The Crash Risk in Individual Stocks Embedded in Skewness Swap Returns

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Abstract

This paper investigates crash risk premiums in individual stocks using skewness swaps. These swaps involve buying a stock's risk-neutral skewness and receiving the realized skewness as a payoff. The strategy's returns, which measure the skewness risk premium, are found to be consistently large and positive. This suggests investors are concerned about potential crashes in individual stocks and require substantial compensation for bearing this risk. Notably, significant results are mainly observed after the 2007/2009 financial crisis, indicating changes in post-crisis option market dynamics. Cross-sectional determinants of skewness swap returns include measures of systematic crash risk and stock overvaluation.

Keywords: Skewness risk premium, skewness swap, financial crisis.

JEL classification codes: G01, G12, G13.

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I. Introduction

Stock markets do crash. Over the past century, the U.S. stock market has experienced at least five of these episodes, two of them just ten years apart: the global financial crisis crash in 2008 and the Covid pandemic crash in 2020.[1] Due to their rarity, they are very hard to predict and investors are left with the only option to hedge against these events, rather than avoiding them. The option market provides an ideal laboratory to study the pricing of crash risk. Investors can protect themselves from market crashes by purchasing put options or more sophisticated option portfolios which provide hedges against variance and skewness risk. Indeed, a significant body of literature focusing on variance risk, skewness risk, and crash risk has examined the returns generated by these hedging option portfolios, particularly variance swaps and skewness swaps (see e.g., Carr and Wu (2009), Kozhan et al. (2013), Schneider and Trojani (2019), Bakshi and Kapadia (2003), Bollerslev and Todorov (2011), Bondarenko (2014b)). These studies have consistently documented highly negative returns, indicating that investors are willing to pay exceptionally high premia to protect themselves against market downturns and turmoils.[2]

Individual stocks do crash as well and they do so even outside periods of market stress.[3] Surprisingly, the evidence on pricing of crash risk for individual stocks is

[1]Other notable events that are commonly considered stock market crashes include the 1929 Great Depression, Black Monday of 1987, and the 2001 dotcom bubble burst.

[2]For example, Carr and Wu (2009) and Kozhan et al. (2013) document that the average absolute monthly return of a variance swap and a skewness swap on the S&P500 index is approximately 66% and 42%, respectively.

[3]In the last twenty years, excluding crisis periods, an average of eighteen S&P500 stocks per month have experienced at least one daily drop greater than -10%.

limited. Early studies agree that single-stock put options are cheap and demand for out-of-the-money puts is low compared to out-of-the-money calls (see e.g., Bollen and Whaley (2004), Garleanu, Pedersen, and Poteshman (2009), Bakshi, Kapadia, and Madan (2003)). Prior studies that have employed option returns to assess the premium paid by investors to be protected against the risk associated with individual stocks have predominantly concentrated on measuring variance risk premiums using variance swaps or variance portfolios (Carr and Wu (2009), Duarte, Jones, and Wang (2023), Heston and Todorov (2023)).

This paper contributes to this literature by studying skewness swap returns on individual stocks within the S&P500 index from 2003 to 2020. Specifically, the paper's contribution can be divided into two key aspects.

First, it proposes a methodology to construct skewness swaps at the individual stock level. These swaps provide a direct exposure to skewness, while being designed to be independent of other moments. The methodology builds on Schneider and Trojani (2019), with the added feature of independence from the fourth moment as well. I provide both convergence results and a simulation analysis, which demonstrate the accuracy of the approach. Skewness swaps are trading strategies in which an investor buys the stock's risk-neutral skewness via an option portfolio at the beginning of the month and receives, as a payoff, the realized skewness of the stock at the end of the month. The strategy is a pure bet on skewness: it performs well when realized returns exhibit high positive skewness and incurs losses when they exhibit low negative skewness.[4] While skewness swaps have been used to study the skewness risk premium on the market index (see, e.g., Schneider and Trojani (2015), Kozhan et al. (2013)), to the best of my knowledge, they have not been employed to investigate the skewness risk premium at the individual stock level.

Empirical implementation of skewness swaps across the cross-section of stock options reveals consistently positive swap returns of remarkable magnitude, with an average Sharpe ratio of 0.53. Despite their overall positivity, the time-series pattern of swap returns displays a strong negative skew, characterized by infrequent but significant losses. These returns resemble the performance of selling crash insurance, which typically generates profits until the triggering event occurs – in this case, a crash of the stock, leading to substantial losses in the strategy. The notably high average returns strongly indicate the presence of a significant crash risk premium priced within individual stock options. Incorporating transaction costs, such as bid-ask spreads, does diminish profits to some extent but does not alter the primary finding: skewness swap returns remain positive for the majority of stocks. These results hold up even when accounting for the discrete nature of option prices and considering the limited range of moneyness. Importantly, these returns cannot be attributed to equity and variance risk premia alone, as their combined effect can only account for approximately 50% of the

[4]Formally, the payoff of a skewness swap is the difference between the realized skewness of the stock (denoted $\mathbb{P}$-skewness) and the risk-neutral skewness (denoted $\mathbb{Q}$-skewness). The payoff of the strategy is thus positive when the realized $\mathbb{P}$-skewness exceeds the $\mathbb{Q}$-skewness. This difference provides a direct and tradable measure of the compensation investors demand for bearing skewness risk, that is, the skewness risk premium. Just as variance swap returns have been used to measure the variance risk premium (e.g., Carr and Wu (2009), Martin (2017)), skewness swap returns capture the skewness risk premium. From a hedging perspective, an investor seeking protection against negative skewness would sell the skewness swap, thereby paying the skewness risk premium.

variability observed in skewness swap returns.

Secondly, skewness swap returns become especially pronounced following the 2007/2009 financial crisis. In fact, when the results are segmented into pre-crisis and post-crisis subsamples, it becomes evident that skewness swap returns experienced a significant right shift in distribution after the financial crisis. Moreover, a greater number of stocks display statistically significant positive swap returns during this post-crisis period. In addition to the increase in crash risk premium measured by skewness swap returns, I also document an increase in the price of crash risk in individual stocks. This is reflected by a more left-skewed implied volatility smile, driven by a higher price of deep out-of-the-money options.

Complementing this evidence, a portfolio sort analysis shows that swap returns are higher for stocks with greater systematic crash risk and overvaluation risk, a relationship that arises specifically in the post-crisis period. This pattern aligns with the broader macroeconomic environment: the years between the financial crisis and the Covid-19 crisis were marked by exceptionally low interest rates, which encouraged reaching-for-yield behavior and drove asset valuations higher (see, e.g., Hau and Lai (2016), Lian, Ma, and Wang (2019)). These findings reinforce the idea that skewness swap returns reflect the stock-level crash risk in the economy.

The paper is related to several strands of literature. The methodology implemented is the skewness swap of Schneider and Trojani (2019), which provide a simple approach to trading skewness in both the stock and option markets. This methodology is situated within an extensive body of research that stems from the findings of Breeden and Litzenberger (1978) and Carr and Madan (2001).[5] On the empirical front, numerous studies have explored the dynamics of variance swaps or variance portfolios applied to the market and individual stocks (see e.g., Heston, Jones, Khorram, Li, and Mo (2022), Duarte, Jones, and Wang (2023), Dew-Becker, Giglio, Le, and Rodriguez (2017), Aït-Sahalia, Karaman, and Mancini (2020), Filipović, Gourier, and Mancini (2016), Johnson (2017)). In contrast, the empirical investigation of skewness swaps has thus far only centered on skewness swaps on the market index (Kozhan et al. (2013), Orłowski, Schneider, and Trojani (2023), Schneider and Trojani (2019)).[6]

Furthermore, this paper is related to the broader literature exploring the risk premium associated with the skewness of individual stocks. Typically, individual stocks exhibit positive skewness (Bessembinder (2018)). However, the ex-ante measurement of skewness has traditionally presented challenges, leading to a variety of methodologies and occasionally conflicting findings. Notably, contradictory outcomes emerge when ex-ante skewness is computed from stock returns versus the option market. For example, Boyer, Mitton, and Vorkink (2010) find that stocks with the lowest expected idiosyncratic skewness outperform those with the highest idiosyncratic skewness. In contrast, Stilger, Kostakis, and Poon (2017) document that stocks exhibiting a greater

[5] This research has led to the formulation of techniques for trading variance or entropy, and, more broadly, higher-order moments using static option portfolios, see e.g., Carr and Wu (2009), Martin (2017), Schneider and Trojani (2019), Bondarenko (2014a), and Driessen et al. (2009) for trading correlation. The methodology of Schneider and Trojani (2019) is nested into the results of Bondarenko (2014a), who focus on variance trading but also provide a general result on how to trade a generic payoff.

[6] There is also a large literature that investigates crash risk and skewness risk in the S&P500 option market with parametric or semi-parametric methodologies other than skewness swaps, see e.g., Andersen et al. (2015b), Todorov (2010), Bates (1991), Bates (2012), Backus, Chernov, and Martin (2011), and Welch (2016).

ex-ante negative skewness in the risk-neutral distribution tend to yield lower subsequent returns.[7] The approach to skewness risk premium analysis outlined in this paper distinguishes itself from prior research by not depending on conventional portfolio-based cross-sectional methods intended to predict stock returns based on ex-ante skewness information. Instead, it employs an individualized trading strategy for each stock, specifically a skewness swap. The returns generated by this strategy directly capture the difference between the $\mathbb{P}$ skewness (skewness in the real-world measure), and $\mathbb{Q}$ skewness (skewness in the risk-neutral measure) of each individual stock. The positive swap returns documented in this paper align with investor preferences for positively skewed payoffs, as proposed in the model by Barberis and Huang (2008) (see also Eraker and Ready (2015)).

Finally, the paper is also related to the literature that connects crash risk and skewness risk with overvaluation risk, as is done for example in Chen, Hong, and Stein (2001), Stilger, Kostakis, and Poon (2017), and Rehman and Vilkov (2012).

The paper is structured as follows. Section II presents the methodology, while Section III provides an overview of the data used. Section IV presents the principal findings: Section IV. A analyzes the distribution of swap returns, Section IV. B compares

[7]Other relevant studies examining the pricing of individual skewness risk include Amaya, Christoffersen, Jacobs, and Vasquez (2015), who demonstrate that stocks with the lowest realized skewness outperformed those with the highest realized skewness. Additionally, Schneider, Wagner, and Zechner (2020) establish a connection between option-implied ex ante skewness and ex post residual coskewness, while Bali and Murray (2013) identify a negative relationship between risk-neutral skewness and skewness in asset returns. Focusing on idiosyncratic risk, Bégin, Dorion, and Gauthier (2020) and Gourier (2016) delve into the significant pricing of idiosyncratic jump risk and variance risk, respectively. In contrast, Langlois (2020) find that the role of idiosyncratic skewness risk is not as robust when compared to systematic skewness risk. Turning attention to the financial crisis, Kelly, Lustig, and Van Nieuwerburgh (2016) and Battalio and Schultz (2011) highlight the elevated pricing of put options on individual stocks during the sample period of 2007-2009.

skewness swap returns with equity and variance swap returns, and Section IV. C explores the post-financial crisis period in detail. Section V includes the model-based and corridor-based versions of skewness swaps as robustness checks. Finally, Section VI concludes. The Appendix includes the methodological details and proofs.

II. Skewness Swaps: Theory and Implementation

A skewness swap is a trading strategy with which an investor can directly buy the skewness of an asset and gain the skewness risk premium. The methodology employed in this paper is an application of Schneider and Trojani (2019) but incorporates two modifications: i) isolating skewness from kurtosis, and ii) considering the early exercise of American options. The econometric details and a convergence analysis are reported in Appendix A and B, while this section only presents the intuition and the main formulas used in the empirical section.

A. Background on Variance Swaps

Before introducing the skewness swaps, this section recalls some known results on variance swaps. As outlined in Carr and Wu (2009), the payoff at maturity T to the long side of the variance swap is equal to the difference between the realized variance over the life of the contract, $RV_{t,T}$, and a constant called the variance swap rate, $SW_{t,T}$:

$$[RV_{t,T} - SW_{t,T}]\, L,$$

where L denotes the notional dollar amount, and t is the start date of the contract. By no arbitrage, $SW_{t,T} = E_t^{\mathbb{Q}}[RV_{t,T}]$, and the variance swap has zero net market value at entry. The return on the strategy thus depends on the difference between the realized variance $RV_{t,T}$ and the risk-neutral variance $SW_{t,T}$ and is a direct measure of the realized variance risk premium.

It is a known result that the swap rate can be measured at time t with the price of a portfolio of options with maturity T (see e.g., Carr and Madan (2001), Carr and Wu (2009), Martin (2017), among others). The realized variance can be measured with the sum of squared daily returns (Carr and Wu (2009)), or by squaring the return over the time period from t to T.[8] The latter can be measured with the payoff of the option portfolio employed in calculating the swap rate (see e.g., the variance portfolios in Heston et al. (2022)).

B. Skewness Swaps: Theory

The skewness swap is defined in a similar way to the variance swap. The payoff at maturity T to the long side of the skewness swap is equal to the difference between the realized skewness over the life of the contract, $RS_{t,T}$,[9] and a constant called the skewness swap rate, $SSW_{t,T}$, which measures the risk-neutral skewness, times the

[8]Because variance has the aggregation property of time, it holds $E_0^{\mathbb{Q}}\left[\sum^T (\delta S)^2\right] = E_0^{\mathbb{Q}}\left[(S_T - S_0)^2\right]$ if prices are martingales (see Neuberger (2012), Bondarenko (2014a)). The third moment, instead, does not have the time-aggregation property.

