Optimal Ownership and Capital Structure with Agency Conflicts

Yunzhi Hu, Zhaojun Yang, and Nanhui Zhu*

Abstract

We develop a continuous-time model examining agency conflicts among controlling shareholders (managers), minority shareholders, and creditors in corporate investment decisions. The manager's private benefits encourage over-investment, while their equity stake and debt overhang lead to under-investment. We show these offsetting incentive effects can achieve optimal investment timing under certain conditions. Agency costs exhibit U-shaped relationships with private benefits, tax rates, volatility, managerial ownership, and leverage. The model reveals how the interplay among agency conflicts, tax benefits, and bankruptcy costs shapes optimal ownership and capital structure, explaining several documented empirical patterns in corporate finance.

*Hu, Yunzhi_Hu@kenan-flagler.unc.edu, University of North Carolina at Chapel Hill Kenan-Flagler Business School; Yang, yangzj@sustech.edu.cn, Southern University of Science and Technology Department of Finance; Zhu (corresponding author), zhunh2022@mail.sustech.edu.cn, University of International Business and Economics China School of Banking and Finance (Primary Affiliation), Southern University of Science and Technology Department of Finance (Secondary Affiliation). We thank an anonymous referee and Jarrad Harford (the editor) for constructive and detailed feedback, comments, and suggestions that significantly improved the article. We thank Christian Ewald, Chao Ying, Mogens Steffensen, Khine Aye Myat Kyaw, and Jun Yu, as well as the participants at the University of Copenhagen, NTNU Business School, Umeå University, Southwestern University of Finance and Economics, Zhongnan University of Economics and Law, and Anhui Polytechnic University for their helpful comments. Yang and Zhu acknowledge the financial support from the National Natural Science Foundation of China (Grant Nos. 72031003, 723B2009, and 72432005) and the China Scholarships Council (Grant No. 202308440548). The work was finished while Zhu was a Ph.D. student at the Southern University of Science and Technology. Earlier versions of this paper were circulated under the titles *"Overcoming Agency Conflicts and Realizing First-Best Corporate Financial Policies"* and *"Optimal Ownership and Capital Structure with Agency Conflicts and Debt Renegotiation"*. All errors are our own.

I. Introduction

Corporate finance theory has long recognized that firm value is shaped not only by traditional factors like taxes and bankruptcy costs but also by complex agency frictions between stakeholders. These relationships create conflicts of interest that can lead to substantial deviations from optimal corporate policies (see, e.g., Morellec (2004), Lambrecht and Myers (2008), and Wang (2011)). Although extensive research has examined these conflicts in isolation, less is known about their interactions and joint effects on corporate decisions. Our paper develops a unified theoretical framework for studying how agency conflicts between managers, shareholders, and creditors jointly influence corporate investment and financing decisions within a dynamic real option setting.

Central to our analysis is the interaction between two key agency problems. First, controlling shareholders (represented by a manager) can extract private benefits by diverting firm resources, creating conflicts with minority shareholders. These private benefits can take various forms, including excessive compensation, perquisite consumption, related-party transactions, and tunneling behaviors. Recent empirical evidence from Morellec, Nikolov, and Schürhoff (2018), analyzing 12,652 firms across 14 countries, shows that such private benefits amount to 2.6% (4.4%) of free cash flows for the median (average) firm. Second, following Myers (1977) and the dynamic extensions in Mauer and Ott (2000) and Hackbarth and Mauer (2012), the issuance of debt introduces conflicts between equity holders and creditors through debt overhang, as equity holders bear the full cost of new investments while sharing the benefits with existing creditors.[1]

[1]In our model, it is assumed that investment is fully equity-financed. As explained in Myers (1977), growth investments are better financed through equity rather than risky debt, because issuing risky debt reduces the present

Building on Mauer and Ott's (2000) framework, we develop a continuous-time model where a firm has both assets in place, generating stochastic cash flows, and a growth option to expand operations by paying a fixed investment cost. The manager, who controls the investment option and makes default decisions, can divert a fraction of net cash flows for private benefits. Before exercising the growth option, the manager can issue debt to exploit tax benefits and sell equity to minority shareholders while retaining partial ownership. This setup creates a rich environment to examine how ownership structure and financing choices interact with investment efficiency.

Our analysis reveals three key forces that shape investment decisions. First, a diversion effect incentivizes investment earlier than the firm-value maximizing threshold (over-investment), as the manager seeks to maximize the combination of his equity value and diverted cash flows (see Jensen (1986), Stulz (1990), and Morellec (2004)). Second, a countervailing cost-sharing effect arises because the manager's equity stake ("skin-in-the-game") means he bears a portion of the investment cost, encouraging him to invest later than optimal (under-investment) (see, e.g., Jensen and Meckling (1976), Morck, Shleifer, and Vishny (1989), and Claessens, Djankov, Fan, and Lang (2002)). Third, the debt overhang effect from the shareholder-creditor conflict typically delays investment beyond the firm-value maximizing threshold (under-investment) because equity holders must bear the full investment cost while sharing benefits with creditors (see, e.g., Mauer and Ott (2000), Pawlina (2010), and Hackbarth and Mauer (2012)).

A key insight of our analysis is that these offsetting incentive effects can achieve a zero-agency cost solution under certain conditions. Our model captures how the previous three

market value of a firm holding growth options by inducing suboptimal investment strategies. Mauer and Ott (2000) and Hackbarth and Mauer (2012) examine cases where the investment is partially debt-financed.

driving forces evolve in response to changes in resource diversion, corporate tax rates, cash flow volatility, managerial ownership, and capital structure. We show that at specific levels of these primary economic factors, the three effects can exactly offset each other, resulting in firm-optimal investment policies. For example, for a given managerial ownership stake, there exists a combination of cash flow diversion and leverage at which the manager's chosen investment policy (manager-optimal) exactly matches the policy that maximizes firm value (firm-optimal). These findings suggest that seemingly inefficient governance structures might actually help achieve efficient investment outcomes by balancing competing distortions.

We examine the optimal ownership and capital structure, along with investment efficiency, and conduct comparative statics with respect to key parameters including managerial diversion, tax rates, and volatility. Our findings reveal that in addition to balancing investment efficiency, the optimal capital structure and ownership structure also weigh the trade-off between tax shield benefits and bankruptcy costs, while simultaneously depending on the initial cash flow level. From the perspective of investment efficiency, as resource diversion increases, the optimal managerial ownership rises to mitigate the enhanced over-investment incentives. In equilibrium, the manager achieves an optimal balance between the investment cost he bears and the cash flow revenue he captures.

We make several novel contributions to the literature. First, our paper shows how agency conflicts can have countervailing effects and may produce firm value-maximizing outcomes. Second, we demonstrate how debt financing and managerial ownership can act as substitutes in mitigating agency conflicts. This substitutability arises because both mechanisms can help counter the over-investment incentives created by private benefits through different channels. Third, we show that agency costs exhibit U-shaped relationships with various parameters, including private benefits, tax rates, volatility, managerial ownership, and leverage. These non-monotonic relationships highlight the complex interactions between different agency problems and suggest that moderate levels of these parameters might be optimal. Fourth, we characterize how optimal ownership and capital structures emerge from the interactions between agency conflicts, tax benefits, and bankruptcy costs.

Our findings help explain several empirical patterns documented in the literature, including the relationship between ownership concentration and firm value (see, e.g., Morck, Shleifer, and Vishny (1988), McConnell and Servaes (1990)) and the financing preferences of family-controlled firms (see, e.g., Romano, Tanewski, and Smyrnios (2001), Wu, Chua, and Chrisman (2007)). The model generates new testable predictions about how agency conflicts affect investment timing and the determination of ownership and capital structure, particularly regarding how these choices should vary with firm characteristics and the institutional environment.

The remainder of this paper is organized as follows. Section II describes the model setup. Section III investigates the investment strategies under two benchmark scenarios, each under a single agency conflict. Section IV explores the ownership structure, capital structure, and investment efficiency in the presence of both agency conflicts. Section V discusses the model extension, empirical relevance, and related literature. Section VI concludes the paper. All mathematical derivations are included in the appendices.

II. Model

Time is continuous and goes to infinity: $t \in [0, +\infty)$. We consider a firm with three types of stakeholders: a manager representing the controlling shareholders, minority shareholders, and creditors. Throughout our analysis, we assume all agents are risk-neutral, deep-pocketed, and are protected by limited liability. All agents discount future cash flows at a rate of r.

The firm's assets generate cash flows at a rate X_t, which follows

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}Z_t, \qquad X_0 = x_0 > 0,$$

where μ represents the expected growth rate, σ denotes the volatility, and Z_t is a standard Brownian motion. We assume $\mu < r$ to ensure the firm's value remains finite. In addition, the firm possesses a one-time growth option: by investing a fixed cost I, the firm can permanently increase its cash flow from X_t to qX_t, where $q > 1$. For the remainder of this paper, we also refer to this growth option as an investment.

A. External Financing

At $t = 0$, the manager issues perpetual debt with a constant coupon payment of c to exploit tax shield benefits. In addition, he sells a fraction $(1 - \lambda)$ of equity to minority shareholders, retaining only a fraction λ for himself. Given tax rate τ, over a short time interval $[t, t + \mathrm{d}t)$, the after-tax cash flow to levered equity before investment is therefore $(1 - \tau)(X_t - c)\mathrm{d}t$. We assume that full loss offsets are available in terms of tax treatment.

Despite selling the securities, the manager remains the sole decision maker for the firm, i.e., he decides whether and when to exercise the growth option, as well as if and when to default.[2] Following Myers' (1977) argument that issuing risky debt diminishes a firm's present market value by inducing suboptimal growth investment strategies, we assume that the growth investment is fully equity-financed.[3] Upon exercising the growth option, the investment cost I is shared between the manager and minority shareholders proportionally to their equity stakes, λ and $(1-\lambda)$, respectively.

B. Private Benefits of Control

The firm operates under imperfect corporate governance, allowing the manager to extract control-related private benefits. We assume that creditors and the government have priority over the manager's diversion privilege (see, e.g., Morellec, Nikolov, and Schürhoff (2012), Morellec et al. (2018)). Specifically, the manager can divert a fraction $\theta \in [0, 1)$ of earnings after interest and taxes for personal consumption.[4] The remaining portion $(1-\theta)$ is distributed as dividends to

[2] As in Morellec (2004) and Grenadier and Wang (2007), we assume the manager's unique combination of professional skills, experience, and fortune in managing the firm's assets grants him exclusive discretion over financial decisions. This specific human capital is associated solely with the manager. His expertise can be viewed as inalienable human capital linked to the project's initiation and operation. Furthermore, outside investors' dispersion makes forming a united front against his investment policy prohibitively costly.

[3] In common real-world practices, prevailing financial covenants in the original debt contracts typically impose restrictions on the issuance of additional debt (see Denis and Wang (2014), Wang (2017)). Mauer and Ott (2000) and Hackbarth and Mauer (2012) study cases where the investment is partially debt-financed.

[4] Cash diversion or resource extraction manifests in various forms, including excessive compensation, perk consumption, nepotistic hiring practices, and other tunneling behaviors (see Morellec et al. (2012), Morellec et al. (2018)).

all shareholders on a pro-rata basis. The diversion rate θ quantifies the deficiency in corporate governance. This potential for cash diversion creates a conflict of interest between the manager and minority shareholders, which we refer to as the "owner-manager conflict."

C. Default

While debt issuance offers tax shield benefits, it also introduces default risk. As standard in the literature, the manager optimally chooses to default when cash flow X_t deteriorates sufficiently. Upon default, a fraction α of the unlevered firm value is lost to bankruptcy costs, while creditors receive the remaining fraction $(1-\alpha)$ of the unlevered firm value, and shareholders get nothing.

D. Equilibrium

Let $\overline{T}$ be the endogenous time that the manager exercises the growth option and $\underline{T}$ be the endogenous time of default. Note that default could occur either before or after the investment. If default happens before the investment, the firm loses the growth option due to its tarnished reputation and the preemption by potential competitors.

Let W_t be the manager's continuation value:

$$(1)\quad W_t = \mathbb{E}_t\Bigg[\int_t^{\overline{T}\wedge\underline{T}} e^{-r(s-t)} \underbrace{(\theta+\lambda(1-\theta))\,(1-\tau)(X_s-c)}_{\text{net cash flow before investment}}\,\mathrm{d}s - e^{-r(\overline{T}-t)} \underbrace{\lambda I}_{\text{investment cost}} + \int_{\overline{T}\wedge\underline{T}}^{\underline{T}} e^{-r(s-t)} \underbrace{(\theta+\lambda(1-\theta))\,(1-\tau)(qX_s-c)}_{\text{net cash flow after investment}}\,\mathrm{d}s\Bigg].$$

Similarly, we define J_t as minority shareholders' continuation value:

$$(2) \quad J_t = \mathbb{E}_t\Bigg[\int_t^{\overline{T}\wedge\underline{T}} e^{-r(s-t)} \underbrace{(1-\lambda)(1-\theta)(1-\tau)(X_s-c)}_{\text{net cash flow before investment}} \mathrm{d}s - e^{-r(\overline{T}-t)} \underbrace{(1-\lambda)I}_{\text{investment cost}} + \int_{\overline{T}\wedge\underline{T}}^{\underline{T}} e^{-r(s-t)} \underbrace{(1-\lambda)(1-\theta)(1-\tau)(qX_s-c)}_{\text{net cash flow after investment}} \mathrm{d}s\Bigg].$$

Creditors receive coupon payments until the firm defaults, after which they receive a fraction $(1-\alpha)$ of the unlevered firm value. Their valuation is therefore:

$$(3) \qquad D_t = \mathbb{E}_t\left[\int_t^{\underline{T}} e^{-r(s-t)} c\mathrm{d}s + e^{-r(\underline{T}-t)}(1-\alpha)\left(\frac{1-\tau}{r-\mu}\right)\left(\mathbb{1}_{\{\overline{T}\le\underline{T}\}} qX_{\underline{T}} + \mathbb{1}_{\{\underline{T}<\overline{T}\}} X_{\underline{T}}\right)\right],$$

where $\mathbb{1}_{\{\cdot\}}$ represents the indicator function. Finally, firm value is the sum of equations (1)-(3) and is given by

$$(4) \quad V_t = \mathbb{E}_t\Bigg[\int_t^{\overline{T}\wedge\underline{T}} e^{-r(s-t)}\left[(1-\tau)X_s+\tau c\right]\mathrm{d}s + \int_{\overline{T}\wedge\underline{T}}^{\underline{T}} e^{-r(s-t)}\left[(1-\tau)qX_s+\tau c\right]\mathrm{d}s - e^{-r(\overline{T}-t)}I + e^{-r(\underline{T}-t)}(1-\alpha)\left(\frac{1-\tau}{r-\mu}\right)\left(\mathbb{1}_{\{\overline{T}\le\underline{T}\}} qX_{\underline{T}} + \mathbb{1}_{\{\underline{T}<\overline{T}\}} X_{\underline{T}}\right)\Bigg].$$

We look for a Markov Perfect Equilibrium where X_t is the payoff-relevant state variable. As such, the valuations satisfy $W_t = W(X_t)$, $J_t = J(X_t)$, $D_t = D(X_t)$, and $V_t = V(X_t)$. We seek an equilibrium in which investment and default both follow threshold strategies: the firm invests when cash flows rise above a pre-determined threshold (x_i) prior to declaring default and defaults when cash flows drop below the endogenous default thresholds. Note that default may occur before or after investment, so there will be two default thresholds. Before making the investment, default happens at x_ℓ, whereas after investment, default happens when X_t falls to x_d.

