Disagreement and Scheduled Announcements: Explaining the Pre-Announcement Drift

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Abstract

This paper proposes a theoretical explanation for the positive *pre-announcement drift* empirically documented ahead of scheduled announcements, using the Federal Open Market Committee (FOMC) meetings as a main example. The framework entails a general equilibrium model of disagreement (differences-of-opinion), where investors interpret a costly signal differently. Investors optimally decide to stop learning when an announcement is imminent, increasing the risk premium ahead of an announcement. The model jointly rationalizes puzzling empirical evidence by generating (1) an upward drift in prices just before scheduled announcements, regardless of the announcement's content, which coexists with (2) low volatility and (3) low trading volume.

Keywords: FOMC announcements, differences of opinion, scheduled announcements, sentiment risk, optimal learning

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I. Introduction

Scheduled announcements, with dates well known in advance, lead investors to anticipate the arrival of information, even without knowing its content. Surprisingly, the empirical literature consistently documents a sizable increase in equity prices before such announcements.[1] Given the extensive documentation on Federal Open Market Committee (FOMC) meetings, this paper uses as its main example what Lucca and Moench (2015) call the pre-FOMC drift, a significant increase in equity prices before scheduled FOMC meetings, accounting for up to 80% of annual U.S. equity returns. Remarkably, this upward drift occurs regardless of the announcement's content and is accompanied by low volatility and trading volume, adding to its puzzling nature. To our knowledge, no existing asset pricing theory fully explains this set of empirical facts.

This paper proposes an economic mechanism that explains the pre-announcement drift and its associated low volatility and trading volume. Building on David (2008) and Dumas, Kurshev, and Uppal (2009), we introduce two key modifications to a disagreement model: (1) scheduled announcements, such that information arrives both continuously and at fixed, well-known points in time, and (2) a costly continuous-time signal, requiring investors to exert effort, however small, to learn from it. These elements, within a disagreement framework, offer a unified explanation for the empirical facts surrounding scheduled announcements.

The pre-announcement drift arises because investors optimally stop learning from the costly signal when an announcement is imminent. Given the cost, investors weigh whether

[1] See Gao, Hu, and Zhang (2020), Savor and Wilson (2016), Donders, Kouwenberg, and Vorst (2000), Chae (2005), and Akbas (2016) for corporate earnings announcements and Lucca and Moench (2015), Savor and Wilson (2013), and Hu, Wang, Pan, and Zhu (2019)for macroeconomic announcements, including FOMC meetings.

learning is worthwhile and choose to forgo it when they foresee an announcement approaching. In a disagreement model, this decision reduces sentiment risk, driving up prices before the announcement. Thus, the reduction in sentiment risk provides a unified explanation for the pre-announcement drift and its related empirical patterns.

Our specific model describes an economy where dividends follow an unknown, unobservable growth rate. Investors learn about this rate from two sources (besides the dividends): scheduled announcements and a costly continuous signal.[2] Both the announcements and the signal are public, meaning investors trade for consumption smoothing rather than speculation.[3] Disagreement is modeled as stemming from unshakable priors: one group, the "Fed believers" (investors B), trust that all relevant information is revealed only through scheduled announcements; the other group, "Fed agnostics" (investors A), view the continuous signal as informative too. Under this structure, investors still form rational expectations, but they do not learn from each other or from prices; they simply "agree to disagree." Since investors interpret signals differently, their estimates of the economy's growth rate diverge. Investors A might be more optimistic or pessimistic depending on the signal content. A change in the beliefs of investors A relative to those of investors B defines *sentiment*, and as both groups trade to clear the market, sentiment is ultimately reflected in prices, with the absolute level of disagreement between the investors determining its riskiness, i.e., stochastic volatility.

[2]Possible micro-foundations for this cost include limited cognitive capacity, as in the optimal inattention work of Abel, Eberly, and Panageas (2007), time spent monitoring information, or transaction and portfolio rebalancing costs incurred upon processing and acting on the information.

[3]This aligns with the empirical evidence that informed trades, though likely present, do not cause the pre-announcement drift; see Appendix A.

The reason why investors decide to stop learning when an announcement is imminent is as follows. While investors A learn about the economy's growth rate from the costly signal, they must weigh its cost against its benefits. As an announcement approaches, the benefits of learning from the signal diminish because new information is expected to arrive soon. The announcement's informational content dwarfs that of the signal, so investors prefer to spare the cost of the signal and wait.[4] Intuitively, when investors anticipate an announcement, they anxiously "freeze up" and stop learning from the costly signal.[5] However, the exact timing of the decision to stop learning is random and endogenous depending on the economy's state.[6] When investors A stop learning, their beliefs stop diverging from those of investors B, thereby reducing disagreement and, in turn, sentiment risk. Since sentiment risk is priced, such a risk reduction increases prices, generating the pre-announcement drift, along with lower volatility (due to reduced risk) and lower trading volume (from decreased hedging demand). Still, the pre-announcement drift cannot be arbitraged away by purchasing equities earlier in an announcement cycle, as this entails exposure to sentiment risk, which deters investors from holding the equity asset. Only a reduction in sentiment risk motivates investors to increase their asset holdings, causing the pre-announcement drift. After an announcement occurs, prices may jump up or down depending on the announcement's content, but no reverse drift occurs. Instead, the differences in investors' beliefs about the announcement generate abnormally high volatility and trading volume, consistent with empirical post-announcement evidence.

[4] The same mechanism is at play in Grossman and Stiglitz (1980), where the information contained in prices and signals are substitutes; if there is too much information in prices, few investors will buy signals.

[5] Ahead of FOMC meetings, expressions such as "Fed watch" or "countdown to FOMC" reflect this phenomenon.

[6] This unique feature of the model aligns with the empirical findings of Kurov, Wolfe, and Gilbert (2020).

The model's generality allows it to encompass various scheduled announcements, including FOMC meetings, without explicitly modeling the Federal Reserve. Announcements are assumed to be imperfectly transparent, leading to disagreements in their interpretation. In the FOMC case, several studies have demonstrated the effects of such an information channel versus the traditional policy channel.[7] Here, announcements are interpreted as forward guidance, particularly through press conferences, since other policy tools, such as short-term rate changes, contain little noise upon release.[8] This interpretation aligns with Stein's (1989) argument that the Fed, aiming to shape expectations while maintaining flexibility, cannot fully and credibly disclose its policy objectives.[9] This lack of disclosure reduces transparency in FOMC announcements, supporting the model's assumption.[10]

Assuming that announcements are noisy does not imply that the pre-announcement drift results from information leakage.[11] Both FOMC and corporate earnings announcements exhibit a consistently positive pre-announcement drift, whereas post-announcement price movements average to zero but vary based on the released information. Surprisingly, the pre-announcement

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[7] See Cieslak and Schrimpf (2019), Gurkaynak, Sack, and Swanson (2005), Jarociński and Karadi (2018), Matheson and Stavrev (2014), and Leombroni, Venter, and Whelan (2016).

