risk in both periods and its expected ability is greater than that of the leading manager from the first period, $E(\mu|\bar{R}) = \frac{\sigma_0^2}{\left(\frac{\sigma_\epsilon^2}{2}\right)+\sigma_0^2}\bar{R} > E(\mu^1|R_1)$. Furthermore, since more returns were observed, its posterior variance is also tighter than that of the first period leader. Hence it will be rational for all investors to invest with the top overall fund once again after two periods.

In case (2) where the first period leader does win the tournament by locking in, the situation is more complicated because locking in also reduces the amount of signal available regarding the leader's ability. Since, $R_1 > \bar{R}$, in most cases we would expect, $E(\mu^1|R_1) > E(\mu|\bar{R}) = \frac{\sigma_0^2}{\left(\frac{\sigma_\epsilon^2}{2}\right)+\sigma_0^2}\bar{R}$, so that expected ability is higher for the top overall fund. However, we can no longer make a stochastic dominance argument because the posterior variance of the winning fund is higher than that of the losing fund. Indeed, there is an anomalous mechanism where even though for the second highest fund has a lower overall return than the top fund from the first period, $\bar{R} < R_1$, because this fund took additional risk it's return history is more informative. In other words, it is possible for the leading manager in the first period to have simply gotten lucky in the first period, and by locking in, she is unable to prove that it was simply due to luck. Hence, it is still possible that $E(\mu|\bar{R}) = \frac{\sigma_0^2}{\left(\frac{\sigma_\epsilon^2}{2}\right)+\sigma_0^2}\bar{R} > E(\mu^1|R_1)$, and investors would flock to the second highest fund, rather than the highest fund that locked in. Because of this possibility, the leader would adjust her lock-in strategy such that she waits longer to lock in her lead. □

Table 1: Distribution of Mutual Funds

This table reports the distribution of mutual funds in our sample. Mutual funds are divided into categories according to their Lipper classifications. The sample is from 1999 to 2022 with 58,419 fund-year observations.

Lipper Classification	Average Number of Funds per Year	Percentage of Total
Large-Cap Value	159.6	6.6%
Large-Cap Core	332.8	13.7%
Large-Cap Growth	271.8	11.2%
Mid-Cap Value	79.8	3.3%
Mid-Cap Core	146.8	6.0%
Mid-Cap Growth	163.9	6.7%
Small-Cap Value	115.8	4.8%
Small-Cap Core	290.3	11.9%
Small-Cap Growth	203.7	8.4%
Multi-Cap Value	170.5	7.0%
Multi-Cap Core	309.8	12.7%
Multi-Cap Growth	189.5	7.8%
Total	2434.4	100%

Table 2: Mutual Fund Risk-Taking

This table reports the summary statistics of mutual fund risk-taking behavior. For each mutual fund, we compute the annualized volatility of daily mutual fund returns for each year in our sample. Mutual fund returns are calculated in excess of the risk-free rate, residuals of CAPM, residuals of 3-Factor Model, and residuals of 4-Factor Model. The sample is from 1999 to 2022 with 58,419 fund-year observations.

Risk Measures	Average	Std Dev	5%-tile	Median	95%-tile
Excess Ret	20.0%	11.6%	9.8%	17.0%	39.0%
CAPM	6.6%	8.5%	2.1%	5.6%	14.1%
3-Factor	4.9%	8.0%	1.7%	4.1%	10.3%
4-Factor	4.7%	8.0%	1.7%	3.9%	9.9%

Table 3: Second Half Tournament Effect

This table reports pooled-panel regression estimates of changes in mutual fund volatility on rankings of mutual fund performance. The dependent variable is VOL_DIFF, which is the annualized percentage volatility of daily mutual fund returns over July to December of each year minus volatility of mutual fund returns over January to June of the same year. Mutual fund returns are calculated in excess of the risk-free rate, residuals of CAPM, residuals of 3-Factor Model, and residuals of 4-Factor Model. The independent variables are indicator functions that equal one if a mutual fund is within the top 20% or bottom 20% rankings among its peers in the same Lipper classification category according to its cumulative returns over January to June of a year. Crisis Period is an indicator variable for year 2008. In Panel B, we interact the indicator functions with the de-meaned absolute value of the difference between a fund's cumulative returns and the 20th (or 80th) percentile breakpoint. The sample is from 1999 to 2022 with 58,419 fund-year observations. All regressions include fixed-effects for each fund category by year. Standard errors are clustered by fund and are shown in parenthesis. Significance is indicated by ** at the 1% level, by * at the 5% level, and by + at the 10% level.