We distinguish between two investment strategies. The firm-optimal strategy aims to maximize firm value subject to the manager's limited liability, while the manager-optimal strategy maximizes the manager's continuation value.[5] The two strategies are respectively captured by two investment thresholds x_i^F and x_i^S. Throughout the paper, we define over-investment as scenarios when $x_i^S < x_i^F$, and under-investment as those when $x_i^S > x_i^F$. In equilibrium, creditors and minority shareholders fully anticipate the manager's future policies and price the securities fairly at issuance. At $t = 0$, the manager chooses capital and ownership structure $\{c, \lambda\}$ to maximize the sum of his continuation value and the proceeds from selling these securities. For the remainder of the paper, we denote by V^F and V^S the initial firm values under firm-optimal and manager-optimal investment strategies, respectively.

Our model integrates two types of agency conflicts: the owner-manager conflict arising from control-related private benefits and the shareholder-creditor conflict stemming from debt. Table 1 presents the baseline parameter values employed for subsequent numerical analysis.[6] Table 2 summarizes the key model variables in the paper.

[Insert Table 1 approximately here]

[Insert Table 2 approximately here]

[5] As we show in Appendix 3, the firm-optimal strategy is equivalent to the setting in which the manager can commit ex-ante (before external financing) to make the investment that maximizes firm value, subject to his limited liability (see footnote 20 of Leland (1994)). In contrast, the manager-optimal strategy is equivalent to the setting in which the manager cannot commit to the investment choices.

[6] The baseline parameters are adopted as standard in the literature. Specifically, the agency-related parameters $\theta = 0.01$ and $\lambda = 0.075$ take reference from Morellec et al. (2012), while other economic parameters follow those in Leland (1994), Mauer and Ott (2000), Lambrecht (2001), Bhanot and Mello (2006), and Hackbarth and Mauer (2012).

III. Two Benchmark Scenarios

Before analyzing the full model, we examine two benchmarks: a pure-equity firm with only owner-manager conflict and a levered firm with only shareholder-creditor conflict. These benchmarks help illustrate how a single agency conflict impacts investment efficiency.

A. Over-Investment and Owner-Manager Conflict

We first study the scenario where the firm is all-equity financed. The model is time-homogeneous; thus, without loss of generality, we abbreviate the expectation operator under pricing measure $\mathbb{E}[\cdot|X_0 = x]$ to $\mathbb{E}[\cdot]$. We use "ˆ" to denote the value functions and investment thresholds in this benchmark.

Define the first-passage time that X_t hits the endogenous investment threshold $\hat{x}_i$ from below by

$$\overline{T}\left(\hat{x}_i\right) := \inf\left\{t \geq 0 : X_t \geq \hat{x}_i\right\}.$$

The manager exercises the investment option once X_t reaches $\hat{x}_i$ from below. Thus the manager's continuation value in equation (1) becomes

$$\widehat{W}(x) = \mathbb{E}\left[\int_0^{\overline{T}(\hat{x}_i)} e^{-rt}\left(\theta + \lambda(1-\theta)\right)(1-\tau)X_t \mathrm{d}t + e^{-r\overline{T}(\hat{x}_i)}\left[\widehat{W}_q\left(\hat{x}_i\right) - \lambda I\right]\right],$$

where

$$\widehat{W}_q(x) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt}\left(\theta + \lambda(1-\theta)\right)(1-\tau)qX_t \mathrm{d}t\right] = \left(\theta + \lambda(1-\theta)\right)\left(\frac{1-\tau}{r-\mu}\right)qx$$

is the manager's post-investment continuation value. Similarly, firm value in equation (4) becomes

$$\widehat{V}(x)=\mathbb{E}\left[\int_0^{\overline{T}(\hat{x}_i)} e^{-rt}(1-\tau)X_t\mathrm{d}t+e^{-r\overline{T}(\hat{x}_i)}\left[\widehat{V}_q\left(\hat{x}_i\right)-I\right]\right],$$

where

$$\widehat{V}_q(x)=\mathbb{E}\left[\int_0^{+\infty} e^{-rt}(1-\tau)qX_t\mathrm{d}t\right]=\left(\frac{1-\tau}{r-\mu}\right)qx$$

is the post-investment firm value.

Let $\hat{x}_i^S$ and $\hat{x}_i^F$ respectively denote the manager-optimal and firm-optimal investment thresholds. The solution steps follow the standard literature on investment as a real option (see McDonald and Siegel (1986), Dixit and Pindyck (1994)). Therefore, we omit the details and leave them in Appendix 2. By the standard smooth-pasting conditions,

$$\left.\frac{\partial\widehat{W}(x)}{\partial x}\right|_{x=\hat{x}_i^S-}=\left.\frac{\partial\widehat{W}_q(x)}{\partial x}\right|_{x=\hat{x}_i^S+},\qquad \left.\frac{\partial\widehat{V}(x)}{\partial x}\right|_{x=\hat{x}_i^F-}=\left.\frac{\partial\widehat{V}_q(x)}{\partial x}\right|_{x=\hat{x}_i^F+},$$

we derive the manager-optimal and firm-optimal investment thresholds:

$$\text{(5)}\qquad \begin{aligned}\hat{x}_i^S&=\left(\frac{\lambda}{\theta+\lambda(1-\theta)}\right)\left(\frac{1}{q-1}\right)\left(\frac{r-\mu}{1-\tau}\right)\left(\frac{\gamma_+}{\gamma_+-1}\right)I,\\ \hat{x}_i^F&=\left(\frac{1}{q-1}\right)\left(\frac{r-\mu}{1-\tau}\right)\left(\frac{\gamma_+}{\gamma_+-1}\right)I,\end{aligned}$$

where γ_+ is given by equation (A2) in Appendix 1.

We can show that

$$\text{(6)}\qquad \frac{\partial\hat{x}_i^S}{\partial\theta}=-\left(\frac{1-\lambda}{\theta+\lambda(1-\theta)}\right)\hat{x}_i^S\leq 0,\qquad \frac{\partial\hat{x}_i^S}{\partial\lambda}=\left(\frac{\theta}{\theta+\lambda(1-\theta)}\right)\left(\frac{\hat{x}_i^S}{\lambda}\right)\geq 0.$$

A higher θ increases the manager's private gains from cash diversion, thereby increasing his incentives for over-investment. For the rest of the paper, we refer to this effect as the *diversion effect*. Conversely, an increase in λ raises the investment cost borne by the manager, which increases his incentives for under-investment. This effect is termed the *cost-sharing effect*. Figure 1 illustrates these patterns.

[Insert Figure 1 approximately here]

It is easily seen that $\hat{x}_i^S = \left(\frac{\lambda}{\theta+\lambda(1-\theta)}\right)\hat{x}_i^F \leq \hat{x}_i^F$, i.e., in this benchmark, the diversion effect always dominates the cost-sharing effect. The ratio $\frac{\lambda}{\theta+\lambda(1-\theta)}$ captures the manager's trade-off in making the investment: he bears only a fraction λ of the investment cost but receives a fraction $\theta + \lambda(1-\theta) \geq \lambda$ of the cash flows. Given that $\theta + \lambda(1-\theta) \geq \lambda$, the manager has incentives to over-invest, which in this context manifests as exercising the investment option too early. Note that $\hat{x}_i^S = \hat{x}_i^F$, i.e., the manager makes the investment that is firm-optimal, holds under two specific conditions: when $\lambda = 1$ (the manager bears the full investment cost) or when $\theta = 0$ (there is no opportunity for cash flow diversion). In either case, the manager's interests align perfectly with maximizing firm value, eliminating his incentives to over-invest.

We compute the derivative of the initial firm value under manager-optimal strategy as:

$$\text{(7)} \qquad \begin{aligned} \frac{\partial \widehat{V}^S(x_0;\lambda)}{\partial \lambda} &= (I\gamma_+)\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{\theta}{\theta+\lambda(1-\theta)}\right)^2\left(\frac{x_0}{\hat{x}_i^S}\right)^{\gamma_+} \geq 0, \\ \frac{\partial \widehat{V}^S(x_0;\lambda)}{\partial \theta} &= -\left(\theta I\gamma_+\right)\left(\frac{1-\lambda}{\theta+\lambda(1-\theta)}\right)^2\left(\frac{x_0}{\hat{x}_i^S}\right)^{\gamma_+} \leq 0. \end{aligned}$$

Firm value increases with λ and decreases with θ, and achieves the value under firm-optimal strategy when $\lambda = 1$. Therefore, at $t = 0$, the manager's optimal ownership strategy is to set

$\lambda = 1$, which eliminates minority shareholders and perfectly aligns managerial incentives with firm interests.

B. Under-Investment and Shareholder-Creditor Conflict

Next, we allow the firm to issue debt, i.e., $c > 0$, but assume away owner-manager conflict by setting either $\theta = 0$ or $\lambda = 1$. This framework follows Mauer and Ott (2000). We use "$\tilde{}$" to denote the value functions and thresholds in this benchmark.

The manager chooses the investment threshold $\tilde{x}_i$, the pre-investment default threshold $\tilde{x}_\ell$, and the post-investment default threshold $\tilde{x}_d$. Let $\tilde{x}_i^F$ and $\tilde{x}_i^S$ represent the firm- and manager-optimal investment thresholds, respectively maximizing firm value and the manager's continuation value. The solution steps closely follow those in Mauer and Ott (2000), and it is a specific case of the equilibrium outlined in the subsequent Section IV. Figure 2 describes the findings.

[Insert Figure 2 approximately here]

Graph A of Figure 2 illustrates how investment thresholds $\tilde{x}_i^F$ and $\tilde{x}_i^S$ vary with coupon payment c. The solid red line indicates that the manager-optimal threshold – chosen by the manager – increases with c. This pattern aligns with the classical *debt overhang* argument: as the firm issues more debt, equity holders have reduced incentives to invest because they must bear full investment cost while more value accrues to debt holders. If $c = 0$, the debt overhang effect disappears, and the two investment thresholds are identical.

Interestingly, the dotted blue line shows that the firm-optimal investment threshold decreases with c. In other words, it is optimal for the firm to exercise the investment option earlier as it takes on more debt. This occurs because more debt leads to an earlier default, as confirmed by Graph B, where the default thresholds increase with c. Once the firm defaults, it loses the investment option. Therefore, it is optimal to exercise the option earlier as the firm takes on more debt. Note that Graph B also shows that the post-investment default cash flow threshold $q \times \tilde{x}_d$ is always above the pre-investment cash flow threshold $\tilde{x}_\ell^F$.[7] This difference, again, is due to the value of the investment option.

At $t = 0$, the manager optimally chooses the coupon payment to maximize firm value under the manager-optimal investment threshold. In this benchmark scenario, the optimal coupon level $\tilde{c}^*$ is determined to be 1.3697, which effectively balances the trade-off between the tax advantages of debt and potential bankruptcy losses. The corresponding leverage ratio $(\widetilde{D}^S/\widetilde{V}^S)$ and yield spread ($\tilde{c}^*/\widetilde{D}^S - r$) are 73.05% and 0.82%, respectively.

IV. Equilibrium under Both Agency Conflicts

Recall that Section III.A shows that owner-manager conflict results in over-investment, whereas Section III.B reveals that shareholder-creditor conflict typically leads to under-investment. This section studies a levered firm with cash flow diversion, where the incentives for both over- and under-investment coexist. We will examine how the three forces—the diversion effect, cost-sharing effect, and debt overhang effect—interact to shape the investment outcomes and how they interplay with the firm's ownership structure and capital structure.

[7]For consistency, we compare the *absolute levels* of operating cash flow at default before and after investment, i.e., we compare $\tilde{x}_\ell^F$ with $q \times \tilde{x}_d$ instead of $\tilde{x}_d$.

A. Model Solution

This subsection solves the model for arbitrary θ. The firm- and manager-optimal investment strategies are defined similarly to those in Section III, respectively maximizing firm value and the manager's continuation value. Let x_i denote the endogenous investment threshold, and $\overline{T}(x_i)$ represent the time to exercise the investment option. We employ a backward induction approach, beginning our analysis after the investment option has been exercised, i.e., $t > \overline{T}(x_i)$. The problem is identical to Leland (1994). Let $W_q(x)$ denote the manager's continuation value function after investment. We have the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rW_q(x) = (\theta + \lambda(1-\theta))(1-\tau)(qx - c) + \mu x \frac{\partial W_q(x)}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 W_q(x)}{\partial x^2},$$

which satisfies the value-matching and smooth-pasting conditions at the default boundary x_d:

$$W_q(x_d) = 0, \qquad \frac{\partial W_q(x)}{\partial x}\Big|_{x=x_d+} = 0.$$

These conditions lead to

$$x_d = \left(\frac{1}{q}\right)\left(\frac{r-\mu}{r}\right)\left(\frac{\gamma_-}{\gamma_- - 1}\right)c, \tag{8}$$

where γ_- is given by equation (A2) in Appendix 1. As argued in Leland (1994), the default threshold x_d is the same for both the firm- and manager-optimal scenarios.[8] Note that x_d is independent of the diversion rate θ, managerial ownership λ, and the tax rate τ.

[8] See footnote 20 there.