[8] This is consistent with Boguth, Gregoire, and Martineau (2019) on the consequences of FOMC press conferences.

[9] See Blinder (2008) and Blinder, Goodhart, Hildebrand, Lipton, and Wyplosz (2001) for discussions on central bank communication strategies.

[10] For a corporate earnings parallel, see Jones (1991) and Balsam, Bartov, and Marquardt (2002) on managing expectations through accruals.

[11] Information leakage refers to any advance revelation of an announcement's content, whether private or public, as in Ai, Bansal, and Han (2021).

drift remains positive even for “bad news” announcements, to which the market reacts negatively (see Appendix A). To the best of our knowledge, no empirical evidence links an announcement’s content to the preceding price drift. Therefore, this paper explicitly rules out insider information leakage as an explanation.

While contributing to the extensive literature on heterogeneous beliefs, this paper introduces a novel dimension: the endogenous decision to learn or stop learning, thereby determining the extent of disagreement. Banerjee, Davis, and Gondhi (2021) explore a related idea, where investors choose to disagree due to higher utility (i.e., utility-based belief biases), while Brunnermeier and Parker (2005) examine its impact on portfolios. In contrast, this paper focuses on how the decision to stop learning influences asset prices, volatility, and trading volume, using scheduled announcements as the key trigger.

Furthermore, this paper contributes to the theoretical understanding of scheduled announcements by incorporating heterogeneous agents into a framework with expected utility, unlike Ai and Bansal (2018), who showed that a “pure news” event cannot generate a risk premium in a single-agent model. Here, sentiment risk, arising from agent heterogeneity, is reduced before an announcement, in contrast to macroeconomic models (see Ai et al. (2021), Wachter and Zhu (2018), and Laarits (2019)) that predict higher pre-announcement risk. These models struggle to explain why high pre-announcement risk would coincide with low realized volatility, while low risk is accompanied by high volatility after the announcement; they would require an unrealistically high (low) price of risk before (after) an announcement to align with the empirical evidence. Additionally, in models such as Laarits (2019) and a modified version of that in Pástor and Veronesi (2013), increased information before the announcement must increase volatility, which again contradicts the empirical evidence. Similarly, in a Kyle (1985) setting, if noise traders were to unexpectedly exit the market before announcements, the price would react more strongly to informed trades and also generate higher volatility.

This paper also contributes to studying the asset pricing effects of scheduled public announcements, as explored by Kim and Verrecchia (1991a,b) and Kondor (2012). These studies examine how price, volume, and volatility depend on factors such as announcement precision, surprise content, and investors' prior knowledge, but focus solely on post-announcement effects. In contrast, this paper provides a unified framework linking asset pricing dynamics both *before* and *after* announcements.

Further empirical research on macroeconomic announcements, particularly FOMC announcements, includes work by Hu et al. (2019), Cieslak, Morse, and Vissing-Jorgensen (2019), and Fisher, Martineau, and Sheng (2022). Cieslak et al. (2019) found that the equity premium is earned during even weeks following the last FOMC meeting. While the current paper focuses on a single announcement (week 0), the model can be extended to include biweekly minor announcements, aligning with Cieslak et al.'s (2019) findings. Fisher et al. (2022), using daily data, documented an increase in attention before macroeconomic announcements, accompanied by high volatility and trading volume. In contrast, the pre-announcement drift occurs with low volatility and trading volume, which could suggest low attention in the hours leading up to the announcement, aligning with our model's prediction of low disagreement. Hu et al. (2019) proposed that macroeconomic announcements increase market uncertainty, which is resolved shortly before the announcement, producing a positive price drift. However, they did not identify the specific uncertainty being reduced. This paper fills that gap by identifying sentiment risk as the uncertainty that declines before the announcement.

The rest of the paper is structured as follows. Section II introduces the model, which features disagreement, scheduled announcements, and a costly signal within a general equilibrium consumption framework. Section III presents the main results, explaining the positive pre-announcement drift along with the empirically observed low volatility and trading volume, patterns that have challenged existing theories. Section IV explores two reduced models: one with a costless signal to isolate the effect of disagreement from its interaction with intermittent learning (i.e., periods of learning and not learning), and another without disagreement or a costly signal, serving a pedagogical purpose by testing whether scheduled announcements alone can generate a pre-announcement drift, which they cannot. Finally, Section V offers concluding remarks.

II. Main Model

This section presents a model of disagreement à la Dumas et al. (2009) with two innovations: scheduled announcements and costly information. To preserve tractability, we include only assumptions essential to the main mechanism; extensions not explicitly modeled are discussed throughout the section.

A. Structure of the Economy

Time t is a continuous variable, with discrete events, called announcements, occurring at a fixed periodicity τ. The market is assumed to be complete and populated by atomistic investors who exhibit power utility over consumption. All investors have the same relative risk aversion, described by the parameter $1-\alpha > 1$ where $\alpha < 0$, and rate of impatience ρ.[12]

[12]The assumption of $1-\alpha > 1$ is important in this model because, with logarithmic utilities, prices are just an average of the prices in representative-agent economies each consisting of investors of one type, A or B. Therefore, in

The economy features a dividend process that follows a geometric Brownian motion with an unobserved growth rate f_t, which is stochastic and follows an Ornstein–Uhlenbeck process. The dividend D_t and the growth rate f_t are governed by the processes

$$\frac{dD_t}{D_t} = f_t\, dt + \sigma_D\, dZ_{D,t}, \tag{1}$$

$$df_t = -\zeta(f_t - \bar{f})\, dt + \sigma_f\, dZ_{f,t}, \tag{2}$$

where the two Brownian motions $\{Z_{D,t}, Z_{f,t}\}$ are uncorrelated with each other. The growth rate f_t is the sole unobserved variable of the economy, and all investors want to estimate it to make optimal consumption decisions. No investor knows the true f_t because it fluctuates before they can ever fully learn it, as in equation (2).

B. Information Assumptions

Investors are divided into two groups with contrasting, unshakable priors about the information content in the announcements and the signal available to learn about the economy's growth rate. Investors A, the "agnostics" to announcements, value the signal as a useful source of information, while investors B, the "believers" in announcements, view the announcements as the only valid source of information and dismiss the signal. Investors' beliefs are assumed to be dogmatic but observable. Therefore, no learning from prices occurs, as all information is public—there is no private information to learn from prices. In what follows, the superscripts A and B indicate investors of groups A and B, respectively.