Panel A: Without distance

Risk Measures:	Excess Ret.	CAPM	3-Factor	4-Factor
Ind(top 20%)	-0.421*	-0.599**	-0.604**	-0.560**
	(0.201)	(0.199)	(0.196)	(0.194)
Ind(bottom 20%)	-0.337**	-0.120**	-0.100*	-0.120**
	(0.048)	(0.043)	(0.042)	(0.041)
Crisis Period	32.439**	5.246**	3.081**	3.196**
	(0.115)	(0.081)	(0.066)	(0.064)
Intercept	-1.981**	-0.210**	-0.128**	-0.132**
	(0.044)	(0.021)	(0.019)	(0.019)
R-squared	0.548	0.043	0.020	0.019

Panel B: Base case

Risk Measures:	Excess Ret.	CAPM	3-Factor	4-Factor
Ind(top 20%)	-0.421**	-0.599**	-0.604**	-0.560**
	(0.150)	(0.147)	(0.145)	(0.144)
Ind(top20%)*	-0.631*	-0.632*	-0.627*	-0.620*
distance above cut-off	(0.264)	(0.262)	(0.260)	(0.258)
Ind(bottom 20%)	-0.337**	-0.120**	-0.100*	-0.121**
	(0.047)	(0.043)	(0.042)	(0.042)
Ind(bottom20%)*	-0.200*	-0.206*	-0.211*	-0.218*
distance below cut-off	(0.087)	(0.096)	(0.096)	(0.096)
Crisis Period	32.439**	5.246**	3.082**	3.196**
	(0.114)	(0.077)	(0.061)	(0.060)
Intercept	-1.981**	-0.210**	-0.128**	-0.132**
	(0.044)	(0.021)	(0.019)	(0.019)
R-squared	0.726	0.437	0.429	0.427

Table 4: Alternative Volatility Measure

This table reports pooled-panel regression estimates of changes in mutual fund volatility on rankings of mutual fund performance. The dependent variable is VOL_RATIO, which is the annualized volatility of daily mutual fund returns over July to December of each year divided by volatility of mutual fund returns over January to June of the same year. Mutual fund returns are calculated in excess of the risk-free rate, residuals of CAPM, residuals of 3-Factor Model, and residuals of 4-Factor Model. The independent variables are indicator functions that equal one if a mutual fund is within the top 20% or bottom 20% rankings among its peers in the same Lipper classification category according to its cumulative returns over January to June of a year. Crisis Period is an indicator variable for year 2008. In Panel B, we interact the indicator functions with the de-meaned absolute value of the difference between a fund's cumulative returns and the 20th (or 80th) percentile breakpoint. The sample is from 1999 to 2022 with 58,419 fund-year observations. All regressions include fixed-effects for each fund category by year. Standard errors are clustered by fund and are shown in parenthesis. Significance is indicated by ** at the 1% level, by * at the 5% level, and by + at the 10% level.

Panel A: Without distance

Risk Measures:	Excess Ret.	CAPM	3-Factor	4-Factor
Ind(top 20%)	-0.015**	-0.015**	-0.017**	-0.013*
	(0.004)	(0.005)	(0.005)	(0.005)
Ind(bottom 20%)	-0.007*	-0.005	-0.004	-0.004
	(0.003)	(0.005)	(0.005)	(0.005)
Crisis Period	1.379**	0.693**	0.514**	0.547**
	(0.005)	(0.008)	(0.008)	(0.008)
Intercept	1.002**	1.025**	1.012**	1.008**
	(0.002)	(0.002)	(0.002)	(0.002)
R-squared	0.877	0.409	0.277	0.276

Panel B: Base case

Risk Measures:	Excess Ret.	CAPM	3-Factor	4-Factor
Ind(top 20%)	-0.017**	-0.017**	-0.015**	-0.010**
	(0.004)	(0.004)	(0.004)	(0.004)
Ind(top20%)*	-0.011*	-0.010*	-0.011**	-0.013**
distance above cut-off	(0.005)	(0.004)	(0.004)	(0.004)
Ind(bottom 20%)	-0.001+	-0.001+	-0.001+	-0.001+
	(0.000)	(0.000)	(0.000)	(0.000)
Ind(bottom20%)*	-0.003*	-0.003*	-0.003*	-0.003**
distance below cut-off	(0.001)	(0.001)	(0.001)	(0.001)
Crisis Period	1.379**	0.693**	0.514**	0.547**
	(0.005)	(0.008)	(0.008)	(0.008)
Intercept	1.002**	1.025**	1.012**	1.008**
	(0.002)	(0.002)	(0.002)	(0.002)
R-squared	0.275	0.126	0.073	0.089