Next turn to $t < \overline{T}(x_i)$. Let $W(x)$ be the manager's continuation value function, which satisfies the following HJB equation:

$$rW(x) = \left(\theta + \lambda(1-\theta)\right)(1-\tau)(x-c) + \mu x \frac{\partial W(x)}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 W(x)}{\partial x^2}.$$

There are two boundaries: the manager chooses to default as $X_t \downarrow x_\ell^S$ and exercises the investment option as $X_t \uparrow x_i^S$. Both boundaries satisfy value-matching and smooth-pasting conditions:

$$\begin{aligned} &W(x_\ell^S) = 0, \qquad W(x_i^S) = W_q(x_i^S) - \lambda I, \\ &\frac{\partial W(x)}{\partial x}\Big|_{x=x_\ell^S+} = 0, \qquad \frac{\partial W(x)}{\partial x}\Big|_{x=x_i^S-} = \frac{\partial W_q(x)}{\partial x}\Big|_{x=x_i^S+}. \end{aligned}$$

The boundary x_i^S characterizes the manager-optimal investment strategy. We will compare it with the firm-optimal investment strategy x_i^F, which maximizes firm value. Specifically, let $V_q(x)$ and $V(x)$ be the firm value functions after and before the investment, satisfying the following HJB equations:

$$\begin{aligned} rV_q(x) &= (1-\tau)qx + \tau c + \mu x \frac{\partial V_q(x)}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V_q(x)}{\partial x^2}, \\ rV(x) &= (1-\tau)x + \tau c + \mu x \frac{\partial V(x)}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V(x)}{\partial x^2}. \end{aligned}$$

Similarly, the above equations satisfy the following value-matching and smooth-pasting conditions at the thresholds x_d, x_ℓ^F, and x_i^F:

$$V_q(x_d) = (1-\alpha)\left(\frac{1-\tau}{r-\mu}\right) q x_d, \qquad V(x_\ell^F) = (1-\alpha)\left(\frac{1-\tau}{r-\mu}\right) x_\ell^F, \qquad V(x_i^F) = V_q(x_i^F) - I,$$

$$\frac{\partial W_q(x)}{\partial x}\Big|_{x=x_d+} = 0, \qquad \frac{\partial W(x)}{\partial x}\Big|_{x=x_\ell^F+} = 0, \qquad \frac{\partial V(x)}{\partial x}\Big|_{x=x_i^F-} = \frac{\partial V_q(x)}{\partial x}\Big|_{x=x_i^F+}.$$

Note that the smooth-pasting conditions for the two default thresholds hold for shareholders/manager value because the firm operates under limited liability. This implies that while the investment decision aims to maximize firm value, default decisions—due to limited liability—are still made to maximize the manager's value.

We proceed to solve the model using standard contingent-claims pricing methods detailed in Appendix 3. The problem requires numerically solving two equations to find x_i and x_ℓ.

B. Baseline Results and Comparative Statics

At $t = 0$, the manager jointly chooses $\{c, \lambda\}$ to maximize the initial firm value $V^S(x_0; c, \lambda)$ under the manager-optimal investment policy, i.e.,

(9) $$\{c^*, \lambda^*\} := \underset{\{c,\lambda\}}{\arg\max} \left\{V^S(x_0; c, \lambda)\right\},$$

for given x_0 and $\theta > 0$. Under base parameters in Table 1, there is a unique set of internal solutions to Problem (9): $c^* = 1.4051$ and $\lambda^* = 0.0975$, associated with a leverage ratio of $D^S/V^S = 74.58\%$ and a yield spread of $\left(c^*/D^S - r\right) = 0.84\%$. Figure 3 shows that firm value exhibits an inverted U-shaped function of either c or λ. In addition to investment efficiency, the optimal coupon level c^* needs to balance tax shield benefits and bankruptcy losses and also depends on cash flow level x_0. Accordingly, the optimal ownership λ^* needs to adjust as well.[9] Note that $\lambda^* < 1$, so the cost-sharing effect is weaker compared to the levered benchmark with $\lambda = 1$ in Section III.B. Recall that both the cost-sharing and debt overhang effects contribute to investment delays. Consequently, a higher debt level ($c^* > \tilde{c}^*$)—and thus a stronger debt overhang effect—is needed to balance investment efficiency, inducing a higher leverage ratio and yield spread.

[Insert Figure 3 approximately here]

Table 3 shows the investment and default thresholds under optimal capital and ownership structure. A few patterns emerge. First, the manager's optimal investment threshold is lower than the firm-optimal threshold: $x_i^S < x_i^F$. This occurs because, under these parameter values, the diversion effect dominates the debt overhang and cost-sharing effects, leading to over-investment. Second, before exercising the investment option, the manager-optimal default threshold is lower than the firm-optimal ($x_\ell^S < x_\ell^F$), while after exercising the investment, the two thresholds are identical (both are $q \times x_d$). This result shows that the value of the investment option reduces the manager's incentives to default.

[Insert Table 3 approximately here]

Interestingly, $x_\ell^S < q \times x_d$, so that the default threshold before investment falls below the one after investment. These findings reveal that the investment option can significantly influence the manager's default decisions.

[9] The optimal ownership level $\lambda^* = 0.0975$ is lower than the controlling shareholder ownership levels reported by Morellec et al. (2018) across various countries and firms because the additional investment option in our model incentivizes the manager to sell more equity to reduce the cost shared.

We now examine how the model solutions—especially the investment and default thresholds, as well as the optimal ownership and capital structure—depend on the model parameters. We will compare the firm-optimal threshold with the manager-optimal one and, therefore, study how over- and under-investment are affected by these parameters.

Figure 4 illustrates how the investment and default thresholds depend on the manager's diversion rate θ. Graph A shows that the manager-optimal investment threshold x_i^S decreases with θ, because a stronger diversion effect increases the incentives for over-investment. By contrast, the firm-optimal threshold x_i^F increases with θ because the manager's incentives to default are diminished at higher values of θ. This latter effect is confirmed by Graph B, which shows that a higher θ also reduces x_ℓ^F, the default threshold before the investment option is exercised. Meanwhile, equation (8) shows that the default threshold x_d after the investment option is exercised, however, is independent of θ. These patterns imply that the manager under-invests when θ is low and over-invests when θ is high. Moreover, there exists a threshold level of θ at which the manager-optimal investment threshold coincides with the firm-optimal threshold.

[Insert Figure 4 approximately here]

Figure 5 shows how the tax rate τ affects investment and default decisions. A higher tax rate τ increases the tax burden of the firm, as the government claims a larger fraction of the cash flows. The reduced after-tax cash flow lowers investment incentives, leading the firms to further delay investment. This is reflected in Graph A, where both firm-optimal and manager-optimal investment thresholds increase with τ. For the same reason, the manager chooses to default earlier, as confirmed by the higher default thresholds in Graph B. When firms default earlier, the market value of the firm's debt decreases, which weakens the debt overhang effect. These combined effects explain why we observe under-investment when τ is low but over-investment when τ is high.

[Insert Figure 5 approximately here]

Figure 6 shows how cash flow volatility σ affects investment and default decisions. A higher volatility σ has two effects. First, following standard real-option logic, higher uncertainty increases the value of waiting and firm value, as confirmed by Graph C. Consistently, Graph A shows both the firm- and manager-optimal investment thresholds increase with σ. For this reason, firms default at lower thresholds when uncertainty is higher, as shown in Graph B. Second, under the same coupon payment c, a higher σ reduces the market value of debt, as illustrated in Graph C. As a result, the debt overhang effect becomes weaker at higher levels of volatility. These two effects combined explain why there is under-investment when σ is low but over-investment when σ is high.

[Insert Figure 6 approximately here]

Figure 7 illustrates how the joint optimal managerial ownership and coupon level determined by Problem (9) vary with diversion rate, tax rate, and volatility. Graph A shows that the optimal managerial ownership λ^* increases with θ. The reason is that when θ increases, the diversion effect becomes stronger. The manager needs to share more investment cost to offset the over-investment incentives. In fact, the ratio $\frac{\lambda^*}{\theta+\lambda^*(1-\theta)}$ stays unchanged, so that the optimal coupon level c^* also stays unchanged.[10] Graph B illustrates that as the tax rate rises, the tax shield benefits increase so the manager prefers to issue more debt. As a result, the manager needs to

[10]The equations (A5), (A6), and (A7) in Appendix 3, which determine the investment and pre-investment default thresholds, all depend critically on the ratio $\frac{\lambda}{\theta+\lambda(1-\theta)}$.

share less investment cost (lower λ^*).[11] Graph C shows that c^* exhibits a U-shaped relationship with σ, consistent with Leland (1994). It also shows that λ^* decreases with σ, which is because by selling more equity, the manager can better diversify the increased cash flow risk.

[Insert Figure 7 approximately here]

C. Optimal Capital or Ownership Structure

In practice, firms often face constraints in simultaneously optimizing their capital and ownership structure. On the one hand, firms may be able to optimize capital structure but face limitations in adjusting ownership structure due to regulatory requirements, insider trading laws, disclosure rules, and shareholder preferences. On the other hand, firms may optimally adjust ownership structure but fail to optimize capital structure due to financial constraints, covenant restrictions, and regulatory rules. This subsection explores scenarios where firms can optimally choose either their capital structure or ownership structure, but not both.

We first investigate the firm-value-maximizing coupon level (c^*) for a given stock ownership (λ) and resource diversion (θ), i.e., we solve the optimization problem[12]

$$c^*(\lambda, \theta) = \arg\max_c \left\{ V^S(x_0; \lambda, \theta) \right\}, \tag{10}$$

[11]If $\tau = 0$, the manager optimally chooses to issue no debt and retain all equity. This can also be derived from the post-investment firm value $V_q(x)$ in equation (A4). Setting the derivative $\frac{\partial V_q(x)}{\partial c}\big|_{c=c_q^*} = 0$ gives the optimal post-investment coupon level $c_q^* = qx\left(\frac{r}{r-\mu}\right)\left(\frac{\gamma_- - 1}{\gamma_-}\right)\left[1 - \gamma_- - \alpha\gamma_-\left(\frac{1}{\tau} - 1\right)\right]^{1/\gamma_-}$. It is straightforward to derive that $\lim_{\tau \downarrow 0} c_q^* = 0$.

[12]Note that θ enters $V^S(x_0)$ only indirectly as θ affects the default and investment thresholds which in turn affect $V^S(x_0)$.

and examine how investment thresholds behave as functions of λ and θ when $c^*(\lambda, \theta)$ endogenously adjusts. Table 4 presents the evolution of leverage ratios, yield spreads, and investment thresholds at the optimal capital structure across different values of λ and θ. The results suggest that as λ increases and thus the cost-sharing effect gets stronger, the needed debt to balance investment efficiency decreases, leading to a lower optimal coupon level and the resulting leverage ratio and yield spread. Conversely, as θ rises, more debt is necessary to mitigate the increased over-investment incentives, so the optimal coupon level and the associated leverage ratio and yield spread increase accordingly.

[Insert Table 4 approximately here]

Table 4 reveals some interesting patterns regarding the default and investment thresholds. First, as λ increases (and thus c^* decreases), the firm faces reduced default risk and can wait longer to invest, reflected by a lower x_ℓ^F and a higher x_i^F. Second, as θ increases (and thus c^* increases), the default risk rises (higher x_ℓ^F), prompting the firm to invest earlier (lower x_i^F). Furthermore, it indicates that the manager-optimal investment threshold increases with λ but decreases with θ. Recall that managerial ownership λ and resource diversion θ influence investment decisions through the critical factor $\frac{\lambda}{\theta+\lambda(1-\theta)}$, as shown in equation (5). This ratio captures the trade-off between the cost-sharing effect and the diversion effect. As λ rises, the ratio increases, implying that the cost-sharing effect dominates (higher x_i^S) and the firm tends to under-invest. Conversely, as θ rises, the ratio decreases, which reflects that the diversion effect prevails (lower x_i^S) and the firm has a tendency toward over-investment. Notably, when λ approaches 0, the manager bears less of investment cost and tends to invest immediately.[13] In this case, the substantive results regarding the effect of diversion rate presented in Table 4 still hold.

[13]In fact, we can prove that $\hat{x}_i^S = 0$ in the extreme case of $\lambda = 0$ from equation (5) in the pure-equity benchmark.

We next study the symmetric counterpart of Problem (10) to solve for the optimal managerial ownership (λ^*) given a coupon level (c) and diversion rate (θ):

$$\lambda^*(c,\theta) = \arg\max_{\lambda} \left\{ V^S(x_0; c, \theta) \right\}.$$

Figure 8 illustrates the optimal ownership, investment thresholds, and key ratio, $\frac{\lambda^*}{\theta+\lambda^*(1-\theta)}$, as c or θ varies. First, note that when $c = 0$, the model reduces to the pure-equity benchmark, where $\lambda^* = 1$ and $x_i^F = x_i^S$. As debt increases and the debt overhang effect gets stronger, the required cost-sharing effect to balance investment efficiency decreases, as reflected by a lower λ^* in Graph A. Graph B shows that the firm-optimal investment threshold x_i^F decreases due to the increased default risk. With a reduced λ^* as c rises, the ratio $\frac{\lambda^*}{\theta+\lambda^*(1-\theta)}$ decreases, implying the diversion effect dominates (lower x_i^S) and the firm tends to over-invest. Finally, as θ rises, the optimal ownership also increases to mitigate the increased diversion effect, as depicted in Graph C. Accordingly, the equilibrium key ratio $\frac{\lambda^*(c,\theta)}{\theta+\lambda^*(c,\theta)(1-\theta)}$, which captures the manager's trade-off between investment cost and cash flow revenue, remains unchanged as θ varies. As a result, the investment and default thresholds also stay constant, as evidenced by equations (A5), (A6), and (A7).

[Insert Figure 8 approximately here]

Furthermore, by setting $\lambda = 0$ in equations (A5) and (A6) in Appendix 3 for a levered firm, it also can be proved that $x_i^S = 0$ and $x_\ell^S = 0$ are the solutions to the simultaneous equations.

D. Investment Efficiency versus Ownership and Capital Structure

This subsection assumes that the firm has no flexibility in choosing its ownership or capital structure, and examines how investment efficiency responds to exogenous changes in λ or c. Recall that the manager's equity ownership λ affects investment decisions through two effects: cost-sharing and diversion. These effects are captured separately by λ and $\theta + \lambda(1 - \theta)$, with their ratio $\frac{\lambda}{\theta+\lambda(1-\theta)}$ being the key factor, as shown in the case of the pure-equity firm in Section III.A. When λ increases, both terms increase, but the cost-sharing effect dominates since the ratio $\frac{\lambda}{\theta+\lambda(1-\theta)}$ increases with λ. This explains why the manager-optimal investment threshold x_i^S increases with λ, as shown in Graph A of Figure 9. In contrast, the firm-optimal investment threshold x_i^F decreases with λ. Note that a stronger cost-sharing effect caused by increased λ reduces the attractiveness of investment to the manager, which reduces his opportunity cost of pre-investment default and thereby incentivizes earlier default, as confirmed by Graph B. Therefore, it is optimal for the firm to exercise the investment option earlier. These patterns imply that there is under-investment when λ is high and over-investment when λ is low.

[Insert Figure 9 approximately here]

Turning to the effect of the coupon payment c, Figure 10 illustrates how the coupon payment c affects investment and default decisions. A higher coupon payment c increases the debt overhang effect, so that the manager finds it optimal to delay investment. This is reflected by the higher manager-optimal investment threshold in Graph A. A larger debt burden also induces earlier default, as confirmed by the higher default thresholds in Graph B. Given that firms default earlier under higher coupon payments, the firm-optimal strategy is to exercise the investment option earlier, which explains why the firm-optimal investment threshold decreases with c in

Graph A. These patterns imply that there is under-investment when c is high and over-investment when c is low.