Announcements: The announcements, denoted by I_t, are noisy information releases

such an economy there would be no risk compensation for the fluctuation in the share of consumption, i.e., no sentiment risk.

about the growth rate f_t that occur with periodicity τ. They are perceived differently by investors A and investors B, and the parameter $\psi \in (0,1)$ determines how informative they are for investors A:

$$(3) \qquad I_t^A = \psi f_t + \sqrt{(1-\psi^2)\gamma_t^A}\,\nu_{I,t}^A + \sigma_I \varepsilon_{I,t}^A \qquad \text{where } \nu_{I,t}^A, \varepsilon_{I,t}^A \sim \mathcal{N}(0,1) \text{ and } \nu_{I,t}^A \perp \varepsilon_{I,t}^A,$$

$$I_t^B = f_t + \sigma_I \varepsilon_{I,t}^B \qquad \text{where } \varepsilon_{I,t}^B \sim \mathcal{N}(0,1).$$

Signal: The signal, denoted by S_t, provides continuous information about shocks to the growth rate, $dZ_{f,t}$. It is also perceived differently by investors A and investors B, and the parameter $\phi_t \in (0,1)$ determines how informative the signal is for investors A:

$$(4) \qquad dS_t^A = \phi_t\, dZ_{f,t} + \sqrt{1-\phi_t^2}\, dZ_{S,t} = dZ_{S,t}^A,$$

$$dS_t^B = dZ_{S,t}^B,$$

where $Z_{S,t}$ is a third Brownian motion, which is uncorrelated with $\{Z_{D,t}, Z_{f,t}\}$. The current model allows for time variation in the parameter ϕ_t, for which we consider the following cases: (i) where ϕ_t changes at an exogenous time t^* (Section III.A), (ii) where t^* is endogenous (Section III.B), and (iii) where ϕ_t remains constant over time (Section III.C). Note that the signal S_t in equation (4) conveys contemporaneous information solely about the *current* growth rate, without providing any information about the upcoming announcement. Thus, the signal is a substitute for the announcement. In contrast, models such as Laarits (2019) and Ying (2020a) incorporate complementary signals, which help investors interpret announcements. These affect only the perceived precision of announcements and do not influence beliefs during the announcement cycle.

The previous assumptions lead to disagreement over announcements when $\psi < 1$ and over signals when $\phi_t > 0$. While disagreement over announcements (ψ) has no effect within an announcement cycle, it is crucial for capturing established post-announcement empirical patterns, such as no average price effect, high volatility, and high trading volume. In contrast, the model's findings stem from the intermittence of ϕ_t, which is usually assumed to be constant in the literature. During an announcement cycle, investors A learn from the signal when $\phi_t > 0$ and ignore it when $\phi_t = 0$. These alternating periods of learning and non-learning drive our main results on prices, volatility, and trading volume in Section III.

Two key parameter restrictions follow. First, disagreement about the signal ($\phi_t > 0$) at some point during an announcement cycle is essential for sentiment risk to affect expected returns (see Section II.G). Second, investors A must foresee a minimal amount of information in announcements ($\psi > 0$); if they fully disregarded future announcements, their posterior variance would converge to a stationary value instead of following a pattern of periodic stationarity, rendering the results in Propositions 2–4 invalid.

In the presence of disagreement, at least one group of investors must be objectively incorrect. Since neither group knows the true state of the economy, the objective measure is not defined on their respective σ-algebra and can be ignored for calculating the equilibrium. However, the objective measure impacts each group's long-term survival through their share of consumption: persistent over- or under-consumption relative to the objective measure depletes a group. To maintain balance, we assume the objective process is that of investors A for the signal and that of investors B for the announcements, making each group correct about one source of information but incorrect about the other.

C. Filtering

Investors need to form an estimate of the growth rate, i.e., a filter, which follows a normal distribution, $f_t \sim \mathcal{N}(\hat{f}_t^i, \gamma_t^i)$, where $\hat{f}_t^i$ and γ_t^i are the mean and posterior variance of the estimated growth rate for each group $i \in \{A, B\}$. The filtering on announcement dates (in discrete time) generates the boundary conditions for the filtering processes between consecutive announcements (in continuous time) where investors use the signal and dividend. A superscript "$-$" denotes quantities immediately before an announcement. From filtering theory, the updated beliefs after observing the announcement are

(5)
$$\begin{aligned} \hat{f}_t^A &= \hat{f}_t^{A-} + \frac{\psi \gamma_t^{A-}(I_k^A - \psi \hat{f}_t^{A-})}{\sigma_I^2 + \gamma_t^{A-}}, \quad & \gamma_t^A &= \frac{\gamma_t^{A-}\big(\sigma_I^2 + \gamma_t^{A-}(1-\psi)\big)}{\sigma_I^2 + \gamma_t^{A-}}, \\ \hat{f}_t^B &= \hat{f}_t^{B-} + \frac{\gamma_t^{B-}(I_k^B - \hat{f}_t^{B-})}{\sigma_I^2 + \gamma_t^{B-}}, \quad & \gamma_t^B &= \frac{\gamma_t^{B-}\sigma_I^2}{\sigma_I^2 + \gamma_t^{B-}}, \end{aligned}$$

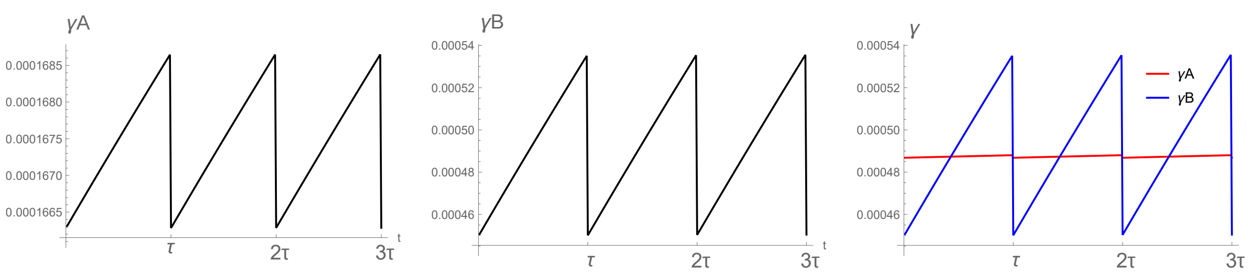
while during an announcement cycle, the conditional expected values and posterior variances obey the stochastic differential equations

(6)
$$\begin{aligned} d\hat{f}_t^A &= -\zeta(\hat{f}_t^A - \bar{f})dt + \frac{\gamma_t^A}{\sigma_D^2}\left(\frac{dD_t}{D_t} - \hat{f}_t^A dt\right) + \phi_t \sigma_f dS_t, \quad & d\gamma_t^A &= \sigma_f^2(1-\phi_t^2) - 2\zeta\gamma_t^A - \frac{(\gamma_t^A)^2}{\sigma_D^2}, \\ d\hat{f}_t^B &= -\zeta(\hat{f}_t^B - \bar{f})dt + \frac{\gamma_t^B}{\sigma_D^2}\left(\frac{dD_t}{D_t} - \hat{f}_t^B dt\right), & d\gamma_t^B &= \sigma_f^2 - 2\zeta\gamma_t^B - \frac{(\gamma_t^B)^2}{\sigma_D^2}. \end{aligned}$$

Although the means $\hat{f}_t^A$ and $\hat{f}_t^B$ of the growth rate are stochastic, the posterior variances γ_t^A and γ_t^B are deterministic functions of time (with closed-form expressions) that exhibit periodic stationarity, illustrated in Figure 1. All figures use the parameter values in Table 1 in Appendix B. The left and middle graphs of Figure 1 show γ_t^A and γ_t^B over three announcement cycles with ϕ_t constant over time. Since ψ and ϕ_t determine how much information investors A extract from announcements and signals, they jointly influence whether investors A are more confident than investors B. With the baseline parameters in Table 1, investors A are more confident, as in Dumas et al. (2009), but this is irrelevant for our model. For instance, with $\phi_t = 0.87$ and $\psi = 1 - \phi_t$, both groups have the same average posterior variance (right graph of Figure 1). Investors B start each cycle more confident, but since they do not learn from the signal between announcements, investors A become more confident as the cycle progresses.