[9]In this context, realized skewness over the life of the contract refers to the cube of the return measured from time t to T, as will be formally defined in Proposition 1.

notional L:

(1) $[RS_{t,T} - SSW_{t,T}]\, L.$

Skewness portfolios can be constructed in different ways, depending on how realized skewness and risk-neutral skewness are measured. For the baseline analysis, I implement the simple swap methodology of Schneider and Trojani (2019), which extends the model-free methodology of Bakshi et al. (2003) to measure the swap returns of every moment of the distribution.[10] I postpone the analyses of more complex model-based and corridor-based alternatives in the robustness Section V.[11]

The structure of the swap strategy of Equation 1 involves a fixed leg ($SSW_{t,T}$) and a floating leg ($RS_{t,T}$), which are exchanged at maturity between two counterparts. These components are defined in Schneider and Trojani (2019) using the following general formulas, which can be applied to trade not only skewness but any moment of the distribution:

(2) $$\text{fixed leg}_{t,T} = \frac{1}{B_{t,T}} \left(\int_0^{F_{t,T}} \Phi''(K) P_{t,T} dK + \int_{F_{t,T}}^{\infty} \Phi''(K) C_{t,T} dK \right)$$

[10]Bakshi et al. (2003) provide a static methodology to measure the standardized and non-standardized risk-neutral moments, while Schneider and Trojani (2019) provide a methodology to trade non-standardized moments. The two methodologies are closely related, and indeed the skewness swap rate employed in this paper has a correlation of more than 90% with the price of the cubic contract of Bakshi et al. (2003).

[11]A different set of skewness swap methodologies are those proposed by Kozhan et al. (2013) and Orłowski et al. (2023), in which the traded realized skewness is the sum of daily cubed returns, i.e., $\sum_t r^3_{t,t+1}$, which is different from the Bakshi et al. (2003) third moment over the full period, i.e., $r^3_{0,T}$. The methodologies that trade $\sum_t r^3_{t,t+1}$ involve a continuous rebalancing of the option portfolio with consequent high trading costs. I therefore opt for the skewness portfolio of Schneider and Trojani (2019) which involves a static option portfolio and a continuos rebalancing only in the underlying asset.

$$
\begin{aligned}
\text{floating leg}_{t,T} = & \underbrace{\left(\int_{0}^{F_{t,T}} \Phi^{''}(K) P_{T,T} dK + \int_{F_{t,T}}^{\infty} \Phi^{''}(K) C_{T,T} dK\right)}_{\text{Payoff of the option portfolio}} \\
& + \underbrace{\sum_{i=1}^{n-1} \left(\Phi^{'}(F_{i-1,T}) - \Phi^{'}(F_{i,T})\right)(F_{T,T} - F_{i,T})}_{\text{Dynamic trading in the underlying}}.
\end{aligned} \tag{3}
$$

$P_{t,T}$ and $C_{t,T}$ are the prices of put and call options at time t with maturity T and strike K, $B_{t,T}$ is the zero-coupon bond with maturity T, and $F_{t,T}$ is the forward price at time t for delivery at time T. $P_{T,T}$ and $C_{T,T}$ denote the put and call option payoff at maturity, defined as $max(0, K - S_T)$ and $max(0, S_T - K)$, respectively.

Equation 2 shows that the fixed leg is an option portfolio with weights given by the function $\Phi^{''}$, which determines the moment of the distribution traded with the swap (e.g., variance or skewness). Equation 3 shows that the floating leg is composed of two parts: the payoff at maturity of the same option portfolio defined in Equation 2, plus a dynamic trading strategy in the forward market, which is rebalanced on the n intermediate dates: $t < t_1 < ... < t_{n-1} < t_n = T$. To simplify the notation, I denote $F_{t_i,T}$ with $F_{i,T}$.

I implement the above swap strategy with a specific choice of the Φ function, which makes the swap a trading strategy specifically designed to target skewness. This is formally demonstrated in the following Proposition:

Proposition 1. *The skewness swap S with the floating leg given by Equation 3 and the fixed leg given by Equation 2 with*

(4)

$$\Phi(x) = \Phi_S\left(\frac{x}{F_{0,T}}\right) = -24\left(\frac{x}{F_{0,T}}\right)^{1/2}\log\left(\frac{x}{F_{0,T}}\right) + 24\left[\left(\frac{x}{F_{0,T}}\right)^{1/2}\left(\log\left(\frac{x}{F_{0,T}}\right)^2 + 8\right) - 8\right]$$

verifies the following property:

$$\text{(5)}\ \textit{fixed leg}_{\Phi_S,0,T} = E_0^{\mathbb{Q}}\left[\left(\log\left(\frac{F_{T,T}}{F_{0,T}}\right)\right)^3 + O\left(\log\left(\frac{F_{T,T}}{F_{0,T}}\right)^5\right)\right].$$

Proof. See Appendix A. □

Proposition 1 formally proves that, under the choice of the Φ function given by Equation 4, the leading term of the floating leg corresponds to the third non-standardized moment of the forward return distribution.[12] This leg is independent of the first, second, and fourth moments, while the error component is a function of the fifth moment.

To better visualize how the option strategy defined by Φ relates to the third moment, Figure 1 plots the payoff of the option portfolio as a function of the forward return $\left(\log\left(\frac{F_{T,T}}{F_{0,T}}\right)\right)$.[13]

[Figure 1 here]

The figure shows that the payoff accurately traces the cube of the return. The figure also displays the payoff of the option portfolio implemented by Schneider and Trojani (2019) and Schneider and Trojani (2015), which relies on a different choice of Φ

[12]The standardized and non-standardized third moments are both used in the literature as measures of skewness. While they both have advantages and disadvantages, the choice of the non-standardized skewness in this paper is dictated by the tradability of the skewness swap. The non-standardized skewness is also consistent with the skewness preferences of Kraus and Litzenberger (1976).

[13]Note that the payoff of the option portfolio equals $\Phi(F_{T,T}) - \Phi(F_{0,T}) - \Phi^{'}(F_{0,T})(F_{T,T} - F_{0,T})$ (see Carr and Madan (2001)).

function based on the Hellinger distance.[14] In this case the accuracy is lower, especially in the tails of the distribution. Numerically, Appendix B shows that the gain in the convergence of the skewness swap implemented in this paper over that implemented in Schneider and Trojani (2019) can be as high as 20%, underscoring the importance of isolating the third moment from the fourth.

C. Skewness Swaps: Implementation and Convergence

The fixed and floating legs of the swap contain a theoretical portfolio with a continuum of options with strikes in the range $[0, +\infty]$. However, only a finite number of strikes is listed on the options market. Thus, I implement the following discrete approximation. Suppose that at time t there are N calls and N puts traded in the market. I order the strikes of the calls such that $K_1 < ... < K_{Mc} \leq F_{t,T} < K_{Mc+1} < ... < K_N$ and the strikes of the puts such that $K_1 < ... < K_{Mp} \leq F_{t,T} < K_{Mp+1} < ... < K_N$. The integrals of the fixed leg in Equation 2 are then approximated with the following quadrature formula:

$$(6)\ \ \text{fixed leg}_{t,T} = \frac{1}{B_{t,T}} \left(\sum_{i=1}^{Mp} \Phi''(K_i) P_{t,T}(K_i) \Delta K_i + \sum_{i=Mc+1}^{N} \Phi''(K_i) C_{t,T}(K_i) \Delta K_i \right)$$

[14]The Hellinger Φ_3 function implemented in Schneider and Trojani (2019) is reported in Equation A.1 in the Appendix. It satisfies fixed $\text{leg}_{\Phi_3,0,T} = E_0^{\mathbb{Q}}\left[\frac{1}{6}y^3 + \frac{1}{12}y^4 + O(y^5)\right]$, where $y = \log(F_{T,T}/F_{0,T})$.

where

$$\Delta K_i = \begin{cases} (K_{i+1} - K_{i-1})/2 & \text{if } 1 < i < N \\ (K_2 - K_1) & \text{if } i = 1 \\ (K_N - K_{N-1}) & \text{if } i = N \end{cases}$$

The floating leg in Equation 3 is composed of two parts: the payoff of the option portfolio plus the delta hedge. The integrals of the option portfolio are approximated with the same quadrature approximation outlined above; the delta hedge is implemented each day t_i, starting from day t_1 (the day after the start date of the swap) until day t_{n-1} (the day before the maturity of the swap).

In Appendix B, I verify the numerical convergence of the discretized swap methodology to the true skewness under the Merton jump-diffusion model. The results, shown in Table B.1, indicate that the value obtained using just four call and four put options deviates from the model-implied skewness by approximately 5%, demonstrating that the methodology achieves a high level of accuracy even with a small number of options.

Another empirical adjustment to the theoretical formulas is necessary because single-stock options are American-style, meaning they can be exercised at any time before maturity. To account for this and ensure the tradability of the skewness swap, I incorporate quoted American option prices in the fixed leg formula and determine the option payoff in the floating leg by evaluating the optimal exercise of these options daily, following the market-based rule introduced by Pool et al. (2008). The accuracy of this American version of the skewness swap in measuring skewness depends on the early exercise premium. In Section IV, I show that in this empirical setting, the error remains negligible, as the skewness swap consists exclusively of out-of-the-money options. For a detailed explanation of the construction of the American skewness swap, see Appendix A.

1. What is the Return on a Skewness Swap?

The payoff of the long side of a skewness swap is defined in dollars by Equation 1. The return is then calculated by standardizing this payoff by the capital required to establish the position, that is, the cost of purchasing the fixed leg of the swap. This cost is given by

$$\text{capital} = \frac{1}{B_{t,T}} \left(\int_0^{F_{t,T}} \mid \Phi^{''}(K) \mid P_{t,T} dK + \int_{F_{t,T}}^{\infty} \mid \Phi^{''}(K) \mid C_{t,T} dK \right)$$

The formula contains the absolute values of the portfolio weights, $\mid \Phi^{''}(K) \mid$, in the integrals to account for the fact that the skewness swap portfolio involves both long and short option positions. This ensures that all positions are properly incorporated in terms of capital requirements and total exposure.

III. Data

This section describes the data sources, the data filtering, and the main variables used in the empirical analysis.

Time Period and Roll-over of the Trading Strategies: I apply the skewness swap to all the components of the S&P500 separately over the period from January 2003 to December 2020. I fix a monthly horizon for the skewness swaps, starting and ending on the third Friday of each month, consistent with the maturity structure of option data. Because the issue of new options sometimes occurs on the Monday after the expiration Friday, I take as the starting day of the swaps the Monday after the third Friday of each month.

Security Data: The list of the actual components of the S&P500 is taken from Compustat. This sample is merged with Optionmetrics and Center for Research in Security Prices (CRSP), with the exclusion of the stocks for which there is not an exact match between the daily close price reported by Optionmetrics, CRSP, and Compustat. The data on the security prices and returns are taken from CRSP, while the data on firm characteristics are taken from Compustat. After this selection, the sample consists of 835 stocks. The methodology requires the calculation of the forward price at time t for delivery of the asset at time T. It is calculated as $F_{t,T} = S_t e^{r(T-t)} - AVD$ according to standard no-arbitrage arguments, where r is the risk-free interest rate, S_t is the stock price at time t, and AVD is the accumulated value of the dividends paid by the stock between time t and time T. The data on the dividend distribution and the risk-free rate are taken from Optionmetrics. I consider only the periods in which stocks do not distribute special dividends in order to avoid special behavior of stocks.

Options Data: The data on option prices and attributes for both single stocks and the

S&P500 index come from Optionmetrics. The following data filtering is applied at the start of the swap: I include only options with positive open interest and exclude those with negative bid-ask spreads, negative implied volatility, or a bid price of zero. The swap is implemented only if at least two call options and two put options pass these filters to construct the fixed leg. After filtering, each stock has an average of 100 monthly swap returns over the full sample period, with approximately 10 to 12 options used in each swap implementation.

FOMC Announcement Data: The information regarding interest rate announcements from the Federal Open Market Committee (FOMC) is sourced from the Federal Reserve Board's website.[15] The analysis exclusively incorporates meetings, both scheduled and unscheduled, that have associated statement files. This selection criteria is applied because these meetings are the ones where discussions regarding potential interest rate adjustments took place.

Start and End of the Financial Crisis 2007/2009: As in Kelly et al. (2016), I consider the start date of the crisis as August 2007 (the asset-backed commercial paper crisis) and June 2009 as the end date of the crisis. The COVID-19 recession is set to begin in February 2020, in line with NBER recession dating.

[15]Source: https://www.federalreserve.gov/monetarypolicy/fomc_historical_year.htm

IV. Empirical Results

The monthly returns from skewness swaps provide a means to assess the time-series and cross-sectional aspects of skewness risk premiums in individual stocks. Section IV. A analyzes the swap return distribution; Section IV. B investigates the relation between skewness swaps and equity and variance swap returns; and Section IV. C analyzes the returns before and after the 2007–2009 financial crisis.

A. Skewness Swap Returns

Skewness swaps are implemented independently each month for every individual stock in the sample. As a result, each stock has its own time series of monthly swap returns. Panel A1 of Table 1 presents the mean, median and annualized Sharpe ratio of the returns of a value-weighted portfolio of skewness swaps on individual stocks. Each month, the portfolio is constructed by including all individual skewness swaps, with weights proportional to the market capitalization of each stock. For comparison, Panel A2 presents the findings for the skewness swaps on the S&P500 index. Panel A3 analyzes the skewness swap returns for each stock separately, and reports cross-sectional averages of mean and median skewness swap returns, along with the number of stocks exhibiting statistically significant positive or negative swap returns.

[Table 1 here]

The results in Panel A are very strong across all three cases: the returns of skewness swaps are positive, significant, and very high.

The average return of the portfolio of skewness swaps is 21.91% per month, yielding a Sharpe ratio of 0.54 (Panel A1). Panel A of Figure 2 presents the histogram of these portfolio returns. The distribution has a positive mean but it also exibits a negative skewness. Indeed, the distribution has a very long left tail, which corresponds to months in which the skewness swap returns are highly negative.

[Figure 2 here]

Panel B of Figure 2 presents the time series of the portfolio's monthly returns, while Panel C illustrates the cumulative growth of a one-dollar investment in a portfolio that allocates each month 95% to T-bills and 5% to the skewness swap portfolio. On average the return is positive, but the dispersion is huge: there are months in which the return is below −100% or above +100%. The gain shares some similarities with the return on selling insurance: on average it is profitable until the trigger event happens (a crash in this case), at which point it generates large losses. For the swap strategy the risk is even higher because the return can be less than −100% due to the short positions in the portfolio of options. Two main crashes stand from the graph: the financial crisis of 2007-2009 and the COVID-19 crisis. These graphs show that the skewness swap is a highly profitable and highly risky strategy. This is also reflected in the difference between the mean and the median of the skewness swap portfolio returns in Table 1: due to these rare crash events, the mean is much lower than the median. We can interpret the median as the average return outside the crashes. From a hedging perspective, an investor who wants to hedge against a drop in the skewness will sell the skewness swap, and the results in Table 1 show that investors are willing to accept deep negative returns for this hedge.

