Price-Path Convexity and Short-Horizon Return Predictability

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Abstract

We document a strong negative relation between the curvature of stock price paths (i.e., price-path convexity) and future short-horizon returns at both the aggregate and firm levels. This relation obtains regardless of the cumulative return during the convexity estimation period. At the aggregate level, convexity is a better predictor of future returns than many commonly used predictors. At the firm level, this effect is not explained by known return predictors, microstructure frictions, or illiquidity. Using survey-based expectations of short-horizon returns, we show that the negative relation between convexity and future returns is driven in part by overextrapolation of past returns.

JEL Classification: G02, G12, G14
Keywords: Price paths, convexity, overextrapolation, short-horizon return predictability

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I. Introduction

Short-horizon stock return predictability – in the form of short-term return reversal – is one of the oldest documented and most robust stock return anomalies (Rosenberg and Rudd (1982), Rosenberg, Reid, and Lanstein (1985), Jegadeesh (1990), Lehmann (1990)). Two potential explanations for this anomaly exist, and they are not necessarily mutually exclusive. One explanation suggests that short-term reversal reflects compensation for liquidity provision (Campbell, Grossman, and Wang (1993), Pástor and Stambaugh (2003), Avramov, Chordia, and Goyal (2006), Nagel (2012)). The other explanation suggests that reversal is driven by overreaction to fads, overreaction to new information, or cognitive biases (Shiller (1984), Black (1986), Stiglitz (1989), Summers and Summers (1989), Subrahmanyam (2005)).

While past returns contain information about future returns, they also serve as a measure of the first-order derivative of the recent price path. However, there is reason to believe that the second-order derivative, or the curvature of the recent price path, also holds predictive power. For example, Greenwood, Shleifer, and You (2019) examine industry-level bubbles and find that price run-ups ending in crashes tend to exhibit greater price-path acceleration than price run-ups that do not.

In this paper, we show that the curvature of the stock price path, which we refer to as 'price-path convexity,' contains information about future returns. Specifically, we find that convexity negatively and significantly predicts future returns at both the aggregate and firm levels. This relation holds regardless of the cumulative return during the convexity estimation period and is not driven by known return predictors, microstructure frictions, or illiquidity. Our evidence suggests that the negative relation between convexity and future returns is at least partially due to the overextrapolation of past returns.

To understand convexity, one can refer to Jensen's inequality.[1] We empirically estimate price-path convexity in three steps. First, we calculate the midpoint of the closing prices on the first and last trading days of the convexity estimation period. Second, we subtract from this midpoint the average of all daily closing prices over the same period. Third, to standardize convexity across stocks, we divide this difference by the midpoint from the first step. Positive convexity reflects a convex price path, and negative convexity reflects a concave price path.

We begin our analysis by demonstrating that convexity has significant return predictive power at the aggregate level. In-sample tests show that a one standard deviation increase in convexity is associated with a 0.40% decrease in future one-month returns. Moreover, convexity's predictive power surpasses that of 15 commonly used predictor variables (Welch and Goyal (2008)). When we exclude the onset of the COVID-19 pandemic, the coefficient on convexity increases by 50%, and its associated R^2 estimate more than doubles. Out-of-sample R^2 estimates for convexity range from 0.39% to 0.61%, depending on the forecast period. In contrast, out-of-sample R^2 estimates for nearly all other predictors are negative. When we exclude the onset of the pandemic, convexity's out-of-sample R^2 estimates roughly quadruple.

After establishing the strength of the convexity-future return relation at the aggregate level, we examine the same relation at the firm level. We find that a

[1] A function $g(x)$ is convex if $g\big(\lambda x + (1-\lambda)y\big) \leq \lambda g(x) + (1-\lambda)g(y), \forall x, y$ and $0 < \lambda < 1$.

value-weighted portfolio that buys stocks in the lowest convexity quintile and sells short stocks in the highest convexity quintile earns an average monthly return of 0.84% (t-statistic of 6.29).

This relation holds across different look-back horizons and holding periods. For instance, a zero-investment portfolio that buys stocks in the lowest ten-day convexity quintile and sells short stocks in the highest ten-day convexity quintile earns an average ten-day return of 0.64% (t-statistic of 7.31). Similarly, for a five-day look-back horizon and five-day holding period, the same strategy yields an average five-day return of 0.37% (t-statistic of 4.94). These results indicate that convexity negatively predicts future returns regardless of the short-term look-back horizon.

Importantly, convexity's return predictability is distinct from that of short-term return reversal. In a double sort of observations into convexity and return quintiles, we find that future average monthly returns decline significantly as convexity increases, regardless of past return quintile. Specifically, the average return difference between the lowest and highest convexity quintiles ranges from 0.78% to 0.97% (t-statistics between 4.60 and 5.69) across return quintiles. We obtain similar results when conducting double sorts using convexity and lagged returns over the final ten days or final five days before portfolio formation.

When we regress monthly returns from the zero-investment convexity portfolio on factor models incorporating market return, size, book-to-market ratio, profitability, asset growth, momentum, short-term reversal, long-term reversal, and illiquidity, the resulting alphas range from 0.81% to 0.96% (t-statistics between 5.63 and 6.41). Not only are the returns of the convexity portfolio uncorrelated with those of commonly used factors, but they also exceed them in magnitude. The portfolio's average return (0.84% per month) and Sharpe ratio (0.23) are more than double those of most factors. During recessions, this outperformance is even more pronounced, with an average return of 1.60% (t-statistic of 3.18) and a Sharpe ratio of 0.34.

To ensure that convexity is not simply capturing information contained in known return predictors, we use a cross-sectional regression approach that controls for firm characteristics (Fama and MacBeth (1973)). The negative convexity-future return relation remains robust after accounting for size, book-to-market ratio, profitability, asset growth, momentum, one-month returns (i.e., short-term reversal), illiquidity, idiosyncratic volatility, skewness, and maximum daily return. When all controls are included in the same specification, a one standard deviation increase in convexity is associated with a 45 basis point decline in future returns (robust t-statistic of -11.15).

Since convexity places greater emphasis on more recent price changes, one concern is that its predictive power stems solely from end-of-month returns. However, we find that the negative relation between convexity and subsequent one-month returns remains significant even after controlling for lagged one-day, five-day, or ten-day returns. These results suggest that convexity's predictive power is not merely short-term return reversal in disguise.

To understand why price-path convexity negatively predicts future returns, we consider three explanations, starting with risk considerations. Although time-varying risk can generate efficient yet predictable returns, it is unlikely to explain our findings. As discussed above, our results (i) persist over very short horizons, during which risk-based explanations are less plausible (Lehmann (1990)); (ii) remain unchanged after accounting for asset pricing models that include systematic risk factors; and (iii) strengthen during economic downturns, contradicting risk-based interpretations (Lakonishok, Shleifer, and Vishny (1994)). Moreover, convexity predicts only short-horizon returns, suggesting that risk is unlikely to drive the negative convexity-future return relation (Baba-Yara, Boons, and Tamoni (2024)).

We then examine whether liquidity provision explains our findings. Returns from the zero-investment convexity portfolio do not strongly correlate with a traded liquidity factor, and convexity's predictive power remains robust after controlling for firm-specific illiquidity. Furthermore, our aggregate results are stronger when illiquidity is low, and our firm-level results remain strong even among the most liquid stocks. These findings suggest that illiquidity is unlikely to be the primary driver.

Finally, we consider the role of mispricing. At the aggregate level, convexity's predictive power strengthens following months in which mispricing is more likely. At the firm level, the negative convexity-future return relation is stronger among younger firms and less-followed firms – groups that tend to be mispriced (Hirshleifer, Hsu, and Li (2013)). These findings suggest that mispricing is a driver of the negative relation between convexity and future returns.

In the final section of the paper, we investigate the behavioral biases that may be driving this mispricing. By decomposing convexity, we show that it is a function of a weighted average of price changes, with more recent price changes receiving greater weight. Since this type of weighted average is commonly used to model the beliefs of extrapolative investors (Barberis (2018)), we then examine whether the negative relation between convexity and future returns is driven by overextrapolation.

To explore this relation, we rely on survey-based expectations of short-term returns.

At the aggregate level, we analyze expectations of one-month returns for the Dow Jones Industrial Average. We find that short-term return expectations are positively related to past returns, suggesting that investors extrapolate past returns when forming these expectations.

Using a non-linear regression framework commonly employed in the literature (e.g., Greenwood and Shleifer (2014), Cassella and Gulen (2018), Da, Huang, and Jin (2021)), we decompose expectations into the predicted component, which represents the extrapolative part of raw expectations, and the residual component.[2] We find that the extrapolative component is negatively associated with future returns, suggesting that greater emphasis on recent returns leads to overextrapolation. This finding indicates that overextrapolation partly explains the negative relation between convexity and future returns. Supporting this conclusion, we find a strong positive relation between convexity and the extrapolative component, but not the residual component, of expectations. In univariate orinary least squares regressions of the extrapolative component on convexity, our R^2 estimates exceed 40%.

At the firm level, we replicate this analysis using rankings from Forcerank, a crowdsourcing platform that ranks stocks based on expected returns for the following week. Consistent with prior research (Da et al. (2021)), we find that Forcerank rankings are positively related to past returns. When regressing future realized returns on each component of raw rankings, we again find a strong, negative relation between the extrapolative component and future returns, with R^2 estimates reaching up to 34%. Taken together, the results at both the aggregate and firm levels suggest that the negative

[2] Our decomposition follows from Cassella, Chen, Gulen, and Petkova (2023).

relation between convexity and future returns is at least partially driven by return overextrapolation.

Our paper contributes to the extensive literature on short-term return predictability. At the aggregate level, we extend the literature on extrapolation and aggregate stock return predictability, which links overextrapolation to long-term returns (e.g., Greenwood and Shleifer (2014), Cassella and Gulen (2018)). We make a novel contribution by being the first to link overextrapolation to short-term returns at the aggregate level. While it is well-established that investors focus on long-term returns (measured quarterly) when forming expectations over horizons ranging from six months to one year, we show that they focus on very short-term returns (i.e., recent weekly returns) when forecasting short-term returns.

At the firm level, we demonstrate that convexity has significant explanatory power for future short-term returns. Specifically, we show that the second-order derivative, or curvature, of recent price movements contains important information about future returns not captured by the first-order derivative, or return, of the recent price path. Our focus on short horizons, during which risk-based explanations are less relevant, and our finding that only the extrapolative component of expectations is related to future short-term returns, provide strong evidence that behavioral biases play a significant role in short-term return predictability.

Since convexity predicts returns at both the aggregate and firm levels, our study also relates to Engelberg, McLean, Pontiff, and Ringgenberg (2023), who study whether cross-sectional predictors contain systematic information. We calculate convexity at the aggregate level using S&P 500 closing prices, but unreported tests show that this measure is over 99% correlated with a value-weighted firm-level measure of convexity. This high correlation suggests that, unlike most cross-sectional predictors in Engelberg et al. (2023), firm-level convexity may indeed contain information about the systematic component of returns.

Finally, we contribute to the literature on extrapolative expectations of stock prices. Much of the prior research has focused on extrapolative expectations over longer time periods.[3] An exception is Da et al. (2021), who examine data on about 300 stocks from Forcerank to study how investors form expectations about very short-term stock returns. They find that investors extrapolate from past returns, and that the extrapolative component of these expectations is strongly negatively related to future returns. Our work complements theirs by providing evidence of overextrapolation of recent price changes over short horizons in the entire U.S. cross-section over a 60-year period.

II. Convexity

In this paper, our goal is to determine whether the curvature of stock price paths contains important information about future returns. To this end, we develop a variable designed to capture this curvature, which we call price-path convexity, or simply convexity.

We calculate convexity in three steps. To fix ideas, assume we are calculating one-month convexity. First, we calculate the midpoint of the closing prices on the first and last trading days of the month. Next, we subtract the average of all daily closing prices in

[3] See Bacchetta, Mertens, and van Wincoop (2009); Amromin and Sharpe (2014); Greenwood and Shleifer (2014); Barberis, Greenwood, Jin, and Shleifer (2015, 2018); Hirshleifer, Li, and Yu (2015); Koijen, Schmeling, and Vrugt (2015); Cassella and Gulen (2018); Choi and Mertens (2019); and Jin and Sui (2022) for examples.

that month from the average in the first step. Lastly, to standardize convexity across all stocks, we divide this difference by the midpoint in the first step.[4] The equation is as follows:

$$(1) \qquad Convexity_{it} = \frac{\dfrac{P_{1_{it}} + P_{N_{it}}}{2} - \dfrac{P_{1_{it}} + P_{2_{it}} + ... + P_{N_{it}}}{N_{it}}}{\dfrac{P_{1_{it}} + P_{N_{it}}}{2}},$$

in which $P_{1_{it}}$ is the first daily closing price of stock i in month t, $P_{N_{it}}$ is the last daily closing price of stock i in month t, and $\frac{P_{1_{it}}+P_{2_{it}}+...+P_{N_{it}}}{N_{it}}$ is the average of all daily closing prices of stock i in month t. When the midpoint is greater (less) than the average, convexity is positive (negative), and the price path is convex (concave).

When calculating convexity, we adjust for stock splits and stock dividends but not cash dividends. When a firm splits its stock or distributes stock dividends, all historical stock prices are adjusted for the corresponding stock split or stock dividend. However, when a firm distributes cash dividends, the stock price is adjusted for the dividend only on its ex-dividend date; historical prices are not adjusted. In this respect, we view stock splits and stock dividends as permanent price changes, which we argue are the most salient.[5]

We estimate convexity using price changes rather than returns for two reasons. First, recent work shows that investors at least partially think about stock price changes in dollar units rather than percentage units (Shue and Townsend (2021)). Second, using price changes better captures the price-path dynamics we are interested in studying.

[4]Alternatively, we could standardize by the average of all daily closing prices of month t. In unreported tests, we find that our results are nearly identical when doing so.

[5]For further support of the salience of permanent price changes, Hartzmark and Solomon (2019, 2022) find that investors tend to focus on capital gains rather than on total returns. Regardless, as shown in the Internet Appendix, we find that our results are nearly identical when we also adjust prices for cash dividends.

Consider the following stylized example. Suppose a stock starts the month at $20, drops $1 per day for ten days, and then increases $1 per day for the next ten days. The stock ends the month at $20, and its estimated convexity is 0.24. If we instead define convexity as in (Greenwood et al. (2019)) (i.e., one-month return minus the return of the first two weeks), convexity would be 0.50 (i.e., 0% one-month return minus -50% return in the first two weeks).

Now, suppose the same $20 stock drops $1.50 per day for the first ten days and then increases $1 per day for the last ten days. The stock ends the month at $15, and its estimated convexity is 0.42. If we use returns, convexity again equals 0.50 (i.e., -25% cumulative return minus -75% return in the first two weeks). Although the price paths differ significantly, the return-based definition fails to capture the sharper decline and recovery in the second scenario, whereas the price-change definition captures these dynamics accurately.