Figure 1: Posterior variances γ_t^A and γ_t^B over time. The left and middle graphs show γ_t^A and γ_t^B for three announcement cycles, respectively, with each exhibiting periodic stationarity. The right graph shows γ_t^A and γ_t^B for $\phi_t = 0.87$ and $\psi = 0.13$ over three announcement cycles.

γ_t^A over three announcement cycles — γ_t^B over three announcement cycles — γ_t^A and γ_t^B



D. Disagreement and Sentiment

Because the two groups of investors have different beliefs about the growth rate f_t, there is disagreement in the economy. A process that captures the disagreement between the two groups from the perspective of investors A can be defined as[13]

$$g_t \equiv \hat{f}_t^A - \hat{f}_t^B,$$

[13]The perspective of either group can be chosen as the reference; here the choice is to focus on investors A, as they find the signal informative.

which follows an Ornstein–Uhlenbeck process with mean reversion to zero:

(7)
$$dg_t = \vartheta_t g_t \, dt + \sigma^g_{D,t} \, dZ^A_{D,t} + \sigma^g_{S,t} \, dZ^A_{S,t},$$
$$\vartheta_t = -\left(\zeta + \frac{\gamma^B_t}{\sigma^2_D}\right), \qquad \sigma^g_{D,t} = \frac{\gamma^A_t - \gamma^B_t}{\sigma_D}, \qquad \sigma^g_{S,t} = \phi_t \sigma_f.$$

From the last equation, a higher ϕ_t increases $\sigma^g_{S,t}$, amplifying the volatility of the disagreement process and causing larger fluctuations. Moreover, the mean reversion in equation (7) occurs independently of the fundamental process in equation (2). Even if $\zeta = 0$, the process of g remains mean-reverting due to its dependence on the posterior variance γ^B_t. Intuitively, extreme belief shifts take time to revert to zero, sustaining disagreement persistence.

Since the probability measures of investors A and B are equivalent, a change of measure η_t from the beliefs of investors B to those of investors A can be specified in continuous time. For any event e_u belonging to the σ-algebra of time u, $E^A_t\left[\frac{\eta_u}{\eta_t}\mathbf{1}_{e_u}\right] = E^B_t\left[\mathbf{1}_{e_u}\right]$.[14] Hence, by the Girsanov theorem,

(8)
$$\frac{d\eta_t}{\eta_t} = \frac{\hat{f}^B_t - \hat{f}^A_t}{\sigma_D} \, dZ^A_{D,t} = -\frac{g_t}{\sigma_D} \, dZ^A_{D,t}.$$

Following Dumas et al. (2009), this change of measure η_t is called the "sentiment" variable.[15] For investors A, η_t represents the extent to which the probability beliefs of investors B differ from their own. When investors A are pessimistic ($g_t < 0$), investors B assign a higher probability to positive dividend innovations. Crucially, η_t and g_t are distinct: the disagreement g_t determines the

[14] This builds on the observational equivalence of the dividend process where $\frac{dD_t}{D_t} = f^A_t \, dt + \sigma_D \, dZ^A_{D,t} = f^B_t \, dt + \sigma_D \, dZ^B_{D,t}$ and the definition of η_t as a martingale under the measure of investors A.

[15] It is also known as the "disagreement weighting process" (see Basak (2005)) or the "disagreement value" (see David (2008)).

stochastic volatility of η_t. The riskiness of the sentiment variable η_t depends on $|g_t|$, making sentiment risk proportional to g_t^2.

Due to the split in filtering (Section II.C), equation (8) holds during an announcement cycle but not at the discrete announcement times. At these points, the likelihood ratio sets the boundary condition for equation (8) without requiring the Girsanov theorem. The sentiment variable η_t jumps up or down based on the announcement realization I_t, but this jump is zero in expectation. Thus, η_t retains its martingale properties throughout the economic timeline, which is essential for deriving a stationary equilibrium pricing kernel.

The Markovian system comprising equations (1), (6), (7), and (8) completely characterizes the evolution of the economy, which is driven by only two Brownian motions, $Z_{D,t}$ and $Z_{S,t}$, because no investor observes $Z_{f,t}$. The diffusion matrix of the four state variables $\{D_t, \hat{f}_t^A, g_t, \eta_t\}$ is a 4×2 matrix

$$M_\sigma = \begin{bmatrix} D_t\sigma_D & 0 \\ \frac{\gamma^A}{\sigma_D} & \phi_t\sigma_f \\ \frac{\gamma^A-\gamma^B}{\sigma_D} & \phi_t\sigma_f \\ -\frac{g}{\sigma_D}\eta & 0 \end{bmatrix}. \tag{9}$$

E. Optimization and Equilibrium

From the perspective of investors A, one can use the change of measure η_t to express the problem that investors B face. This results in the following two optimization problems:

For investors A:

$$\sup_{c_t, t^*} E^A \int_0^\infty e^{-\rho t} \frac{1}{\alpha} (c_t^A)^\alpha \, dt$$

subject to

$$E^A \int_0^\infty \xi_t^A c_t^A \, dt = E^A \int_0^\infty \xi_t^A \theta^A D_t \, dt$$

For investors B:

$$\sup_{c_t} E^A \int_0^\infty \eta_t e^{-\rho t} \frac{1}{\alpha} (c_t^B)^\alpha \, dt$$

subject to

$$E^A \int_0^\infty \xi_t^A c_t^B \, dt = E^A \int_0^\infty \xi_t^A \theta^B D_t \, dt$$

where t^* represents the point in time in an announcement cycle at which investors A stop learning from the signal. Taking first-order conditions with respect to c_t^A and c_t^B, with Lagrange multipliers λ^A and λ^B, and imposing the condition that the dividend must be fully consumed, $c_t^A + c_t^B = D_t$, the state price density ξ_t^A is obtained as

$$\xi_t^A = e^{-\rho t} \left[\left(\frac{1}{\lambda^A} \right)^{\frac{1}{1-\alpha}} + \left(\frac{\eta_t}{\lambda^B} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} D_t^{\alpha - 1}, \tag{10}$$

and the optimal levels of consumption for the two investor groups are

$$c_t^A = \omega_{\eta_t} D_t, \qquad c_t^B = (1 - \omega_{\eta_t}) D_t \tag{11}$$

$$\text{where} \quad \omega_{\eta_t} = \frac{\left(\frac{1}{\lambda^A}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{1}{\lambda^A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\eta_t}{\lambda^B}\right)^{\frac{1}{1-\alpha}}}.$$