The results for the skewness swap on the S&P500, reported in Panel A2, are consistent with those documented in the existing literature,[16] and demonstrate that the magnitude of swap returns in individual stocks is considerably lower, approximately around half, than the corresponding swap return on the S&P500. These results are consistent with the hypothesis that investors are more concerned about market crash risk than crash risk in individual stocks.

Panel A3 analyzes the cross-sectional distribution of skewness swap returns, confirming that S&P500 stocks predominantly exhibit positive and substantial skewness swap returns. The average monthly swap return is 20.55%, comparable to the portfolio returns in Panel A1. The mean return is significantly positive for 146 stocks, while the median return is significantly positive for 673 stocks. Notably, no stocks display a statistically significant negative mean or median swap return. Consistent with the portfolio results, the median return exceeds the mean, due to occasional instances of extreme negative returns.

Altogether, the results from Panels A1, A2, and A3 support the idea that investors have a preference for positive skewness not only at the market level but also at the individual stock level.

[16]The average monthly skewness swap return on the S&P500 implemented by Kozhan et al. (2013) during the period 1996-2012 amounted to -42% (variable xs in their Table 1). The negative sign results from their standardization using the risk-neutral skewness, which is negative, while my standardization employs the absolute value of the risk-neutral skewness. Similarly, according to Schneider and Trojani (2015), the Hellinger skewness swap applied to the S&P500 in the sample period 1990-2014 yielded an average monthly payoff of approximately 0.0042 and an implied leg of -0.0075 (results in their Figure 9 for $x = 0$). This corresponds to a monthly return of approximately 56%.

The table also provides the outcomes of three robustness checks. Panel B computes the return of a skewness swap without the dynamic trading in the underlying asset in the floating leg of Equation 3. The results show that, even without delta-hedging, the returns of the skewness swaps are still highly positive and significant.

Panel C calculates the swap return while accounting for the bid-ask spread incurred by investors. Optionmetrics provides quoted bid-ask spreads for each option at the close of each trading day. Based on prior research showing that investors typically pay 40% to 60% of the quoted option spread (see e.g., Muravyev and Pearson (2020)), I adopt a conservative approach to transaction costs. Specifically, for this analysis, I assume investors pay 60% of the option bid-ask spread and 100% of the stock bid-ask spread. Incorporating transaction costs significantly reduces the average return, underscoring the bid-ask spread as a key friction that some investors pay while others profit from. Nonetheless, for those fully paying the transaction costs, the mean swap return remains 10.58% for the swap portfolio (Panel C1) and 4.95% across the cross-section of stocks (Panel C3). Even if the return for the portfolio is not statistically significant, 62 stocks display significantly positive swap returns and only one significantly negative. Overall, these results emphasize the positive returns of the strategy even after accounting for transaction costs.

Finally, Panel D computes the return of a skewness swap that uses synthetic European option prices instead of the actual American option prices.[17] Also in this case

[17] The synthetic European option prices are recovered with the Black–Scholes formula applied to the implied volatility of the American option prices provided by Optionmetrics. This European swap is not tradable, but its return is useful to measure the importance of the early exercise component in the tradable American swap for individual stocks.

the returns are still positive and significant.

B. Relation between Skewness Swaps, Variance Swaps, and Stock Returns

In this section, I examine the interplay among skewness swaps, variance swaps, and stock returns. The objective is to determine whether skewness swaps are fully explained by variance swaps and equity returns or if they provide supplementary insights.

[Figure 3 here]

Figure 3 depicts the time-series of three cross-sectional quantiles (10%, 50%, and 90%) of skewness swap and variance swap returns for individual stocks.[18] The figure also highlights three significant crashes that resulted in deep negative returns for skewness swaps: September 2008 (Lehman default), August 2011 (US credit rating downgrade), and March 2020 (Covid-19 crisis). Corresponding to these events, variance swap returns display major positive spikes. This is expected since, during crashes, volatility rises while skewness generally drops. However, outside of these pivotal events, the negative correlation between the two series is less evident.

[Table 2 here]

[18]The variance swap is a trading strategy with the same structure as the skewness swap but with portfolio weights such that fixed $\text{leg}_{t,T,\Phi_2} \simeq E_t^{\mathbb{Q}}\left[\left(\log\left(\frac{F_{T,T}}{F_{t,T}}\right)\right)^2\right]$. Following Schneider and Trojani (2015), I implement the variance swap of Equation 2 with Φ function equal to $\Phi_2(x/F_{0,T}) = -4((x/F_{0,T})^{0.5} - 1)$.

Table 2 compares skewness swap returns with equity returns (Panel A), and variance swap returns (Panel B). It reports (i) the percentage of months in which equity or variance swap returns share the same sign as skewness swap returns, and (ii) the R-squared from regressing skewness swap returns on equity or variance swap returns. The analysis is computed for the portfolio of swaps (Panels A1 and B1) and for the individual swaps, for which the table reports cross-sectional averages and quantiles (Panels A2 and B2). Finally, Panel C presents the R-squared from a regression of skewness swap returns on equity returns, variance swap returns, and, to capture potential nonlinearities, the square and cube of equity returns. Only the stocks with at least 100 swaps are included in this analysis (401 stocks).

As expected, skewness swap returns tend to align with the same sign as equity returns in most instances, 74% of the months on average for individual swaps and 82.71% for the portfolio (see Panel A). The average cross-sectional R squared is 21.08% for individual swaps and 45.90% for the portfolio, indicating that there is a substantial portion of the variation in skewness swap returns unexplained by equity returns.

Similar observations hold when examining the relationship between skewness swap returns and variance swap returns (Panel B). As expected, the variables display a negative comovement, since they have the same sign for less than 30% of the months for both the individual and portfolio of swaps. While the R-squared are higher than in Panel A (around 61% for the portfolio and an average of 41.2% for individual swaps), there is still substantial variation left unexplained.

In the most stringent specification in Panel C, the R-squared are about 88% for the skewness swap portfolio and an average of 67% for individual swaps. This is not surprising, given that the specification also includes r_i^2 and r_i^3 which are proxies for realized variance and skewness, respectively.

Figure 3 suggests that the R-squared reported above may be inflated by a few major market crashes, during which skewness swap returns (and equity returns) decline sharply, while variance swap returns surge.[19] Excluding the six market crashes in the sample (August 2008, October 2008, August 2011, February 2018, and March 2020) results in a substantial decline in explanatory power, with the R-squared in the most stringent specification dropping from 88% to 54%.[20]

Overall, this analysis highlights that, while skewness swap, variance swap, and equity returns are correlated with the expected sign, especially during extreme market crashes, the information in skewness swap returns is not spanned by these other variables.

C. Skewness Swap Returns After the Financial Crisis

The time-series graphs of Panels B and C in Figure 2 suggest that there is a substantial increase in the skewness swap returns after the financial crisis.

The period between the end of the financial crisis and the onset of the Covid recession represents a distinct phase in financial markets. In response to the crisis, policymakers, most notably the Federal Open Market Committee (FOMC), lowered

[19] It is well known (see, e.g., Aït-Sahalia and Xiu (2016)) that the correlation between two time series can differ substantially depending on whether jumps are included or excluded, with the latter capturing the correlation in the continuous component of the series.

[20] Results available upon requests.

interest rates to near zero and maintained them at those levels for an extended period. This prolonged low-rate environment, as widely documented in the literature (e.g., Hau and Lai (2016), Lian et al. (2019)), encouraged increased risk-taking and contributed to elevated asset valuations. Indeed, following the crisis, the U.S. stock market surged, with valuations rising well above historical levels, as also documented in the Federal Reserve's 2018 Financial Stability Report,[21] and reflected in the time-series of Tobin's q for the U.S. economy (see top graph of Figure D.1 in the Appendix).

This section examines the distribution of skewness swap returns during this peculiar period. First, it formally tests for differences in the distribution of returns between the pre-crisis and post-crisis periods (Section 1). Second, it also documents changes in the implied volatility smile (Section 2). Finally, it investigates whether post-crisis skewness swap returns are particularly linked to systematic and firm-specific measures of crash risk and overvaluation risk (Section 3).

1. Pre- versus Post-Crisis Swap Returns

[Table 3 here]

Table 3 reports the mean and median skewness swap returns for the periods before the financial crisis (January 2003–July 2007) and after the financial crisis (June 2009–February 2020). The average return of the skewness swap portfolio (Panel A) increases by almost 50% in relative terms, from 26.17% to 38.32% after the crisis. The increase in the median return is even more pronounced, from 29.27% to 51.43%. These

[21]Source: 2018 Financial Stability Report, Board of Governors of the Federal Reserve System, https://federalreserve.gov/publications/files/financial-stability-report-201811.pdf

results suggest a post-crisis shift in the swap return distribution, characterized by a higher mean and greater left skewness. The kernel densities, reported in the top graph of Figure 4, visually confirm this change in the swap return distribution. A formal test for distribution differences is presented in the last column of Panel A in Table 3, which shows a statistically significant Kolmogorov-Smirnov t-statistic.

The results for the S&P500 index swap, reported in Panel B, document a more modest post-crisis increase of approximately 12% (from 71.20% to 80.28%).

[Figure 4 here]

Panel C completes the analysis and presents the cross-sectional averages of the mean and median skewness swap returns for individual stocks. The mean rises from 6.98% to 24.29%, while the median increases from 28.64% to 49.38%. Additionally, the numbers in parentheses indicate that before the financial crisis, only 172 (302) stocks had a significantly positive mean (median) skewness swap return, whereas after the crisis, this number rises to 311 (614).

Overall, these results indicate that, after the financial crisis, investors demanded a much higher compensation for crash risk in individual stocks.

2. Pre- versus Post-Crisis Implied Volatility Smile

The crash of October 1987 (i.e. the Black Monday) is considered a landmark event in the history of the option market. The literature documented that following the Black Monday, the implied volatility smile of the S&P500 index became asymmetric (see Bates (2000) and Rubinstein (1994), among others), due to the disproportionate increase in the price of out-of-the-money put options compared to the price of out-of-the-money call options. This new asymmetry, known as "smirk", is commonly interpreted as evidence of a pronounced aversion of investors to market crashes (see e.g., Bates (2000)).

On the other hand, for individual stocks, early studies (see e.g., Bollen and Whaley (2004)) using samples ending prior to the 2007-2009 financial crisis, document an overall simmetry of the implied volatility smile. If the change previously documented in skewness swap returns genuinely reflect a change in investors preferences and beliefs about stock-specific crashes, I should also observe a smirk in individual stocks following the financial crisis.

To test whether this is the case, I create pre- and post-crisis average smiles, following the methodology in Bollen and Whaley (2004).[22] The results, displayed in the bottom graphs of Figure 4, distinctly reveal that, after the financial crisis, the implied volatility smile became more asymmetric for individual stocks due to a pronounced increase in the price of deep out-of-the-money put options (moneyness category 1). Indeed, the average difference between the implied volatility of the most

[22] In detail, for each stock and day, put and call options with maturities up to one year are categorized into five moneyness groups based on their deltas. Category 1 includes call options with $0.875 < \Delta_C \leq 0.980$ and $-0.125 < \Delta_P \leq -0.020$. Category 2 includes options with $0.625 < \Delta_C \leq 0.875$ and $-0.375 < \Delta_P \leq -0.125$, category 3 includes options with $0.375 < \Delta_C \leq 0.625$ and $-0.625 < \Delta_P \leq -0.375$, category 4 includes options with $0.125 < \Delta_C \leq 0.375$ and $-0.875 < \Delta_P \leq -0.625$, and finally, category 5 includes options with $0.020 < \Delta_C \leq 0.125$ and $-0.980 < \Delta_P \leq -0.875$. Implied volatilities are averaged within each category to obtain average daily implied volatility smiles. Finally, these daily smiles are averaged across stocks in the pre-crisis and post-crisis periods to create the average pre-crisis and post-crisis smiles depicted in the bottom graphs of Figure 4. To better illustrate the variation in slopes, the pre-crisis smiles are shifted within the graph to align with the post-crisis smiles.

out-of-the-money puts and calls increases from 5% to about 11% for individual stocks (more than a 100% increase), and from 11% to 15% for S&P500 options (about a 35% increase). While the change is statistically significant at the 1% level in both cases, it is relatively smaller in magnitude for S&P500 options.

These results are consistent with those based on skewness swap returns, and further confirm an increased concern among investors for stock-specific crashes.

3. The Cross-Section of Swap Returns After the Crisis

This section investigates the cross-sectional dynamics of skewness swap returns across the pre- and post-crisis subsamples. Given the distinctive features of the post-crisis environment, characterized by persistently low interest rates and elevated asset valuations, the analysis focuses on whether exposures to systematic crash risk and firm-specific overvaluation risk are reflected in skewness swap returns during this period.[23] Specifically, it tests whether stocks with higher exposure to these risks earn higher average swap returns, and how these relationships differ across the two periods.

I consider the following measures of systematic crash risk exposure. First, for each stock i, I regress the skewness swap return time series $r_{sk,i,t}$ on the market skewness swap return $r_{sk,S\&P500,t}$ and its square $r^2_{sk,S\&P500,t}$. The regression coefficients $\beta_{i,swap,1}$ and $\beta_{i,swap,2}$ serve as the first two systematic risk exposures considered. Following Duan and Wei (2009), I also consider the regression R squared, which

[23] Table E.3 in the Appendix shows a general increase in skewness swap returns following Federal Reserve rate cuts, further supporting the view that interest rate levels have an important effect on individual swap returns.

captures the proportion of systematic variance in the total variance of each swap, denoting it as $SysRiskProp_i$.[24] I then consider the tail beta proposed by De Jonghe (2010), which quantifies the probability of a stock price crash conditional on a crash in the market index.[25] I calculate these systematic crash risk measures for each stock separately for the pre- and post-crisis periods, and I sort the stocks into quartiles according to each measure.