To be clear, we do not disagree that the return-based definition successfully captures the acceleration of price paths of bubbles. However, since we are interested in studying the price paths of stocks with positive returns or negative returns, we argue that a convexity measure based on price changes is better suited for our study.[6]

III. Aggregate-Level Results

In this section, we examine the relation between convexity and future returns at the aggregate level using the S&P 500 index. Specifically, we calculate convexity using S&P

[6] A similar argument can be made for a return-based measure that more heavily weighs more recent returns.

500 closing prices from CRSP (`spindx`) and obtain data for the other 15 predictor variables from Amit Goyal's website. Our sample covers the period from July 1963 to December 2022.

Table IA1 in the Internet Appendix provides summary statistics for convexity and its correlations with the other predictor variables. Panel A shows that convexity is approximately symmetric, with a mean and median slightly below zero. Panel B indicates that, aside from its correlation with volatility, convexity exhibits little correlation with the other variables.[7]

A. In-Sample

Panel A of Table 1 presents results from a standard aggregate stock return predictive regression of the following form:

$$r_{t+1} = \alpha + \beta x_t + \varepsilon_{t+1}, \tag{2}$$

in which r_{t+1} is the S&P 500 log excess return next month, and x_t is a predictor variable.

Theory often provides guidance on the sign of β under predictability, so we follow Inoue and Kilian (2005) and use a one-sided alternative hypothesis. To ensure comparability across predictors, we take two steps. First, we take the negative of CON (convexity), NTIS (net equity expansion), TBL (interest rate on three-month T-bill), LTY (long-term government bond yield), and INFL (inflation) so that the hypotheses for all

[7]Unreported tests show that the positive correlation between convexity and volatility is driven by months with negative S&P 500 returns and positive convexity. That is, these months are characterized by steep declines early in the month followed by partial recoveries, reflecting heightened volatility.

predictors are consistent (i.e., H_0: $\beta = 0$ against H_A: $\beta > 0$). Second, we standardize all predictors to have a mean of zero and a standard deviation of one.

We estimate equation (2) using OLS and adjust t-statistics (reported below the coefficients) for heteroskedasticity and autocorrelation (Newey and West (1987)). To determine the number of lags for standard error adjustment, we apply the rule $4(T/100)^{2/9}$, in which T is the number of months in the sample (Andrews (1991), Newey and West (1994)). We use this rule throughout the paper, which typically results in setting the number of lags to six. In unreported tests, we confirm that our findings are highly robust to alternative lag selections.

[Insert Table 1 Here]

The first column in Panel A of Table 1 shows that a one-standard-deviation increase in convexity is associated with a 0.40% decrease in future one-month returns. TBL, LTR (return on long-term government bonds), TMS (long-term government bond yield minus T-bill rate), and INFL also exhibit significant in-sample (IS) return predictability. However, the coefficient on convexity is the largest among all predictors in Panel A (tied with LTR). Moreover, the bottom row of Panel A indicates that the R^2 for convexity is 0.85%, the highest among all predictors. Although all R^2 values are small in absolute terms, any estimate of 0.50% or higher is considered economically significant for predicting monthly returns (Campbell and Thompson (2008)).

B. Out-of-Sample

Strong in-sample predictive performance does not necessarily translate into strong out-of-sample (OS) predictive performance (Welch and Goyal (2008)). To assess OS performance, Panel B of Table 1 reports OS R^2 estimates. For each predictor, we estimate the following predictive regression:

$$\hat{r}_{t+1} = \hat{\alpha}_t + \hat{\beta}_t x_t, \tag{3}$$

in which $\hat{\alpha}$ and $\hat{\beta}$ are the OLS estimates of α and β from equation (2) based on data available through month t. We compare each predictor's performance to that of the unconditional mean excess return through month t. OS R^2 is calculated as follows (Campbell and Thompson (2008)):

$$R^2{}_{OS} = 1 - \frac{\sum_{t=1}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=1}^{T}(r_t - \bar{r}_t)^2}, \tag{4}$$

in which $\hat{r}_t$ is the fitted value from a predictive regression estimated through period t-1, and $\bar{r}_t$ is the unconditional mean excess return through period t-1. A predictor has a positive OS R^2 if its mean squared prediction error (MSPE) is lower than that of the unconditional mean excess return. We use the MSPE-adjusted test statistic of Clark and West (2007) to test the null hypothesis that OS R^2 is not positive (i.e., $H_0 : R^2{}_{OS} \leq 0$ against $H_A : R^2{}_{OS} > 0$).

Panel B of Table 1 presents OS R^2 estimates for each predictor. A range of window sizes should be used for determining OS significance (Hansen and Timmerman (2012), Rossi and Inoue (2012)), so we present results for forecast periods beginning January 1975, January 1980, and January 1985. Across all forecast periods, OS R^2 estimates for convexity are positive. Although they are not significant at conventional levels, all three estimates are significant at least at the 88% level. In contrast, nearly all OS R^2 estimates for other predictors are negative, and none of the positive estimates approach significance.

C. Forecast Encompassing

Next, we use forecast encompassing tests to determine whether convexity contains stronger return predictive information than each of the other 15 predictor variables. We define the optimal combination forecast of aggregate returns as a weighted average of the OS forecast based on convexity and the OS forecast based on one of the other predictors:

$$\hat{r}_{t+1} = (1-\lambda)\hat{r}^{i}_{t+1} + \lambda\hat{r}^{CON}_{t+1}, \tag{5}$$

in which $\hat{r}^{i}_{t}$ is the predictive regression forecast based on one of the 15 predictors, $\hat{r}^{CON}_{t}$ is the predictive regression forecast based on convexity, and $0 \leq \lambda \leq 1$. If $\lambda = 0$, then the forecast based on predictor i fully encompasses the forecast based on convexity, implying that convexity provides no additional predictive information. If $\lambda > 0$, then convexity contains information about aggregate returns beyond what predictor i captures.

Harvey, Leybourne, and Newbold (1998) develop a test statistic for evaluating whether the forecast based on predictor i encompasses the forecast based on convexity (i.e., $H_0 : \lambda = 0$ against $H_A : \lambda > 0$). We use the modified version of this statistic (Rapach, Strauss, and Zhou (2010)) to assess the significance of λ and report $\hat{\lambda}$ estimates in Panel C of Table 1. With only one exception, $\hat{\lambda}$ estimates are highly significant across all predictors and forecast periods. When the forecast period begins in January 1985 and predictor i is LTY, $\hat{\lambda} = 0.72$, which is significant at the 89% level. These results indicate that convexity provides unique information about future returns.

D. COVID-19 Pandemic

Despite the strong significance of our results in nearly every instance in the first three panels of Table 1, the last three panels indicate that price action during the onset of the COVID-19 pandemic had a disproportionately negative effect. Specifically, these panels show that our results become substantially stronger when excluding March and April 2020 from the sample.

Panel A2 demonstrates that in IS tests, the coefficient on convexity increases by nearly 50%, while both the associated t-statistic and R^2 estimate more than double compared to Panel A. Panel B2 shows that all out-of-sample R^2 estimates increase by a factor of approximately four when omitting the first two months of the pandemic. Panel C2 reveals that in nearly every forecast encompassing test, 100% of the weight should be assigned to convexity with almost all estimated weights highly significant at the 99% level.

In summary, Table 1 provides strong evidence that convexity is negatively associated with future aggregate returns and serves as a more effective predictor than a range of commonly studied return predictors.

IV. Firm-Level Results

In this section, we examine the relation between convexity and future returns at the firm level. We obtain price, return, and price-adjustment factor data from CRSP and firm-specific accounting data from the CRSP/Compustat Merged annual file. Our analysis includes only domestic ordinary common shares (`shrcd` 10 or 11) traded on the NYSE, AMEX, or NASDAQ (`exchcd` 1, 2, or 3). The primary sample period spans July 1963 through December 2022. We follow Shumway (1997) in accounting for missing delisting returns and exclude observations with an unadjusted month-end price below five dollars. To ensure there is a full month's worth of trading, we eliminate observations in the portfolio formation month in which a firm first enters or exits the database or if any daily closing price is missing.

Table IA2 in the Internet Appendix presents summary statistics for convexity and its correlations with control variables. Panel A shows that convexity is approximately symmetrically distributed, with a mean and median near zero. Panel B indicates that convexity exhibits low correlations with control variables, suggesting it provides unique information beyond those variables.

A. Portfolio Returns

To assess the predictive power of convexity at the firm level, we sort stocks into quintiles based on convexity at the end of each month, beginning in June 1963. We then form value-weighted portfolios held for one month, ten days, or five days. The lowest (highest) convexity quintile is labeled CON1 (CON5).

Panel A of Table 2 reports returns and Sharpe ratios for portfolios sorted by one-month convexity. Stocks in the lowest convexity quintile earn an average return of 1.24% per month, and stocks in the highest convexity quintile earn 0.41%. The zero-investment portfolio that buys CON1 and shorts CON5 yields an average return of 0.84% per month (t-statistic of 6.29) and a Sharpe ratio of 0.23, which annualizes to 0.80.

[Insert Table 2 Here]

Panels B and C of Table 2 demonstrate that convexity retains predictive power over shorter horizons. The zero-investment portfolio earns 0.64% per ten-day period (t-statistic of 7.31) and 0.37% per five-day period (t-statistic of 4.94). These returns annualize to 16% and 18.5%, respectively. In the Internet Appendix, Table IA3 shows that our results are nearly identical when also adjusting convexity for cash dividends, and Table IA4 shows that our results are even stronger when forming equal-weighted portfolios.

Overall, Table 2 shows that price-path convexity contains significant information about future returns across multiple horizons.

B. Short-Term Reversal

In this test, we examine how the negative relation between convexity and future returns varies between the best-performing (winner) and worst-performing (loser) stocks. Table 3 presents future returns after sorting stocks into convexity and lagged return quintiles.

Panel A of Table 3 reports future one-month returns for portfolios sorted by one-month convexity and lagged one-month returns. CON1 (SREV1) represents the future returns of stocks in the lowest convexity (lagged return) quintile, while CON5 (SREV5) represents those in the highest convexity (lagged return) quintile. Within each lagged return quintile, future returns decline nearly monotonically as convexity increases. The zero-investment portfolio, CON1–CON5, which buys CON1 and shorts CON5, exhibits a highly significant decline in future returns. Stocks with the lowest convexity earn between 0.78% and 0.97% (t-statistics between 4.60 and 5.69) more per month than those with the highest convexity.

The relation between lagged returns and future returns is less consistent. The first row in Panel A shows that across lagged return quintiles, future returns are relatively similar. In particular, SREV1–SREV5, which captures the future returns of the zero-investment portfolio that buys losers (SREV1) and shorts winners (SREV5), indicates that among stocks in CON1, lagged returns do not predict future returns. Some relation emerges in CON2, CON3, and CON4, but none is evident in CON5. Thus, when sorting on convexity, lagged one-month returns are not always negatively associated with future one-month returns.

[Insert Table 3 Here]

Panels B and C of Table 3 confirm that the negative relation between convexity and future returns holds among both winners and losers when considering shorter horizons. These panels show that when stocks are double-sorted on ten-day convexity and lagged ten-day returns (Panel B) or five-day convexity and lagged five-day returns (Panel C), future returns vary significantly across convexity quintiles.

In summary, Table 3 shows that the second-order derivative, or the curvature of the recent price path, contains important information about future returns that is distinct from that in the first-order derivative, or the return, of the recent price path.

C. Factor-Model Adjusted Returns

To ensure that our results are not driven by factors known to predict returns, Table 4 presents factor model-adjusted returns of value-weighted portfolios. CAPM, FF3, and FF5 are average monthly alphas from regressing excess portfolio returns on excess market return, the Fama and French (1993) three-factor model (excess market return plus size (SMB) and value (HML)), and the Fama and French (2015) five-factor model (three-factor model plus profitability (RMW) and asset growth (CMA)). Other alphas are from regressions that augment the five-factor model with factors for momentum (UMD) (Carhart (1997)), short-term reversal (SREV) (Lehmann (1990), Jegadeesh (1990)), long-term reversal (LREV) (De Bondt and Thaler (1985)), or liquidity (LIQ) (traded liquidity factor of Pástor and Stambaugh (2003), which begins January 1968). This table shows that regardless of the factor model used, the zero-investment portfolio earns at least 0.81% per month, which is just a few basis points below the average raw return of the zero-investment portfolio. Table IA5 in the Internet Appendix shows that our alphas remain similarly unchanged when regressing returns from equal-weighted portfolios on the same factor models. In summary, these factor models explain essentially none of the returns earned by the zero-investment portfolio.

[Insert Table 4 Here]

D. Portfolio Characteristics

Next, we compare the characteristics of the zero-investment convexity portfolio with those of several commonly studied factors. Panel A of Table 5 shows that the average return of the convexity portfolio exceeds that of the best-performing factor (UMD) by 20 basis points and is more than twice the return of six other factors (SMB, HML, RMW, CMA, LREV, and LIQ). Although the standard deviation of the convexity portfolio is higher than that of most factors, its Sharpe ratio is more than 50% higher than the second-highest Sharpe ratio and more than double that of four factors.

[Insert Table 5 Here]

Understanding how the zero-investment convexity portfolio performs in downturns is particularly important. Panel B of Table 5 reports its characteristics during recessions. The first row shows that its average return is 1.60% per month in recessions, nearly double its full-sample average and substantially higher than that of commonly studied factors during recessions. Again, while its standard deviation exceeds that of most factors, its Sharpe ratio remains the highest and at least double that of seven factors.

Overall, Table 5 demonstrates that the zero-investment convexity portfolio delivers higher returns and a superior Sharpe ratio than those of several commonly studied factors.

E. Robustness

In this subsection, we briefly describe the results of a battery of robustness tests.

1. Subsamples

Table IA6 in the Internet Appendix reports the future one-month returns of the zero-investment convexity portfolio across various subsamples. Panel A shows that the portfolio's returns remain highly significant in both the first and second halves of the sample period. Panel B confirms that the results hold in an earlier U.S. sample (July 1926 through June 1963), mitigating concerns about data snooping (Schwert (2003)). Panel C indicates that while returns are highest among smaller firms, the portfolio still earns 0.76% per month (t-statistic of 5.29) among large firms, which is only slightly below its full-sample average. Panel D shows that although the effect is stronger in January – consistent with short-term reversal patterns (Jegadeesh (1990)) – the portfolio continues to generate 0.75% per month (t-statistic of 5.40) from February through December. Collectively, these results confirm that our findings are robust across different sample periods and firm sizes.