Equation (10) explicitly states that the sentiment η_t is priced. Intuitively, investors A are concerned about the beliefs of investors B because collectively they share the aggregate dividend, thereby clearing the market. Since the sentiment η_t determines the consumption share of investors A, as per equation (11), fluctuations in this share of consumption, i.e., the riskiness of η_t, are priced.[16] Using equation (11), the utility of investors A is defined as

$$U_t^A(D_t, \hat{f}_t^A, g_t, \eta_t, t) = \int_t^\infty e^{-\rho(u-t)} \frac{1}{\alpha} E^A \left[(\omega(\eta_u) D_u)^\alpha \right] du. \tag{12}$$

The ratio of pricing densities, used to price all assets in Section II.F, depends on the power $1 - \alpha$. Assuming this power is an integer,[17] the binomial formula can be used to write the ratio as

$$\frac{\xi_u^A}{\xi_t^A} = e^{-\rho(u-t)} \left(\omega_{\eta_t} \right)^{1-\alpha} \sum_{j=0}^{1-\alpha} \frac{(1-\alpha)!}{j!(1-\alpha-j)!} \left(\frac{1}{\omega_{\eta_t}} - 1 \right)^j \left(\frac{D_u}{D_t} \right)^{\alpha-1} \left(\frac{\eta_u}{\eta_t} \right)^{\frac{j}{1-\alpha}}. \tag{13}$$

Note that the ratio of pricing densities in equation (13) depends on the expectation of the joint distribution of η_u and D_u to certain powers. The moment-generating function of this distribution can be expressed in closed form as follows.

Proposition 1. *The characteristic function of the joint distribution of η_u and D_u takes the form*

$$E^A \left[\left(\frac{D_u}{D_t} \right)^\varepsilon \left(\frac{\eta_u}{\eta_t} \right)^\chi \middle| Y_t, t \right] = H_f(\hat{f}^A, t, u; \varepsilon) \times H_g(g, t, u; \varepsilon, \chi), \tag{14}$$

where the functions $H_f(\cdot)$ and $H_g(\cdot)$ are given in closed form in Appendix C.1, and $H_g(\cdot)$ is hump-shaped in g and decreasing in g^2.

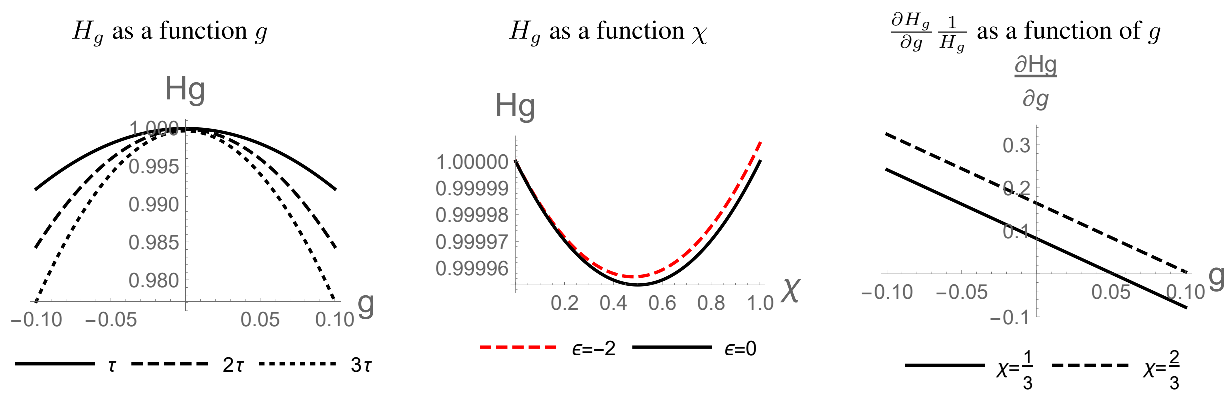
Intuitively, prices depend on both the expected growth rate $\hat{f}^A$ and the absolute level of investor disagreement (g^2). As disagreement decreases, risk declines, prompting investors to hold more risky assets and driving up prices. Notably, the price density reflects only continuous risk since scheduled announcements, being predictable, cause jumps only on those discrete dates. Consequently, the risk before an announcement stems solely from continuous Brownian motions if there are no potential jumps in the next instant.

[16] This is true for time-additive utility; with recursive utility, g can also be priced as in Boroviécka (2011).

[17] The integer assumption, though standard in the literature, is not necessary; see Bhamra and Uppal (2014) for a general case.

In Figure 2, the left graph illustrates the hump shape of H_g with respect to g (the riskiness of η_t) when the correlation effect is ignored, i.e., $\varepsilon = 0$. The middle graph shows both the variance ($\varepsilon = 0$) and the correlation ($\varepsilon = -2$) effects when $g = 0$. The right graph shows the logarithmic derivative $\frac{\partial H_g}{\partial g} \frac{1}{H_g}$, which is decreasing in g.

Figure 2: Properties of the function H_g**.** The left graph shows H_g as a function g at different points in the future: $u - t = \tau$, $u - t = 2\tau$, and $u - t = 3\tau$, with $\varepsilon = 0$. The middle graph shows H_g as a function of χ, when isolating the variance effect ($\varepsilon = 0$) and when including the correlation effect ($\varepsilon = -2$). The right graph shows $\frac{\partial H_g}{\partial g} \frac{1}{H_g}$ as a function of g for two values of χ.



F. Asset Prices

Since agents care about two Brownian motions, three linearly independent securities are needed to complete the financial market and implement equilibrium during an announcement cycle. The choice of securities is arbitrary, and the selection here consists of (i) a riskless, instantaneous bank deposit, (ii) an equity that pays the aggregate dividend D_t perpetually, and (iii) a perpetual bond providing one unit of consumption per unit time. The prices of the equity and bond are

$$P(D_t, \hat{f}_t, t) = D_t \int_t^{\infty} E\left[\frac{\xi_u}{\xi_t} \frac{D_u}{D_t} \,\middle|\, \hat{f}_t, t \right] du,$$

$$B(\hat{f}_t, t) = \int_t^{\infty} E\left[\frac{\xi_u}{\xi_t} \,\middle|\, \hat{f}_t, t \right] du.$$

Furthermore, the noise of the announcements generates an additional source of risk with periodicity τ, requiring an additional security for hedging. To prevent distortions in the stock and bond markets, the model includes a tailored futures contract in zero net supply, fluctuating solely with announcement releases. This futures contract serves exclusively as a hedge against announcement risk while remaining orthogonal to stocks and bonds. Since the market value of a future contract is always adjusted to equal zero, no capital flow is induced, so the positions in the risky stock and bond remain unaffected.