[Table 4 here]

Panel A of Table 4 reports the average skewness swap return for each portfolio and for the difference portfolio. T-statistics for the difference portfolio are computed using the Newey and West (1987) correction method with the optimal lag length suggested by Andrews and Monahan (1992). The results show that, while none of the systematic crash risk measures are correlated with skewness swap returns in the pre-crisis period, two measures, systematic risk proportion and tail beta, are significantly correlated with skewness swap returns in the post-crisis period. Stocks with higher systematic risk proportion and higher tail beta earn higher skewness swap returns, with a clear monotonic pattern.

Panel B of Table 4 presents the results for firm specific carachteristics that are most likely to reflect the stock's crash risk. First, following the literature linking

[24] Duan and Wei (2009) show that, under a standard factor model for stock returns, the level and slope of the implied volatility smile for individual stocks are more closely related to the proportion of systematic risk than to individual betas. This builds on results from Bakshi et al. (2003), who link the higher-order risk-neutral moments of individual stocks to those of the market.

[25] I estimate the tail beta for each stock using the extreme value theory approach of De Jonghe (2010). This method involves transforming stock and market returns into unit Pareto marginals, then estimating the tail index of their joint probability distribution using the Hill estimator. The tail region is defined by a crash probability of $p = 0.04\%$, following De Jonghe (2010).

overvaluation to increased crash risk (see, e.g., Abreu and Brunnermeier (2003), Hong and Stein (2003)), I consider three standard measures of overvaluation: i) the logarithm of the book-to-market ratio (BM), ii) the logarithm of company's Tobin's q, and iii) the maximum daily return of the stock over the preceding month (MAX), as outlined in Bali et al. (2011).[26] Second, following the literature linking the put option market to crash risk (see e.g., Carr and Wu (2011)), I consider (i) the number of put options traded at the start of the swap, denoted as Np, and (ii) the moneyness of the most out-of-the-money put option traded, which I calculate as $min(K/F_{0,T})$, where K is the strike price of the put option and $F_{0,T}$ is the forward price at time 0 for delivery at time T. The first measure reflects the overall activity in the put market, while the second captures the depth of downside protection sought by investors, with lower values indicating a higher demand for deep out-of-the-money puts. Finally, I consider the risk-neutral variance of the stock, denoted as $Qvar$, which is computed using a portfolio of options as described in Section IV. B.

Each month, I sort stocks into quartiles based on the variables above calculated at the start of the month. Panel B of Table 4 reports average swap returns for each portfolio and the difference portfolio in the pre- and post-crisis subsamples. The results show that, in the post-crisis period only, overvaluation measures emerge as significantly correlated with skewness swap returns: stocks with lower book-to-market ratios and higher Tobin's q earn higher returns, with a clear monotonic pattern. $MaxRet$ is mildly

[26]The Tobin's q and book-to-market ratio are computed using the Compustat variables as follows: i) Tobin_q = (atq+(prccq*cshoq)-(seqq+txditcq-pstkq))/atq, and ii) BM = (seqq+txditcq-pstkq)/(prccq*cshoq). As these measures are recorded quarterly in Compustat, I consider the last value recorded prior to the start of the swap.

negatively correlated with swap returns, consistent with Bali et al. (2011), suggesting it captures short-term overpricing that is corrected in the following month. Instead, BM and Tobin's q appear to reflect more persistent overvaluation. Trading activity in the put option market is significantly related to skewness swap returns in both samples: stocks with a higher number of put options traded and deeper out-of-the-money puts exhibit higher swap returns, reinforcing the idea that the put option market provides meaningful information about a firm's crash risk. Finally, risk-neutral variance is related to skewness swap returns in the post-crisis period only and with a negative sign, further highlighting that variance and skewness capture different dimensions of risk.

In summary, the findings of this section indicate that skewness swap returns are particularly high during the post-crisis period, where they appear to be positively correlated with exposures to both systematic and firm-specific crash risk, as well as overvaluation risk. These results suggest that skewness swap returns capture the stock-level crash risk prevailing in the economic environment.

V. Robustness Checks

Options are not available for all strike values, and moneyness coverage varies across months. This section addresses this limitation by implementing two robustness checks to examine skewness swap returns before and after the financial crisis while maintaining a constant moneyness range. First, I compute the return of a model-based skewness swap with a fixed moneyness range and a fixed number of options. Second, following the literature on corridor swaps, I construct a corridor-version of the skewness swap using different corridor values.[27]

In both of these alternative specifications, the results align consistently with the earlier findings: swap returns exhibit positivity, notably high values, and a post-financial crisis increase.

A. The Model-based Skewness Swap

As a first robustness check, I introduce a model-based version of the skewness swap. In this approach, rather than relying on actual option prices, I utilize option prices derived from a fitted model. I chose the Merton jump-diffusion model of Merton (1976) as a benchmark model due to its well-documented ability to fit short-term options and its mathematical tractability (see e.g., Hagan, Kumar, Lesniewski, and Woodward (2002)).

The dynamics of the stock under the Merton jump-diffusion model is as follows:

$$(7)\quad ds_t = \left(r - \lambda\kappa - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t + \log(\psi) dq_t$$

where s_t is the logarithm of the stock price, r is the risk-free interest rate, σ is the instantaneous variance, and q_t is a Poisson process, independent from W_t, which equals one when a jump occurs. I follow the standard assumptions that jumps occur within dt with probability λdt, and that the jump size follows $\log(\psi) \sim N(\mu, \delta^2)$. Finally,

[27] For a review on the literature on corridor swaps see, for example, Carr and Lewis (2004), Lee (2010), and Andersen, Bondarenko, and Gonzalez-Perez (2015a).

$\kappa = E[\psi - 1]$ represents the mean jump size.

The implementation of the skewness swap based on the Merton jump-diffusion model consists of two main steps: (i) calibrating the model parameters to option prices at the swap inception date, and (ii) constructing the swap using a regular grid of option prices generated from the calibrated model. The details of the implementation are provided in Appendix C. This approach ensures a consistent moneyness range and a fixed number of options across different stocks and time periods. However, a key limitation is that the model-based skewness swap is not tradeable.

[Table 5 here]

Panel A of Table 5 reports the mean and median return of the model-based portfolio of swaps in the pre- and post- crisis samples. The results indicate that the mean (median) model-based skewness swap return is 22.97% (32.59%) before the financial crisis and 29.43% (42.62%) after, representing a 20%–30% increase. The Kolmogorov-Smirnov statistic confirms a shift in the distribution of swap returns between these periods. These findings are consistent with the baseline analysis, though slightly smaller, likely due to the smoother nature of model-derived option prices compared to actual market prices.

B. Corridor Skewness Swaps

In this section, I apply the corridor variant of Andersen et al. (2015a) to the skewness swaps. Corridor swaps focus on price changes within a fixed moneyness range, applying a consistent truncation rule to both swap legs. Varying the corridor range enables analysis of deep far out-of-the-money options' impact on skewness swap returns.

In formulas, given a corridor $[a, b]$, and a generating Φ function, the fixed leg and floating leg of the corridor swap are defined as follows:

$$\text{(8)} \quad \text{Corridor fixed leg}_{t,T} = \frac{1}{B_{t,T}} \left(\int_{a}^{\min(b,F_{t,T})} \Phi^{''}(K) P_{t,T} dK + \int_{\max(F_{t,T},a)}^{b} \Phi^{''}(K) C_{t,T} dK \right)$$

$$\text{(9)} \quad \begin{aligned} \text{Corridor floating leg}_{t,T} = & \left(\int_{a}^{\min(b,F_{t,T})} \Phi^{''}(K) P_{T,T} dK + \int_{\max(F_{t,T},a)}^{b} \Phi^{''}(K) C_{T,T} dK \right) \\ & + \sum_{i=1}^{n-1} \left(\Phi^{'}_{a,b}(F_{i-1,T}) - \Phi^{'}_{a,b}(F_{i,T}) \right) (F_{T,T} - F_{i,T}). \end{aligned}$$

The option portfolio in the fixed and floating leg has weights given by $\Phi^{''}(K)$ inside the corridor and zero outside of the corridor, that is, out-of-the-money calls (puts) with strike $K > b$ ($K < a$) have no contribution in the skewness swap. The dynamic trading in the underlying is rebalanced according to the function $\Phi^{'}_{a,b}$, which is equal to $\Phi^{'}(a)$ if $x < a$, $\Phi^{'}(x)$ if $a \leq x \leq b$, and $\Phi^{'}(b)$ if $x > b$. In other words, the dynamic trading is rebalanced only on price changes inside the corridor or on price changes from regions inside (outside) the corridor to regions outside (inside) the corridor.

Panel B of Table 5 presents the mean and median returns for five corridor-based skewness swaps. These corridors are defined as nested intervals around $F_{t,T}$, ranging from one to five standard deviations away.[28] The results consistently show positive returns across all corridors except the first, which covers only one standard deviation.[29]

[28]Mathematically, the n-th corridor, for $n = 1, 2, 3, 4, 5$, is given by $[F_{t,T}e^{-n\sigma\sqrt{T-t}}, F_{t,T}e^{+n\sigma\sqrt{T-t}}]$, where $F_{t,T}$ is the forward price, σ is the at-the-money volatility, and $T - t$ represents time to maturity. Each corridor includes all options with strike prices within n standard deviations from $F_{t,T}$.

[29]The returns of the corridor swap covering one standard deviation are very noisy due to the limited

Skewness swap returns increase as the corridor widens, highlighting the crucial role of out-of-the-money options in measuring the skewness risk premium. Additionally, for corridors spanning two to five standard deviations, returns are higher in the post-crisis subsample. The widest corridor, covering five standard deviations, aligns most closely with the baseline skewness swap returns without corridor restrictions (Panel A of Table 3), as expected.

VI. Conclusion

This paper examines the crash risk premium in individual stocks through the returns of skewness swaps.

Similar to variance swaps, a skewness swap strategy involves taking positions in the skewness of a stock by buying and selling out-of-the-money put and call options. The return of this strategy captures the difference between the stock's realized skewness and its risk-neutral skewness, providing a tradable measure of the compensation investors require for exposure to the risk of a sudden decline in skewness. Notably, this return is independent of the first, second, and fourth moments, offering a pure bet on skewness.

I apply this strategy to the S&P500 index constituents from 2003 to 2020 and show that skewness swap returns are positive and statistically significant. The findings are robust across different implementations of the skewness swap strategy and indicate that investors demand significant compensation for skewness risk in individual stocks,

number of options within the narrow corridor.

which is not subsumed by compensation for variance risk.

The crash risk premium, measured by skewness swap returns, becomes particularly pronounced after the 2007/2009 global financial crisis. This shift also coincides with a rise in the price of crash risk, as reflected in a more left-skewed implied volatility smile, driven by higher prices for deep out-of-the-money options. A portfolio sort analysis further shows that, in the post-crisis period only, skewness swap returns are positively correlated with measures of systematic crash risk and overvaluation. These findings reinforce the idea that skewness swap returns reflect the stock-level crash risk in the economy.

These results have broad implications for asset pricing. They highlight the importance of measuring higher-order moments of return distributions and suggest that models incorporating skewness and tail risk provide a more complete framework for understanding investor preferences.

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Figure 1. Payoff of the Skewness Swap Option Portfolio

The figure illustrates the payoff of the skewness swap option portfolio as a function of the forward return $log\left(\frac{F_{T,T}}{F_{0,T}}\right)$. For comparison, it also displays the payoff of the Hellinger skewness swap option portfolio, as implemented in Schneider and Trojani (2019), along with the cubic function $log\left(\frac{F_{T,T}}{F_{0,T}}\right)^3$ as a benchmark.

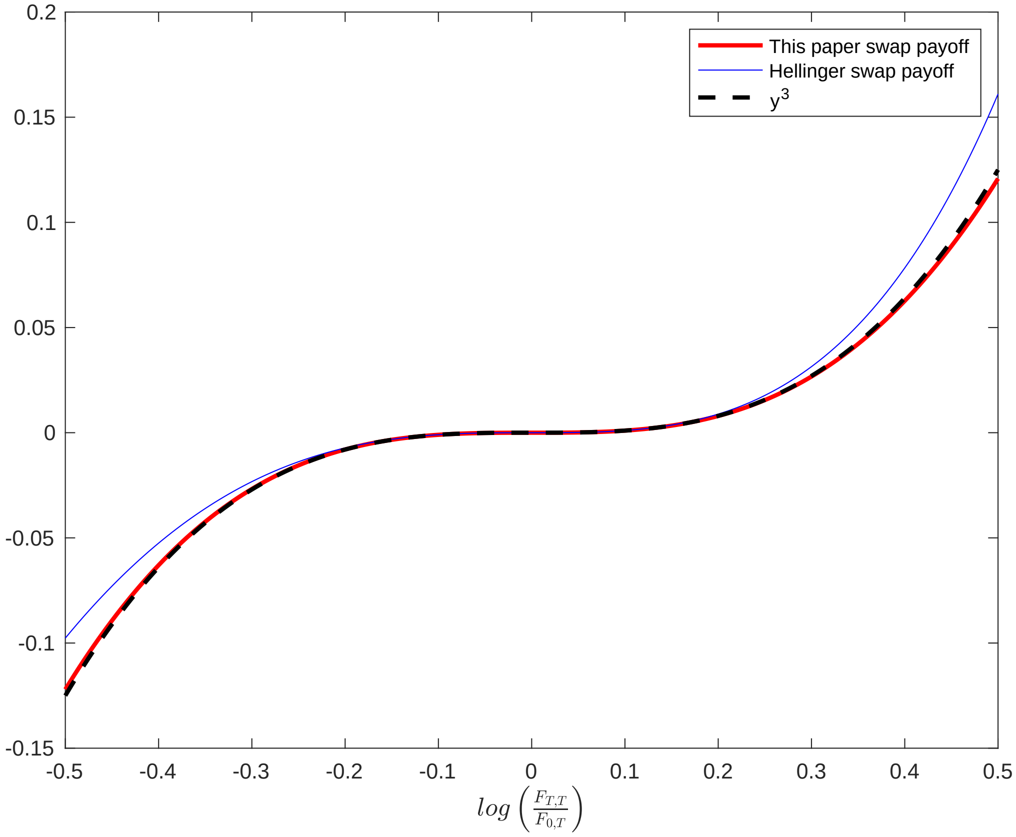
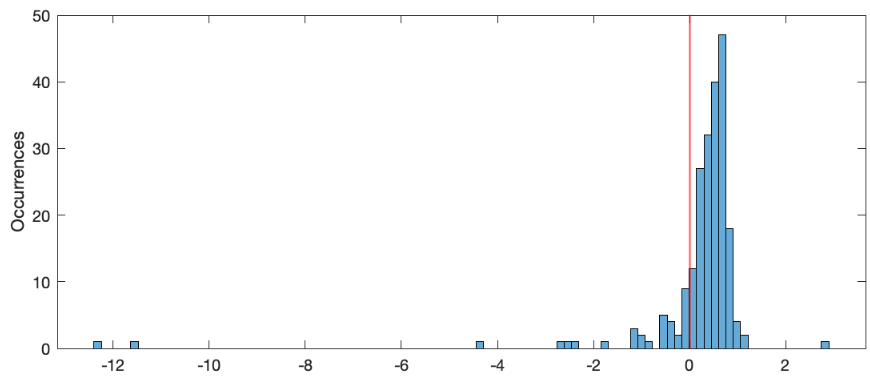