[Insert Figure 10 approximately here]

1. Investment Efficiency

Previous figures have shown that firm-optimal and manager-optimal investment thresholds can coincide so that under- and over-investment effects exactly offset each other. In this case, the investment policy is efficient. Figure 11 plots the combinations of ownership (λ) and capital structure (c) that achieve this efficiency, which is referred to as the *Firm-Optimal Investment* (*FOI*) curve. Several interesting patterns emerge. First, there exists a negative relationship between c and λ along the efficiency curve. This substitutability arises because both debt overhang and cost-sharing effects contribute to under-investment. Second, the curve shifts upward with the diversion rate θ, as higher diversion increases incentives for over-investment. For any given c, a higher λ is needed to curb the over-investment incentives. Similarly, the curve shifts upward with tax rate τ because higher default thresholds reduce the market value of debt and weaken the debt overhang effect. For a given λ, a higher c is needed to curb over-investment. Interestingly, Graph C shows that the substitutability between c and λ decreases with volatility (σ). This occurs because higher σ, by reducing the market value of debt, reduces the incremental debt overhang effect from an increase in c, requiring a smaller reduction in λ to maintain investment efficiency.

[Insert Figure 11 approximately here]

2. Measuring Efficiency Loss in Investment

Recall that V^F and V^S are the initial firm values under firm-optimal and manager-optimal investment policies, respectively. We measure agency cost as the percentage efficiency loss in initial firm value due to suboptimal investment policies, i.e., $\left(V^F - V^S\right)/V^F$. Under $\{c^*, \lambda^*\}$ and other parameters in Table 1, agency costs are 0.02%. This is lower than 0.07% in the levered benchmark where $\theta = 0$ or $\lambda = 1$, and considerably lower than 0.90% reported by Mauer and Ott (2000) and 0.61% by Hackbarth and Mauer (2012). The relatively low agency costs in our model arise because the diversion effect introduces over-investment incentives, which partially offset the debt overhang effect. Figure 12 illustrates how agency costs vary with parameters $\{\theta, \tau, \sigma, \lambda, c\}$. All graphs exhibit U-shaped patterns with a minimum value of zero.

[Insert Figure 12 approximately here]

V. Extension, Empirical Relevance, and Related Literature

A. Extension: Ownership-Dependent Resource Diversion

This section extends the model by assuming θ is a decreasing function of λ.[14] This extension is motivated by the rationale that as λ increases, managers are less inclined to divert resources because they share with minority shareholders the loss in firm value as resources are diverted (see Jensen and Meckling (1976)).[15]

[14]The authors thank the anonymous referee for suggesting this extension.

[15]This is analogous, but in reverse, to the Myers' (1977) argument for why equity holders may choose to under-invest in positive-NPV investments when they have to put up the full cost of the investment but share the benefits with risky debt.

Specifically, we assume $\theta = f(\lambda)$, satisfying $f'(\lambda) < 0$, $f(1) = 0$, and $f(0) = b$, where $b \in (0, 1)$ stands for the maximum diversion rate. This formulation implies that when the manager retains all the equity, he has no incentive to divert. The derivative $f'(\lambda)$ reflects the sensitivity of the manager's diversion rate to his equity stake. In the numerical analysis, we model $f(\cdot)$ as an exponential function,[16] i.e.,

$$\theta = f(\lambda) := (1 - b)^{\lambda} + b - 1,$$

where $b = 0.01$. The solution steps remain unchanged.

Compared to the case of fixed diversion rate, when θ decreases with λ, an increase in ownership has an additional effect of diminishing the diversion effect, which further reduces the manager's incentives to over-invest. We can see this effect clearly in the case of a pure-equity firm. By substituting $\theta = f(\lambda)$ into equation (5), we derive that

$$\frac{\mathrm{d}\hat{x}_i^S}{\mathrm{d}\lambda} = \left(\frac{f(\lambda) \overbrace{- \lambda\,(1 - \lambda)f'(\lambda)}^{\text{diminished diversion}}}{f(\lambda) + \lambda\,(1 - f(\lambda))} \right) \left(\frac{\hat{x}_i^S}{\lambda} \right) \geq 0. \tag{11}$$

The incremental delay in investment is captured by the additional positive term $(-\lambda(1 - \lambda)f'(\lambda))$ in equation (11), which is equal to 0 in equation (6).

For a levered firm, the qualitative results continue to hold when the diversion rate depends on ownership. Table 5 further compares two scenarios: one where the diversion rate θ is fixed, and another where θ decreases as ownership λ increases. When θ decreases with ownership, we find

[16] Alternatively, $f(\cdot)$ can be modeled in other forms, such as a linear function $f(\lambda) := -b\lambda + b$, or a logarithmic function $f(\lambda) := -\frac{b}{\ln 2}\ln(\lambda + 1) + b$, or a power function $f(\lambda) := \frac{1}{\lambda + a} - \frac{1}{a} + b$, where $a = \frac{1}{2b}\sqrt{b(b + 4)} - \frac{1}{2}$. Numerically, these different functional forms barely change the results.

that a lower ownership stake λ is needed to achieve the firm-optimal investment level compared to the case of fixed θ. Moreover, this difference in needed ownership levels becomes less pronounced as coupon payment c increases.

[Insert Table 5 approximately here]

Finally, the diminished diversion effect due to higher ownership contributes to the increase in firm value. We can see this effect from the pure-equity case. By equations (5) and (A3), and applying $\theta = f(\lambda)$, we derive that

$$\frac{\mathrm{d}\widehat{V}^S(x_0;\lambda)}{\mathrm{d}\lambda} = (I\gamma_+)\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{[f(\lambda)]^2 \overbrace{-\lambda(1-\lambda)f'(\lambda)f(\lambda)}^{\text{diminished diversion}}}{[f(\lambda)+\lambda(1-f(\lambda))]^2}\right)\left(\frac{x_0}{\hat{x}_i^S}\right)^{\gamma_+} \geq 0. \tag{12}$$

The additional positive term $(-\lambda(1-\lambda)f'(\lambda)f(\lambda))$ in equation (12) reflects the incremental firm value from the reduced diversion effect as λ increases, which equals 0 in equation (7).

B. Empirical Relevance

Our model integrates two types of conflicts, which allows us to identify the diversion effect and cost-sharing effect that impact investment efficiency alongside the debt overhang effect. We show that the diversion effect incentivizes over-investment, while the cost-sharing effect more closely aligns managerial incentives with firm interests, thereby reducing the tendency to over-investment. These findings are consistent with Morck et al. (1989), who assert that there is a natural tendency for the manager to allocate company resources in ways that maximize his own interests, often conflicting with the interests of external shareholders. Moreover, their empirical research indicates that granting a certain level of equity to the manager can effectively mitigate this conflict of interest.

We have demonstrated that higher managerial ownership mitigates over-investment in a pure-equity firm. This result is supported by empirical evidence indicating that pure-equity family firms are generally reluctant to get access to external equity financing, as reported in Romano et al. (2001), Blanco-Mazagatos, De Quevedo-Puente, and Castrillo (2007), Wu et al. (2007), and Croci, Doukas, and Gonenc (2011). Further, We have shown that in a levered firm, there is under-investment when λ is high and over-investment when λ is low. This finding is consistent with Hermalin and Weisbach (1991), Griffith (1999), and Davies, Hillier, and McColgan (2005), who provide empirical evidence of a significant nonlinear relationship between corporate value and managerial ownership, and is also supported by Morck et al. (1988) and McConnell and Servaes (1990), who document an inverted U-shaped relationship between firm performance and insider ownership, as illustrated in Graph B of Figure 3.

Our model generates several new testable predictions. First, we have shown how agency conflicts and market conditions, described by the parameters $\{\theta, \tau, \sigma, \lambda, c\}$, affect investment timing and agency costs. Second, we provide predictions on how ownership structure and capital structure vary with firm characteristics and the institutional environment represented by $\{\theta, \tau, \sigma\}$.

C. Relation to the Literature

Our model builds upon the growth option framework of Mauer and Ott (2000) and Hackbarth and Mauer (2012), who emphasize the under-investment problem and the agency costs of debt. Beyond the traditional trade-off between tax benefits and bankruptcy costs (see Leland

(1994)), we enhance their framework by incorporating the manager's private benefits and show how different forces interact and shape the firm's investment and financing policies.

Our work extends existing work on agency frictions (see, e.g., Jensen and Meckling (1976), Jensen (1986), Stulz (1990), and Morellec (2004)). Jensen and Meckling (1976) suggest that increasing managerial ownership can align the interests of managers and shareholders, thereby reducing agency costs. Jensen (1986) argues that debt can mitigate the agency costs of free cash flow by limiting the cash flow available for managerial discretion. In a two-period model, Stulz (1990) examines how optimal financing policies can mitigate agency frictions and investment inefficiencies. The friction in Stulz (1990) is the managers' free cash flow problem, where managers may invest in negative-NPV projects to derive perquisites from investment. In that model, shareholders cannot observe cash flows or investment decisions; therefore, managers tend to over-invest. Debt serves as a useful tool to force managers to pay out cash and curb incentives for over-investment. However, this hard-claim nature of debt can lead to under-investment when the realized cash flows are low. Morellec (2004) further extends Stulz (1990) to a continuous-time framework. In his model, the manager tends to over-invest in negative-NPV projects due to his ability to derive utility from investment. He then shows how debt financing can mitigate managerial diversion by restricting free cash flow.

Our framework differs in several key ways. Unlike Stulz (1990), there is no asymmetric information in our model. Most fundamentally, Jensen (1986), Stulz (1990), and Morellec (2004) all focus on managers' free cash flow problem where managers may invest in negative-NPV projects; however, our model has no free cash flow problem since investments always have positive NPV and the manager (along with equity holders) has deep pockets, following the standard approach as in the Leland (1994) model. In our model, over-investment incentives arise from managers' ability to divert cash flows, and firms issue debt to exploit the tax shield benefit of debt.[17] Indeed, we show that when the tax rate is zero, the firm optimally never issues any debt.

Moreover, the dynamic nature of our model reveals important interactions between investment and default decisions that are absent from the Stulz's (1990) static framework. Specifically, the uncertain timing between investment and default highlights how investment options influence default incentives, similar to the results in Mauer and Ott (2000) and Hackbarth and Mauer (2012). For instance, we find the post-investment default threshold is independent of agency frictions (θ, λ) and tax shield benefits (τ). However, the pre-investment default threshold varies with these parameters, highlighting how investment options interact with default incentives and agency friction severity.

Our dynamic model also generates different predictions from Stulz (1990) regarding the effects of cash flow volatility. While Stulz (1990) finds that higher volatility universally reduces firm value across debt levels, our model, consistent with Dixit and Pindyck (1994), shows that increased volatility can enhance the value of investment options and potentially increase firm value. As shown in Figure 6, higher volatility mitigates under-investment but exacerbates over-investment.

Morellec et al. (2012) and Morellec et al. (2018) study the impact of managerial diversion on leverage decisions. Our work has a different focus: we study how joint ownership and capital structure decisions interact to mitigate and eliminate investment inefficiency associated with the timing of investment. Kanagaretnam and Sarkar (2011) study how managerial ownership

[17] Of course, the issuance of debt also introduces the potential of debt overhang and hence mitigates the over-investment problem.

addresses under-investment in a growth option model, but there is no incentive for over-investment, and they do not study optimal financing structures.

VI. Conclusion

This paper analyzes investment and financing decisions through a continuous-time model that considers both owner-manager and shareholder-creditor conflicts. We study how agency conflicts interact with investment, ownership, and capital structure using analytical and numerical approaches. Our analysis reveals three key forces shaping firm investment: diversion, cost-sharing, and debt overhang. The interplay of these forces can potentially lead firms to achieve firm-optimal investment policies. Our findings show that agency costs have U-shaped relationships with the diversion rate, tax rate, volatility, managerial ownership, and coupon payment. The combination of ownership and capital structure choices can reduce or eliminate agency costs. We reveal how agency conflicts, tax benefits, and bankruptcy costs interact to shape optimal ownership and capital structure, providing explanations for several documented empirical results.

For model simplicity, we only allow the manager to choose capital and ownership structure at $t = 0$. Recent studies (see DeMarzo and He (2021), Hu, Varas, and Ying (2021), and Benzoni, Garlappi, Goldstein, and Ying (2022)) have explored dynamic capital and debt structures with flexible debt issuance and varying maturities. Additionally, real-world debt renegotiation often mitigates debt overhang concerns. Future research could extend our model to incorporate these dynamic elements.

Appendix. Derivations

1. Method of Pricing Corporate Securities

Following the general method of pricing corporate securities summarized in Tan and Yang (2017), here we present a sketch of deriving the manager's continuation value before investment $\widehat{W}(X_t)$ in a pure-equity firm. The value functions of all other securities can be obtained similarly.

Given the investment threshold $\hat{x}_i$, the manager's continuation value before investment is

$$\widehat{W}(x) = \mathbb{E}\left[\int_0^{\overline{T}(\hat{x}_i)} e^{-rt}(\theta+\lambda(1-\theta))(1-\tau)X_t \mathrm{d}t + e^{-r\overline{T}(\hat{x}_i)}\left[\widehat{W}_q\left(\hat{x}_i\right)-\lambda I\right]\right],$$

where $\overline{T}(\hat{x}_i) := \inf\{t \geq 0 : X_t \geq \hat{x}_i\}$. By the dynamic programming method, $\widehat{W}(x)$ satisfies the following ordinary differential equation

$$\text{(A1)} \qquad r\widehat{W}(x) = (\theta+\lambda(1-\theta))(1-\tau)x + \mu x\frac{\partial \widehat{W}(x)}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 \widehat{W}(x)}{\partial x^2},$$

the general solution to which is of the form

$$\widehat{W}(x) = (\theta+\lambda(1-\theta))\left(\frac{1-\tau}{r-\mu}\right)x + B_1\left(x\right)^{\gamma_-} + B_2\left(x\right)^{\gamma_+}, \qquad \text{for } 0 < x < \hat{x}_i,$$

where B_1 and B_2 are constants to be determined, and $\gamma_+(\gamma_-)$ is the positive (negative) root of equation $\frac{1}{2}\sigma^2\gamma\left(\gamma-1\right)+\mu\gamma-r=0$, given by

$$\text{(A2)} \quad \gamma_+ := \frac{1}{2}-\frac{\mu}{\sigma^2}+\sqrt{\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} > 1, \quad \gamma_- := \frac{1}{2}-\frac{\mu}{\sigma^2}-\sqrt{\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} < 0.$$

The equation (A1) for $\widehat{W}(x)$ is subject to the boundary condition

$$\widehat{W}(\hat{x}_i) = \widehat{W}_q(\hat{x}_i) - \lambda I,$$

in addition to $B_1 = 0$ due to $\gamma_- < 0$ and finite valuation. Then we can solve the value of B_2 and thus derive the value function $\widehat{W}(x)$, given by

$$\widehat{W}(x) = (\theta + \lambda(1-\theta)) \left(\frac{1-\tau}{r-\mu}\right) x + \left[\widehat{W}_q\left(\hat{x}_i\right) - \lambda I - (\theta + \lambda(1-\theta)) \left(\frac{1-\tau}{r-\mu}\right) \hat{x}_i\right] \left(\frac{x}{\hat{x}_i}\right)^{\gamma_+}.$$