G. Contemporaneous Excess Returns

The pricing measure in equation (10) encompasses the equilibrium's risk-free rate and market prices of risk. As in Cox and Huang (1989), the risk-free rate r on the instantaneous bank deposit is defined as the drift, and the market prices of risk, denoted by the vector κ, define the diffusion of the risk-neutral measure. By applying Itô's lemma to equation (10), one obtains the risk-free rate and market prices of risk as

$$r = \rho + (1-\alpha)\hat{f}_t - \frac{1}{2}(2-\alpha)(1-\alpha)\sigma_D^2 - g_t(1-\omega_\eta)\left(\frac{\alpha g_t \omega_\eta}{2(1-\alpha)\sigma_D^2} + (1-\alpha)\right), \tag{15}$$

$$\kappa = \begin{bmatrix} (1-\alpha)\sigma_D + \frac{g_t(1-\omega_\eta)}{\sigma_D} \\ 0 \end{bmatrix}. \tag{16}$$

From equation (15), the risk-free rate increases with the expected dividend growth rate but responds to the disagreement g_t in a non-monotonic and asymmetric way, as shown in David (2008). This is because g_t affects both the average of $\hat{f}_t^A$ and $\hat{f}_t^B$ and the gap between them. If one defines a weighted average belief $\hat{f}_t^M = \omega_{\eta t}\hat{f}_t^A + (1-\omega_{\eta t}\hat{f}_t^B)$, the role of disagreement in the risk-free rate can be isolated: it depends solely on the disagreement squared—that is, on the degree of belief dispersion. Intuitively, the square of the disagreement increases the risk-free rate because, as a priced source of risk, it reduces all stochastic discount factors.

The contemporaneous (expected) excess returns are given by $\mu - r = \sigma_R \kappa$, where the return volatility σ_R is obtained by pre-multiplying the diffusion matrix of the state variables (M_σ in equation (9)) by the gradient of the price function, resulting in

$$(17)\quad \sigma_R = \left[\underbrace{\sigma_D + \frac{\gamma^A}{\sigma_D}\frac{\partial P}{\partial \hat{f}_t}\frac{1}{P} + \frac{(\gamma_t^A - \gamma_t^B)}{\sigma_D}\frac{\partial P}{\partial g_t}\frac{1}{P}}_{\text{fundamental component} = \bar{\sigma}_R} \quad \underbrace{- \frac{g_t \eta_t}{\sigma_D}\frac{\partial P}{\partial \eta_t}\frac{1}{P}}_{\text{sentiment component}} \quad \phi_t \sigma_f \left(\frac{\partial P}{\partial \hat{f}_t}\frac{1}{P} + \frac{\partial P}{\partial g_t}\frac{1}{P} \right) \right].$$

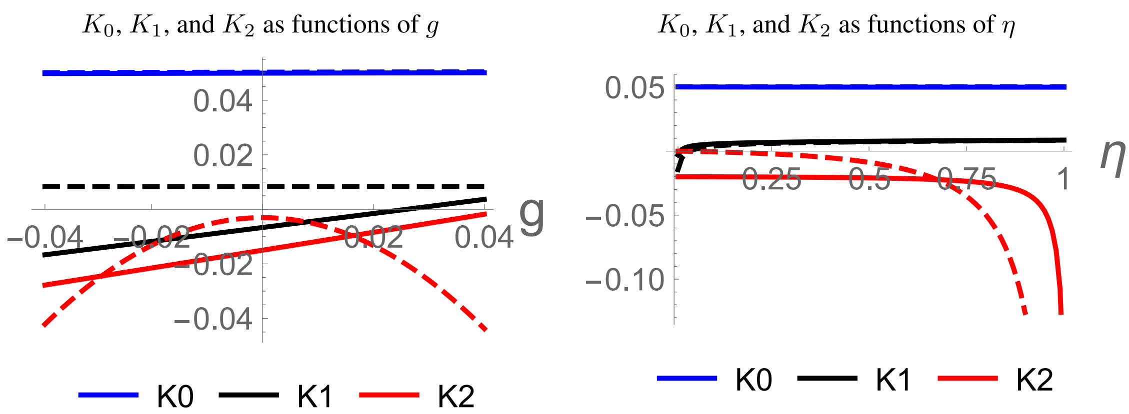
Here $\frac{\partial P}{\partial \hat{f}_t} < 0$, $\frac{\partial P}{\partial g_t} > 0$, and $\frac{\partial P}{\partial \eta_t} > 0$. Sentiment risk introduces excess volatility, as demonstrated in Scheinkman and Xiong (2003). The intuition behind $\frac{\partial P}{\partial g_t} > 0$ is that an increase in disagreement stems from greater optimism among investors A, which also prompts them to buy more of the stock and push prices up. Similarly, $\frac{\partial P}{\partial \eta_t} > 0$ holds because a rise in η reduces the consumption share of investors A (see equation (11)), making their consumption riskier. This heightened risk raises their hedging demand, which also drives up prices.

Therefore, the excess rates of return conditionally expected by investors are

$$(18)\quad \mu - r = \bar{\sigma}_R(1-\alpha)\sigma_D + g_t \left(\frac{(1-\omega_{\eta_t})}{\sigma_D}\bar{\sigma}_R - (1-\alpha)\eta_t \frac{\partial P}{\partial \eta_t}\frac{1}{P} \right) - g_t^2 \frac{\eta_t(1-\omega_{\eta_t})}{\sigma_D^2}\frac{\partial P}{\partial \eta_t}\frac{1}{P}$$
$$= K_0 + K_1 g_t + K_2 g_t^2,$$

where the signs of $K_0 > 0$, $K_1 \approx 0$, and $K_2 < 0$ follow from the price sensitivities. The hump-shaped relationship between $\mu - r$ and the disagreement g arises from the latter being the variance of sentiment; see equation (8). Since sentiment determines investors' consumption share (see equation (11)), greater disagreement-squared amplifies consumption fluctuations. Such fluctuations make the risky asset essential for hedging, resulting in a negative risk premium. Figure 3 shows the different components of the relationship between contemporaneous (expected) excess returns and disagreement. Solid lines include the correlation effect between dividends and sentiment ($\varepsilon = -2$), while dashed lines isolate the impact of disagreement as the variance of sentiment ($\varepsilon = 0$).

Figure 3: Components of contemporaneous (expected) excess returns as functions of the disagreement. The left graph shows K_0, K_1, and K_2 from equation (18) as functions of the disagreement g, and the right graph shows them as functions of η; the dashed lines isolate the variance effect ($\varepsilon = 0$) while the solid ones include the correlation effect ($\varepsilon = -2$).