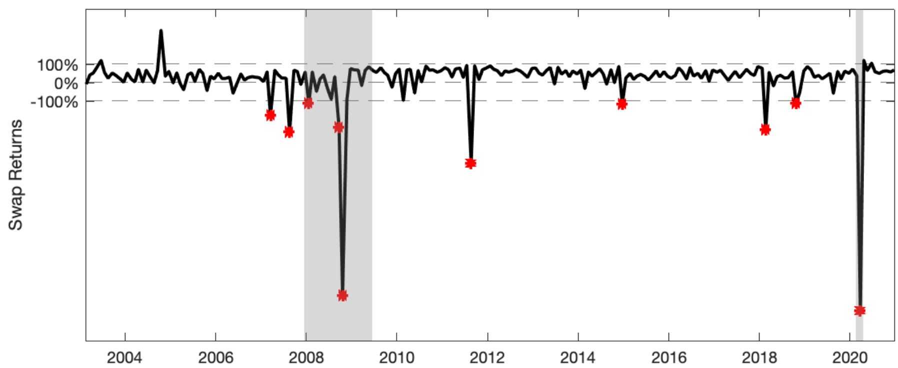
Figure 2. Returns of the Portfolio of Skewness Swaps

Panel A shows the histogram of returns for the skewness swap portfolio in individual stocks, as analyzed in Panel A1 of Table 1, while Panel B presents the time series of the same portfolio returns. The sample period runs from January 1, 2003, to December 31, 2020. In Panel B, red markers indicate months when returns dropped below -100%. Panel C depicts the growth of a one-dollar investment in a portfolio partially allocated to the risk-free rate and skewness swaps. Each month, 95% of the portfolio is allocated to one-month T-bills and 5% to the swap portfolio strategy, with returns compounding over time. Shaded gray regions represent the financial crisis and COVID-19 crisis periods.

Panel A: Histogram



Panel B: Monthly Time-Series



Panel C: Compound Return

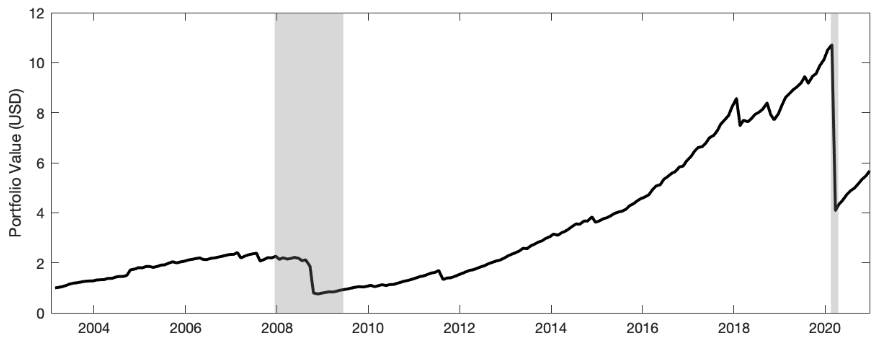


Figure 3. Time Series of Skewness Swaps and Variance Swaps in Individual Stocks

The figure shows the cross-sectional times series of skewness swap returns and variance swap returns for the stocks which are part of the S&P500 index. For each month, the picture shows the cross-sectional 10% quantile, 50% quantile, and 90% quantile. The figure highlights the timing of three major events: i) The default of Lehman Brother in September 2008, ii) the US credit rating downgrade in August 2011, and iii) the Covid-19 pandemic in March 2020.

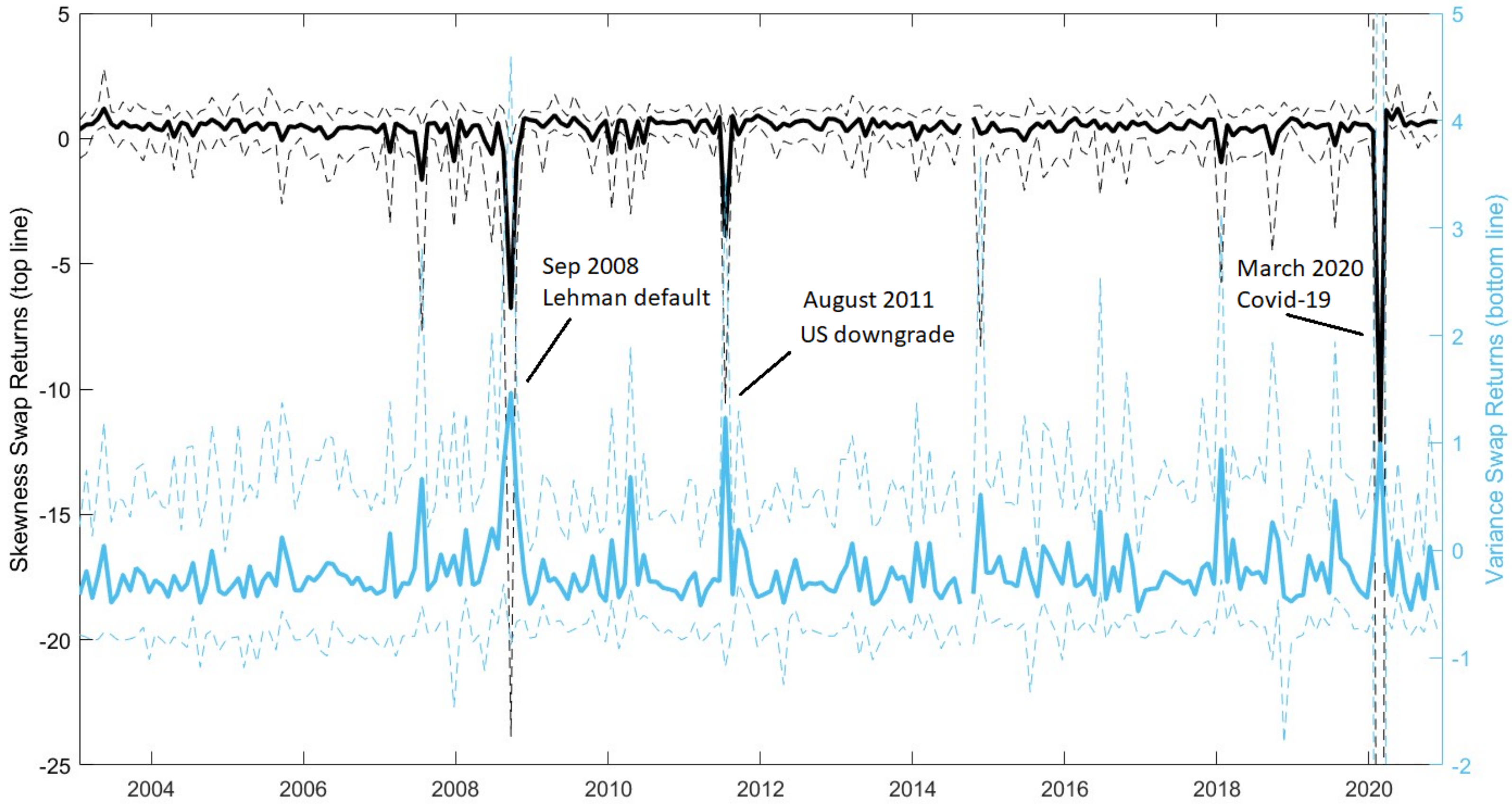


Figure 4. The Option Market Before and After the 2007-2009 Financial Crisis

The top graph presents the kernel density estimation of the return distribution for the portfolio of skewness swaps before the financial crisis (Jan 2003–Aug 2007) and after the financial crisis (June 2009–Feb 2020). The bottom graphs show the average implied volatility smile before and after the financial crisis for both the cross-section of individual stocks (left graph) and the S&P500 index (right graph). The implied volatility smile is constructed by grouping options into five moneyness categories based on their deltas, following Bollen and Whaley (2004), and averaging implied volatilities within each category. To better illustrate differences in slope, the pre-crisis smile curves are vertically shifted so that the implied volatility of the at money options (category 3) overlaps with that of the post-crisis smile, allowing for a clearer comparison of their relative shapes.

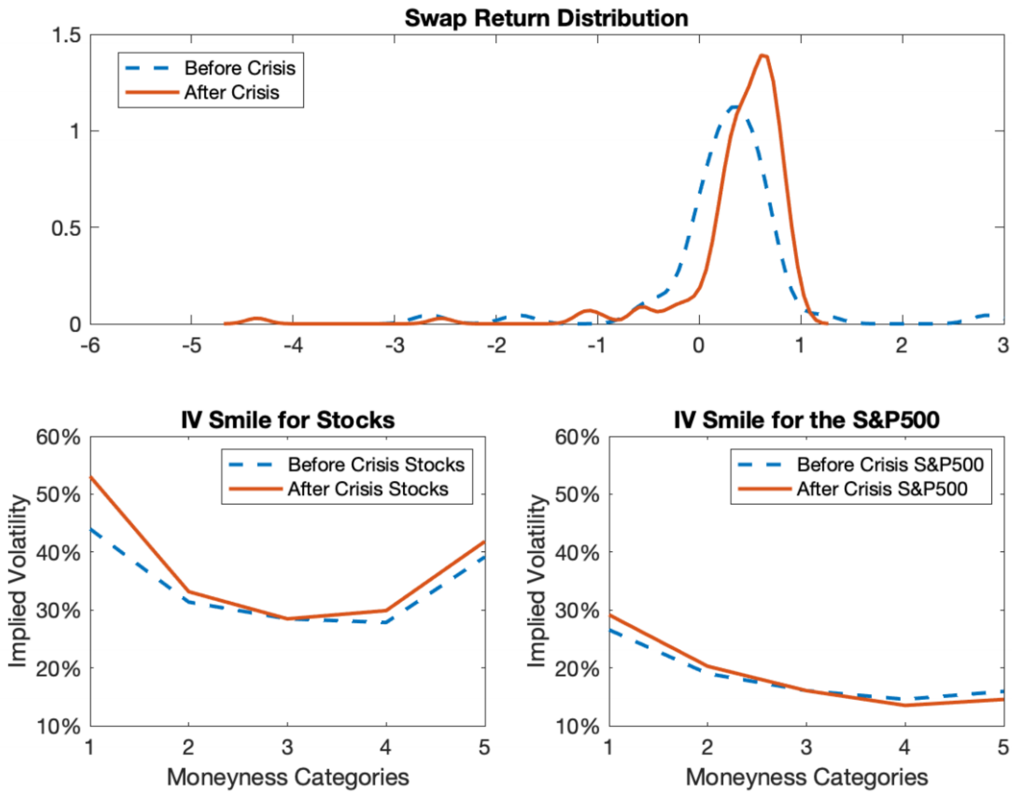


Table 1. Skewness Swaps on Individual Stocks.

Panel A of the table presents the mean and median monthly returns for a value-weighted portfolio of skewness swaps on individual stocks (A1) and the skewness swap for the S&P500 index (A2). Below the median, the table reports a confidence interval estimated using a bootstrap technique with 2,000 bootstrap samples, while the t-statistic is shown below the mean. The table also reports the annualized Sharpe ratio. Panel A3 reports the mean and median skewness swap returns across individual stocks. The numbers in parentheses indicate the count of stocks with statistically significant positive (N pos) or negative (N neg) swap returns. Panels B, C, and D present variations of the baseline skewness swap calculation: Panel B excludes dynamic trading in the underlying stock, Panel C incorporates transaction costs, and Panel D constructs skewness swaps using synthetic European options (i.e., excluding early exercise), which is only applicable to options on individual stocks. Only the stocks with at least 10 skewness swaps are included in the analysis.