2. Solution to a Pure-Equity Firm

We employ a backward induction method. After investment, the manager's cash flow is $(\theta + \lambda(1-\theta))(1-\tau)qX_t$, including private benefits $\theta(1-\tau)qX_t$ from diversion and equity stake $\lambda(1-\theta)(1-\tau)qX_t$. Minority shareholders' cash flow is $(1-\lambda)(1-\theta)(1-\tau)qX_t$. Hence, after investment, the manager's value is

$$\widehat{W}_q(x) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt}(\theta + \lambda(1-\theta))(1-\tau)qX_t \mathrm{d}t\right] = (\theta + \lambda(1-\theta)) \left(\frac{1-\tau}{r-\mu}\right) qx,$$

and minority shareholders' value is

$$\widehat{J}_q(x) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt}(1-\lambda)(1-\theta)(1-\tau)qX_t \mathrm{d}t\right] = (1-\lambda)(1-\theta) \left(\frac{1-\tau}{r-\mu}\right) qx.$$

Firm value after investment equals the sum of $\widehat{W}_q(x)$ and $\widehat{J}_q(x)$, given by

$$\widehat{V}_q(x) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt}(1-\tau)qX_t\mathrm{d}t\right] = \left(\frac{1-\tau}{r-\mu}\right)qx.$$

Before investment, the instantaneous cash flows to the manager and minority shareholders are $(\theta+\lambda(1-\theta))(1-\tau)X_t$ and $(1-\lambda)(1-\theta)(1-\tau)X_t$, respectively. Once the cash flow hits the investment threshold $\hat{x}_i$ from below, the firm exercises its investment option, with the manager paying a fraction of the sunk cost λI and minority shareholders paying the remaining portion $(1-\lambda)I$. Therefore, the manager's value before investment is

$$\begin{aligned}\widehat{W}(x) &= \mathbb{E}\left[\int_0^{\overline{T}(x_i)} e^{-rt}(\theta+\lambda(1-\theta))(1-\tau)X_t\mathrm{d}t + e^{-r\overline{T}(\hat{x}_i)}\left[\widehat{W}_q\left(\hat{x}_i\right)-\lambda I\right]\right] \\ &= (\theta+\lambda(1-\theta))\left(\frac{1-\tau}{r-\mu}\right)x + \left[\widehat{W}_q\left(\hat{x}_i\right)-\lambda I-(\theta+\lambda(1-\theta))\left(\frac{1-\tau}{r-\mu}\right)\hat{x}_i\right]\left(\frac{x}{\hat{x}_i}\right)^{\gamma_+}.\end{aligned}$$

In the same way, minority shareholders' value is

$$\begin{aligned}\widehat{J}(x) &= \mathbb{E}\left[\int_0^{\overline{T}(\hat{x}_i)} e^{-rt}(1-\lambda)(1-\theta)(1-\tau)X_t\mathrm{d}t + e^{-r\overline{T}(\hat{x}_i)}\left[\widehat{J}_q\left(\hat{x}_i\right)-(1-\lambda)I\right]\right] \\ &= (1-\lambda)(1-\theta)\left(\frac{1-\tau}{r-\mu}\right)x + \left[\widehat{J}_q\left(\hat{x}_i\right)-(1-\lambda)I-(1-\lambda)(1-\theta)\left(\frac{1-\tau}{r-\mu}\right)\hat{x}_i\right]\left(\frac{x}{\hat{x}_i}\right)^{\gamma_+},\end{aligned}$$

and firm value is

$$\begin{aligned}\widehat{V}(x) &= \mathbb{E}\left[\int_0^{\overline{T}(\hat{x}_i)} e^{-rt}(1-\tau)X_t\mathrm{d}t + e^{-r\overline{T}(\hat{x}_i)}\left[\widehat{V}_q\left(\hat{x}_i\right)-I\right]\right] \\ &= \left(\frac{1-\tau}{r-\mu}\right)x + \left[\widehat{V}_q\left(\hat{x}_i\right)-I-\left(\frac{1-\tau}{r-\mu}\right)\hat{x}_i\right]\left(\frac{x}{\hat{x}_i}\right)^{\gamma_+}.\end{aligned} \tag{A3}$$

The smooth-pasting condition for manager-optimal investment threshold $\hat{x}_i^S$ is

$$\frac{\partial \widehat{W}(x)}{\partial x}\Big|_{x=\hat{x}_i^S-} = \frac{\partial \widehat{W}_q(x)}{\partial x}\Big|_{x=\hat{x}_i^S+},$$

leading to

$$\hat{x}_i^S = \left(\frac{\lambda}{\theta + \lambda(1-\theta)}\right)\left(\frac{1}{q-1}\right)\left(\frac{r-\mu}{1-\tau}\right)\left(\frac{\gamma_+}{\gamma_+ - 1}\right) I.$$

Similarly, for firm-optimal investment threshold $\hat{x}_i^F$, we have

$$\frac{\partial \widehat{V}(x)}{\partial x}\Big|_{x=\hat{x}_i^F-} = \frac{\partial \widehat{V}_q(x)}{\partial x}\Big|_{x=\hat{x}_i^F+},$$

leading to

$$\hat{x}_i^F = \left(\frac{1}{q-1}\right)\left(\frac{r-\mu}{1-\tau}\right)\left(\frac{\gamma_+}{\gamma_+ - 1}\right) I.$$

3. Solution to a Levered Firm

We now consider a levered firm with an investment option. Debt provides tax shields but incurs financial distress costs. Once default occurs, the firm is directly liquidated and the absolute priority rule applies. The manager has the right to make the investment and default policies.

After investment, if the cash flow decreases and hits the default boundary x_d from above, the firm chooses to go bankrupt. Upon liquidation, shareholders get nothing, and debtholders collect the liquidation value $(1-\alpha)\left(\frac{1-\tau}{r-\mu}\right) q x_d$, equal to the recovery rate times the unlevered firm value. The cash flows to debtholders, the manager, and minority shareholders after investment but prior to bankruptcy are c, $(\theta + \lambda(1-\theta))(1-\tau)(qX_t - c)$, and $(1-\lambda)(1-\theta)(1-\tau)(qX_t - c)$, respectively. Thus, after investment but before default, debt value is

$$\begin{aligned}
D_q(x) &= \mathbb{E}\left[\int_0^{\underline{T}(x_d)} e^{-rt} c\mathrm{d}t + e^{-r\underline{T}(x_d)}(1-\alpha)\left(\frac{1-\tau}{r-\mu}\right) q x_d\right] \\
&= \frac{c}{r} + \left[(1-\alpha)\left(\frac{1-\tau}{r-\mu}\right) q x_d - \frac{c}{r}\right]\left(\frac{x}{x_d}\right)^{\gamma_-},
\end{aligned}$$

where $\underline{T}(x_d) := \inf\{t \geq 0 : X_t \leq x_d\}$. The manager's value is

$$\begin{aligned}
W_q(x) &= \mathbb{E}\left[\int_0^{\underline{T}(x_d)} e^{-rt}(\theta + \lambda(1-\theta))(1-\tau)(qX_t - c)\mathrm{d}t\right] \\
&= (\theta + \lambda(1-\theta))(1-\tau)\left[\left(\frac{qx}{r-\mu} - \frac{c}{r}\right) - \left(\frac{qx_d}{r-\mu} - \frac{c}{r}\right)\left(\frac{x}{x_d}\right)^{\gamma_-}\right].
\end{aligned}$$

Minority shareholders' value is

$$\begin{aligned}
J_q(x) &= \mathbb{E}\left[\int_0^{\underline{T}(x_d)} e^{-rt}(1-\lambda)(1-\theta)(1-\tau)(qX_t - c)\mathrm{d}t\right] \\
&= (1-\lambda)(1-\theta)(1-\tau)\left[\left(\frac{qx}{r-\mu} - \frac{c}{r}\right) - \left(\frac{qx_d}{r-\mu} - \frac{c}{r}\right)\left(\frac{x}{x_d}\right)^{\gamma_-}\right],
\end{aligned}$$

and firm value is

$$\begin{aligned}
V_q(x) &= \mathbb{E}\left[\int_0^{\underline{T}(x_d)} e^{-rt}\left[(1-\tau)qX_t + \tau c\right]\mathrm{d}t + e^{-r\underline{T}(x_d)}(1-\alpha)\left(\frac{1-\tau}{r-\mu}\right) q x_d\right] \\
&= \left(\frac{1-\tau}{r-\mu}\right) qx + \frac{\tau c}{r} - \left[\alpha\left(\frac{1-\tau}{r-\mu}\right) q x_d + \frac{\tau c}{r}\right]\left(\frac{x}{x_d}\right)^{\gamma_-}.
\end{aligned} \tag{A4}$$

Before both investment and bankruptcy, debtholders receive coupon payments c, the manager receives $(\theta + \lambda(1-\theta))(1-\tau)(X_t - c)$, and minority shareholders get $(1-\lambda)(1-\theta)(1-\tau)(X_t - c)$. Once the cash flow deteriorates and hits the bankruptcy boundary x_ℓ from above, debtholders get the liquidation value while shareholders receive nothing.

Conversely, once the cash flow grows and hits the investment threshold x_i from below, we take it that the debtholders, the manager, minority shareholders, and the firm get nothing but a lump-sum payment, amounting to $D_q(x_i)$, $W_q(x_i) - \lambda I$, $J_q(x_i) - (1-\lambda)I$, and $V_q(x_i) - I$, respectively. Thus the debt value is

$$\begin{aligned} D(x) &= \mathbb{E}\left[\int_0^{\widehat{T}} e^{-rt} c \mathrm{d}t + e^{-r\widehat{T}} G_D\left(X_{\widehat{T}}\right)\right] \\ &= \frac{c}{r} + \Delta_\ell(x)\left[(1-\alpha)\left(\frac{1-\tau}{r-\mu}\right)x_\ell - \frac{c}{r}\right] + \Delta_i(x)\left[D_q(x_i) - \frac{c}{r}\right], \end{aligned}$$

where $\widehat{T} := \inf\{t \geq 0 : X_t \notin (x_\ell, x_i)\}$, $G_D(x_\ell) = (1-\alpha)\left(\frac{1-\tau}{r-\mu}\right)x_\ell$, $G_D(x_i) = D_q(x_i)$, $\Delta_\ell(x) := \frac{(x_i)^{\gamma_+}(x)^{\gamma_-} - (x_i)^{\gamma_-}(x)^{\gamma_+}}{(x_i)^{\gamma_+}(x_\ell)^{\gamma_-} - (x_i)^{\gamma_-}(x_\ell)^{\gamma_+}}$, and $\Delta_i(x) := \frac{(x_\ell)^{\gamma_-}(x)^{\gamma_+} - (x_\ell)^{\gamma_+}(x)^{\gamma_-}}{(x_i)^{\gamma_+}(x_\ell)^{\gamma_-} - (x_i)^{\gamma_-}(x_\ell)^{\gamma_+}}$. The manager's value is

$$\begin{aligned} W(x) &= \mathbb{E}\left[\int_0^{\widehat{T}} e^{-rt}(\theta + \lambda(1-\theta))(1-\tau)(X_t - c)\mathrm{d}t + e^{-r\widehat{T}} G_W\left(X_{\widehat{T}}\right)\right] \\ &= (\theta + \lambda(1-\theta))(1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) - \Delta_\ell(x)(\theta + \lambda(1-\theta))(1-\tau)\left(\frac{x_\ell}{r-\mu} - \frac{c}{r}\right) \\ &\quad + \Delta_i(x)\left[W_q(x_i) - \lambda I - (\theta + \lambda(1-\theta))(1-\tau)\left(\frac{x_i}{r-\mu} - \frac{c}{r}\right)\right], \end{aligned}$$

where $G_W(x_\ell) = 0$ and $G_W(x_i) = W_q(x_i) - \lambda I$. Minority shareholders' value is

$$\begin{aligned} J(x) &= \mathbb{E}\left[\int_0^{\widehat{T}} e^{-rt}(1-\lambda)(1-\theta)(1-\tau)(X_t - c)\mathrm{d}t + e^{-r\widehat{T}} G_J\left(X_{\widehat{T}}\right)\right] \\ &= (1-\lambda)(1-\theta)(1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) - \Delta_\ell(x)(1-\lambda)(1-\theta)(1-\tau)\left(\frac{x_\ell}{r-\mu} - \frac{c}{r}\right) \\ &\quad + \Delta_i(x)\left[J_q(x_i) - (1-\lambda)I - (1-\lambda)(1-\theta)(1-\tau)\left(\frac{x_i}{r-\mu} - \frac{c}{r}\right)\right], \end{aligned}$$

where $G_J(x_\ell) = 0$ and $G_J(x_i) = J_q(x_i) - (1-\lambda)I$. Firm value is

$$
\begin{aligned}
V(x) &= \mathbb{E}\left[\int_0^{\widehat{T}} e^{-rt}\left[(1-\tau)X_t + \tau c\right]\mathrm{d}t + e^{-r\widehat{T}} G\left(X_{\widehat{T}}\right)\right] \\
&= \left(\frac{1-\tau}{r-\mu}\right)x + \frac{\tau c}{r} - \Delta_\ell(x)\left[\alpha\left(\frac{1-\tau}{r-\mu}\right)x_\ell + \frac{\tau c}{r}\right] \\
&\quad + \Delta_i(x)\left[V_q(x_i) - I - \left(\frac{1-\tau}{r-\mu}\right)x_i - \frac{\tau c}{r}\right],
\end{aligned}
$$

where $G(x_\ell) = (1-\alpha)\left(\frac{1-\tau}{r-\mu}\right)x_\ell$ and $G(x_i) = V_q(x_i) - I$.

The post-investment default threshold x_d and the pre-investment one x_ℓ are determined by the smooth-pasting conditions, respectively given by

$$
\left.\frac{\partial W_q(x)}{\partial x}\right|_{x=x_d+} = 0, \quad \left.\frac{\partial W(x)}{\partial x}\right|_{x=x_\ell+} = 0,
$$

leading to

$$
x_d = \left(\frac{1}{q}\right)\left(\frac{r-\mu}{r}\right)\left(\frac{\gamma_-}{\gamma_- - 1}\right)c,
$$

and the default threshold x_ℓ determined by

$$
\text{(A5)} \quad \Gamma_1\left[(1-\tau)\left[\left(\frac{q-1}{r-\mu}\right)x_i - \left(\frac{c}{r}\right)\left(\frac{1}{\gamma_- - 1}\right)\left(\frac{x_i}{x_d}\right)^{\gamma_-}\right] - \frac{\lambda I}{\theta + \lambda(1-\theta)}\right]
= \Gamma_2(1-\tau)\left(\frac{x_\ell}{r-\mu} - \frac{c}{r}\right) - \left(\frac{1-\tau}{r-\mu}\right)x_\ell,
$$

where $\Gamma_1 := \frac{(\gamma_+ - \gamma_-)(x_\ell)^{(\gamma_+ + \gamma_-)}}{(x_i)^{\gamma_+}(x_\ell)^{\gamma_-} - (x_i)^{\gamma_-}(x_\ell)^{\gamma_+}}$ and $\Gamma_2 := \frac{\gamma_-(x_i)^{\gamma_+}(x_\ell)^{\gamma_-} - \gamma_+(x_i)^{\gamma_-}(x_\ell)^{\gamma_+}}{(x_i)^{\gamma_+}(x_\ell)^{\gamma_-} - (x_i)^{\gamma_-}(x_\ell)^{\gamma_+}}$.