H. Portfolio Allocation

Under the complete market assumption, the portfolio allocation problem is reduced to the positions making up the exposure that investors A desire to the shocks $Z_{D,t}$ and $Z_{S,t}$. All state variables are driven by these two Brownian motions, with the diffusion matrix M_σ given in equation (9). Defining the aggregate utility of consumption as in equation (12), investors A seek a vector $\theta^\top = [\theta_P \;\; \theta_B]$ of asset allocations that satisfies the system

$$(19) \qquad \begin{bmatrix} U_t^A & \frac{\partial U_t^A}{\partial \hat{f}_t^A} & \frac{\partial U_t^A}{\partial g_t} & \frac{\partial U_t^A}{\partial \eta_t} \end{bmatrix} M_\sigma = \begin{bmatrix} \theta_P & \theta_B \end{bmatrix} \begin{bmatrix} P & \frac{\partial P}{\partial \hat{f}_t^A} & \frac{\partial P}{\partial g_t} & \frac{\partial P}{\partial \eta_t} \\ 0 & \frac{\partial B}{\partial \hat{f}_t^A} & \frac{\partial B}{\partial g_t} & \frac{\partial B}{\partial \eta_t} \end{bmatrix} M_\sigma.$$

The left-hand side represents the target exposures based on consumption utility, while the right-hand side shows the exposure to available securities. Thus, $\theta^\top$ reflects how investors achieve their desired allocation. The allocation defined in equation (19) is used to calculate trading volume, which is defined as the square of the change in an investor's allocation to each asset (turnover), matching the other group's change in allocation due to market clearing.

III. Main Results

This section examines the impact of the signal's information content. When $\phi_t = 0$, investors A ignore the signal and stop learning until an announcement occurs. We investigate the implications of such alternating periods of learning and non-learning for prices, volatility, and trading volume. The analysis is divided into two parts: first, we assume an exogenous pattern for ϕ_t over an announcement cycle, and then we endogenize ϕ_t, demonstrating that the optimal pattern that emerges mirrors the exogenously imposed one. Finally, this section presents additional analyses linking the model's findings to empirical evidence. Our main results use the following definition.

Definition 1: A positive pre-announcement drift is an increase in the expected excess return on the risky asset before the announcement.

A. Exogenous ϕ_t

Assume that investors A stop extracting information from the signal exogenously when a certain percentage of time t^* in each cycle is reached. For example, investors A might extract information from the signal ($\phi_t > 0$) during the first $t^* = 95\%$ of the announcement cycle, but ignore the signal ($\phi_t = 0$) for the 5% of time remaining.[18] The implications of ceasing to learn in the run-up to an announcement are discussed below.

1. State Variables

Ceasing to learn from the signal primarily impacts the filtering process of investors A. Since they rely on the signal to infer the economy's growth rate, ignoring it increases their uncertainty γ_t^A. Furthermore, when investors A ignore the signal, their beliefs align with those of investors B, who always disregard the signal, thus reducing the absolute difference in their posterior variances.[19] Finally, as investors A become more similar to investors B, fluctuations in the change of measure η decrease, i.e., the riskiness of the sentiment (g^2) decreases. These effects are formalized as follows:[20]

Proposition 2. *Investors A ceasing to learn from the signal (i.e., setting $\phi_t = 0$) has the following effects:*

(a) *It increases the posterior variance γ_t^A of investors A.*

(b) *It decreases the absolute difference between the posterior variances, $|\gamma_t^A - \gamma_t^B|$, while increasing the difference $\gamma_t^A - \gamma_t^B$.*

(c) *It decreases the variance of the sentiment, g_t^2, while the disagreement g_t continues to follow an Ornstein–Uhlenbeck process with mean reversion to zero.*

The changes in state variables summarized in Proposition 2 do not generate jumps, which would require an additional premium in the state price density. Although the diffusion structure of

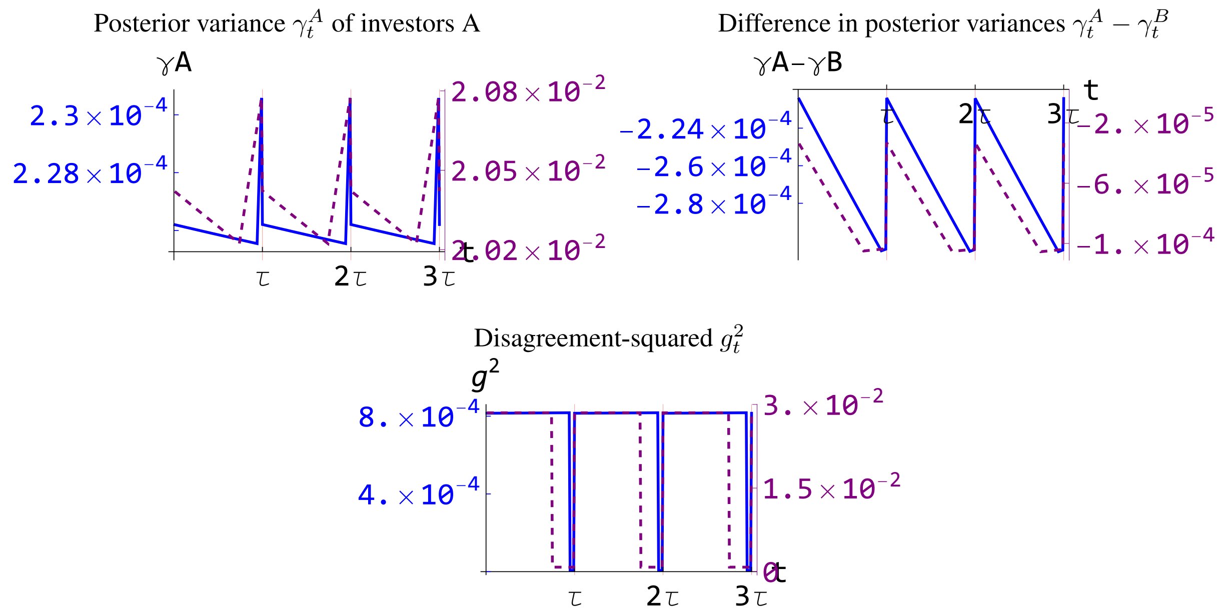
[18]For the FOMC case, the drift occurs over the two days before the meeting, so $t^* = \frac{31-2}{31} \approx 94\%$.

[19]This is referred to as differences in overconfidence, which can be shut down as in Xiong and Yan (2010).

[20]The proof is given in Appendix C.2.

the economy changes when learning stops, all processes remain continuous, so no jump premium arises. Figure 4 illustrates Proposition 2 for two choices of t^*: 95% (solid lines) and 75% (dashed lines). The top left graph shows the posterior variance γ_t^A of investors A, which decreases during an announcement cycle until t^* is reached. After t^*, γ_t^A increases (part (a) of Proposition 2), before dropping each time an announcement occurs. The top right graph shows the difference $\gamma_t^A - \gamma_t^B$ between the posterior variances of investors A and investors B. It decreases until t^* and exhibits a slight increase thereafter (part (b) of Proposition 2) that precedes the jump due to disagreement each time an announcement occurs. The bottom graph shows the disagreement-squared, which dramatically decreases after t^* (part (c) of Proposition 2), thereby reducing the riskiness of the sentiment η_t.

Figure 4: Posterior variance of investors A, difference in posterior variances, and disagreement-squared over three announcement cycles. The top left graph shows the posterior variance γ_t^A of investors A, the top right graph shows the difference $\gamma_t^A - \gamma_t^B$ between the posterior variances of investors A and investors B, and the bottom graph shows the disagreement-squared, g_t^2. Three announcements are marked at times τ, 2τ, and 3τ. The solid lines are the results for $t^* = 95\%$ and the dashed lines for $t^* = 75\%$.