Panel A: Skewness Swap Returns

A1: Portfolio of Swaps				A2: S&P500 Swaps				A3: Individual Swaps		
	Mean	Median	Sharpe R.		Mean	Median	Sharpe R.		Mean	Median
	21.91%**	47.99%***	0.54		54.37%***	91.06%***	0.79		20.55%	51.06%
tstat/CI	(2.36)	[39.12, 55.45]		tstat/CI	(3.35)	[88.03, 93.77]		N pos	(146)	(673)
								N neg	(0)	(0)

Panel B: Without Dynamic Trading in the Underlying

B1: Portfolio of Swaps				B2: S&P500 Swaps				B3: Individual Swaps		
	Mean	Median	Sharpe R.		Mean	Median	Sharpe R.		Mean	Median
	28.70%**	57.89%***	0.55		67.02%***	92.55%***	1.08		18.81%	49.11%
tstat/CI	(2.19)	[50.01, 65.78]		tstat/CI	(4.40)	[89.97, 94.88]		N pos	(177)	(690)
								N neg	(0)	(0)

Panel C: With Transaction Costs

C1: Portfolio of Swaps				C2: S&P500 Swaps				C3: Individual Swaps		
	Mean	Median	Sharpe R.		Mean	Median	Sharpe R.		Mean	Median
	10.58%	39.53%***	0.23		41.98%**	84.04%***	0.55		4.95%	39.80%
tstat/CI	(1.02)	[28.75, 44.92]		tstat/CI	(2.31)	[80.59, 89.46]		N pos	(62)	(611)
								N neg	(1)	(0)

Panel D: Without Early Exercise

D1: Portfolio of Swaps				D2: S&P500 Swaps				D3: Individual Swaps		
	Mean	Median	Sharpe R.		Mean	Median	Sharpe R.		Mean	Median
	18.98%*	47.02%***	0.39		N/A	N/A	N/A		14.63%	50.64%
tstat/CI	(1.66)	[38.87, 54.99]		tstat/CI	N/A	N/A		N pos	(139)	(672)
								N neg	(0)	(0)

Table 2. Skewness Swap Returns, Equity Returns, and Variance Swap Returns.

The table presents statistics comparing skewness swap returns with equity returns (Panel A) and variance swap returns (Panel B). Specifically, it reports: (i) the percentage of months in which equity or variance swap returns share the same sign as skewness swap returns, and (ii) the R-squared from regressing skewness swap returns on equity or variance swap returns. Panel C extends this analysis by presenting the R-squared from a regression of skewness swap returns on equity returns, its square, its cube, and variance swap returns. The analysis is conducted at both the portfolio level (Panels A1, B1, and C1) and the individual stock level, where the table reports cross-sectional averages and quantiles (Panels A2, B2, and C2). The analysis includes stocks with at least 100 swaps, resulting in a total of 401 stocks.

Panel A: Skewness Swap Returns $r_{sk,i}$ and Equity Returns r_i					
	A1: Portfolio of Swap	**A2: Individual Swaps**			
		Mean	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$
% of months in which $r_{sk,i}r_i > 0$	82.71%	74.00%	71.43%	74.19%	76.67%
R squared of $r_{sk,i}$ on r_i (%)	45.90%	21.08%	12.19%	21.04%	29.57%
Panel B: Skewness Swap Returns $r_{sk,i}$ and Variance Swap Returns $r_{var,i}$					
	B1: Portfolio of Swap	**B2: Individual Swaps**			
		Mean	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$
% of months in which $r_{sk,i}r_{var,i} > 0$	15.89%	27.82%	24.34%	27.67%	30.87%
R squared of $r_{sk,i}$ on $r_{var,i}$ (%)	61.52%	41.12%	15.78%	38.17%	64.59%
Panel C: Regression of $r_{sk,i}$ on r_i, r_i^2, r_i^3, and $r_{var,i}$					
	C1: Portfolio of Swap	**C2: Individual Swaps**			
		Mean	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$
R squared of $r_{sk,i}$ on r_i, r_i^2, r_i^3, and $r_{var,i}$ (%)	88.82%	67.50%	52.63%	67.81%	84.01%

Table 3. Skewness Swap Returns Before and After the Financial Crisis

The table reports the mean and median returns for the portfolio of skewness swaps (Panel A), the skewness swap on the S&P500 index (Panel B), and the cross-sectional mean and median of individual swap returns (Panel C). These statistics are reported separately for the periods before the financial crisis (Jan 2003 – Aug 2007) and after the financial crisis (June 2009 – Feb 2020). Below the mean, the corresponding t-statistic is provided, while below the median, the 1% confidence interval is reported, computed using a bootstrap technique with 2000 bootstrap samples. The last column in Panels A and B reports the Kolmogorov-Smirnov test statistic, assessing differences in the return distributions between the pre- and post-crisis periods. In Panel C, the numbers in parentheses indicate the count of stocks with statistically significant positive and negative swap returns.

Panel A: Portfolio of Skewness Swaps

	Before FC		**After FC**		
	Mean	Median	Mean	Median	KS Test Statistic
	26.17%***	29.27%***	38.32%***	51.43%***	0.30***
tstat/CI	(2.86)	[20.56, 50.11]	(6.88)	[40.85, 62.00]	

Panel B: S&P500 Skewness Swaps

	Before FC		**After FC**		
	Mean	Median	Mean	Median	KS Test Statistic
	71.2%***	82.93%***	80.28%***	95.72%***	0.50***
tstat/CI	(11.09)	[73.37, 87.18]	(13.57)	[92.11, 97.35]	

Panel C: Individual Swaps

	Before FC		**After FC**	
	Mean	Median	Mean	Median
	6.98%	28.64%	24.29%	49.38%
N pos	(172)	(302)	(311)	(614)
N neg	(17)	(20)	(1)	(1)

Table 4. Systematic and Firm-Specific Crash Risk

The table reports the results of a portfolio sort analysis of skewness swap returns based on exposures to systematic crash risk, *SysRiskProp*, β_1, β_2, and tail beta (Panel A), as well as firm-specific characteristics (Panel B), including the logarithm of the book-to-market ratio at the beginning of each month (BM), Tobin's Q, maximum return over the previous month (Max Ret), minimum moneyness traded, number of put options traded (Np), and risk-neutral variance (Qvar). The analysis is conducted separately for the pre-crisis period (Jan 2003–Aug 2007) and the post-crisis period (June 2009–Feb 2020). The table displays the average skewness swap return for each portfolio, along with the return of the difference portfolio. T-statistics for the difference portfolio are computed using the Newey and West (1987) correction method, with the optimal lag length selected according to Andrews and Monahan (1992).

Panel A: Systematic measures

	Before FC					After FC				
Measure	ptf1	ptf2	ptf3	ptf4	ptf4 - ptf1	ptf1	ptf2	ptf3	ptf4	ptf4 - ptf1
SysRiskProp	0.24	0.34	0.35	0.21	-0.03	0.19	0.29	0.31	0.33	0.14**
Tail beta	0.29	0.28	0.27	0.25	-0.04	0.19	0.27	0.31	0.36	0.17***
$\boldsymbol{\beta_1}$	0.28	0.35	0.29	0.20	-0.08	0.25	0.34	0.31	0.20	-0.06
$\boldsymbol{\beta_2}$	0.13	0.35	0.29	0.31	0.17	0.24	0.32	0.30	0.24	0.01

Panel B: Firm-specific measures

	Before FC					After FC				
Measure	ptf1	ptf2	ptf3	ptf4	ptf4 - ptf1	ptf1	ptf2	ptf3	ptf4	ptf4 - ptf1
BM	0.31	0.25	0.25	0.28	-0.03	0.34	0.31	0.25	0.21	-0.13**
Tobin Q	0.22	0.32	0.29	0.27	0.05	0.21	0.26	0.30	0.34	0.13**
Max Ret	0.26	0.23	0.25	0.29	0.03	0.34	0.28	0.24	0.27	-0.07*
Min Mon Traded	0.41	0.27	0.23	0.19	-0.23***	0.38	0.29	0.22	0.25	-0.13***
Np	0.24	0.25	0.22	0.41	0.18***	0.18	0.27	0.34	0.38	0.19***
Qvar	0.26	0.26	0.26	0.33	0.07	0.36	0.31	0.23	0.23	-0.12**

Table 5. Robustness: The Model-based Skewness Swaps and the Corridor Skewness Swaps.

The table presents the mean and median returns for the model-based portfolio of swaps (Panel A) and the corridor-based portfolios of swaps (Panel B), separately for the periods before the financial crisis (Jan 2003 – Aug 2007) and after the financial crisis (June 2009 – Feb 2020). The last column reports the Kolmogorov-Smirnov test statistic, which tests the difference in return distributions between the pre- and post-crisis periods. The model-based skewness swap is constructed at each swap start date using option prices generated from the Merton jump-diffusion model that best fits the data. The corridor swaps are implemented following Andersen et al. (2015a), with five corridor choices defined as $K \in [F_{t,T}e^{-SD\sigma\sqrt{T-t}}, F_{t,T}e^{SD\sigma\sqrt{T-t}}]$, where SD ranges from 1 to 5, K is the strike price, $F_{t,T}$ is the forward price, $T-t$ is the time to maturity, and σ is the at-the-money volatility. Below the mean, the corresponding t-statistic is reported, while below the median, the 1% confidence interval is provided, computed using a bootstrap technique with 2000 bootstrap samples.

Panel A: Model-based Portfolio of Swaps

	Before FC		**After FC**		
	Mean	Median	Mean	Median	KS Test
	22.97%***	32.59%***	29.43%***	42.62%***	0.22*
tstat/CI	(3.31)	[21.88, 43.16]	(5.32)	[32.68, 50.51]	

Panel B: Corridor-based Portfolio of Swaps

Corridor Range	**Before FC**		**After FC**		
	Mean	Median	Mean	Median	KS Test
1SD	44.14%	10.07%	-2.09%	16.91%	0.12
	(1.20)	[-3.72, 32.72]	(-0.22)	[-3.85, 22.19]	
2SD	14.82%**	23.57%***	17.62%***	35.37%***	0.22*
	(1.99)	[14.00, 35.50]	(3.01)	[22.60, 44.71]	
3SD	17.14%**	27.17%***	25.5%***	40.94%***	0.21*
	(2.06)	[19.82, 41.44]	(4.02)	[29.55, 51.86]	
4SD	19.87%**	28.94%***	31.34%***	47.31%***	0.26***
	(2.51)	[17.85, 43.83]	(5.19)	[35.25, 55.89]	
5SD	21.99%***	29.17%***	34.95%***	49.78%***	0.28***
	(2.83)	[20.18, 44.10]	(6.06)	[38.22, 58.42]	

A. The Skewness Swaps

Background and Theory on Skewness Swaps

The skewness swap is a contract through which an investor can buy the skewness of an asset by taking positions in options. At the contract's initiation, the investor purchases a portfolio of options, and upon the options' expiration, she receives the payoff associated with this option portfolio. The fundamental concept underpinning this contract is that the price of the option portfolio quantifies the risk-neutral skewness of the asset, while the option portfolio's payoff plus a continuous hedging in the underlying stock market quantifies the realized skewness of the asset. In essence, it functions akin to a swap contract in which two parties agree to exchange a fixed leg, determined by the price of the option portfolio, for a floating leg, determined by the payoff of the option portfolio plus the hedge, at the contract's maturity.

I build on the general divergence trading strategies of Schneider and Trojani (2019) and Schneider and Trojani (2015) to construct the skewness swap implemented in this paper. Schneider and Trojani (2019) introduce a new class of swap trading strategies with which an investor can take a position in the generalized Bregman (1967) divergence of the asset. The skewness can be seen as a special type of divergence, and Schneider and Trojani (2015) propose a Hellinger skew swap for trading skewness. The swap developed in this paper builds on the Hellinger skewness swap of Schneider and Trojani (2015) with the following differences: (a) this skewness swap is a pure bet on the third moment of the stock returns while being independent of the first, second, and fourth moments, and (b) the swap is applied directly to American options.

The swap consists of a fixed leg and a floating leg, which investors exchange at maturity. These are defined by Equations 2 and 3 in the main text. A key part of the formulas is the function $\Phi : \mathbb{R} \longrightarrow \mathbb{R}$, which is a twice-differentiable generating function that defines the moment of the distribution we want to trade. For example, if $\Phi(x) = \Phi_2(x) = -4((x/F_{0,T})^{0.5} - 1)$, then fixed $\text{leg}_{0,T} = E_0^{\mathbb{Q}}\left[\log(F_{T,T}/F_{0,T})^2 + O(\log(F_{T,T}/F_{0,T})^3)\right]$. In this example, Φ_2 captures the second-order variation of the returns. The fixed $\text{leg}_{0,T}$ quantifies the risk-neutral moment and is established at time 0, whereas the floating $\text{leg}_{0,T}$ measures the realization of the moment between time 0 and time T, with its value known at time T. It is worth noting that dividends do not affect the methodology because the modeled return is the forward return $y = \log(F_{T,T}/F_{0,T})$ in which the dividends are included in the calculation of $F_{0,T}$. Equation 3 shows that the floating leg is composed of two parts: the payoff of the option portfolio at maturity plus dynamic trading in the forward market, which is rebalanced at the intermediate dates i. All the payments of the swap are made at maturity, when the investors exchange the fixed leg with the floating leg. The value of the swap at time 0 is zero, as $E_0^{\mathbb{Q}}[\text{floating leg}_{0,T}] = \text{fixed leg}_{0,T}$.

If the function Φ is

$$\text{(A.1)}\quad \Phi(x) := \Phi_3\left(\frac{x}{F_{0,T}}\right) = -4\left(\frac{x}{F_{0,T}}\right)^{1/2}\log\left(\frac{x}{F_{0,T}}\right)$$

then

$$\text{(A.2)}\quad \text{fixed leg}_{\Phi_3,0,T} = E_0^{\mathbb{Q}}\left[\frac{1}{6}y^3 + \frac{1}{12}y^4 + O(y^5)\right],$$

where $y = \log(F_{T,T}/F_{0,T})$. This is the Hellinger skewness swap proposed by Schneider and Trojani (2015) for studying the third moment of the returns. However, the formula A.2 shows that the swap depends theoretically on the fourth moment as well, and in Section B of the Appendix I will show that this dependence could potentially lead to a biased measure of the third moment.

The Skewness Swap Implemented in this Paper

To mitigate the reliance on the fourth moment, as discussed earlier, and enhance the isolation of the third moment, I introduce a new skewness swap denoted as S. The function Φ_S for this swap is a fusion of Φ_3 and Φ_4, with Φ_4 being the function that characterizes the kurtosis swap in Schneider and Trojani (2015). In detail, $\Phi_4\left(\frac{x}{F_{0,T}}\right) = -4\left[(x/F_{0,T})^{1/2}(\log(x/F_{0,T})^2 + 8) - 8\right]$ and verifies fixed $\text{leg}_{\Phi_4,0,T} = E_0^{\mathbb{Q}}[\frac{1}{12}y^4 + O(y^5)]$, where $y = \log(F_{T,T}/F_{0,T})$. By taking a long position in 6 times Φ_3 and a short position in 6 times Φ_4, I can construct a new simple skewness swap which does not depend on the fourth moment anymore. The formal result is stated in Proposition 1 in the main text, and I outline the proof below.