2. Bid-Ask Bounce

Since the relation between convexity and future returns is short term in nature, one concern is that the observed negative relation could be driven by a bid-ask bounce. However, our analysis suggests this is not the case.

Table IA7 presents results from two tests designed to address this concern. In Panel A, we examine portfolio returns after skipping the first day following portfolio formation. Even after excluding first-day returns, the zero-investment portfolio continues to earn 0.66% per month (t-statistic of 4.93). In Panel B, we compute returns using the bid-ask midpoint instead of closing prices and find only a slight reduction in the magnitude of the relation.

3. Acceleration

In Section II, we distinguish convexity from the acceleration variable in Greenwood et al. (2019). However, since they argue that acceleration "measures the convexity of the price path," it is important to clarify the relation between these two measures.[8]

As an initial step, we examine the correlation between convexity and acceleration. We compute one-month acceleration as the cumulative one-month return minus the cumulative ten-day return at the beginning of the month. At the firm level, these two measures exhibit a correlation of 54.3%. Given this high correlation, we analyze future returns after sorting stocks into convexity quintiles and acceleration quintiles.

Table IA8 in the Internet Appendix shows that the future returns of these 25 portfolios closely resemble those obtained from double sorting stocks into convexity and lagged return quintiles (Table 3). While convexity strongly predicts future returns across all acceleration quintiles, acceleration exhibits predictive power within only a few convexity quintiles. Overall, although convexity and acceleration are conceptually related and exhibit strong correlation, they capture distinct information about future returns.

F. Fama-MacBeth Regressions

To determine whether price-path convexity predicts future returns beyond firm-level characteristics, we estimate cross-sectional regressions for each month in our sample (Fama and MacBeth (1973)). The dependent variable is the future one-month return, and

[8]We thank an anonymous referee for urging us to explore this relation.

independent variables are standardized to have a mean of zero and a standard deviation of one. To reduce the influence of small firms, we use weighted least squares with market capitalization as the weight (Hou, Xue, and Zhang (2020)).

Panel A of Table 6 shows that when future one-month returns are regressed on convexity alone, the coefficient is -0.42 (t-statistic of -6.91) This estimate indicates that a one standard deviation increase in convexity is associated with a 0.42% decline in future returns. The second column shows that adding controls for firm-level characteristics that are commonly used in factor models (i.e., market capitalization, book-to-market ratio, profitability, asset growth, and momentum) increases the coefficient's magnitude to -0.44 (t-statistic of -9.12). The third column adds lagged one-month returns, yet the coefficient on convexity remains virtually unchanged. This result reinforces the above finding that convexity's predictive power is distinct from short-term reversal.

[Insert Table 6 Here]

In the next four columns, we include controls for price movements from the previous month (illiquidity, idiosyncratic volatility, skewness, and maximum daily return). Including these controls does not affect the significance of the convexity coefficient. The last column shows that when including all controls, the coefficient on convexity is at its most significant.

Panel B of Table 6 demonstrates that the predictive power of convexity is distinct from that of one-day, five-day, and ten-day returns at the end of the month. Although controlling for one-day returns slightly weakens the negative relation between convexity and future returns, the coefficient on convexity remains highly significant. Specifically, a one standard deviation increase in convexity is associated with a 0.35% decline in future one-month returns. Similar patterns appear when controlling for other short-term returns. The modest reduction in magnitude is expected given that convexity partially reflects month-end price changes.

Overall, Table 6 shows that convexity maintains a strong negative relation with future returns even after accounting for firm-level characteristics known to predict returns. While convexity is influenced by price movements in the latter half of the month, controlling for very short-term returns does not eliminate its predictive power.

V. Potential Explanations

Thus far, we have shown that convexity is negatively associated with future returns at both the aggregate and firm levels. In this section, we explore three potential explanations for this negative relation.

A. Risk

We first consider time-varying risk, which can lead to returns that are both efficient and predictable.[9] This explanation is often applied to return predictability at the aggregate level (e.g., Fama and French (1988), Campbell and Cochrane (1999)). However, we argue that it is unlikely to account for our findings for several reasons.

First, our results obtain even over very short-horizons (Table 2), during which systematic changes in firm valuations are less likely (Lehmann (1990)). Second, our results

[9] Models consistent with this explanation include, for example, rational learning (Timmermann (1993), Pastor and Veronesi (2009)), changing risk aversion (Campbell and Cochrane (1999)), long-run risk (Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012)), risk-sharing opportunities (Lustig and Van Nieuwerburgh (2005)), and rare-disaster risk (Gabaix (2008), Gabaix (2012), Wachter (2013)). See Campbell (2018) for a review of rational models of asset prices.

are largely unaffected when regressed on factors typically argued to be associated with risk (Table 4). Third, the stronger relative outperformance of the zero-investment portfolio during bad states of the world (Table 5) contradicts a risk-based explanation (Lakonishok et al. (1994)). Lastly, Table IA9 in the Internet Appendix shows that our results do not hold over long horizons, where risk – rather than mispricing – is more likely to explain return predictability (Baba-Yara et al. (2024)). Overall, we find it unlikely that risk considerations drive our results.

B. Illiquidity and Mispricing

The next two potential explanations we consider are illiquidity and mispricing. We consider each explanation at the aggregate level before doing so at the firm level.

1. Aggregate Level

To test these explanations at the aggregate level, we regress future S&P 500 log excess return on convexity after interacting convexity with dummies that indicate whether proxies for illiquidity or mispricing are below or above their time-series median. Following Da, Liu, and Schaumburg (2014), we use a detrended Amihud (2002) measure and realized volatility (RV) as illiquidity proxies. For investor optimism, which can contribute to mispricing, we use the number of initial public offerings (NIPO) and the monthly equity share in new issues (S).

[Insert Table 7 Here]

The first two columns of Table 7 focus on illiquidity proxies. The results suggest that the negative relation between convexity and future log excess returns is stronger in months with below-median, not above-median, illiquidity. In contrast, the last two columns show that this negative relation holds only during months of above-median investor optimism. Taken together, Table 7 suggests that the negative relation between convexity and future returns at the aggregate level is driven by mispricing rather than illiquidity.

2. Firm Level

To test these explanations at the firm level, we independently double sort observations into convexity quintiles and focal proxy quintiles and analyze the future one-month returns of the zero-investment convexity portfolio.

Panels A and B of Table 8 present results based on illiquidity proxies. In Panel A, we measure illiquidity using the Amihud (2002) measure, and in Panel B, we use the bid-ask spread. The bottom two rows of each panel show that although the relation between convexity and future returns is strongest among the most illiquid stocks, it remains highly significant even among the most liquid stocks. This finding contrasts somewhat with our above results, which indicate that the negative convexity–future return relation is unaffected when controlling for the traded liquidity factor of Pástor and Stambaugh (2003) in a portfolio setting (Table 4) or for the Amihud (2002) illiquidity measure in a regression setting (Table 6). Considering these results together, we conclude that while illiquidity is not the primary driver of our findings, it may play a secondary or even tertiary role.

[Insert Table 8 Here]

Panels C and D of Table 8 present results based on mispricing proxies. In Panel C, we focus on investor attention. Stocks that receive less attention are more likely to be mispriced, for their prices may not fully reflect available information (Hirshleifer et al. (2013)). Using the total number of IBES analyst estimates over the previous twelve months as a proxy for investor attention, we find that returns of the zero-investment convexity portfolio are indeed higher for stocks that garner less attention.

In Panel D, we examine whether the negative convexity-future return relation varies with firm age, as measured by the number of months since entering the CRSP database. Due to limited publicly available information, younger firms typically have higher valuation uncertainty (Kumar (2009), Hirshleifer et al. (2013)). We find that returns of the zero-investment convexity portfolio decrease with firm age, and thus, increase with valuation uncertainty.

Overall, the results in Table 8 support the view that mispricing contributes to the negative relation between convexity and future returns.

VI. Behavioral Biases

The preceding results suggest that mispricing contributes to the negative relation between convexity and future returns. This finding naturally raises the question: what behavioral biases might drive this mispricing? To explore this question, we first examine a decomposition of convexity.

A. Decomposition

Ignoring subscripts, convexity can be loosely defined as the difference between two average daily price changes:

$$
\begin{aligned}
\text{(6)} \qquad \text{Convexity} &= \frac{\frac{P_1+P_N}{2} - \frac{P_1+P_2+\cdots+P_N}{N}}{\frac{P_1+P_N}{2}} \\
&= \frac{P_N - \frac{P_1+P_2+\cdots+P_N}{N} - \frac{P_N-P_1}{2}}{\frac{P_1+P_N}{2}} \\
&= \frac{\frac{N\Delta P_N+(N-1)\Delta P_{N-1}+\cdots+\Delta P_1}{N} - \frac{P_N-P_1}{2}}{\frac{P_1+P_N}{2}} \\
&= \frac{N}{P_1+P_N}\left[\underbrace{\frac{N\Delta P_N+(N-1)\Delta P_{N-1}+\cdots+\Delta P_1}{N+(N-1)+\cdots+1}}_{\text{weighted avg } \Delta P} - \underbrace{\frac{P_N-P_1}{N}}_{\text{avg } \Delta P}\right]
\end{aligned}
$$

The first average is a weighted average of daily price changes, in which the weights decrease linearly from more recent to older days. This type of weighted average is commonly used to model the beliefs of an extrapolative investor (Barberis (2018)). The second average is a simple mean of daily price changes, which approximates the cumulative return over the same period. Since convexity is a function of a weighted average often used to represent extrapolative beliefs, we next examine its relation to survey-based expectations, which prior research has identified as extrapolative (e.g., Greenwood and Shleifer (2014), Da et al. (2021)).

B. Aggregate Level

Before discussing the relation between convexity and expectations, we first need to characterize the extent to which expectations about short-horizon returns are (i) extrapolative and (ii) associated with future returns. We begin at the aggregate level. Specifically, we focus on survey expectations of one-month returns for the Dow Jones Industrial Average (DJIA), which we obtain from the Yale International Center for Finance.[10] The survey question posed to respondents is as follows: "How much of a change in percentage terms do you expect in the following one month?"

Data on individual investor expectations begin in January 1999, and data for institutional investor expectations begin in August 1993. Both datasets extend to February 2022. For both groups, there are some days with multiple observations. We consider any observation with an absolute expected return of 50% or more to be erroneous and exclude these observations (0.14% of the total). Our return data, which we calculate for the DJIA, are derived from historical price data obtained from the Wall Street Journal website.[11]

Table IA10 in the Internet Appendix presents summary statistics of our expectations data. Panel A displays statistics for all expectations. For both investor types, the mean expected one-month return is just above 0%, with the median slightly higher than the mean. Panel B shows statistics for expectations after averaging those on days with multiple observations. The statistics for average expectations are similar to those for raw expectations. We use the averaged expectations for all subsequent tests. In unreported tests, we find that our results remain qualitatively similar when using all expectations.

[10]We thank Robert Shiller for kindly providing us the data.
[11]See `https://www.wsj.com/market-data/quotes/index/DJIA/historical-prices` for more details.

1. Expectations and Past Returns

Building on previous research on survey expectations and extrapolation (Greenwood and Shleifer (2014), Cassella and Gulen (2018), Da et al. (2021)), we use an exponential decay function to estimate the extent to which investors extrapolate past returns when forming expectations about future returns. Specifically, we estimate a non-linear least squares regression of daily average expectations on past weekly returns using the following model:

$$Exp_t = a + b\sum_{j=1}^{N} w_j R^W_{t-j,t-j+1} + \varepsilon^{Exp}_{it} \qquad (7)$$

$$w_j = \frac{\lambda^j}{\sum_{k=1}^{N}\lambda^k},$$

in which Exp_t is the average one-month return expectation on day t, N is the number of weekly lags, and $R^W_{t-j,t-j+1}$ is the j-lagged weekly return. The weight parameter w_j is defined by λ, which estimates the extent to which past returns are weighted. A higher b suggests that investor expectations respond strongly to past returns, and a higher λ implies that relatively less weight is placed on recent returns. We estimate this model using weekly returns of the DJIA over periods of four, eight, twelve, and sixteen weeks.

The results in Table 9 show that both individual and institutional investors extrapolate past returns when forming expectations about future short-horizon returns. Specifically, the positive and significant estimates of b in each column indicate that investors significantly respond to past returns when forming expectations. Instead of presenting λ, which is inversely related to the weight placed on recent returns, we report

DOX. DOX is defined as $1 - \lambda$, and it is positively related to the weight placed on recent returns (Cassella and Gulen (2018)). Our findings show that DOX is positive and significant for both investor types and for all lag periods. These results suggest that investors place relatively more weight on recent returns when forming expectations about future returns.

[Insert Table 9 Here]

Another notable result is the difference in the estimates of b and DOX between investor types. Since b captures a level effect and DOX captures a slope effect, Da et al. (2021) argue that the appropriate measure for the degree of extrapolation is $b \times$ DOX. Using this measure, we find that regardless of the number of lags, individual investors extrapolate more than institutional investors.

Overall, the results from the non-linear regression model in Table 9 indicate that both individual and institutional investors extrapolate past returns when forming expectations about future one-month returns.

2. Expectations and Future Returns

Next, we analyze how these expectations relate to future realized returns. To do so, we regress future one-month DJIA returns, estimated over a 21-day period, on average expectations from the prior day.

The first three columns in Panel A of Table 10 show that raw average expectations are negatively associated with future returns, albeit insignificantly so. In the last six columns of Panel A, we use the non-linear regression model discussed above to decompose raw expectations into fitted values and a residual component. For this panel, we decompose raw expectations using the past four weekly returns. As in Cassella et al. (2023), we interpret the fitted values (i.e., predicted expectations) as the extrapolative component of raw expectations. By decomposing raw expectations, we can identify which component drives the slightly negative relation between expectations and future returns.[12]

The middle three columns in Panel A of Table 10 present results from regressions of future returns on the extrapolative component of expectations. All three coefficients are statistically significant, and each is at least twice as large in magnitude as their counterparts in the first three columns. The R^2 estimates in the middle three columns are also at least six times higher than those in the first three columns. In contrast, the last three columns in Panel A show that the residual component of expectations is not associated with future returns. All coefficients and R^2 estimates are close to zero.

[Insert Table 10 Here]

In Panels B, C, and D of Table 10, we estimate the same specifications but vary the number of past weekly returns used to estimate the two components of raw expectations. The middle three columns in these panels show that the coefficients and R^2 estimates associated with the extrapolative component are qualitatively similar to those in Panel A. The last three columns show that, regardless of the number of past weekly returns used to decompose raw expectations, the coefficients and R^2 estimates associated with the residual component remain close to zero.