We assert that the derived default thresholds are incentive compatible; that is, they maximize both the manager's value and firm value subject to the manager's limited liability. This assertion aligns with the arguments and approaches in Leland (1994), Mauer and Ott (2000), and Hackbarth and Mauer (2012), among others. To see this analytically, we examine the determination of post-investment default threshold x_d. First, the manager-optimal default strategy is formulated to solve the problem:

$$\max_{x_d} W_q(x), \qquad \forall x \geq x_d.$$

By the first-order optimality condition $\frac{\partial W_q(x)}{\partial x_d} = 0$, we get $x_d = \left(\frac{1}{q}\right)\left(\frac{r-\mu}{r}\right)\left(\frac{\gamma_-}{\gamma_- - 1}\right)c$. Note that

$$\left.\frac{\partial^2 W_q(x)}{\partial x_d^2}\right|_{x_d = \left(\frac{1}{q}\right)\left(\frac{r-\mu}{r}\right)\left(\frac{\gamma_-}{\gamma_- - 1}\right)c} = (\theta + \lambda(1-\theta))(1-\tau)\left(\frac{q}{x_d}\right)\left(\frac{\gamma_- - 1}{r-\mu}\right)\left(\frac{x}{x_d}\right)^{\gamma_-} < 0.$$

Therefore, for any $x \geq x_d$, $W_q(x)$ can be maximized by setting $x_d = \left(\frac{1}{q}\right)\left(\frac{r-\mu}{r}\right)\left(\frac{\gamma_-}{\gamma_- - 1}\right)c$. Next, consider the firm-optimal default threshold, which solves

$$\max_{x_d} V_q(x), \qquad \text{subject to: } W_q(x) \geq 0, \qquad \forall x \geq x_d.$$

We derive that

$$\frac{\partial V_q(x)}{\partial x_d} = \left[q\alpha(\gamma_- - 1)\left(\frac{1-\tau}{r-\mu}\right) + \frac{\tau c \gamma_-}{r x_d}\right]\left(\frac{x}{x_d}\right)^{\gamma_-} < 0$$

since $\gamma_- < 0$. Thus, the lower x_d, the larger $V_q(x)$. However, the choice of x_d must take into account the manager's limited liability: for any $x \geq x_d$, $W_q(x)$ must be non-negative. We get

$$\begin{aligned}
\left.\frac{\partial W_q(x)}{\partial x}\right|_{x=x_d} &= (\theta + \lambda(1-\theta))(1-\tau)\left[q\left(\frac{1-\gamma_-}{r-\mu}\right) + \frac{c\gamma_-}{r x_d}\right], \\
\frac{\partial^2 W_q(x)}{\partial x^2} &= (\theta + \lambda(1-\theta))(1-\tau)(1-\gamma_-)\left(\frac{\gamma_-}{x^2}\right)\left(\frac{q x_d}{r-\mu} - \frac{c}{r}\right)\left(\frac{x}{x_d}\right)^{\gamma_-}.
\end{aligned}$$

Given x_d, we have $W_q(x_d) = 0$. So if $\frac{\partial W_q(x)}{\partial x}\big|_{x=x_d} < 0$, then

$$\exists\, \epsilon > 0,\ W_q(x_d + \epsilon) < W_q(x_d) = 0,$$

violating the manager's limited liability. Thus, we must have $\frac{\partial W_q(x)}{\partial x}\big|_{x=x_d} \geq 0$, equivalent to

$$x_d \geq \left(\frac{1}{q}\right)\left(\frac{r-\mu}{r}\right)\left(\frac{\gamma_-}{\gamma_- - 1}\right)c.$$

In this region of x_d, if $x_d \leq \frac{(r-\mu)c}{qr}$, we get $\frac{\partial^2 W_q(x)}{\partial x^2} \geq 0$ and then $\frac{\partial W_q(x)}{\partial x} \geq \frac{\partial W_q(x)}{\partial x}\big|_{x=x_d} \geq 0$; if $x_d > \frac{(r-\mu)c}{qr}$, then $\frac{\partial W_q(x)}{\partial x} > 0$. In both cases we can derive $W_q(x) \geq W_q(x_d) = 0$ for any $x \geq x_d$, meaning the manager's limited liability holds. Thus, we look for a x_d in the region $x_d \geq \left(\frac{1}{q}\right)\left(\frac{r-\mu}{r}\right)\left(\frac{\gamma_-}{\gamma_- - 1}\right)c$ that maximizes $V_q(x)$. Since $V_q(x)$ decreases with x_d, the firm-optimal default threshold x_d is therefore

$$x_d = \left(\frac{1}{q}\right)\left(\frac{r-\mu}{r}\right)\left(\frac{\gamma_-}{\gamma_- - 1}\right)c \qquad \Leftrightarrow \qquad \frac{\partial W_q(x)}{\partial x}\Big|_{x=x_d} = 0.$$

Thus, the smooth-pasting condition $\frac{\partial W_q(x)}{\partial x}\big|_{x=x_d} = 0$ is essential for maximizing the post-investment firm value subject to the manager's limited liability constraint. Given that both the manager and minority shareholders receive zero salvage value in default, they agree on the firm's default policies, as discussed in Morellec et al. (2012) and Morellec et al. (2018). Additionally, the smooth-pasting condition for pre-investment default threshold x_ℓ involves complexities, which are elaborated upon in Chapter 4 of Dixit and Pindyck (1994).

The manager-optimal investment threshold x_i^S that maximizes the manager's ex-post value is determined by

$$\frac{\partial W(x)}{\partial x}\Big|_{x=x_i-} = \frac{\partial W_q(x)}{\partial x}\Big|_{x=x_i+},$$

which gives

$$\text{(A6)}\quad (1-\tau)\left[\Gamma_3\left(\frac{x_\ell}{r-\mu}-\frac{c}{r}\right)+\left(\frac{q-1}{r-\mu}\right)x_i-\left(\frac{c}{r}\right)\left(\frac{\gamma_-}{\gamma_--1}\right)\left(\frac{x_i}{x_d}\right)^{\gamma_-}\right]$$
$$=\Gamma_4\left[(1-\tau)\left[\left(\frac{q-1}{r-\mu}\right)x_i-\left(\frac{c}{r}\right)\left(\frac{1}{\gamma_--1}\right)\left(\frac{x_i}{x_d}\right)^{\gamma_-}\right]-\frac{\lambda I}{\theta+\lambda(1-\theta)}\right],$$

where $\Gamma_3 := \frac{(\gamma_- - \gamma_+)(x_i)^{(\gamma_+ + \gamma_-)}}{(x_i)^{\gamma_+}(x_\ell)^{\gamma_-} - (x_i)^{\gamma_-}(x_\ell)^{\gamma_+}}$ and $\Gamma_4 := \frac{\gamma_+(x_i)^{\gamma_+}(x_\ell)^{\gamma_-} - \gamma_-(x_i)^{\gamma_-}(x_\ell)^{\gamma_+}}{(x_i)^{\gamma_+}(x_\ell)^{\gamma_-} - (x_i)^{\gamma_-}(x_\ell)^{\gamma_+}}$. The firm-optimal investment threshold x_i^F that maximizes firm value is determined by

$$\frac{\partial V(x)}{\partial x}\Big|_{x=x_i-} = \frac{\partial V_q(x)}{\partial x}\Big|_{x=x_i+},$$

which leads to

$$\text{(A7)}\quad \Gamma_3\left[\alpha\left(\frac{1-\tau}{r-\mu}\right)x_\ell+\frac{\tau c}{r}\right]-\gamma_-\left(\frac{c}{r}\right)\left[\left(\frac{\alpha\gamma_-}{\gamma_--1}\right)(1-\tau)+\tau\right]\left(\frac{x_i}{x_d}\right)^{\gamma_-}$$
$$=\Gamma_4\left[V_q(x_i)-I-\left(\frac{1-\tau}{r-\mu}\right)x_i-\frac{\tau c}{r}\right]-\left(\frac{1-\tau}{r-\mu}\right)(q-1)x_i.$$

We can solve equations (A5), (A6), and (A7) numerically. Specifically, solving equations (A5) and (A6) simultaneously yields the manager-optimal investment and default polices $\{x_i^S, x_\ell^S\}$, and solving equations (A5) and (A7) simultaneously produces the firm-optimal polices $\{x_i^F, x_\ell^F\}$.

References

Benzoni, L.; L. Garlappi; R. S. Goldstein; and C. Ying. "Debt dynamics with fixed issuance costs." Journal of Financial Economics, 146 (2022), 385–402.

Bhanot, K., and A. S. Mello. "Should corporate debt include a rating trigger?" Journal of Financial Economics, 79 (2006), 69–98. ISSN 0304-405X.

Blanco-Mazagatos, V.; E. De Quevedo-Puente; and L. A. Castrillo. "The trade-off between financial resources and agency costs in the family business: An exploratory study." Family Business Review, 20 (2007), 199–213.

Claessens, S.; S. Djankov; J. P. Fan; and L. H. Lang. "Disentangling the incentive and entrenchment effects of large shareholdings." Journal of Finance, 57 (2002), 2741–2771.

Croci, E.; J. A. Doukas; and H. Gonenc. "Family control and financing decisions." European Financial Management, 17 (2011), 860–897.

Davies, J.; D. Hillier; and P. McColgan. "Ownership structure, managerial behavior and corporate value." Journal of Corporate Finance, 11 (2005), 645–660.

DeMarzo, P. M., and Z. He. "Leverage dynamics without commitment." Journal of Finance, 76 (2021), 1195–1250.

Denis, D. J., and J. Wang. "Debt covenant renegotiations and creditor control rights." Journal of Financial Economics, 113 (2014), 348–367.

Dixit, A. K., and R. S. Pindyck. Investment under Uncertainty. Princeton University Press: Princeton, NJ (1994).

Grenadier, S. R., and N. Wang. "Investment under uncertainty and time-inconsistent preferences." Journal of Financial Economics, 84 (2007), 2–39.

Griffith, J. M. "CEO ownership and firm value." Managerial and Decision Economics, 20 (1999), 1–8.

Hackbarth, D., and D. C. Mauer. "Optimal priority structure, capital structure, and investment." The Review of Financial Studies, 25 (2012), 747–796.

Hermalin, B. E., and M. S. Weisbach. "The effects of board composition and direct incentives on firm performance." Financial Management, 101–112.

Hu, Y.; F. Varas; and C. Ying. "Debt maturity management." Working Paper (2021).

Jensen, M. C. "Agency costs of free cash flow, corporate finance, and takeovers." American Economic Review, 76 (1986), 323–329.

Jensen, M. C., and W. H. Meckling. "Theory of the firm: Managerial behavior, agency costs and ownership structure." Journal of Financial Economics, 3 (1976), 305–360.

Kanagaretnam, K., and S. Sarkar. "Managerial compensation and the underinvestment problem." Economic Modelling, 28 (2011), 308–315.

Lambrecht, B. M. "The impact of debt financing on entry and exit in a duopoly." The Review of Financial Studies, 14 (2001), 765–804.

Lambrecht, B. M., and S. C. Myers. “Debt and managerial rents in a real-options model of the firm.” Journal of Financial Economics, 89 (2008), 209–231.

Leland, H. E. “Corporate debt value, bond covenants, and optimal capital structure.” Journal of Finance, 49 (1994), 1213–1252.

Mauer, D. C., and S. H. Ott. “Agency costs, underinvestment, and optimal capital structure.” Project flexibility, agency, and competition: New developments in the theory and application of real options. Oxford, 151–179.

McConnell, J. J., and H. Servaes. “Additional evidence on equity ownership and corporate value.” Journal of Financial Economics, 27 (1990), 595–612.

McDonald, R., and D. Siegel. “The value of waiting to invest.” The Quarterly Journal of Economics, 101 (1986), 707–727.

Morck, R.; A. Shleifer; and R. W. Vishny. “Management ownership and market valuation: An empirical analysis.” Journal of Financial Economics, 20 (1988), 293–315.

Morck, R.; A. Shleifer; and R. W. Vishny. “Alternative mechanisms for corporate control.” American Economic Review, 79 (1989), 842–852.

Morellec, E. “Can managerial discretion explain observed leverage ratios?” The Review of Financial Studies, 17 (2004), 257–294.

Morellec, E.; B. Nikolov; and N. Schürhoff. “Corporate governance and capital structure dynamics.” Journal of Finance, 67 (2012), 803–848.

Morellec, E.; B. Nikolov; and N. Schürhoff. “Agency conflicts around the world.” The Review of Financial Studies, 31 (2018), 4232–4287.

Myers, S. C. “Determinants of corporate borrowing.” Journal of Financial Economics, 5 (1977), 147–175.

Pawlina, G. “Underinvestment, capital structure and strategic debt restructuring.” Journal of Corporate Finance, 16 (2010), 679–702.

Romano, C. A.; G. A. Tanewski; and K. X. Smyrnios. “Capital structure decision making: A model for family business.” Journal of Business Venturing, 16 (2001), 285–310.

Stulz, R. “Managerial discretion and optimal financing policies.” Journal of Financial Economics, 26 (1990), 3–27.

Tan, Y., and Z. Yang. “Growth option, contingent capital and agency conflicts.” International Review of Economics and Finance, 51 (2017), 354–369.

Wang, H. “Managerial entrenchment, equity payout and capital structure.” Journal of Banking and Finance, 35 (2011), 36–50.

Wang, J. “Debt covenant design and creditor control rights: Evidence from the tightest covenant.” Journal of Corporate Finance, 44 (2017), 331–352.

Wu, Z.; J. H. Chua; and J. J. Chrisman. “Effects of family ownership and management on small business equity financing.” Journal of Business Venturing, 22 (2007), 875–895.

TABLE 1

Baseline Parameter Values Used for Numerical Analysis

Parameter	Symbol	Value	Parameter	Symbol	Value
Initial cash flow level	x_0	1.9	Risk-free interest rate	r	0.07
Expected growth rate	μ	0	Volatility	σ	0.1
Corporate tax rate	τ	0.35	Bankruptcy loss rate	α	0.6
Investment cost	I	20	Investment scaling factor	q	2
Manager's diversion rate	θ	0.01			

TABLE 2

Summary of Key Model Variables

This table summarizes the symbols for key model variables in the paper. Panels A, B, and C each report on two benchmarks and the general scenario. In the text, the superscripts " F " and " S " over variables indicate firm-optimal and manager-optimal values, respectively.