Proof of Proposition 1.

For every Φ:

$$\text{fixed leg}_{\Phi,0,T} = E_0^{Q}[\text{floating leg}_{\Phi_3,0,T}] = E_0^{Q}\left[\int_0^{F_{0,T}} \Phi''(K)P_{E,T,T}dK + \int_{F_{0,T}}^{\infty} \Phi''(K)C_{E,T,T}dK\right]$$

because $E_i^Q[F_{T,T} - F_{i,T}] = 0$ for every i, and hence the conditional expectation of the second term of the floating leg defined by Equation 3 is zero.

By applying the result of Carr and Madan (2001) to our case, I obtain the following:

$$\int_0^{F_{0,T}} \Phi^{''}(K) P_{E,T,T}(K) dK + \int_{F_{0,T}}^{\infty} \Phi^{''}(K) C_{E,T,T}(K) dK = \Phi(F_{T,T}) - \Phi(F_{0,T}) - \Phi^{'}(F_{0,T})(F_{T,T} - F_{0,T}).$$

I substitute the definition of Φ of Equation 4, and with some calculations I obtain

$$\begin{aligned}
&\Phi(F_{T,T}) - \Phi(F_{0,T}) - \Phi^{'}(F_{0,T})(F_{T,T} - F_{0,T}) = \\
&= 72 - \frac{(72F_{T,T})}{F_{0.T}} - 24\left(\frac{F_{T,T}}{F_{0.T}}\right)^{0.5} \log\left[\frac{F_{T,T}}{F_{0.T}}\right] + 24\left(-8 + \left(\frac{F_{T,T}}{F_{0.T}}\right)^{0.5}\left(8 + \log\left[\frac{F_{T,T}}{F_{0.T}}\right]^2\right)\right) = \\
&= 72 - 72e^y - 24e^{y/2}y + 24(-8 + e^{y/2}(8 + y^2)) = \\
&= y^3 + O(y^5),
\end{aligned}$$

where $y = \log(F_{T,T}/F_{0,T})$, and the last equality is obtained by substituting e^y with its power series expansion $e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!}$. □

American Versus European Skewness Swap

The preceding formulas for the skewness swap pertain to options with a European-style exercise, meaning they can only be exercised at maturity. However, in the equity market, European options are available only for indexes, while options on individual stocks are American, allowing exercise at any point before maturity.

This section illustrates a straightforward adjustment to Equations 2 and 3 for crafting a tradable skewness swap involving American options. The discrepancy stemming from the use of American options instead of European options is a simple function of the early exercise premium, which, in this context, is negligible.

I start by defining the American call option payoff at time T:

$$C_{A,T,T} = \frac{(S_{t^*} - K)}{B_{t^*,T}}$$

where $t^* = \min\{0 \leq t \leq T : (S_{t^*} - K) > C(t^*, S_{t^*}, K, T - t^*)\}$, and analogously, the American put option payoff:

$$P_{A,T,T} = \frac{(K - S_{t^*})}{B_{t^*,T}}$$

where $t^* = \min\{0 \leq t \leq T : (K - S_{t^*}) > P(t^*, S_{t^*}, K, T - t^*)\}$. The idea is that the investor exercises the American options optimally, and the final payoff at maturity is given by the compounded optimal exercise proceeds.[30]

I define a new swap whose floating leg is given by

$$\begin{aligned} \text{floating leg}_{A,0,T} = & \left(\int_0^{F_{0,T}} \Phi''(K) P_{A,T,T} dK + \int_{F_{0,T}}^{\infty} \Phi''(K) C_{A,T,T} dK \right) \\ & + \sum_{i=1}^{n-1} \left(\Phi'(F_{i-1,T}) - \Phi'(F_{i,T}) \right) (F_{T,T} - F_{i,T}) \end{aligned} \tag{A.3}$$

and whose fixed leg is given by the expectation of the floating leg

(A.4)

[30]Many studies show that investors actually do not optimally exercise their stock options, and in particular they miss most of the advantageous exercise opportunities (see, e.g., Pool et al. (2008), Barraclough and Whaley (2012), Cosma et al. (2020), Bryzgalova et al. (2022)). This issue is important for in-the-money options, while here the skewness swap is constructed using out-of-the-money options, for which the early exercise is less relevant. In the empirical section, I show that, in my analysis, the early exercise value is very small, and hence an alteration of the early exercise proceeds given by a suboptimal behavior would not alter the main results of the paper.

$$\text{fixed leg}_{A,0,T} = E_0^{\mathbb{Q}}[\text{floating leg}_{A,0,T}] = \frac{1}{B_{0,T}} \left(\int_0^{F_{0,T}} \Phi''(K) P_{A,0,T} dK + \int_{F_{0,T}}^{\infty} \Phi''(K) C_{A,0,T} dK \right).$$

The subscript A indicates that the prices are American option prices. The next proposition shows that fixed $\text{leg}_{A,0,T}$ equals fixed $\text{leg}_{0,T}$ plus the price of the early exercise and that floating $\text{leg}_{A,0,T}$ equals floating $\text{leg}_{0,T}$ plus the realization of the early exercise.

Proposition 2. *The swap with fixed leg given by Equation A.4 and floating leg given by Equation A.3 verifies the following properties:*

$$\textit{fixed leg}_{A,0,T} = \textit{fixed leg}_{0,T} + \frac{1}{B_{0,T}} \left(\int_0^{F_{0,T}} \Phi''(K)(P_{A,0,T} - P_{E,0,T}) dK \right) + \\ + \frac{1}{B_{0,T}} \left(\int_{F_{0,T}}^{\infty} \Phi''(K)(C_{A,0,T} - C_{E,0,T}) dK \right)$$

$$\textit{floating leg}_{A,0,T} = \textit{floating leg}_{0,T} + \left(\int_0^{F_{0,T}} \Phi''(K)(P_{A,T,T} - P_{E,T,T}) dK \right) + \\ + \left(\int_{F_{0,T}}^{\infty} \Phi''(K)(C_{A,T,T} - C_{E,T,T}) dK \right)$$

The difference between the American and European prices $(P_{A,0,T} - P_{E,0,T})$ *and* $(C_{A,0,T} - C_{E,0,T})$ *measures the price of the early exercise. The difference between the payoff of American and European options* $(P_{A,T,T} - P_{E,T,T})$ *and* $(C_{A,T,T} - C_{E,T,T})$ *measures the realization of the early exercise.*

Proof. The proof can be readily derived by simply adding and subtracting the European option prices and payoffs in Equations A.4 and A.3, respectively.

□

B. Numerical Analysis: Convergence of the Skewness Swap

This section delves into an examination of the accuracy of skewness swaps in measuring the third moment of stock returns within the Merton jump-diffusion model.[31] Within this section, two convergence exercises are conducted. Firstly, I apply Equation 6 in the main text with an increasing number of options to verify numerically that the skewness swap accurately estimates the skewness of the stock as the number of options approaches infinity. Secondly, while keeping a constant number of ten options (which aligns with the average number used in empirical analyses), I assess the error magnitude when these ten options span a wider range of moneyness levels.

The dynamics of the Merton (1976) model is described by Equation 7 in the main text. The characteristic function is given by:

$$\phi(u) = E[e^{iu(s_t - s_0)}] = e^{t(i(r-0.5\sigma^2-\lambda\kappa)u-0.5\sigma^2u^2+\lambda(e^{i\mu u-0.5\delta^2u^2}-1))}$$

and the moments can be recovered from the property of the characteristic function:

$$E[(s_t - s_0)^k] = (-i)^k \frac{d^k}{du^k}\phi(u)_{|u=0}.$$

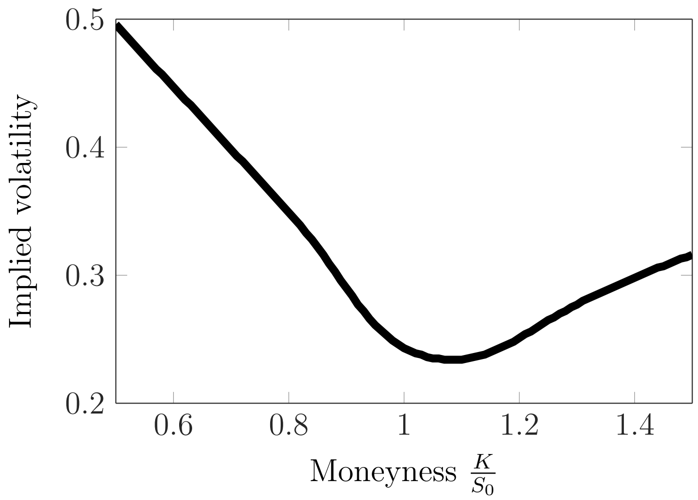
As a result, the third moment can be expressed in closed form by the following formula:

$$E[(s_t-s_0)^3] = 3\delta^2\lambda\mu t+\lambda\mu^3 t-3(-\delta^2\lambda-\lambda\mu^2-\sigma^2)(r+(-\kappa\lambda+\lambda\mu-0.5\sigma^2))t^2+(r-\kappa\lambda+\lambda\mu-0.5\sigma^2)^3t^3.$$

[31] It's worth noting that the selection of the Merton model is not limiting, as demonstrated in the work of Hagan et al. (2002), where it is shown that, for time horizons less than one year, the implied volatility smile generated by the Merton model satisfactorily mirrors empirical data.

Figure B.1. Merton Implied Volatility Smile

The figure displays the one-month implied volatility smile generated by the Merton model with the following set of representative parameters: $r = 0, \mu = -0.05, \delta = 0.08, \sigma = 0.2, \lambda = 3$.



I chose the following standard parameter values for the simulation study: $r = 0$, $\mu = -0.05$, $\delta = 0.08$, $\sigma = 0.2$, $\lambda = 3$, $t = 30/365$. With these parameters, the Merton implied volatility smile is left skewed, as shown in Figure B.1.

Convergence in the Number of Options

Table B.1 shows the convergence of the fixed leg of the swap to the true model-based third moment when the number of options used increases from 8 to 100. The fixed leg of the swap is evaluated using Equation 6, with option prices derived analytically from the Merton jump-diffusion model. The price of a call option is given by (see Merton (1976)):

$$C_{MRT}(0, t, K) = \sum_{n=0}^{\infty} e^{-\lambda' t + n \log(\lambda' t) - \sum_{i=1}^{n} \log n} C(S_0, K, r_n, \sigma_n)$$

where $\lambda' = \lambda(1 + k)$, $k = e^{\mu + \frac{1}{2}\delta^2} - 1$, $C(S_0, K, r_n, \sigma_n)$ is the Black-Scholes price of an

European call with volatility $\sigma_n = \sqrt{\sigma^2 + \frac{n\delta^2}{t}}$ and risk-free rate $r_n = r - \lambda k + \frac{n \log(1+k)}{t}$. The price of a put option is defined analogously. The initial stock price, denoted as S_0, is set at 100, and the options cover an evenly spaced range of strike prices from 50 to 150.

I evaluate the precision of two skewness swaps in this analysis. The first is the skewness swap utilized in the empirical study of this paper, labeled as Φ_S, where the Φ function is defined by Equation 4. The second is the skewness swap originally introduced by Schneider and Trojani (2015), referred to as Φ_3, where the Φ function is defined by Equation A.1.

Table B.1 illustrates the measurement errors as a percentage of the true skewness. The third column reveals a consistent reduction in measurement errors as the number of options increases, eventually stabilizing at around 0.7% to 0.8% when using twenty or more options. It's important to note that the error cannot reach zero due to its dependence on the fifth moment in Equation 5. Nevertheless, the error magnitude remains quite low, with an error of approximately 1.7% even with just 10 options. On the other hand, the second column demonstrates the error of the skewness swap defined by Φ_3, which displays a more pronounced bias that persists even as the number of options increases. This discrepancy arises from the error term's dependence on the fourth moment, which naturally holds greater significance than the fifth moment.

In summary, this analysis underscores the convergence of the skewness swap implemented in this paper to the stock's skewness within the Merton jump diffusion model. It emphasizes the importance of isolating the third moment from the fourth

Table B.1. Convergence in the Number of Options.

This table shows the convergence of the fixed leg of the trading strategy to the third moment of the asset returns when the number of options increases. The error is computed as (|True moment − Strategy fixed leg|)/(|True moment|) and is displayed as a percentage. The returns are assumed to follow a Merton jump-diffusion process with standard parameters, i.e., $\mu = -0.05$, $\delta = 0.08$, $\sigma = 0.2$, $\lambda = 3$, $r = 0$, $t = 30/365$. The true moment is computed in closed form and is equal to $-3.12 \cdot 10^{-4}$. In the second column, I consider the skewness swap of Schneider and Trojani (2015) with $\Phi = \Phi_3$, while in the third column I consider the skewness swap introduced in this paper with $\Phi = \Phi_S$.

Number of options	Error (%)	
	Φ_3	Φ_S
8	18.81%	5.16%
10	21.10%	1.70%
20	22.70%	0.72%
50	22.80%	0.86%
100	22.80%	0.87%

moment for achieving a precise measurement.

Convergence in the Range of Moneyness

As a second convergence exercise, I check how the precision of the methodology depends on the range of moneyness available. In this context, the moneyness of an option is defined in standard deviations (SD) as $\log(K/F_{0,T})/(\sigma\sqrt{T})$, where K is the strike price, $F_{0,T}$ is the forward price, σ is the at-the-money implied volatility, and T is the time to maturity. In this exercise I maintain a constant number of options, set at 10, and assess the error of the swap when these ten options span moneyness ranges of

$\pm 1SD$, $\pm 2SD$, $\pm 3SD$, and $\pm 4SD$.[32]

Figure B.2 illustrates the results. Every quadrant i, where $i = 1, ..4$, illustrates the part of the Merton implied volatility smile spanning $\pm iSD$. Each plot provides a zoomed out perspective compared to the previous one by 1SD. Positioned at the top of each graph is the measurement error (expressed as a percentage of the true skewness) of a skewness swap calculated using ten options spanning the corresponding moneyness range of $\pm iSD$ as described above. The results show that the precision changes considerably depending on the moneyness range available. At least three standard deviations are needed in order to have an error around 10%, and four standard deviations are needed to have an error around 1%.