[12] To ensure that no future information is used in the decomposition, we use an expanding window of observations through the date of the expectation to obtain the parameters used to estimate the extrapolative and residual components. To alleviate concerns related to small sample sizes, the first estimation window includes the first 500 observations for individual and institutional investor expectations and the first 700 observations for all investor expectations (i.e., about 20% of the overall sample for each).

Overall, the results in Table 10 show that the extrapolative component of expectations is negatively associated with future one-month returns. This negative relation suggests that the extrapolative component is overextrapolative in nature. Since convexity is also negatively associated with future returns, the results imply that overextrapolation of past returns may also be partially responsible for the negative relation between convexity and future returns.

3. Expectations and Convexity

To assess whether the relation between convexity and future returns aligns with overextrapolation, we regress expectations on convexity. In Panel A of Table 11, we estimate convexity using daily data in the four weeks (i.e., 21 days) leading up to the record date of expectations in the survey. In the first three columns, we find a positive relation between convexity and raw expectations. The coefficient in the third column suggests that a one standard deviation change in convexity is associated with a 0.26 percentage point change in expectations.

In the last six columns of Panel A, we present regression results for the extrapolative and residual components of expectations, which we estimate using four lags of weekly returns. The middle three columns in Panel A show that convexity has substantially greater explanatory power for the extrapolative component than for raw expectations. For instance, the fourth column indicates that convexity explains over 9% of the variation in the extrapolative component of individual investor expectations, which is nearly nine times more than in raw expectations. In contrast, the last three columns show that convexity has little explanatory power for the residual component of expectations.

[Insert Table 11 Here]

In the bottom three panels of Table 11, we present similar results with two changes: (1) we adjust the number of past weekly returns used to estimate the two components of raw expectations, and (2) we extend the estimation of convexity to the past eight, twelve, and sixteen weeks to match the lag structure used for decomposing raw returns. These panels show that the relation between the extrapolative component and convexity strengthens with the number of lags. For example, Panels B and C show R^2 estimates climb to 32.44% and 42.85% when using eight and twelve weekly lags, respectively. The last three columns in these panels confirm that despite using more weekly lags, there is still no relation between the residual component and convexity.

In summary, Table 11 shows that convexity is strongly positively correlated with the extrapolative component of expectations. Since overextrapolation often coincides with future return reversals (e.g., Barberis et al. (2015)), this finding supports the idea that overextrapolation may partly explain the negative relation between convexity and future returns.

C. Firm Level

We now examine the relation between expectations and convexity at the firm level. Since we are unaware of any survey data on one-month ahead returns at the individual stock level, we use rankings from Forcerank, which is an online crowdsourcing platform. Forcerank organizes weekly contests in which participants rank ten stocks based on their expected returns over the following week.

These data include nearly 250,000 rankings across 1,433 unique contests (or over 14,000 average rankings per contest) and span from March 2016 to February 2018.

1. Rankings and Past Returns

We again use an exponential decay function to estimate the extent to which investors extrapolate past returns. However, we substitute return expectations with contest average Forcerank rankings:

$$(8) \qquad Rank_{itc} = a + b \sum_{j=1}^{N} w_j AR^{W}_{t-j,t-j+1} + \varepsilon^{Exp}_{itc}$$

$$w_j = \frac{\lambda^j}{\sum_{k=1}^{N} \lambda^k},$$

in which $Rank_{itc}$ represents the contest average Forcerank ranking of stock i with start date t in contest c. Since these rankings are contest specific, we follow Da et al. (2021) and use contest-adjusted past returns ($AR^{W}_{t-j,t-j+1}$). We estimate our model using past weekly returns spanning four, eight, and twelve weeks.

[Insert Table 12 Here]

The results from this non-linear regression, presented in Table 12, show that Forcerank participants extrapolate past returns when forming expectations about future one-week rankings. Specifically, the estimates for both DOX and b are positive and significant across all three lag levels. Importantly, our coefficient estimates align qualitatively with those of Da et al. (2021).[13]

[13] The high t-statistic for the coefficient on a reflects that a represents the average ranking for each stock, which is always between one and ten, ensuring it is significantly far from zero.

2. Rankings and Future Returns

Next, we analyze the relation between these rankings and future one-week returns by regressing future returns on contest average rankings.

The first column in Panel A of Table 13 shows that raw contest average rankings are not significantly related to future one-week returns. In the last two columns of Panel A, we decompose raw rankings into the extrapolative and residual components using the non-linear model described above. We begin with the past four weekly returns for the decomposition and use an expanding window of observations up to the contest start date to estimate the parameters for each component. The first estimation window includes the first ten start dates (i.e., 920 observations). The second column shows that the extrapolative component is significantly negatively associated with future returns, and the third column shows that the residual component has no relation to future returns. These results are consistent with those observed at the aggregate level.

[Insert Table 13 Here]

In Panels B and C of Table 13, we use eight and twelve lags to decompose rankings into their extrapolative and residual components. The results for the extrapolative component in these panels are not as significant as those in Panel A, but they are qualitatively similar.

In summary, the results in Table 13 show that the extrapolative component of Forcerank rankings is negatively associated with future one-week returns. The negative sign of this relation suggests that the extrapolative component is overextrapolative in nature. These findings suggest that the negative relation between convexity and future returns at the firm level may also be driven, in part, by overextrapolation of past returns.

3. Rankings and Convexity

We now use the non-linear regression model to examine the relation between convexity and the extrapolative component of Forcerank rankings. Table 14 presents results from regressions of Forcerank rankings on convexity. As before, we estimate convexity using daily data over the same time period used to decompose the rankings.

In Panel A of Table 14, we estimate convexity using daily data from the month (i.e., 21 days) leading up to the start of the Forcerank contest and decompose rankings using the past four weeks. The first column shows that convexity is positively and significantly associated with raw rankings. In the second column, we present results from a regression of the extrapolative component of Forcerank rankings on convexity. The coefficient on convexity in this column is the same as in the first column, but its statistical significance and explanatory power are much larger. For example, the average R^2 estimate in the second column (27.96%) is almost ten times higher than in the first column (2.93%). The last column shows no relation between the residual component of Forcerank rankings and convexity.

[Insert Table 14 Here]

In Panels B and C of Table 14, we estimate the same specification but construct convexity and decompose Forcerank rankings using the past eight or twelve weeks of data. The relations between convexity and raw rankings, the extrapolative component, and the residual component are roughly the same as in Panel A. If anything, the relation between convexity and the extrapolative component is stronger in these panels.

Overall, the results in Table 14 show that convexity is highly correlated with the extrapolative component of Forcerank rankings. Although these expectations are for one-week ahead returns and are ordinal, we argue that these results still provide evidence that convexity captures the extrapolative component of return expectations at the firm level. This component is negatively associated with future returns, which suggests that it is overextrapolative in nature.

VII. Conclusion

In this paper, we show that the curvature of stock price paths contains information about future returns at both the aggregate and firm levels. Specifically, we find that price-path convexity, which we construct using daily closing prices, is negatively related to future returns. At the aggregate level, convexity outperforms many commonly used predictors. At the firm level, this relation holds regardless of the cumulative return during the period in which convexity is estimated and remains robust across a range of tests.

We argue that this relation is driven primarily by mispricing rather than risk considerations or illiquidity. Our empirical evidence suggests that this mispricing largely stems from investors overextrapolating past returns. We conclude that behavioral biases play a significant role in short-horizon return predictability.

References

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets 5*(1), 31–56.

Amromin, G. and S. A. Sharpe (2014). From the Horse's Mouth: Economic Conditions and Investor Expectations of Risk and Return. *Management Science 60*(4), 845–866.

Andrews, D. W. K. (1991). Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation. *Econometrica 59*(3), 817–858.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The Cross-Section of Volatility and Expected Returns. *The Journal of Finance 61*(1), 259–299.

Avramov, D., T. Chordia, and A. Goyal (2006). Liquidity and Autocorrelations in Individual Stock Returns. *The Journal of Finance 61*(5), 2365–2394.

Baba-Yara, F., M. Boons, and A. Tamoni (2024). Persistent and transitory components of firm characteristics: Implications for asset pricing. *Journal of Financial Economics 154*, 103808.

Bacchetta, P., E. Mertens, and E. van Wincoop (2009). Predictability in financial markets: What do survey expectations tell us? *Journal of International Money and Finance 28*(3), 406–426.

Bali, T. G., N. Cakici, and R. F. Whitelaw (2011). Maxing out: Stocks as lotteries and the cross-section of expected returns. *Journal of Financial Economics 99*(2), 427–446.

Bansal, R., D. Kiku, and A. Yaron (2012). An Empirical Evaluation of the Long-Run Risks Model for Asset Prices. *Critical Finance Review 1*(1), 183–221.

Bansal, R. and A. Yaron (2004). Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *The Journal of Finance 59*(4), 1481–1509.

Barberis, N. (2018). Psychology-based Models of Asset Prices and Trading Volume. In B. D. Bernheim, S. DellaVigna, and D. Laibson (Eds.), *Handbook of Behavioral Economics*, Chapter 2, pp. 79–175. North Holland: Elsevier Science.

Barberis, N., R. Greenwood, L. Jin, and A. Shleifer (2015). X-CAPM: An extrapolative capital asset pricing model. *Journal of Financial Economics 115*(1), 1–24.

Barberis, N., R. Greenwood, L. Jin, and A. Shleifer (2018). Extrapolation and bubbles. *Journal of Financial Economics 129*(2), 203–227.

Black, F. (1986). Noise. *The Journal of Finance 41*(3), 528–543.

Campbell, J. Y. (2018). *Financial Decisions and Markets.* Princeton University Press.

Campbell, J. Y. and J. H. Cochrane (1999). By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy 107*(2), 205–251.

Campbell, J. Y., S. J. Grossman, and J. Wang (1993). Trading Volume and Serial Correlation in Stock Returns. *The Quarterly Journal of Economics 105*(4), 905–939.

Campbell, J. Y. and S. B. Thompson (2008). Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average? *The Review of Financial Studies 21*(4), 1509–1531.

Carhart, M. M. (1997). On Persistence in Mutual Fund Performance. *The Journal of Finance 52*(1), 57–82.

Cassella, S., Z. Chen, H. Gulen, and R. Petkova (2023). Extrapolators at the Gate: Market-wide Misvaluation and the Value Premium. Working paper.

Cassella, S. and H. Gulen (2018). Extrapolation Bias and the Predictability of Stock Returns by Price-Scaled Variables. *The Review of Financial Studies 31*(11), 4345–4397.

Choi, J. J. and T. M. Mertens (2019). Extrapolative Expectations and the Equity Premium. Working paper.

Clark, T. E. and K. D. West (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics 138*(1), 291–311.

Cooper, M. J., H. Gulen, and M. J. Schill (2008). Asset Growth and the Cross-Section of Stock Returns. *The Journal of Finance 63*(4), 1609–1651.

Da, Z., X. Huang, and L. J. Jin (2021). Extrapolative beliefs in the cross-section: What can we learn from the crowds? *Journal of Financial Economics 140*(1), 175–196.

Da, Z., Q. Liu, and E. Schaumburg (2014). A Closer Look at the Short-Term Return Reversal. *Management Science 60*(3), 658–674.

Davis, J. L., E. F. Fama, and K. R. French (2000). Characteristics, Covariances, and Average Returns: 1929 to 1997. *The Journal of Finance 55*(1), 389–406.

De Bondt, W. and R. Thaler (1985). Does the Stock Market Overreact? *The Journal of Finance 40*(3), 793–805.

Engelberg, J., R. D. McLean, J. Pontiff, and M. C. Ringgenberg (2023). Do Cross-Sectional Predictors Contain Systematic Information? *Journal of Financial and Quantitative Analysis 58*(3), 1172–1201.

Fama, E. F. and K. R. French (1988). Dividend yields and expected stock returns. *Journal of Financial Economics 22*(1), 3–25.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of Financial Economics 116*(1), 1–22.

Fama, E. F. and J. D. MacBeth (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy 81*(3), 607–636.

Gabaix, X. (2008). Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-finance. *American Economic Review 98*(2), 64–67.

Gabaix, X. (2012). Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance. *The Quarterly Journal of Economics 127*(2), 645–700.

Greenwood, R. and A. Shleifer (2014). Expectations of Returns and Expected Returns. *The Review of Financial Studies 27*(3), 714–746.

Greenwood, R., A. Shleifer, and Y. You (2019). Bubbles for Fama. *Journal of Financial Economics 131*(1), 20–43.

Hansen, P. R. and A. Timmerman (2012). Choice of Sample Split in Out-of-Sample Forecast Evaluation. Working paper, European University Institute.

Hartzmark, S. M. and D. H. Solomon (2019). The Dividend Disconnect. *The Journal of Finance 74*(5), 2153–2199.

Hartzmark, S. M. and D. H. Solomon (2022). Reconsidering Returns. *The Review of Financial Studies 35*(1), 343–393.

Harvey, D. I., S. J. Leybourne, and P. Newbold (1998). Tests for Forecast Encompassing. *Journal of Business & Economics Statistics 16*(2), 254–259.

Hirshleifer, D., P.-H. Hsu, and D. Li (2013). Innovative efficiency and stock returns. *Journal of Financial Economics 107*(3), 632–654.

Hirshleifer, D., J. Li, and J. Yu (2015). Asset pricing in production economies with extrapolative expectations. *Journal of Monetary Economics 76*, 87–106.

Hou, K., C. Xue, and L. Zhang (2015). Digesting Anomalies: An Investment Approach. *The Review of Financial Studies 28*(3), 650–705.

Hou, K., C. Xue, and L. Zhang (2020). Replicating Anomalies. *The Review of Financial Studies 33*(5), 2019–2133.

Inoue, A. and L. Kilian (2005). In-Sample or Out-of-Sample Tests of Predictability: Which One Should We Use? *Econometric Review 23*, 371–402.

Jegadeesh, N. (1990). Evidence of Predictable Behavior of Security Returns. *The Journal of Finance 45*(3), 881–898.

Jegadeesh, N. and S. Titman (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *The Journal of Finance 48*(1), 65–91.

Jegadeesh, N. and S. Titman (2001). Profitability of Momentum Strategies: An Evaluation of Alternative Explanations. *The Journal of Finance 56*(2), 699–720.

Jin, L. J. and P. Sui (2022). Asset pricing with return extrapolation. *Journal of Financial Economics 145*(2), 273–295.

Koijen, R. S., M. Schmeling, and E. B. Vrugt (2015). Survey Expectations of Returns and Asset Pricing Puzzles. Working paper.

Kumar, A. (2009). Hard-to-Value Stocks, Behavioral Biases, and Informed Trading. *Journal of Financial and Quantitative Analysis 44*(6), 1375–1401.

Lakonishok, J., A. Shleifer, and R. W. Vishny (1994). Contrarian Investment, Extrapolation, and Risk. *The Journal of Finance 49*(5), 1541–1578.