Variable	Symbol	Variable	Symbol
Panel A. pure-equity benchmark under only owner-manager conflict			
Post-investment minority shareholders' value	$\widehat{J}_q$	Post-investment firm value	$\widehat{V}_q$
Post-investment manager's value	$\widehat{W}_q$		
Pre-investment minority shareholders' value	$\widehat{J}$	Pre-investment firm value	$\widehat{V}$
Pre-investment manager's value	$\widehat{W}$	Investment threshold	$\hat{x}_i$
Panel B. levered benchmark under only shareholder-creditor conflict			
Pre-investment debt value	$\widetilde{D}$	Pre-investment firm value	$\widetilde{V}$
Post-investment default threshold	$\tilde{x}_d$	Investment threshold	$\tilde{x}_i$
Pre-investment default threshold	$\tilde{x}_\ell$		
Panel C. general levered scenario under both types of agency conflicts			
Post-investment manager's value	W_q	Post-investment debt value	D_q
Post-investment minority shareholders' value	J_q	Post-investment firm value	V_q
Pre-investment manager's value	W	Pre-investment debt value	D
Pre-investment minority shareholders' value	J	Pre-investment firm value	V
Post-investment default threshold	x_d	Investment threshold	x_i
Pre-investment default threshold	x_ℓ		

TABLE 3

Investment and Default Thresholds in a Levered Firm When There Are Both Under- and Over-Investment Incentives

This table presents the investment and default thresholds in a levered firm with both agency conflicts under optimal capital and ownership structure $\{c^*, \lambda^*\}$. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, $\theta = 0.01$, $c^* = 1.4051$, and $\lambda^* = 0.0975$.

x_i^F	x_i^S	x_ℓ^F	x_ℓ^S	$q \times x_d$
2.6760	2.6092	1.0537	1.0536	1.0764

TABLE 4

Optimal Capital Structure and Associated Firm Characteristics

Given managerial ownership λ and diversion rate θ, the optimal coupon level c^* is determined as a function of λ and θ: $c^*(\lambda, \theta) = \arg\max_c \left\{V^S(x_0; \lambda, \theta)\right\}$. The associated leverage ratio is $L^S := D^S(x_0; c^*) / V^S(x_0; c^*)$, and the associated yield spread is $YS^S := c^*/D^S(x_0; c^*) - r$. The corresponding pre-investment firm-optimal default threshold is x_ℓ^F, and the firm-optimal and manager-optimal investment thresholds are x_i^F and x_i^S, respectively. *Parameters*: $x_0 = 1.9$, $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, $\theta = 0.01$, and $\lambda = 0.0975$.

	$c^*(\lambda, \theta)$	L^S	YS^S	x_ℓ^F	x_i^F	x_i^S	Outcome
$\lambda = 0.10$	1.4039	74.52%	0.841%	1.0530	2.6765	2.6148	Over-invest
$\lambda = 0.15$	1.3891	73.85%	0.831%	1.0444	2.6820	2.6917	Under-invest
$\lambda = 0.20$	1.3827	73.57%	0.827%	1.0408	2.6843	2.7323	Under-invest
$\theta = 0.004$	1.3817	73.53%	0.826%	1.0402	2.6847	2.7398	Under-invest
$\theta = 0.008$	1.3964	74.18%	0.836%	1.0486	2.6793	2.6509	Over-invest
$\theta = 0.012$	1.4147	75.03%	0.848%	1.0594	2.6721	2.5691	Over-invest

TABLE 5

The Ownership Needed to Achieve Firm-Optimal Investment

This table reports the needed ownership to achieve firm-optimal investment for various coupon levels. The upper panel presents the case where the diversion rate is fixed at $\theta \equiv 0.01$, while the lower panel presents the case where the diversion rate is modeled as $\theta = f(\lambda) := (1-b)^{\lambda} + b - 1$, with $b = 0.01$. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, and $q = 2$.

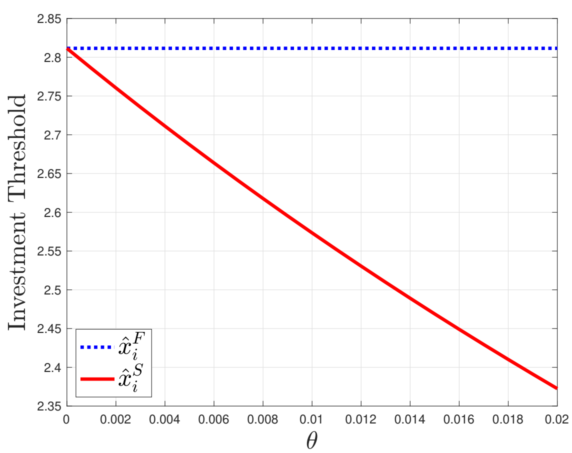
Coupon payment c	1.3	1.4	1.5	1.6
Needed ownership λ if $\theta \equiv 0.01$	0.1862	0.1369	0.0991	0.0700
Needed ownership λ if $\theta = f(\lambda)$	0.1610	0.1222	0.0909	0.0657
Consequent $\theta = f(\lambda)$	0.0084	0.0088	0.0091	0.0093

FIGURE 1

Investment Thresholds in Pure-Equity Benchmark under Only Owner-Manager Conflict

In this benchmark, the firm issues no debt ($c = 0$). Graph A varies the diversion rate θ, with $\hat{x}_i^S = \hat{x}_i^F$ at $\theta = 0$. Graph B varies managerial ownership λ, with $\hat{x}_i^S = \hat{x}_i^F$ at $\lambda = 1$. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $I = 20$, $q = 2$, $\theta = 0.01$, and $\lambda = 0.075$.

Graph A. $\hat{x}_i$ **vs.** θ



Graph B. $\hat{x}_i$ **vs.** λ

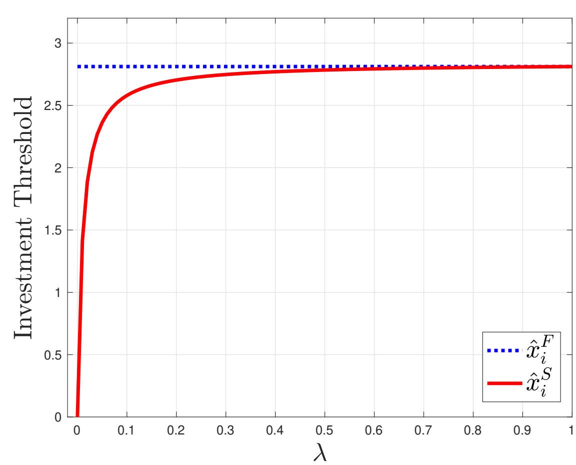
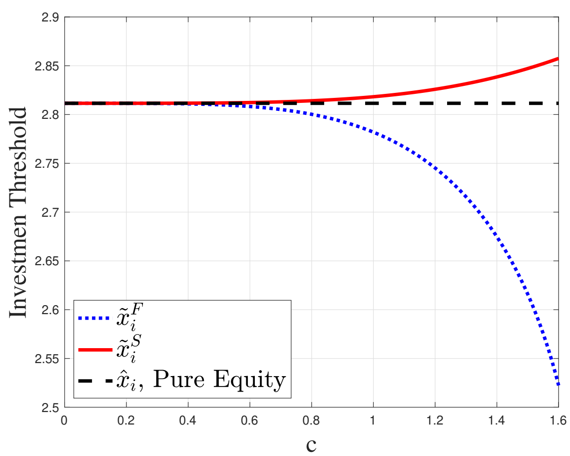


FIGURE 2

Investment and Default Thresholds in Levered Benchmark under Only Shareholder-Creditor Conflict

This benchmark scenario assumes no private benefits ($\theta = 0$) or full managerial ownership ($\lambda = 1$). Graph A displays investment thresholds as a function of coupon payment c, with $\tilde{x}_i^S = \tilde{x}_i^F = \hat{x}_i$ at $c = 0$. Graph B illustrates default thresholds versus coupon payment c, with $q \times \tilde{x}_d > \tilde{x}_\ell^F$. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, and $q = 2$.

Graph A. $\tilde{x}_i$ **vs.** c



Graph B. $\tilde{x}_\ell^F$ **and** $q \times \tilde{x}_d$ **vs.** c

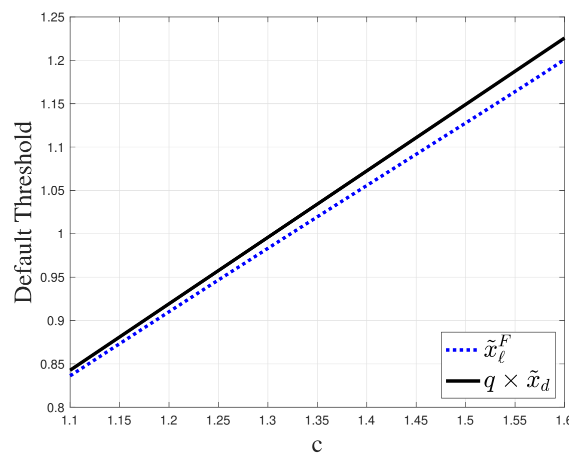


FIGURE 3

Firm Value versus Coupon Payment and Managerial Ownership

Under various initial cash flow levels x_0, Graph A varies c while fixing $\lambda^* = 0.0975$, and Graph B varies λ while fixing $c^* = 1.4051$. Each curve in the two graphs exhibits an inverted U-shaped function with a unique maximum point, indicated by the vertical dashed line. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, and $\theta = 0.01$.

Graph A. $V^S(x_0; c, \lambda^*)$ **vs.** c

Graph B. $V^S(x_0; c^*, \lambda)$ **vs.** λ

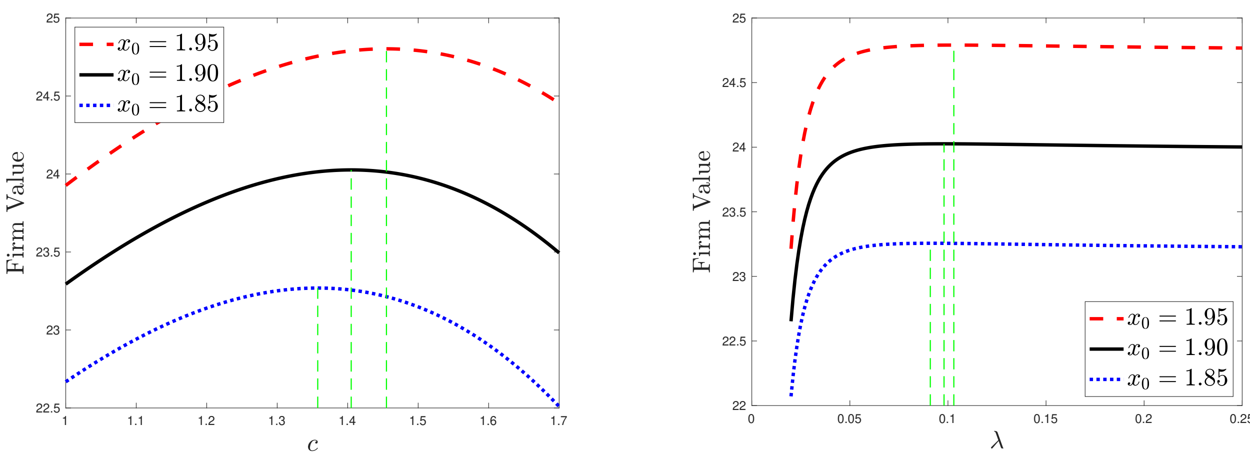
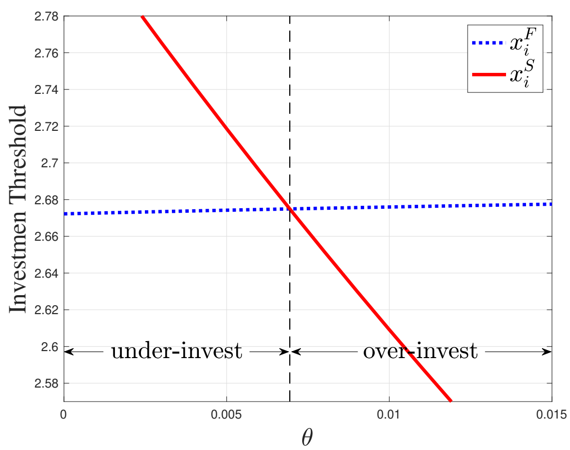


FIGURE 4

Investment and Default Thresholds under Both Conflicts versus Diversion Rate

Graph A highlights an intersection point between x_i^S and x_i^F. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, $c^* = 1.4051$, and $\lambda^* = 0.0975$.

Graph A. x_i **vs.** θ



Graph B. x_ℓ **vs.** θ

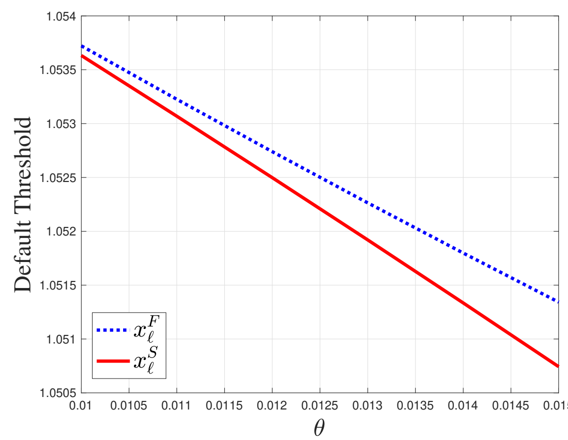


FIGURE 5

Investment and Default Thresholds and Debt Value under Both Conflicts versus Tax Rate

Graph A highlights an intersection point between x_i^S and x_i^F. *Parameters*: $x_0 = 1.9$, $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\alpha = 0.6$, $I = 20$, $q = 2$, $\theta = 0.01$, $c^* = 1.4051$, and $\lambda^* = 0.0975$.