C. Model-based Skewness Swap

This section provides a comprehensive breakdown of the procedures entailed in constructing the model-based skewness swap, as employed in the robustness Section 5.1. The implementation of the model-based skewness swap consists of two principal steps: i) calibration of the Merton model, and ii) implementation of the swap based on the calibrated model. These steps are detailed below.

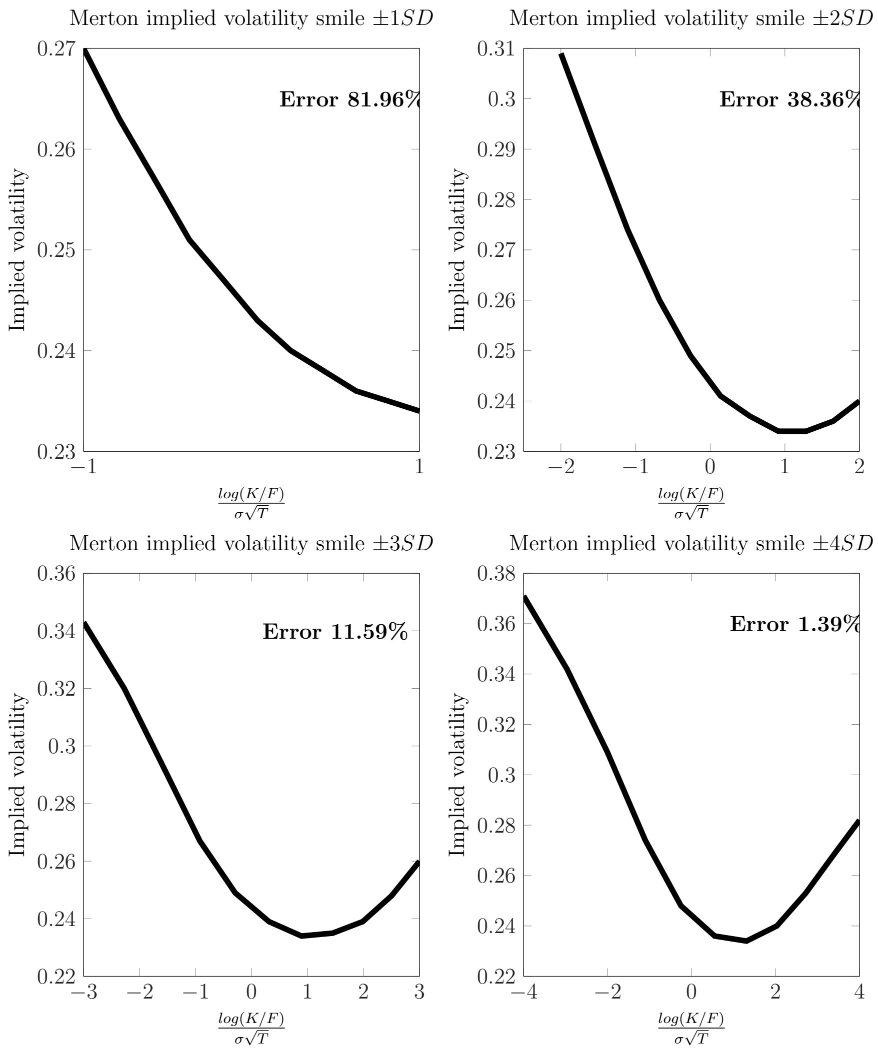
Calibration

The model-based skewness swaps are implemented monthly, as the tradable

[32]To illustrate, if the ten options encompass a moneyness range of $\pm 1SD$, their strike prices are evenly distributed within the strike range of $[F_{0,T}e^{-\sigma\sqrt{T}}, F_{0,T}e^{+\sigma\sqrt{T}}]$. Similarly, if the ten options cover a moneyness range of $\pm 2SD$, their strike prices are evenly spaced within the strike range of $[F_{0,T}e^{-2\sigma\sqrt{T}}, F_{0,T}e^{+2\sigma\sqrt{T}}]$, and so forth.

Figure B.2. Convergence in the Moneyness Range

This figure illustrates the volatility smile derived from the Merton jump-diffusion process, with the following parameters: $\mu = -0.05$, $\delta = 0.08$, $\sigma = 0.2$, $\lambda = 3$, $r = 0$, and $t = 30/365$. The illustration covers increasing moneyness ranges. In the first plot, a moneyness range of $[-1SD, 1SD]$ is considered, followed by the second plot with $[-2SD, 2SD]$, the third with $[-3SD, 3SD]$, and finally, the fourth with $[-4SD, 4SD]$. Here, SD is defined as $SD = \log(K/F_{0,T})/(\sigma\sqrt{T})$, where K represents the strike price, $F_{0,T}$ is the forward price, σ is the at-the-money implied volatility, and T is the time to maturity. Each plot provides a zoomed-out perspective compared to the previous one by $1SD$. Within each moneyness range, the fixed leg of the skewness swap is computed and compared to the true skewness derived through a closed-form expression. The measurement error is presented at the top of each graph.



skewness swaps, and they start and end on the third Friday of each month. The months in which the stocks pay dividends are excluded in order to simplify the calculation of the model-based option prices. The model is recalibrated at each start date of the swap and for each stock separately. In detail, at each start date of the swap t and for each stock S_t, I consider all the out-of-the-money options with a maturity 30 days provided by the Optionmetrics implied volatility surface file. This sample constitutes the calibration sample. The benchmark model is the Merton jump-diffusion model with Gaussian jump-size distribution, whose dynamics is given by Equation 7 in the main text. I then calibrate the parameters of the model by minimizing the implied volatility mean squared error (IVMSE) as

$$IVMSE(\chi) = \sum_{i=1}^{n} (\sigma_i - \sigma_i(\chi))^2$$

where $\chi = \{\lambda, \mu, \delta, \sigma\}$ is the set of parameters to estimate, $\sigma_i = BS^{-1}(O_i, T_i, K_i, S, r)$ is the market implied volatility provided by Optionmetrics, and

$\sigma_i(\chi) = BS^{-1}(O_i(\chi), T_i, K_i, S, r)$ is the model implied volatility, where $O_i(\chi)$ is the Merton model price of the option i. The model implied volatility is obtained by inverting the Black–Scholes formula, where the option price is given by the Merton model price. The choice of the implied volatility mean squared error (IVMSE) loss function follows the argumentation of Christoffersen and Jacobs (2004), who show that the calibration made on implied volatilities is more stable out of sample. Table C.1 displays the average calibrated parameters for the cross-section of stocks.

Implementation of the Model-based Swap

Table C.1. Calibrated Parameters of the Merton Jump-diffusion Model.

This table displays the average calibrated parameters of the Merton jump-diffusion model for the cross-section of stocks. The model is calibrated separately for each stock, and it is recalibrated monthly at each start date of the swap. The calibration sample includes all the out-of-the-money options with a maturity 30 days quoted by the Optionmetrics interpolated volatility surface file on the calibration day. The numbers displayed are the average calibrated parameters across months and across stocks.

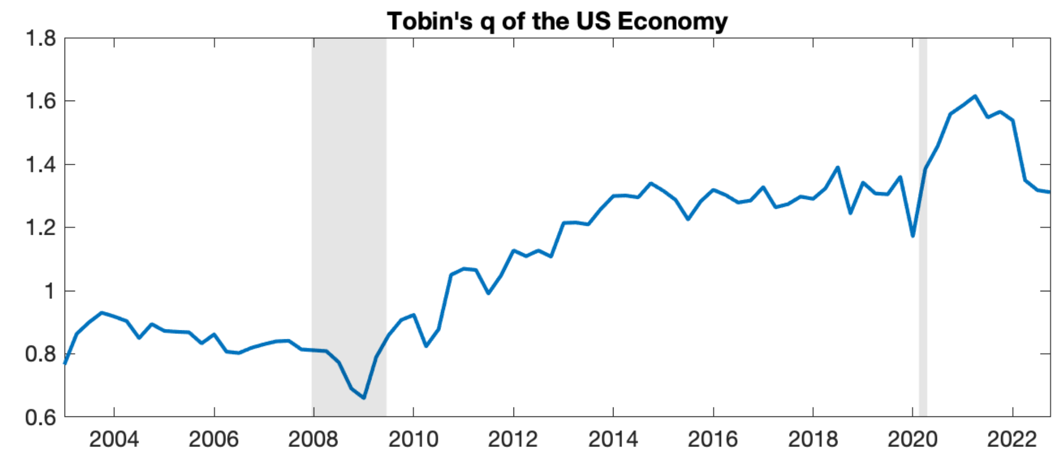
	λ	μ	δ	σ
Average across stocks	4.01	-0.13	0.20	0.24

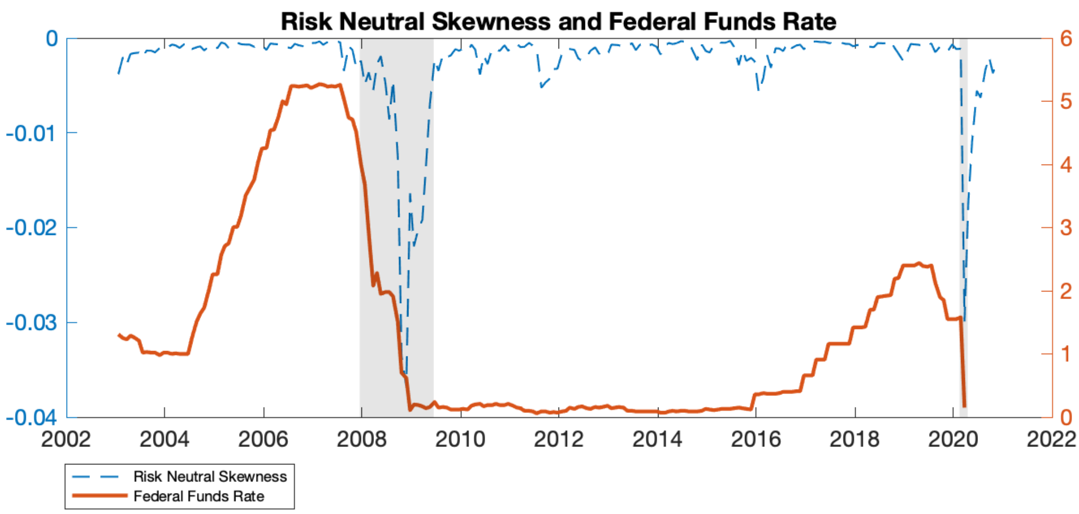
Following the calibration procedure outlined in the preceding step, the parameters $\widehat{\lambda}, \widehat{\mu}, \widehat{\delta}, \widehat{\sigma}$ of the Merton jump-diffusion model are estimated individually for each stock S_t on each starting date of the swaps, denoted as t. Subsequently, Merton option prices are computed for a regularly spaced grid of strikes, covering the moneyness range $[S_t e^{-4\sigma\sqrt{T-t}}, S_t e^{4\sigma\sqrt{T-t}}]$. In this formula, σ is determined as the implied volatility of an at-the-money option, represented as $\sigma = BS^{-1}(C_{MRT}, T-t, S_t, S_t, r)$, where C_{MRT} corresponds to the Merton price of a call option with a strike equal to S_t. The equispaced grid is designed to include twenty out-of-the-money puts with strikes spanning the range $[S_t e^{-4\sigma\sqrt{T-t}}, S_t]$, along with twenty out-of-the-money calls having strikes covering the range $[S_t, S_t e^{4\sigma\sqrt{T-t}}]$. The fixed leg of the swap, as defined in Equation 6, is then calculated using these model-derived option prices. The floating leg of the swap comprises the total of the payoff from the same option portfolio and a continuous delta-hedge executed in the forward market. The return of the swap is computed as outlined in Section 1 in the main text.

D. Additional Figures

Figure D.1. Overvaluation of the Stock Market After the Financial Crisis.

The upper chart presents the time-series of the Tobin's q metric for the US economy. The data is sourced from the FRED database operated by the Federal Reserve Bank of St. Louis, specifically from the variable denoted as (Nonfinancial Corporate Business; Corporate Equities; Liability, Level/1000)/(Nonfinancial Corporate Business; Net Worth, Level). The lower chart presents the time-series of the average value of the fixed leg of the swaps across stocks, accompanied by the corresponding time-series for the Federal Funds Rate. The data source for the Federal Funds Rate is the H15 Report of the Federal Reserve accessed from Wharton Research Data Service (WRDS). The regions in gray highlight the financial crisis and Covid-19 recessions, as officially defined by the National Bureau of Economic Research (NBER).





E. Additional Tables

Table E.1. Skewness Swap Returns Around FOMC Announcements.

The table reports the results of a panel regression examining changes in skewness swap returns following FOMC announcements. The analysis covers announcements from 2004 to 2020, including 151 instances where interest rates were on the agenda. The dependent variable $\Delta r_{sk,i,FOMC_t}$ measures, for each stock i, the change in the return of the first swap following the FOMC date relative to the last swap before the announcement. This variable is regressed on the interest rate decision, IR_{FOMC_t}, and its interaction with two dummy variables: $1_{IR_{FOMC_t}<0}$, which equals one when the Federal Reserve lowers rates, and $1_{IR_{FOMC_t}>=0}$, which equals one when rates are held constant or increased. Standard errors are clustered at both the stock and time levels.

Panel Regression of $\boldsymbol{\Delta r_{sk,i,FOMC_t}}$		
IR_{FOMC_t}	-0.026**	
	(-2.28)	
$1_{IR_{FOMC_t}>=0} \cdot IR_{FOMC_t}$		-0.006
		(-0.70)
$1_{IR_{FOMC_t}<0} \cdot IR_{FOMC_t}$		-0.029**
		(-2.16)
Stock fixed effect	Y	Y
Year fixed effect	Y	Y
R squared (%)	1.33	1.52