Lehmann, B. N. (1990). Fads, Martingales, and Market Efficiency. *The Quarterly Journal of Economics 105*(1), 1–28.

Lustig, H. N. and S. G. Van Nieuwerburgh (2005). Housing Collateral, Consumption Insurance, and Risk Premia: An Empirical Perspective. *The Journal of Finance 60*(3), 1167–1219.

Nagel, S. (2012). Evaporating Liquidity. *The Review of Financial Studies 25*(7), 2005–2039.

Newey, W. K. and K. D. West (1987). A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica 55*(3), 703–708.

Newey, W. K. and K. D. West (1994). Automatic Lag Selection in Covariance Matrix Estimation. *Review of Economic Studies 61*(4), 631–653.

Pástor, L. and R. F. Stambaugh (2003). Liquidity Risk and Expected Stock Returns. *Journal of Political Economy 111*(3), 642–685.

Pastor, L. and P. Veronesi (2009). Learning in Financial Markets. *Annual Review of Financial Economics 1*(1), 361–381.

Rapach, D. E., J. K. Strauss, and G. Zhou (2010). Out-of-Sample Equity Premium Prediction: Combination Forecasts and Links to the Real Economy. *The Review of Financial Studies 23*(2), 821–862.

Rosenberg, B., K. Reid, and R. Lanstein (1985). Persuasive Evidence of Market Inefficiency. *Journal of Portfolio Management 11*, 9–16.

Rosenberg, B. and A. Rudd (1982). Factor-Related and Specific Returns of Common Stocks: Serial Correlation and Market Inefficiency. *The Journal of Finance 37*, 543–554.

Rossi, B. and A. Inoue (2012). Out-of-Sample Forecast Tests Robust to the Choice of Window Size. *Journal of Business & Economics Statistics 30*(3), 432–453.

Schwert, G. W. (2003). Anomalies and market efficiency. In G. Constantinides, M. Harris, and R. Stulz (Eds.), *Handbook of the Economics of Finance*, Volume 1, Chapter 15, pp. 939–974. North Holland: Elsevier Science.

Shiller, R. J. (1984). Stock Prices and Social Dynamics. *Brookings Papers on Economic Activity 1984*(2), 457–510.

Shue, K. and R. R. Townsend (2021). Can the Market Multiply and Divide? Non-Proportional Thinking in Financial Markets. *The Journal of Finance 76*(5), 2307–2357.

Shumway, T. (1997). The Delisting Bias in CRSP Data. *The Journal of Finance 52*(1), 327–340.

Stiglitz, J. E. (1989). Using Tax Policy To Curb Speculative Short-Term Trading. *Journal of Financial Services Research 3*, 101–115.

Subrahmanyam, A. (2005). Distinguishing Between Rationales for Short-Horizon Predictability of Stock Returns. *The Financial Review 40*(1), 11–35.

Summers, L. H. and V. P. Summers (1989). When Financial Markets Work Too Well: A Cautious Case For a Securities Transactions Tax. *Journal of Financial Services Research 3*, 261–286.

Timmermann, A. G. (1993). How Learning in Financial Markets Generates Excess Volatility and Predictability in Stock Prices. *The Quarterly Journal of Economics 108*(4), 1135–1145.

Wachter, J. A. (2013). Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility? *The Journal of Finance 68*(3), 987–1035.

Welch, I. and A. Goyal (2008). A Comprehensive Look at The Empirical Performance of Equity Premium Prediction. *The Review of Financial Studies 21*(4), 1455–1508.

Table 1. Aggregate Stock Returns

This table presents results associated with predicting one-month aggregate stock returns. Panels A and A2 present in-sample results from ordinary least squares regressions of future excess returns (ERET) on predictor variables. Independent variables are standardized to mean zero and unit standard deviation. Panels B and B2 present out-of-sample R^2 estimates (Campbell and Thompson (2008)) for forecast periods beginning January 1975, January 1980, or January 1985. Statistical significance of the out-of-sample R^2 estimate is determined by the mean squared prediction error-adjusted test statistic of Clark and West (2007). Panels C and C2 present the estimated weight placed on a predictive regression forecast based on convexity and a predictive regression forecast based on one of the other 15 predictors (i.e., forecast encompassing tests). Statistical significance of λ, which is the weight placed on convexity in the forecast encompassing tests, is determined by the forecast encompassing test statistic of Harvey et al. (1998). CON (convexity) is estimated using one month of S&P 500 daily closing prices, ERET is the S&P 500 log excess return, DP is the log dividend-to-price ratio, DY is the log dividend yield, EP is the log earnings-to-price ratio, DE is the log dividend payout ratio, BM is the book-to-market value ratio for the Dow Jones Industrial Average, VOL is the sum of squared daily returns on the S&P 500, NTIS is net equity expansion, TBL is the interest rate on a three-month Treasury bill, LTY is the long-term government bond yield, LTR is the return on long-term government bonds, TMS is the long-term government bond yield minus the Treasury bill rate, DFY is the difference between Moody's BAA- and AAA-rated corporate bond yields, DFR is the long-term corporate bond return minus the long-term government bond return, and INFL is inflation calculated from the CPI for all urban consumers. For all panels, holding periods are from July 1963 through December 2022, but in Panels A2, B2, and C2, holding periods exclude the onset of the COVID-19 pandemic (i.e., March and April 2020). t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)). *, **, and *** indicate significance at the 90%, 95%, and 99% levels. All R^2 estimates are percentages.

Panel A: In-Sample

	CON	ERET	DP	DY	EP	DE	BM	VOL	NTIS	TBL	LTY	LTR	TMS	DFY	DFR	INFL
$\hat{\beta}$	0.40*	0.05	0.14	0.14	0.04	0.13	0.02	-0.10	0.15	0.29**	0.19	0.40**	0.28**	-0.21	0.15	0.31*
	(1.59)	(0.25)	(0.78)	(0.81)	(0.16)	(0.61)	(0.09)	(-0.26)	(0.67)	(1.78)	(1.14)	(2.43)	(1.66)	(-0.87)	(0.67)	(1.53)
$R^2{}_{IS}$	0.85	0.01	0.10	0.10	0.01	0.09	0.00	0.05	0.12	0.45	0.19	0.84	0.42	0.24	0.12	0.51

Panel B: Out-of-Sample

	CON	ERET	DP	DY	EP	DE	BM	VOL	NTIS	TBL	LTY	LTR	TMS	DFY	DFR	INFL
$R^2{}_{OS}$ (1975)	0.39	-0.36	-0.68	-0.70	-1.29	-2.28	-0.88	-3.41	-0.35	-1.52	-1.54	-0.57	-0.38	-0.38	-0.65	-0.77
$R^2{}_{OS}$ (1980)	0.53	-0.35	-0.72	-0.73	-0.51	-0.92	-0.53	-3.58	-0.45	-0.36	-0.35	-0.41	-0.14	-0.83	-0.66	-0.10
$R^2{}_{OS}$ (1985)	0.61	-0.36	-0.84	-0.87	-0.51	-1.08	-0.61	-4.08	-0.92	0.00	0.03	-0.65	-0.65	-0.89	-0.60	-0.28

Panel C: Forecast Encompassing

	CON	ERET	DP	DY	EP	DE	BM	VOL	NTIS	TBL	LTY	LTR	TMS	DFY	DFR	INFL
$\hat{\lambda}$ (1975)	-	0.87*	0.80**	0.80**	1.00**	1.00***	1.00**	1.00*	0.64**	0.91***	1.00***	0.62***	0.62**	0.65**	0.98**	0.72**
$\hat{\lambda}$ (1980)	-	0.90*	0.88**	0.88**	0.89*	0.93**	0.93**	1.00**	0.67**	0.72**	0.83*	0.62***	0.62**	0.80**	1.00**	0.62**
$\hat{\lambda}$ (1985)	-	0.90*	0.94**	0.95**	0.88*	0.96**	0.96**	1.00**	0.75***	0.67*	0.72	0.71**	0.75**	0.93**	0.97*	0.66**

Panel A2: In-Sample (Excluding March and April 2020)

	CON	ERET	DP	DY	EP	DE	BM	VOL	NTIS	TBL	LTY	LTR	TMS	DFY	DFR	INFL
$\hat{\beta}$	0.59***	0.06	0.13	0.14	0.03	0.13	0.01	-0.39	0.15	0.29**	0.19	0.40***	0.27**	-0.20	0.23	0.29*
	(3.62)	(0.33)	(0.74)	(0.79)	(0.15)	(0.59)	(0.07)	(-2.13)	(0.67)	(1.76)	(1.14)	(2.48)	(1.61)	(-0.81)	(1.03)	(1.44)
$R^2{}_{IS}$	1.90	0.02	0.09	0.10	0.01	0.09	0.00	0.84	0.12	0.45	0.19	0.86	0.40	0.21	0.29	0.45

Panel B2: Out-of-Sample (Excluding March and April 2020)

	CON	ERET	DP	DY	EP	DE	BM	VOL	NTIS	TBL	LTY	LTR	TMS	DFY	DFR	INFL
$R^2{}_{OS}$ (1975)	1.79***	-0.33	-0.71	-0.73	-1.32	-2.35	-0.90	-1.59	-0.36	-1.58	-1.74	-0.56	-0.42	-0.44	-0.35	-0.88
$R^2{}_{OS}$ (1980)	2.09***	-0.32	-0.75	-0.76	-0.53	-0.97	-0.55	-1.56	-0.46	-0.38	-0.35	-0.40	-0.17	-0.91	-0.32	-0.19
$R^2{}_{OS}$ (1985)	2.38***	-0.33	-0.89	-0.91	-0.53	-1.12	-0.64	-1.81	-0.96	-0.02	0.05	-0.65	-0.71	-0.98	-0.21	-0.40

Panel C2: Forecast Encompassing (Excluding March and April 2020)

	CON	ERET	DP	DY	EP	DE	BM	VOL	NTIS	TBL	LTY	LTR	TMS	DFY	DFR	INFL
$\hat{\lambda}$ (1975)	-	1.00***	1.00***	1.00***	1.00***	1.00***	1.00***	1.00**	0.90***	1.00***	1.00***	0.80***	0.84***	0.91***	1.00***	1.00***
$\hat{\lambda}$ (1980)	-	1.00***	1.00***	1.00***	1.00***	1.00***	1.00***	1.00**	0.93***	1.00***	1.00***	0.82***	0.90***	1.00***	1.00***	0.93***
$\hat{\lambda}$ (1985)	-	1.00***	1.00***	1.00***	1.00***	1.00***	1.00***	1.00**	1.00***	1.00***	1.00***	0.98***	1.00***	1.00***	1.00***	1.00***

Table 2. Portfolio Returns

This table presents future returns of convexity portfolios. Stocks are sorted into convexity quintiles (CON1, CON2, etc.) at the end of each month. Panel A presents future one-month returns after sorting by one-month convexity. Panel B presents future ten-day returns after sorting by ten-day convexity. Panel C presents future five-day returns after sorting by five-day convexity. All returns are from value-weighted portfolios. Holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)).

Panel A: One-Month Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	1.24	1.06	0.97	0.83	0.41	0.84
	(5.88)	(6.44)	(6.10)	(4.84)	(1.79)	(6.29)
Sharpe ratio	0.16	0.16	0.15	0.11	0.01	0.23

Panel B: Ten-Day Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	0.81	0.62	0.57	0.48	0.17	0.64
	(4.72)	(4.63)	(4.50)	(3.59)	(0.99)	(7.31)
Sharpe ratio	0.16	0.14	0.13	0.10	0.00	0.26

Panel C: Five-Day Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	0.54	0.45	0.41	0.39	0.17	0.37
	(4.65)	(5.00)	(4.89)	(4.18)	(1.55)	(4.94)
Sharpe ratio	0.16	0.16	0.15	0.13	0.03	0.19

Table 3. Short-Term Reversal

This table presents future returns of portfolios that are independently double sorted into convexity quintiles (CON1, CON2, etc.) and lagged return quintiles (SREV1, SREV2, etc.) at the end of each month. Panel A presents future one-month returns after sorting by one-month convexity and lagged one-month returns. Panel B presents future ten-day returns after sorting by ten-day convexity and lagged ten-day returns. Panel C presents future five-day returns after sorting by five-day convexity and lagged five-day returns. CON1-CON5 (SREV1-SREV5) presents future returns of the zero-investment portfolio that buys CON1 (SREV1) and sells short CON5 (SREV5) within each column (row). All returns are from value-weighted portfolios. Holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)).

Panel A: One-Month Convexity and One-Month Returns

	SREV1	SREV2	SREV3	SREV4	SREV5	SREV1-SREV5
CON1	1.25	1.37	1.28	1.26	1.14	0.10
	(4.55)	(5.68)	(6.00)	(5.86)	(4.53)	(0.52)
CON2	1.34	1.17	1.15	1.12	0.67	0.67
	(6.11)	(6.17)	(7.01)	(6.19)	(3.21)	(4.10)
CON3	1.40	1.24	0.95	0.81	0.79	0.61
	(6.15)	(7.20)	(5.76)	(4.93)	(4.08)	(3.28)
CON4	1.00	1.01	0.91	0.76	0.43	0.57
	(4.52)	(5.44)	(5.29)	(3.97)	(2.14)	(3.36)
CON5	0.27	0.54	0.48	0.35	0.34	-0.06
	(0.94)	(2.16)	(2.32)	(1.51)	(1.40)	(-0.35)
CON1-CON5	0.97	0.83	0.78	0.92	0.81	
	(5.56)	(5.01)	(4.60)	(5.69)	(5.08)	

Panel B: Ten-Day Convexity and Ten-Day Returns

	SREV1	SREV2	SREV3	SREV4	SREV5	SREV1-SREV5
CON1	1.20	1.05	0.81	0.70	0.57	0.63
	(5.77)	(5.93)	(4.76)	(4.15)	(3.16)	(5.14)
CON2	1.10	0.83	0.66	0.44	0.18	0.93
	(5.92)	(5.96)	(5.18)	(2.97)	(1.14)	(7.36)
CON3	1.00	0.72	0.60	0.41	0.10	0.91
	(5.77)	(5.35)	(4.76)	(3.06)	(0.60)	(6.81)
CON4	0.81	0.70	0.45	0.40	-0.01	0.82
	(4.66)	(4.90)	(3.36)	(2.88)	(-0.07)	(6.48)
CON5	0.39	0.34	0.22	0.11	-0.19	0.58
	(1.86)	(1.90)	(1.24)	(0.62)	(-1.03)	(4.38)
CON1-CON5	0.81	0.71	0.59	0.59	0.76	
	(6.37)	(5.83)	(4.90)	(5.65)	(6.65)	

Panel C: Five-Day Convexity and Five-Day Returns

	SREV1	SREV2	SREV3	SREV4	SREV5	SREV1-SREV5
CON1	1.00	0.65	0.65	0.43	0.23	0.76
	(7.12)	(5.08)	(5.63)	(3.65)	(1.74)	(7.49)
CON2	0.88	0.57	0.43	0.35	0.04	0.85
	(7.39)	(6.05)	(4.66)	(3.69)	(0.37)	(9.19)
CON3	0.80	0.52	0.39	0.29	0.06	0.74
	(6.93)	(5.66)	(4.66)	(3.12)	(0.65)	(8.36)
CON4	0.80	0.53	0.38	0.34	0.06	0.74
	(6.55)	(5.47)	(3.99)	(3.72)	(0.48)	(7.82)
CON5	0.56	0.40	0.18	0.12	-0.12	0.68
	(4.19)	(3.31)	(1.55)	(1.01)	(-1.01)	(6.92)
CON1-CON5	0.44	0.25	0.47	0.32	0.35	
	(5.02)	(2.49)	(5.37)	(3.55)	(3.78)	

Table 4. Factor-Model Adjusted Returns

This table presents future factor model-adjusted returns of convexity portfolios. Stocks are sorted into convexity quintiles (CON1, CON2, etc.) at the end of each month, and value-weighted portfolios are held for one month. Convexity is estimated using one month of daily closing prices. CAPM, FF3, and FF5 are average monthly alphas from regressing excess portfolio returns on excess market return, the Fama and French (1993) three-factor model (excess market return plus size (SMB) and value (HML)), and the Fama and French (2015) five-factor model (three-factor model plus profitability (RMW) and asset growth (CMA)). Other alphas are from regressions that augment the five-factor model with factors for momentum (UMD), short-term reversal (SREV), long-term reversal (LREV), or liquidity (LIQ). Holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)).