Graph A. x_i **vs.** τ Graph B. x_ℓ **vs.** τ Graph C. D^S **vs.** τ

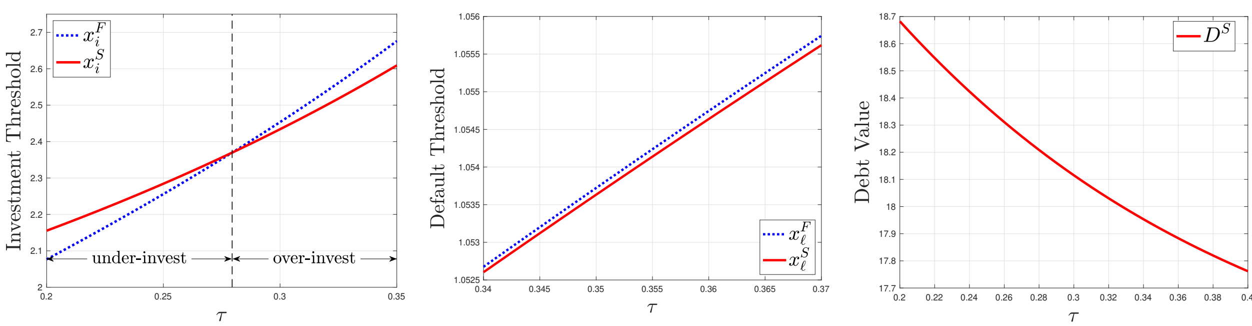
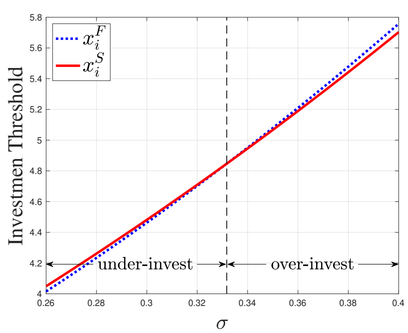


FIGURE 6

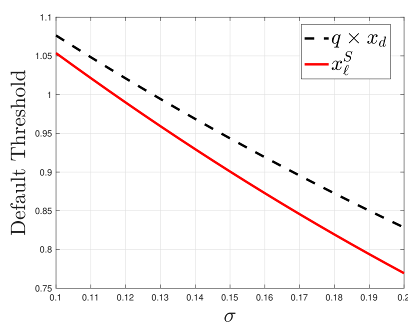
Investment and Default Thresholds and Valuations under Both Conflicts versus Volatility

Graph A highlights an intersection point between x_i^S and x_i^F. *Parameters*: $x_0 = 1.9$, $r = 0.07$, $\mu = 0$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, $\theta = 0.01$, $c^* = 1.4051$, and $\lambda^* = 0.0975$.

Graph A. x_i **vs.** σ



Graph B. x_ℓ^S **and** $q \times x_d$ **vs.** σ



Graph C. V^S **and** D^S **vs.** σ

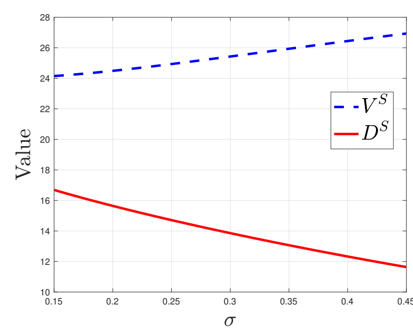


FIGURE 7

Optimal Capital and Ownership Structure versus Diversion Rate, Tax Rate, and Volatility

The ratio $\frac{\lambda^*}{\theta+\lambda^*(1-\theta)}$ and optimal c^* in Graph A remain constant across various θ. *Parameters*: $x_0 = 1.9$, $r = 0.07$, $\mu = 0$, $\sigma = 0.15$, $\tau = 0.35$, $\alpha = 0.5$, $I = 20$, $q = 2$, and $\theta = 0.01$.

Graph A. c^* **and** λ^* **vs.** θ Graph B. c^* **and** λ^* **vs.** τ Graph C. c^* **and** λ^* **vs.** σ

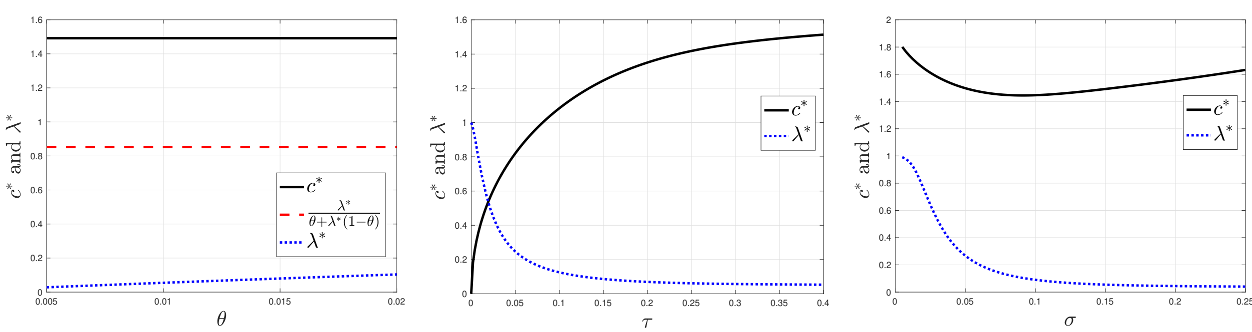


FIGURE 8

Optimal Ownership Structure and Associated Firm Characteristics

Given coupon level c and diversion rate θ, the optimal managerial ownership level λ^* is determined as a function of c and θ: $\lambda^*(c, \theta) = \arg\max_{\lambda} \left\{ V^S(x_0; c, \theta) \right\}$. The corresponding firm-optimal and manager-optimal investment thresholds are x_i^F and x_i^S, respectively. The ratio $\frac{\lambda^*(c,\theta)}{\theta+\lambda^*(c,\theta)(1-\theta)}$ in Graph C remains constant across various θ. *Parameters*: $x_0 = 1.9$, $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, $\theta = 0.01$, and $c = 1.4051$.

Graph A. $\lambda^*(c, \theta)$ **vs.** c — Graph B. x_i^F **and** x_i^S **vs.** c — Graph C. $\lambda^*(c, \theta)$ **vs.** θ

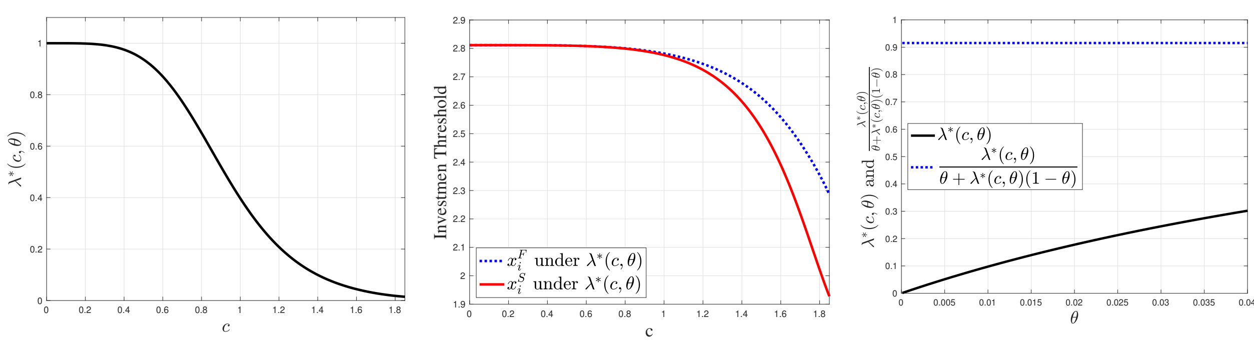
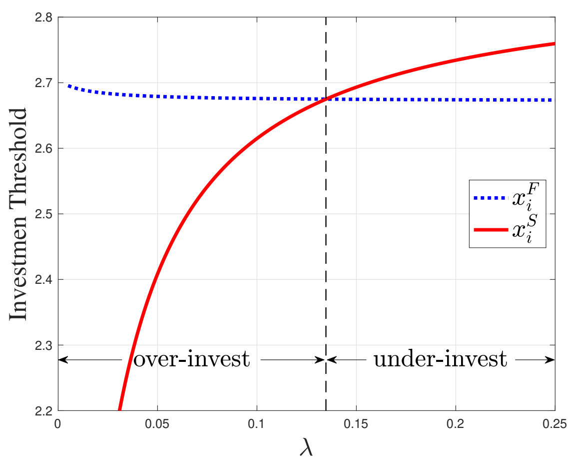


FIGURE 9

Investment and Default Thresholds under Both Conflicts versus Managerial Ownership

Graph A highlights an intersection point between x_i^S and x_i^F. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, $\theta = 0.01$, and $c^* = 1.4051$.

Graph A. x_i **vs.** λ



Graph B. x_ℓ **vs.** λ

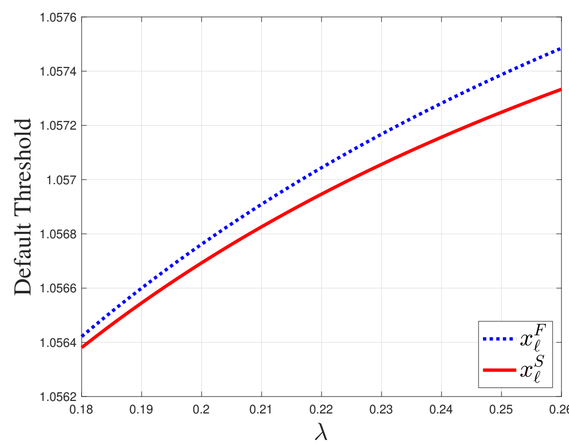
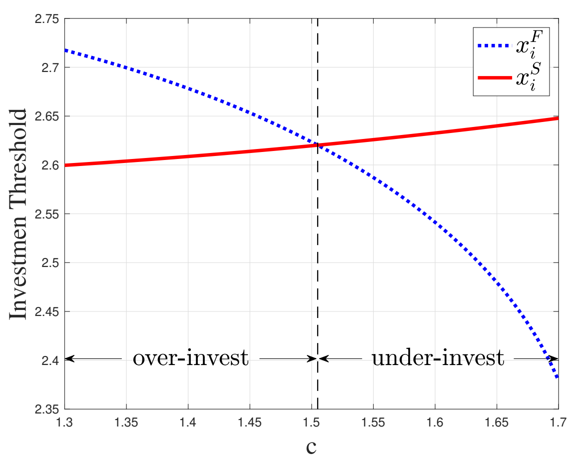


FIGURE 10

Investment and Default Thresholds under Both Conflicts versus Coupon Payment

Graph A highlights an intersection point between x_i^S and x_i^F. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, $\theta = 0.01$, and $\lambda^* = 0.0975$.

Graph A. x_i **vs.** c



Graph B. x_ℓ **vs.** c

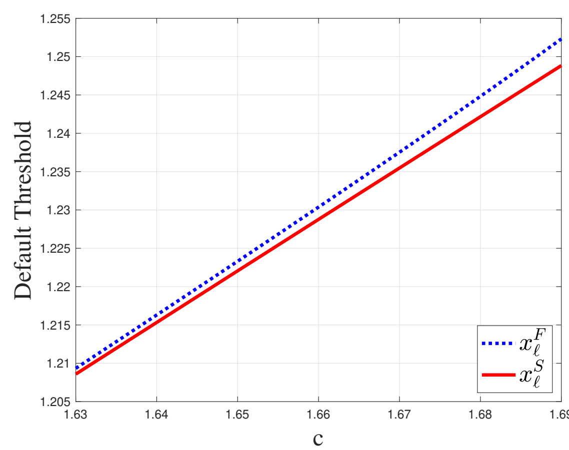


FIGURE 11

***Firm-Optimal Investment* (*FOI*) Curve versus Diversion Rate, Tax Rate, and Volatility**

The combination $\{c, \lambda\}$ on each curve achieves the firm-optimal investment. *Parameters*: $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, and $\theta = 0.01$.

Graph A. ***FOI* Curve vs.** θ Graph B. ***FOI* Curve vs.** τ Graph C. ***FOI* Curve vs.** σ

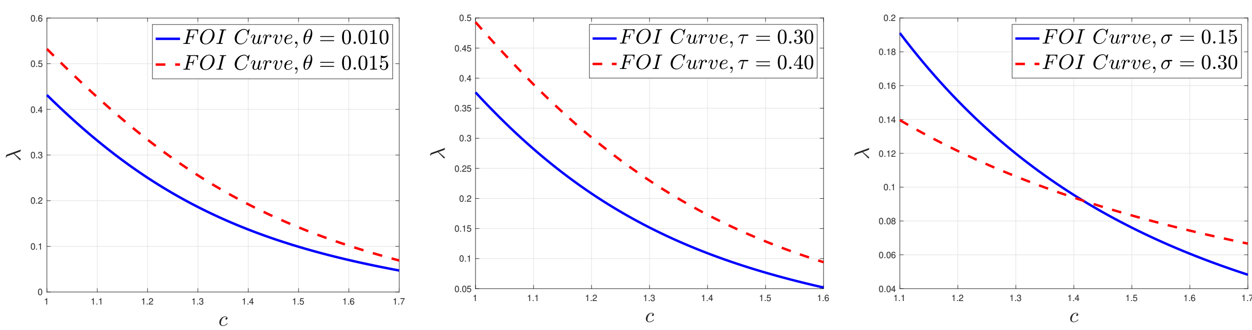
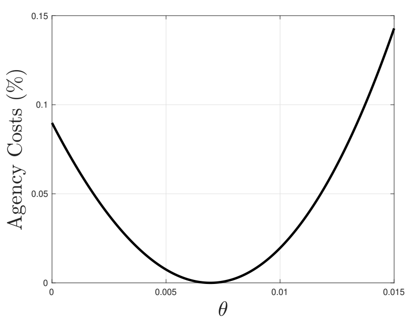


FIGURE 12

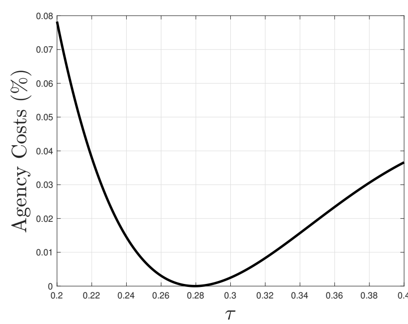
Agency Costs under Both Conflicts

Agency costs are the efficiency loss in initial firm value, measured as $\left(V^F - V^S\right)/V^F$ and expressed in percentage (%). Five graphs respectively vary (A) diversion rate, (B) tax rate, (C) volatility, (D) managerial ownership, and (E) coupon payment, each exhibiting a U-shaped curve with a minimum value of zero. *Parameters*: $x_0 = 1.9$, $r = 0.07$, $\mu = 0$, $\sigma = 0.1$, $\tau = 0.35$, $\alpha = 0.6$, $I = 20$, $q = 2$, $\theta = 0.01$, $c^* = 1.4051$, and $\lambda^* = 0.0975$.

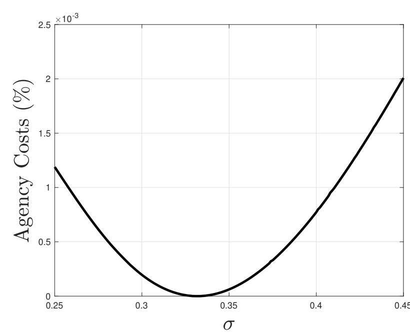
Graph A. **Agency Costs vs.** θ



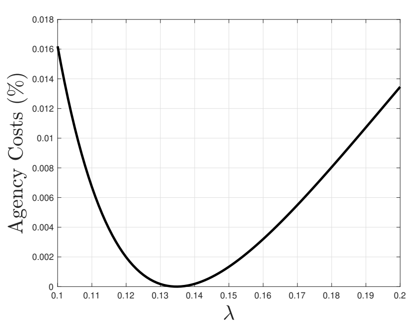
Graph B. **Agency Costs vs.** τ



Graph C. **Agency Costs vs.** σ



Graph D. **Agency Costs vs.** λ



Graph E. **Agency Costs vs.** c