Table 5: Tournament Effect across Time

This table reports pooled-panel regression estimates of changes in mutual fund volatility on rankings of mutual fund performance. The dependent variable is VOL_DIFF, which is the annualized percentage volatility of daily mutual fund returns over the starting month to December of each year minus volatility of mutual fund returns over January to June of the same year. We vary the starting month across specifications. Mutual fund returns are calculated in excess of the risk-free rate and residuals of 4-Factor Model. The independent variables are indicator functions that equal one if a mutual fund is within the top 20% or bottom 20% rankings among its peers in the same Lipper classification category according to its cumulative percentage returns over January to the end of the month before. Crisis Period is an indicator variable for year 2008. We interact the indicator functions with the de-meaned absolute value of the difference between a fund's cumulative returns and the 20th (or 80th) percentile breakpoint. The sample is from 1999 to 2022 with 58,419 fund-year observations. All regressions include fixed-effects for each fund category by year. Standard errors are clustered by fund and are shown in parenthesis. Significance is indicated by ** at the 1% level, by * at the 5% level, and by + at the 10% level.

Panel A: With Distance Measure; Excess Return

Risk Measures:	July	August	September	October	November	December
Ind(top 20%)	-0.421**	-0.526**	-0.523**	-0.722**	-0.774**	-0.821**
	(0.150)	(0.147)	(0.148)	(0.150)	(0.150)	(0.149)
Ind(top20%)*	-0.631*	-0.621*	-0.602*	-0.584*	-0.562*	-0.551**
distance above cut-off	(0.264)	(0.250)	(0.242)	(0.236)	(0.226)	(0.213)
Ind(bottom 20%)	-0.337**	-0.231**	-0.109*	0.059	0.196**	0.093+
	(0.047)	(0.050)	(0.054)	(0.054)	(0.059)	(0.049)
Ind(bottom20%)*	-0.200*	-0.154*	-0.11	-0.103*	-0.104*	-0.085*
distance below cut-off	(0.087)	(0.077)	(0.081)	(0.051)	(0.048)	(0.043)
Crisis Period	32.439**	36.360**	42.430**	46.377**	41.485**	33.229**
	(0.114)	(0.121)	(0.137)	(0.146)	(0.151)	(0.152)
Intercept	-1.981**	-1.873**	-2.136**	-2.274**	-3.174**	-4.406**
	(0.044)	(0.043)	(0.042)	(0.042)	(0.039)	(0.046)
R-squared	0.726	0.752	0.748	0.761	0.755	0.760

Panel B: With Distance Measure; Residuals of 4-Factor Model

Risk Measures:	July	August	September	October	November	December
Ind(top 20%)	-0.560**	-0.588**	-0.556**	-0.556**	-0.599**	-0.678**
	(0.144)	(0.140)	(0.142)	(0.144)	(0.144)	(0.144)
Ind(top20%)*	-0.620*	-0.613*	-0.594*	-0.576*	-0.556*	-0.542**
distance above cut-off	(0.258)	(0.244)	(0.236)	(0.230)	(0.220)	(0.209)
Ind(bottom 20%)	-0.121**	-0.095*	-0.034	-0.01	-0.003	-0.036
	(0.042)	(0.044)	(0.048)	(0.047)	(0.051)	(0.036)
Ind(bottom20%)*	-0.218*	-0.176*	-0.136	-0.150**	-0.145**	-0.123**
distance below cut-off	(0.096)	(0.084)	(0.087)	(0.056)	(0.050)	(0.044)
Crisis Period	3.196**	3.550**	4.156**	4.386**	3.527**	2.699**
	(0.060)	(0.062)	(0.068)	(0.073)	(0.076)	(0.073)
Intercept	-0.132**	-0.160**	-0.119**	-0.103**	-0.240**	-0.434**
	(0.019)	(0.017)	(0.019)	(0.019)	(0.014)	(0.018)
R-squared	0.427	0.438	0.424	0.414	0.417	0.421