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
CAPM	0.25	0.17	0.11	-0.05	-0.59	0.83
	(2.97)	(3.54)	(2.90)	(-1.02)	(-6.60)	(5.99)
FF3	0.24	0.16	0.09	-0.05	-0.57	0.81
	(3.08)	(3.42)	(2.46)	(-1.10)	(-6.17)	(5.63)
FF5	0.34	0.15	0.05	-0.08	-0.54	0.88
	(4.35)	(3.18)	(1.17)	(-1.53)	(-6.05)	(6.16)
FF5+UMD	0.35	0.15	0.02	-0.09	-0.52	0.87
	(4.21)	(3.13)	(0.40)	(-1.60)	(-5.51)	(5.64)
FF5+SREV	0.38	0.15	0.04	-0.09	-0.58	0.96
	(4.65)	(3.26)	(1.00)	(-1.69)	(-6.31)	(6.41)
FF5+LREV	0.34	0.15	0.05	-0.08	-0.54	0.88
	(4.35)	(3.17)	(1.17)	(-1.53)	(-6.11)	(6.21)
FF5+LIQ	0.34	0.19	0.05	-0.09	-0.60	0.94
	(4.14)	(3.73)	(1.09)	(-1.69)	(-6.45)	(6.31)

Table 5. Portfolio Characteristics

This table presents one-month return characteristics of the zero-investment convexity portfolio and several commonly-studied factors. The zero-investment convexity portfolio is value-weighted and is formed by sorting convexity into quintiles at the end of each month, buying stocks in the lowest convexity quintile, and selling short stocks in the highest convexity quintile. Convexity is estimated using one month of daily closing prices. Factors include excess market return (Market), size (SMB), value (HML), profitability (RMW), asset growth (CMA), momentum (UMD), short-term reversal (SREV), long-term reversal (LREV), and liquidity (LIQ). Panel A presents results for all holding periods. Panel B presents results from holding periods during recessions (as identified by the NBER). Holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags in Panel A and four lags in Panel B (Newey and West (1987)).

Panel A: Full Sample

Portfolio	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Kurtosis
Convexity	0.84	3.57	0.23	-0.30	7.08
	(6.29)				
Market	0.55	4.50	0.12	-0.50	4.74
	(3.19)				
SMB	0.22	3.02	0.07	0.34	6.12
	(1.82)				
HML	0.31	2.97	0.10	0.12	5.33
	(2.30)				
RMW	0.28	2.22	0.13	-0.28	14.24
	(3.06)				
CMA	0.30	2.06	0.15	0.32	4.38
	(3.38)				
UMD	0.64	4.19	0.15	-1.28	12.88
	(3.99)				
SREV	0.46	3.13	0.15	0.44	8.81
	(4.29)				
LREV	0.25	2.65	0.09	0.64	5.50
	(2.20)				
LIQ	0.33	3.49	0.10	-0.15	4.22
	(2.32)				

Panel B: Recession Periods

Portfolio	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Kurtosis
Convexity	1.60	4.65	0.34	-0.03	3.19
	(3.18)				
Market	-0.88	6.51	-0.13	0.07	2.67
	(-1.22)				
SMB	0.06	3.66	0.02	0.59	4.11
	(0.17)				
HML	0.56	3.71	0.15	-0.61	3.66
	(1.51)				
RMW	0.37	2.24	0.17	0.36	3.22
	(1.40)				
CMA	0.83	2.58	0.32	-0.12	2.63
	(2.55)				
UMD	0.54	6.59	0.08	-1.80	10.06
	(0.76)				
SREV	0.48	4.61	0.10	0.52	3.86
	(1.07)				
LREV	0.94	3.54	0.26	0.48	4.42
	(2.60)				
LIQ	0.30	5.08	0.06	-0.44	3.51
	(0.51)				

Table 6. Fama-MacBeth Regressions

This table presents time-series averages of coefficient estimates from monthly cross-sectional regressions of future one-month returns on convexity and controls. Regressions are estimated using weighted least squares with market capitalization as the weight. Convexity is estimated using one month of daily closing prices. Size is the natural log of market capitalization, which is stock price multiplied by shares outstanding. Book-to-market is book equity divided by market capitalization (Davis, Fama, and French (2000)). Profitability is income before extraordinary items scaled by book equity (Hou, Xue, and Zhang (2015)). Asset growth is the percentage change in total assets over the last two fiscal years (Cooper, Gulen, and Schill (2008)). Momentum is the cumulative raw return beginning twelve months ago through the month before last (Jegadeesh and Titman (1993, 2001)). 1M ret. is one-month return (i.e., short-term reversal). Illiquidity is the absolute stock return in the month divided by trading volume in the same month (Amihud (2002)). IV is idiosyncratic volatility, which is the standard deviation of residuals from a regression of daily stock returns in excess of the risk-free rate on the Fama and French (1993) three-factor model in the previous month (Ang, Hodrick, Xing, and Zhang (2006)). Skewness is the total skewness of daily stock returns over the previous twelve months. Max return is the maximum daily return in the month (Bali, Cakici, and Whitelaw (2011)). 1D return, 5D return, and 10D return in Panel B are lagged one-day, five-day, and ten-day returns. Specifications in Panel B include all control variables from the last column in Panel A. Holding periods are from July 1963 through December 2022. All independent variables are as of the end of the previous month, winsorized at the 1% and 99% levels each month, and standardized to mean zero and unit standard deviation. t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)). All R^2 estimates are percentages.

Panel A: Firm-Specific Characteristics

	1	2	3	4	5	6	7	8
Convexity	-0.42	-0.44	-0.45	-0.44	-0.44	-0.44	-0.43	-0.45
	(-6.91)	(-9.12)	(-9.89)	(-9.05)	(-9.85)	(-9.32)	(-9.78)	(-11.15)
Size		-0.10	-0.11	-0.12	-0.17	-0.10	-0.14	-0.19
		(-1.55)	(-1.65)	(-1.68)	(-2.82)	(-1.55)	(-2.24)	(-3.07)
Book-to-market		0.21	0.17	0.21	0.19	0.21	0.19	0.15
		(3.76)	(3.03)	(3.72)	(3.34)	(3.65)	(3.38)	(2.68)
Profitability		0.19	0.17	0.19	0.18	0.19	0.18	0.15
		(4.74)	(4.06)	(4.73)	(4.54)	(4.57)	(4.58)	(3.86)
Asset growth		-0.07	-0.08	-0.08	-0.06	-0.08	-0.07	-0.07
		(-1.81)	(-1.90)	(-1.92)	(-1.50)	(-2.01)	(-1.67)	(-1.85)
Momentum		0.48	0.47	0.48	0.51	0.51	0.49	0.52
		(4.44)	(4.13)	(4.41)	(4.95)	(4.54)	(4.81)	(4.77)
1M return			-0.37					-0.44
			(-6.70)					(-6.90)
Illiquidity				0.00				0.16
				(0.00)				(0.54)
IV					-0.23			-0.48
					(-3.24)			(-5.82)
Skewness						-0.08		-0.07
						(-2.24)		(-2.09)
Max return							-0.19	0.36
							(-3.02)	(4.35)
Average R^2	1.62	10.86	12.16	11.13	11.86	11.44	11.82	14.50
Observations	2,436,978	1,875,358	1,875,358	1,782,120	1,875,357	1,875,355	1,875,357	1,782,118

Panel B: Partial-Month Return Controls

	1	2	3
Convexity	-0.35	-0.26	-0.42
	(-8.36)	(-4.98)	(-6.96)
1D return	-0.31		
	(-7.81)		
5D return		-0.30	
		(-4.38)	
10D return			-0.04
			(-0.47)
Controls	Yes	Yes	Yes
Average R^2	15.02	15.21	15.17
Observations	1,782,118	1,782,118	1,782,118

Table 7. Aggregate-Level Explanations

This table presents results from regressions of future one-month aggregate stock returns on convexity after interacting convexity with dummies indicating whether the illiquidity proxy (Amihud or RV) or investor optimism proxy (NIPO or S) is below or above its time-series median. Future one-month return is the S&P 500 log excess return. Convexity is estimated using one month of S&P 500 daily closing prices. Amihud is the difference between the average Amihud (2002) illiquidity measure and its moving average in the previous 12 months. RV is the realized volatility on the S&P 500 index. NIPO is the number of initial public offerings. S is the share of equity issues in total equity and debt issues. Low is a dummy that equals one if the value of the proxy is below the time-series median and zero otherwise. High is one minus Low. t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)). All R^2 estimates are percentages.

	1	2	3	4
Convexity × low	-1.05	-0.68	-0.26	-0.14
	(-3.34)	(-1.83)	(-0.98)	(-0.33)
Convexity × high	-0.12	-0.34	-0.65	-0.63
	(-0.38)	(-1.20)	(-1.61)	(-2.53)
Proxy	Amihud	RV	NIPO	S
R^2	1.78	0.94	1.77	1.16
Observations	714	714	714	714

Table 8. Firm-Level Explanations

This table presents future one-month returns of portfolios that are independently double sorted on convexity (CON1, CON2, etc.) and a proxy for illiquidity or mispricing. In Panel A, we sort stocks into convexity quintiles and illiquidity quintiles (ILLIQ1, ILLIQ2, etc.). We proxy for illiquidity using the measure of Amihud (2002). In Panel B, we sort stocks into convexity quintiles and bid-ask spread quintiles (SPRD1, SPRD2, etc.). We estimate the bid-ask spread as the difference between the ask and the bid scaled by price. In Panel C, we sort stocks into convexity quintiles and attention quintiles (ATT1, ATT2, etc.). We proxy for attention with the number of IBES analyst estimates over the previous twelve months. In Panel D, we sort stocks into convexity quintiles and valuation uncertainty quintiles (VU1, VU2, etc.). We proxy for valuation uncertainty with the number of months since entering the CRSP database. CON1-CON5 presents future one-month returns of the zero-investment portfolio that buys portfolio CON1 and sells short portfolio CON5 within each column. All returns are from value-weighted portfolios. Holding periods are from July 1963 through December 2022 in Panels A and D, January 1993 through December 2022 in Panel B, and January 1984 through December 2022 in Panel C. t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)).

Panel A: Amihud

	ILLIQ1	ILLIQ2	ILLIQ3	ILLIQ4	ILLIQ5
CON1	1.16	1.35	1.39	1.48	1.44
	(5.46)	(5.98)	(6.05)	(6.30)	(6.33)
CON2	1.04	1.11	1.14	1.23	1.25
	(6.35)	(6.10)	(5.61)	(5.81)	(5.90)
CON3	0.95	1.03	0.97	1.12	1.03
	(6.02)	(5.90)	(4.91)	(5.54)	(5.06)
CON4	0.84	0.79	0.85	0.88	0.78
	(4.90)	(4.33)	(4.21)	(4.20)	(3.52)
CON5	0.50	0.39	0.32	0.01	-0.23
	(2.18)	(1.66)	(1.31)	(0.05)	(-0.87)
CON1-CON5	0.66	0.95	1.07	1.46	1.67
	(4.40)	(6.41)	(8.29)	(10.99)	(9.90)

Panel B: Bid-Ask Spread

	SPRD1	SPRD2	SPRD3	SPRD4	SPRD5
CON1	1.06	0.91	0.93	1.09	1.05
	(3.14)	(2.82)	(2.76)	(3.26)	(3.07)
CON2	1.04	0.98	0.81	0.87	1.10
	(4.24)	(4.07)	(2.75)	(3.00)	(4.34)
CON3	0.76	0.99	1.01	1.01	0.74
	(2.76)	(3.99)	(3.21)	(3.63)	(2.82)
CON4	0.82	0.75	0.64	0.60	0.30
	(3.08)	(2.77)	(2.09)	(1.87)	(1.04)
CON5	0.50	0.21	0.21	-0.05	-0.49
	(1.41)	(0.57)	(0.57)	(-0.12)	(-1.27)
CON1-CON5	0.57	0.69	0.72	1.14	1.55
	(2.32)	(2.84)	(3.27)	(3.82)	(5.58)

Panel C: Attention

	ATT1	ATT2	ATT3	ATT4	ATT5
CON1	1.27	1.25	1.58	1.20	1.17
	(4.16)	(4.19)	(5.37)	(4.65)	(4.42)
CON2	1.08	1.22	1.10	1.04	1.18
	(4.77)	(5.25)	(4.85)	(5.19)	(5.94)
CON3	1.10	1.07	1.01	0.97	1.04
	(5.11)	(4.88)	(4.80)	(5.09)	(5.16)
CON4	0.57	0.73	0.79	0.88	0.88
	(2.11)	(2.93)	(3.53)	(4.40)	(4.05)
CON5	-0.10	0.03	0.32	0.30	0.56
	(-0.26)	(0.09)	(1.08)	(1.05)	(1.92)
CON1-CON5	1.37	1.22	1.26	0.90	0.61
	(4.33)	(5.67)	(5.43)	(4.81)	(3.36)

Panel D: Valuation Uncertainty

	VU1	VU2	VU3	VU4	VU5
CON1	1.28	1.30	1.33	1.23	1.21
	(6.30)	(5.52)	(5.73)	(4.49)	(4.00)
CON2	1.02	1.12	1.14	1.25	1.00
	(6.47)	(5.93)	(5.38)	(5.47)	(4.20)
CON3	0.95	1.01	1.11	0.95	1.06
	(6.21)	(5.40)	(5.54)	(4.37)	(4.69)
CON4	0.75	0.88	0.90	0.90	0.89
	(4.68)	(4.44)	(4.03)	(3.80)	(3.45)
CON5	0.51	0.48	0.36	0.18	0.13
	(2.43)	(2.06)	(1.28)	(0.68)	(0.40)
CON1-CON5	0.77	0.82	0.97	1.05	1.08
	(4.84)	(5.25)	(5.51)	(6.31)	(6.47)

Table 9. DJIA Expectations and Past Returns

This table presents results from non-linear least squares regressions of daily average expectations of one-month returns on past weekly returns (both of the Dow Jones Industrial Average). Our non-linear model is of the following form:

$$Exp_t = a + b \sum_{j=1}^{N} w_j R^W_{t-j,t-j+1} + \varepsilon^{Exp}_{it}$$
$$w_j = \frac{\lambda^j}{\sum_{k=1}^{N} \lambda^k},$$

in which Exp_t is the average one-month return expectation on day t, N is the number of weekly lags, and $R^W_{t-j,t-j+1}$ is the j-lagged weekly return. The weight parameter w_j is defined by λ, which estimates the extent to which past returns are weighted. Weekly returns are estimated using returns over a five-day period. DOX is the degree of extrapolative weighting (Cassella and Gulen (2018)). Individual expectations are from January 1999 through February 2022. Institutional expectations are from August 1993 through February 2022. t-statistics below coefficients are adjusted for clustering standard errors by month. *, **, and *** indicate significance at the 90%, 95%, and 99% levels. All R^2 estimates are percentages.

	Individual				Institutional				All			
	1	2	3	4	5	6	7	8	9	10	11	12
DOX (1-λ)	0.29***	0.18***	0.13***	0.11***	0.40***	0.36***	0.28***	0.28***	0.35***	0.23***	0.15***	0.15***
	(2.92)	(5.07)	(5.06)	(5.77)	(3.40)	(4.42)	(3.42)	(3.50)	(4.50)	(4.78)	(4.74)	(5.29)
a	0.0004	-0.0003	-0.0009	-0.0013*	0.0003	0.0001	-0.0000	-0.0000	0.0005	0.0001	-0.0003	-0.0005
	(0.45)	(-0.32)	(-1.14)	(-1.66)	(0.41)	(0.18)	(-0.04)	(-0.04)	(0.86)	(0.18)	(-0.60)	(-0.90)
b	0.69***	1.10***	1.46***	1.73***	0.37***	0.44***	0.54***	0.54***	0.50***	0.74***	1.00***	1.11***
	(7.14)	(7.83)	(8.95)	(10.20)	(4.78)	(4.77)	(4.41)	(4.20)	(6.51)	(7.99)	(8.70)	(8.83)
Lags	4	8	12	16	4	8	12	16	4	8	12	16
R^2	4.05	5.33	6.14	6.50	2.21	2.30	2.42	2.41	3.53	4.20	4.81	4.92
Observations	2,538	2,538	2,538	2,538	2,529	2,529	2,529	2,529	3,572	3,572	3,572	3,572

Table 10. DJIA Expectations and Future Returns

This table presents results from regressions of realized future one-month returns on daily average expectations of future one-month returns (both of the Dow Jones Industrial Average). The first three columns of each panel present results using raw expectations. The middle three columns of each panel present results using the predicted component of expectations of future one-month returns from the non-linear regression model described in Table 9. The last three columns present results using the residual between the raw expectation and the predicted component of expectations. In Panels A, B, C, and D, we use four, eight, twelve, and sixteen lags of weekly returns, respectively, to estimate the extrapolative and residual components. Individual expectations are from January 1999 through February 2022. Institutional expectations are from August 1993 through February 2022. t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using eight lags (Newey and West (1987)). *, **, and *** indicate significance at the 90%, 95%, and 99% levels. All R^2 estimates are percentages.

Panel A: Past Four Weekly Returns

	Raw			Extrapolative			Residual		
	Individual	Institutional	All	Individual	Institutional	All	Individual	Institutional	All
	1	2	3	4	5	6	7	8	9
Forecast	-0.25	-0.12	-0.14	-0.66**	-0.53*	-0.57**	-0.07	-0.03	-0.02
	(-1.54)	(-0.99)	(-1.10)	(-2.13)	(-1.72)	(-2.10)	(-0.55)	(-0.27)	(-0.14)
R^2	0.32	0.07	0.11	2.22	1.55	1.81	0.03	0.00	0.00
Observations	2,039	2,030	2,873	2,039	2,030	2,873	2,039	2,030	2,873

Panel B: Past Eight Weekly Returns

	Raw			Extrapolative			Residual		
	Individual	Institutional	All	Individual	Institutional	All	Individual	Institutional	All
	1	2	3	4	5	6	7	8	9
Forecast	-0.25	-0.12	-0.14	-0.76**	-0.50	-0.58*	-0.02	-0.03	-0.00
	(-1.54)	(-0.99)	(-1.10)	(-2.37)	(-1.57)	(-1.95)	(-0.21)	(-0.28)	(-0.02)
R^2	0.32	0.07	0.11	3.01	1.37	1.82	0.00	0.01	0.00
Observations	2,039	2,030	2,873	2,039	2,030	2,873	2,039	2,030	2,873

Panel C: Past Twelve Weekly Returns

	Raw			Extrapolative			Residual		
	Individual	Institutional	All	Individual	Institutional	All	Individual	Institutional	All
	1	2	3	4	5	6	7	8	9
Forecast	-0.25	-0.12	-0.14	-0.77**	-0.47	-0.59**	0.01	-0.03	0.01
	(-1.54)	(-0.99)	(-1.10)	(-2.40)	(-1.47)	(-1.96)	(0.06)	(-0.31)	(0.09)
R^2	0.32	0.07	0.11	3.08	1.21	1.88	0.00	0.01	0.00
Observations	2,039	2,030	2,873	2,039	2,030	2,873	2,039	2,030	2,873

Panel D: Past Sixteen Weekly Returns

	Raw			Extrapolative			Residual		
	Individual	Institutional	All	Individual	Institutional	All	Individual	Institutional	All
	1	2	3	4	5	6	7	8	9
Forecast	-0.25	-0.12	-0.14	-0.68**	-0.48	-0.54*	-0.02	-0.03	-0.00
	(-1.54)	(-0.99)	(-1.10)	(-2.00)	(-1.50)	(-1.75)	(-0.13)	(-0.29)	(-0.02)
R^2	0.32	0.07	0.11	2.39	1.28	1.60	0.00	0.01	0.00
Observations	2,039	2,030	2,873	2,039	2,030	2,873	2,039	2,030	2,873

Table 11. DJIA Expectations and Convexity

This table presents results from regressions of expectations of future one-month returns on convexity (both of the Dow Jones Industrial Average). The first three columns of each panel present results using raw expectations. The middle three columns of each panel present results using the predicted component of expectations of future one-month returns from the non-linear regression model described in Table 9. The last three columns present results using the residual between the raw expectation and the predicted component of expectations. In Panels A, B, C, and D, we estimate convexity and decompose expectations using four, eight, twelve, and sixteen lags of weekly data and returns, respectively. Individual expectations are from January 1999 through February 2022. Institutional expectations are from August 1993 through February 2022. t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using eight lags (Newey and West (1987)). *, **, and *** indicate significance at the 90%, 95%, and 99% levels. All R^2 estimates are percentages.

Panel A: Past Four Weekly Returns

	Raw			Extrapolative			Residual		
	Individual	Institutional	All	Individual	Institutional	All	Individual	Institutional	All
	1	2	3	4	5	6	7	8	9
Convexity	0.34***	0.21**	0.26***	0.27***	0.13***	0.17***	0.07	0.08	0.09
	(2.82)	(2.51)	(3.60)	(4.06)	(5.09)	(4.45)	(0.62)	(0.97)	(1.53)
R^2	1.10	0.55	0.65	9.56	9.39	7.63	0.05	0.08	0.08
Observations	2,039	2,030	2,873	2,039	2,030	2,873	2,039	2,030	2,873

Panel B: Past Eight Weekly Returns

	Raw			Extrapolative			Residual		
	Individual	Institutional	All	Individual	Institutional	All	Individual	Institutional	All
	1	2	3	4	5	6	7	8	9
Convexity	0.43***	0.25**	0.31***	0.43***	0.26***	0.32***	-0.01	-0.01	-0.02
	(3.26)	(2.18)	(2.66)	(4.94)	(7.09)	(6.16)	(-0.09)	(-0.10)	(-0.17)
R^2	1.72	0.86	1.28	18.89	32.44	24.10	0.00	0.00	0.00
Observations	2,039	2,030	2,873	2,039	2,030	2,873	2,039	2,030	2,873

Panel C: Past Twelve Weekly Returns

	Raw			Extrapolative			Residual		
	Individual	Institutional	All	Individual	Institutional	All	Individual	Institutional	All
	1	2	3	4	5	6	7	8	9
Convexity	0.47***	0.32***	0.35***	0.57***	0.31***	0.40***	-0.11	0.01	-0.05
	(3.08)	(3.61)	(3.44)	(5.47)	(7.87)	(6.67)	(-0.85)	(0.08)	(-0.58)
R^2	2.09	1.37	1.70	28.70	42.85	33.19	0.12	0.00	0.04
Observations	2,039	2,030	2,873	2,039	2,030	2,873	2,039	2,030	2,873

Panel D: Past Sixteen Weekly Returns

	Raw			Extrapolative			Residual		
	Individual	Institutional	All	Individual	Institutional	All	Individual	Institutional	All
	1	2	3	4	5	6	7	8	9
Convexity	0.55***	0.29***	0.40***	0.62***	0.30***	0.44***	-0.07	-0.02	-0.03
	(3.96)	(3.70)	(4.92)	(6.21)	(8.01)	(7.83)	(-0.56)	(-0.23)	(-0.39)
R^2	2.90	1.11	2.23	31.28	41.79	38.32	0.05	0.00	0.02
Observations	2,039	2,030	2,873	2,039	2,030	2,873	2,039	2,030	2,873

Table 12. Forcerank Rankings and Past Returns

This table presents results from non-linear least squares regressions of contest average Forcerank rankings of one-week returns on past weekly returns. Our non-linear model is of the following form:

$$Rank_{itc} = a + b \sum_{j=1}^{N} w_j AR^{W}_{t-j,t-j+1} + \varepsilon^{Exp}_{itc}$$
$$w_j = \frac{\lambda^j}{\sum_{k=1}^{N} \lambda^k},$$

in which $Rank_{itc}$ represents the contest average Forcerank ranking of stock i with start date t in contest c. N is the number of weekly lags, and $R^{W}_{t-j,t-j+1}$ is the j-lagged weekly return. The weight parameter w_j is defined by λ, which estimates the extent to which past returns are weighted. Weekly returns are estimated using returns over a five-day period. DOX is the degree of extrapolative weighting (Cassella and Gulen (2018)). Forcerank rankings are from March 2016 through February 2018. t-statistics below coefficients are adjusted for clustering standard errors by contest start date. *, **, and *** indicate significance at the 90%, 95%, and 99% levels. All R^2 estimates are percentages.

	1	2	3
DOX (1-λ)	0.55***	0.46***	0.45***
	(11.20)	(11.23)	(11.40)
a	5.51***	5.51***	5.51***
	(6666)	(6665)	(6665)
b	9.79***	11.63***	11.93***
	(12.28)	(12.21)	(11.88)
Lags	4	8	12
R^2	4.34	4.46	4.47
Observations	14,202	14,202	14,202

Table 13. Forcerank Rankings and Future Returns

This table presents time-series averages of coefficient estimates from weekly cross-sectional regressions of realized future one-week returns on contest average Forcerank rankings of the same. The first column of each panel presents results using raw expectations. The middle column of each panel presents results using the predicted component of expectations of future one-week returns from the non-linear regression model described in Table 12. The last column in each panel presents results using the residual between the raw expectation and the predicted component of expectations. In Panels A, B, and C, we use four, eight, and twelve lags of weekly returns to estimate the extrapolative and residual components. Forcerank rankings are from March 2016 through February 2018. t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using four lags (Newey and West (1987)). *, **, and *** indicate significance at the 90%, 95%, and 99% levels. All R^2 estimates are percentages.

Panel A: Past Four Weekly Returns

	Raw	Extrapolative	Residual
	1	2	3
Forecast	-0.03	-0.66**	-0.02
	(-0.77)	(-2.21)	(-0.52)
Average R^2	1.12	1.58	1.03
Observations	13,462	13,462	13,462

Panel B: Past Eight Weekly Returns

	Raw	Extrapolative	Residual
	1	2	3
Forecast	-0.03	-0.48	-0.03
	(-0.77)	(-1.59)	(-0.69)
Average R^2	1.12	1.67	2.22
Observations	13,462	13,462	13,462

Panel C: Past Twelve Weekly Returns

	Raw	Extrapolative	Residual
	1	2	3
Forecast	-0.03	-0.48	-0.03
	(-0.77)	(-1.58)	(-0.69)
Average R^2	1.12	1.67	1.00
Observations	13,462	13,462	13,462

Table 14. Forcerank Rankings and Convexity

This table presents time-series averages of coefficient estimates from weekly cross-sectional regressions of contest average Forcerank rankings of future one-week returns on convexity. The first column of each panel presents results using raw expectations. The middle column of each panel presents results using the predicted component of expectations of future one-week returns from the non-linear regression model described in Table 12. The last column presents results using the residual between the raw expectation and the predicted component of expectations. In Panels A, B, and C, we estimate convexity and decompose expectations using four, eight, and twelve lags of weekly data and returns, respectively. Forcerank rankings are from March 2016 through February 2018. t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using four lags (Newey and West (1987)). *, **, and *** indicate significance at the 90%, 95%, and 99% levels. All R^2 estimates are percentages.

Panel A: Past Four Weekly Returns

	Raw	Extrapolative	Residual
	1	2	3
Convexity	0.12***	0.12***	0.00
	(3.76)	(13.67)	(0.03)
Average R^2	2.93	27.96	1.74
Observations	13,462	13,462	13,462

Panel B: Past Eight Weekly Returns

	Raw	Extrapolative	Residual
	1	2	3
Convexity	0.11***	0.13***	-0.02
	(4.58)	(13.30)	(-0.84)
Average R^2	2.36	34.21	1.32
Observations	13,462	13,462	13,462

Panel C: Past Twelve Weekly Returns

	Raw	Extrapolative	Residual
	1	2	3
Convexity	0.11***	0.13***	-0.02
	(5.34)	(13.58)	(-0.73)
Average R^2	2.20	31.34	1.02
Observations	13,462	13,462	13,462

Internet Appendix

Table IA1. Summary Statistics: Aggregate-Level

This table presents summary statistics of convexity and its correlations with the other 15 predictor variables at the aggregate level. Convexity (multiplied by 100) is estimated using one month of S&P 500 daily closing prices, ERET is the S&P 500 log excess return, DP is the log dividend-to-price ratio, DY is the log dividend yield, EP is the log earnings-to-price ratio, DE is the log dividend payout ratio, BM is the book-to-market value ratio for the Dow Jones Industrial Average, VOL is the sum of squared daily returns on the S&P 500, NTIS is net equity expansion, TBL is the interest rate on a three-month Treasury bill, LTY is the long-term government bond yield, LTR is the return on long-term government bonds, TMS is the long-term government bond yield minus the Treasury bill rate, DFY is the difference between Moody's BAA- and AAA-rated corporate bond yields, DFR is the long-term corporate bond return minus the long-term government bond return, and INFL is inflation calculated from the CPI for all urban consumers. Sample is from July 1963 through December 2022.

Panel A: Convexity

P1	P10	P25	P50	P75	P90	P99	Mean
-3.01	-1.43	-0.74	-0.12	0.65	1.39	3.71	-0.03

Panel B: Correlations with Convexity

ERET	DP	DY	EP	DE
-0.15	0.00	-0.01	0.00	0.00

BM	VOL	NTIS	TBL	LTY
-0.03	0.35	-0.07	0.01	0.02

LTR	TMS	DFY	DFR	INFL
0.12	0.03	-0.07	-0.18	-0.07

Table IA2. Summary Statistics: Firm-Level

This table presents summary statistics of convexity and its correlations with our control variables. Convexity (multiplied by 100) is estimated using one month of daily closing prices. Size is the natural log of market capitalization, which is stock price multiplied by shares outstanding. Book-to-market is book equity divided by market capitalization (Davis et al. (2000)). Profitability is income before extraordinary items scaled by book equity (Hou et al. (2015)). Asset growth is the percentage change in total assets over the last two fiscal years (Cooper et al. (2008)). Momentum is the cumulative raw return beginning twelve months ago through the month before last (Jegadeesh and Titman (1993, 2001)). 1M ret. is one-month return (i.e., short-term reversal). Illiquidity is the absolute stock return in the month divided by trading volume in the same month (Amihud (2002)). IV is idiosyncratic volatility, which is the standard deviation of residuals from a regression of daily stock returns in excess of the risk-free rate on the Fama and French (1993) three-factor model in the previous month (Ang et al. (2006)). Skewness is the total skewness of daily stock returns over the previous twelve months. Max return is the maximum daily return in the month (Bali et al. (2011)). Sample is from July 1963 through December 2022. All independent variables are winsorized at the 1% and 99% levels each month.

Panel A: Convexity

P1	P10	P25	P50	P75	P90	P99	Mean
-8.88	-3.43	-1.54	0.04	1.60	3.44	8.55	0.01

Panel B: Correlations with Convexity

Size	Book-to-market	Profitability	Asset growth	Momentum
0.00	-0.01	0.00	0.01	0.01

1M ret.	Illiquidity	IV	Skewness	Max return
-0.02	0.02	0.01	0.00	0.01

Table IA3. Cash Dividends-Adjusted Convexity

This table presents future returns of convexity portfolios after also adjusting for cash dividends. Stocks are sorted into convexity quintiles (CON1, CON2, etc.) at the end of each month. Panel A presents future one-month returns after sorting by one-month convexity. Panel B presents future ten-day returns after sorting by ten-day convexity. Panel C presents future five-day returns after sorting by five-day convexity. All returns are from value-weighted portfolios. Holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)).

Panel A: One-Month Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	1.24	1.08	0.96	0.83	0.40	0.84
	(5.84)	(6.61)	(6.04)	(4.79)	(1.77)	(6.33)
Sharpe ratio	0.16	0.16	0.14	0.10	0.01	0.23

Panel B: Ten-Day Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	0.81	0.63	0.57	0.48	0.16	0.65
	(4.75)	(4.66)	(4.49)	(3.59)	(0.93)	(7.47)
Sharpe ratio	0.16	0.14	0.13	0.10	0.00	0.26

Panel C: Five-Day Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	0.54	0.46	0.41	0.39	0.17	0.37
	(4.68)	(5.05)	(4.75)	(4.19)	(1.56)	(4.93)
Sharpe ratio	0.16	0.16	0.15	0.13	0.03	0.19

Table IA4. Portfolio Returns: Equal-Weighted Portfolios

This table presents future returns of convexity portfolios. Stocks are sorted into convexity quintiles (CON1, CON2, etc.) at the end of each month, at which point equal-weighted portfolios are formed. Panel A presents future one-month returns after sorting by one-month convexity. Panel B presents future ten-day returns after sorting by ten-day convexity. Panel C presents future five-day returns after sorting by five-day convexity. Holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)).

Panel A: One-Month Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	1.44	1.26	1.20	1.03	0.36	1.07
	(5.84)	(6.08)	(6.10)	(4.91)	(1.38)	(9.21)
Sharpe ratio	0.17	0.18	0.18	0.13	0.00	0.38

Panel B: Ten-Day Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	0.98	0.69	0.64	0.55	0.08	0.90
	(5.27)	(4.54)	(4.25)	(3.43)	(0.40)	(10.35)
Sharpe ratio	0.18	0.15	0.14	0.11	-0.02	0.43

Panel C: Five-Day Returns

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
Raw returns	0.71	0.47	0.38	0.39	0.14	0.58
	(6.11)	(5.01)	(4.37)	(4.11)	(1.15)	(7.96)
Sharpe ratio	0.21	0.16	0.14	0.13	0.02	0.37

Table IA5. Factor-Model Adjusted Returns: Equal-Weighted portfolios

This table presents future factor model-adjusted returns of convexity portfolios. Stocks are sorted into convexity quintiles (CON1, CON2, etc.) at the end of each month. Convexity is estimated using one month of daily closing prices. CAPM, FF3, and FF5 are average monthly alphas from regressing excess portfolio returns on excess market return, the Fama and French (1993) three-factor model (excess market return plus size (SMB) and value (HML)), and the Fama and French (2015) five-factor model (three-factor model plus profitability (RMW) and asset growth (CMA)). Other alphas are from regressions that augment the five-factor model with factors for momentum (UMD), short-term reversal (SREV), long-term reversal (LREV), or liquidity (LIQ). All returns are from equal-weighted portfolios. Holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)).

	CON1	CON2	CON3	CON4	CON5	CON1-CON5
CAPM	0.41	0.35	0.32	0.11	-0.67	1.08
	(3.05)	(3.26)	(3.20)	(1.16)	(-5.07)	(9.21)
FF3	0.32	0.20	0.16	-0.02	-0.75	1.08
	(4.85)	(4.35)	(3.75)	(-0.48)	(-9.78)	(8.70)
FF5	0.41	0.17	0.11	-0.04	-0.69	1.10
	(5.80)	(3.66)	(2.47)	(-0.77)	(-8.65)	(8.44)
FF5+UMD	0.43	0.19	0.13	-0.00	-0.65	1.08
	(5.50)	(4.52)	(3.41)	(-0.08)	(-8.91)	(8.12)
FF5+SREV	0.42	0.15	0.08	-0.09	-0.74	1.16
	(5.94)	(3.24)	(1.79)	(-1.51)	(-8.74)	(8.55)
FF5+LREV	0.41	0.17	0.11	-0.04	-0.69	1.10
	(5.77)	(3.74)	(2.57)	(-0.80)	(-8.99)	(8.56)
FF5+LIQ	0.43	0.17	0.11	-0.06	-0.71	1.13
	(5.63)	(3.57)	(2.31)	(-1.05)	(-8.44)	(8.28)

Table IA6. Subsamples

This table shows the future one-month returns of the zero-investment convexity portfolio across various subsample cuts. The zero-investment convexity portfolio is value-weighted and is formed by sorting convexity into quintiles at the end of each month, buying stocks in the lowest convexity quintile, and selling short stocks in the highest convexity quintile. Convexity is estimated using one month of daily closing prices. In Panels A and B, holding periods are as defined in that row. In Panels C and D, holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation (Newey and West (1987)). The number of lags used is determined using the rule $4(T/100)^{2/9}$, in which T is the number of months in the sample.

Panel A: Main Sample

Time period	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Kurtosis
July 1963 - December 2022	0.84	3.57	0.23	-0.30	7.08
	(6.45)				
July 1963 - December 1992	0.97	2.78	0.35	0.03	4.23
	(6.66)				
January 1993 - December 2022	0.70	4.21	0.17	-0.32	6.46
	(3.33)				

Panel B: Earlier U.S. Sample

Time period	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Kurtosis
July 1926 - June 1963	0.62	4.91	0.13	0.25	24.57
	(2.74)				
July 1926 - December 1945	0.71	6.49	0.11	0.17	15.28
	(1.75)				
January 1946 - June 1963	0.51	2.03	0.25	0.08	4.33
	(3.46)				

Panel C: Size

Time period	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Kurtosis
Small	1.11	3.02	0.37	0.55	10.59
	(8.27)				
Medium	0.98	3.28	0.30	0.04	6.82
	(7.76)				
Large	0.76	3.94	0.19	-0.35	6.46
	(5.29)				

Panel D: Season

Time period	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Kurtosis
January	1.83	4.06	0.45	-0.18	4.07
	(3.79)				
February-December	0.75	3.51	0.21	-0.34	7.54
	(5.40)				

Table IA7. Bid-Ask Bounce

This table presents future one-month returns of the zero-investment convexity portfolio after accounting for a bid-ask bounce. The zero-investment convexity portfolio is value-weighted and is formed by sorting convexity into quintiles at the end of each month, buying stocks in the lowest convexity quintile, and selling short stocks in the highest convexity quintile. Convexity is estimated using one month of daily closing prices. Panel A presents results after skipping the first day after portfolio formation. Panel B presents results using the bid-ask midpoint. Holding periods are from July 1963 through December 2022 in Panel A and January 1993 through December 2022 in Panel B. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags in Panel A and and five lags in Panel B (Newey and West (1987)).

Panel A: Skip First Day

Portfolio	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Kurtosis
Full month	0.84	3.57	0.23	-0.30	7.08
	(6.29)				
Skip first day	0.66	3.59	0.18	-0.44	7.99
	(4.93)				

Panel B: Bid-Ask Bounce

Portfolio	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Kurtosis
Close-price returns	0.70	4.21	0.17	-0.32	6.46
	(3.33)				
Bid-ask midpoint returns	0.62	4.30	0.14	-0.27	6.34
	(2.97)				

Table IA8. Acceleration

This table presents future one-month returns of portfolios that are independently double sorted into convexity quintiles (CON1, CON2, etc.) and acceleration quintiles (ACCEL1, ACCEL2, etc.) at the end of each month. Acceleration is cumulative one-month return minus the cumulative ten-day return to begin the month. CON1-CON5 (ACCEL1-ACCEL5) presents future returns of the zero-investment portfolio that buys CON1 (ACCEL1) and sells short CON5 (ACCEL5) within each column (row). All returns are from value-weighted portfolios. Holding periods are from July 1963 through December 2022. t-statistics below returns are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)).

	ACCEL1	ACCEL2	ACCEL3	ACCEL4	ACCEL5	ACCEL1-ACCEL5
CON1	1.34	1.30	1.24	1.00	0.93	0.42
	(5.74)	(6.26)	(5.81)	(3.90)	(3.11)	(1.75)
CON2	1.32	1.16	0.98	0.99	0.82	0.50
	(6.43)	(6.89)	(5.73)	(5.33)	(3.37)	(2.66)
CON3	1.37	1.20	1.04	0.76	0.86	0.51
	(5.88)	(6.94)	(6.33)	(4.55)	(4.33)	(2.73)
CON4	0.77	1.24	0.89	0.81	0.57	0.20
	(2.75)	(6.22)	(4.97)	(4.33)	(2.93)	(0.84)
CON5	0.49	0.53	0.59	0.58	0.25	0.24
	(1.17)	(1.78)	(2.27)	(2.70)	(1.06)	(0.80)
CON1-CON5	0.85	0.77	0.65	0.42	0.68	
	(2.86)	(3.91)	(3.17)	(2.27)	(2.92)	

Table IA9. Long-Run Returns

This table presents results from regressions of future one-month returns on convexity. In Panel A, we present results from ordinary least squares regressions of the future one-month S&P 500 log excess return on convexity estimated using one month of S&P 500 daily closing prices. In Panel B, we present time-series averages of coefficient estimates from monthly cross-sectional regressions of future one-month returns on convexity. Cross-sectional regressions are estimated using weighted least squares with market capitalization as the weight. Sample is from July 1963 through December 2022. t-statistics below coefficients are adjusted for heteroskedasticity and autocorrelation using six lags (Newey and West (1987)). All R^2 estimates are percentages.

Panel A: Aggregate Stock Returns

	1Q ahead	2Q ahead	3Q ahead	4Q ahead
	1	2	3	4
Convexity	-0.24	0.02	0.11	-0.01
	(-1.97)	(0.17)	(1.01)	(-0.14)
R^2	0.94	0.00	0.20	0.00
Observations	712	709	706	703

Panel B: Firm Stock Returns

	1Q ahead	2Q ahead	3Q ahead	4Q ahead
	1	2	3	4
Convexity	-0.12	0.01	0.005	-0.06
	(-3.31)	(0.34)	(0.14)	(-1.47)
Average R^2	1.40	1.44	1.43	1.33
Observations	2,410,620	2,373,713	2,337,970	2,301,595

Table IA10. Expectations of One-Month Returns

This table presents summary statistics of expectations of future one-month returns of the Dow Jones Industrial Average. Panel A presents statistics on all expectations. Panel B presents statistics on daily average expectations.

Panel A: All Expectations

	Period	Mean	Median	Minimum	Maximum	Obs.
Individual	January 1999 - February 2022	0.06%	0.50%	-30%	25%	6,407
Institutional	August 1993 - February 2022	0.05%	0.10%	-30%	20%	5,686
All	August 1993 - February 2022	0.05%	0.50%	-30%	25%	12,093

Panel B: Daily Average Expectations

	Period	Mean	Median	Minimum	Maximum	Obs.
Individual	January 1999 - February 2022	0.15%	0.50%	-30%	22%	2,538
Institutional	August 1993 - February 2022	0.09%	0.33%	-20%	15%	2,529
All	August 1993 - February 2022	0.13%	0.33%	-20%	22%	3,572