2. Returns

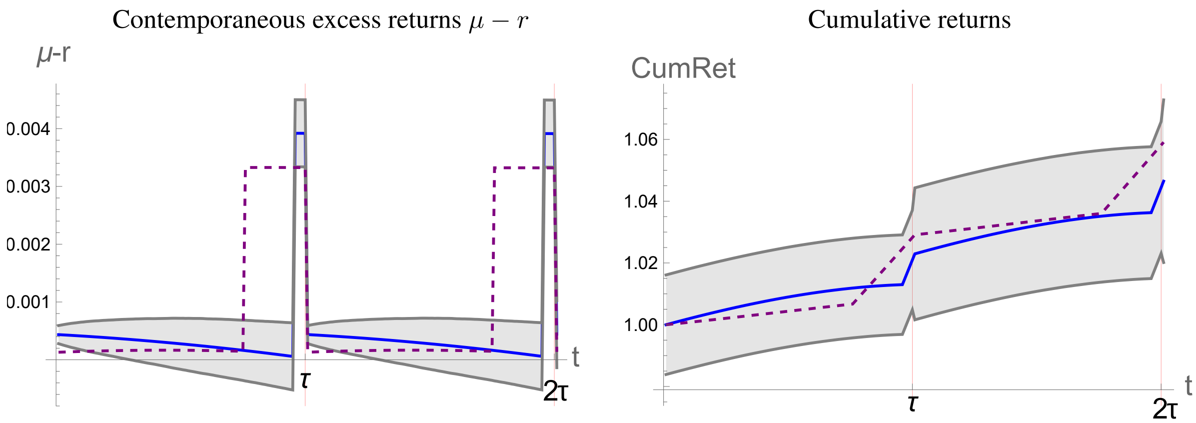
The key impact of stopping signal learning is an increase in excess returns, from Proposition 2(c), which drives the pre-announcement drift defined in Definition 1. Parts (a) and (b) of the proposition have offsetting effects on the fundamental volatility ($\bar{\sigma}_R$), leaving no consistent impact on excess returns. As equation (18) shows, the sharp drop in disagreement-squared after t^* raises excess returns, given that $K_2 < 0$:

$$\underbrace{\mu - r}_{\uparrow} = K_0 + K_1 g_t + \underbrace{K_2}_{<0} \underbrace{g_t^2}_{\downarrow}.$$

While the drift of the price process, i.e., the expected return, increases sharply, the process itself stays continuous and free of jumps. As the announcement approaches and investors A stop learning from the signal, fluctuations in g_t become smaller, reducing g_t^2 and, hence, sentiment risk. Recall that sentiment risk charges a premium from the vagaries of others by forcing investors to hedge and hold fewer shares of equity than would be optimal in a market without sentiment risk (see equation (11)). Therefore, when sentiment risk is reduced, investors are more willing to hold the risky asset, thus increasing its price.

Figure 5 displays the excess returns and cumulative returns over two announcement cycles, together with confidence intervals. There is a considerable increase in returns ahead of the announcement, due to the dramatic decrease in disagreement-squared when investors A stop learning from the signal. If the stopping of learning occurs at $t^* = 95\%$ (solid blue lines), about half of the returns in an announcement cycle are earned in the 5% of the cycle just before the announcement. If learning stops earlier, at $t^* = 75\%$ (dashed purple lines), about two-thirds of the returns in an announcement cycle are earned in the 25% of time just ahead of the announcement.

Figure 5: Excess returns and cumulative returns around announcements. The left graph shows the contemporaneous (expected) excess returns $\mu - r$, and the right graph shows the cumulative returns. Two announcements are marked at times τ and 2τ. The solid lines are for $t^* = 95\%$ and the dashed lines for $t^* = 75\%$.



3. Volatility and Volume

As in many information models, volatility and trading volume go hand in hand. To understand both volatility and volume in an announcement cycle, one must identify the components that drive the price changes and hedging needs. Each variable is decomposed into two components: one that persists regardless of the information extracted, ϕ_t, denoted by the subscript $(\mathcal{RL})$, and another that appears only when investors A learn from the signal, i.e., when $\phi_t > 0$.

Beginning with volatility, an application of Itô's lemma allows the terms in equation (17) to be grouped as

$$dP_t = \mu_{(\mathcal{RL})}\, dt + \left[\underbrace{\sigma_{(\mathcal{RL})}}_{\text{independent of learning}} \quad \underbrace{\phi_t \sigma_f \left(\frac{\partial P}{\partial \hat{f}_t} \frac{1}{P} + \frac{\partial P}{\partial g_t} \frac{1}{P} \right)}_{\text{present only if learning}} \right] \begin{bmatrix} dZ^A_{D,t} \\ dZ^A_{S,t} \end{bmatrix},$$

where $\mu_{(\mathcal{RL})}$ and $\sigma_{(\mathcal{RL})}$ are present regardless of whether investors learn from the signal or not. The term related to the signal shock $dZ^A_{S,t}$ is present only when investors A learn from the signal.

Thus, learning from the signal injects additional volatility into the returns because of the shock $dZ_{S,t}$, which disappears once investors A stop learning at t^*. Hence, volatility decreases as an announcement approaches and investors stop learning.

Turning now to trading volume, equation (19), which defines the portfolio allocations, can be decomposed into

$$\begin{bmatrix} M^U_{(\mathcal{RL})} & \phi_t \sigma_f \left(\frac{\partial U^A_t}{\partial \hat{f}^A_t} + \frac{\partial U^A_t}{\partial g_t} \right) \end{bmatrix} = \begin{bmatrix} \theta_P & \theta_B \end{bmatrix} \begin{bmatrix} M^P_{(\mathcal{RL})} & \phi_t \sigma_f \left(\frac{\partial P_t}{\partial \hat{f}^A_t} + \frac{\partial P_t}{\partial g_t} \right) \\ M^B_{(\mathcal{RL})} & \phi_t \sigma_f \left(\frac{\partial B_t}{\partial \hat{f}^A_t} + \frac{\partial B_t}{\partial g_t} \right) \end{bmatrix},$$

where $M^U_{(\mathcal{RL})}$, $M^P_{(\mathcal{RL})}$, and $M^B_{(\mathcal{RL})}$ are present regardless of whether investors learn from the signal or not.[21] Trading volume is calculated as the squared turnover over time, $(d\theta_p)^2$. Thus, the change in position $d\theta_p$ arises from applying Itô's lemma to the previous equation, resulting in

$$d\theta_p = \mu_{(\mathcal{RL})}\, dt + \begin{bmatrix} \underbrace{\sigma_{(\mathcal{RL})}}_{\text{independent of learning}} & \underbrace{\phi_t \sigma_f \mathcal{F}(\hat{f}^A_t, g_t, U^A_t, B_t, P_t)}_{\text{present only if learning}} \end{bmatrix} \begin{bmatrix} dZ^A_{D,t} \\ dZ^A_{S,t} \end{bmatrix},$$

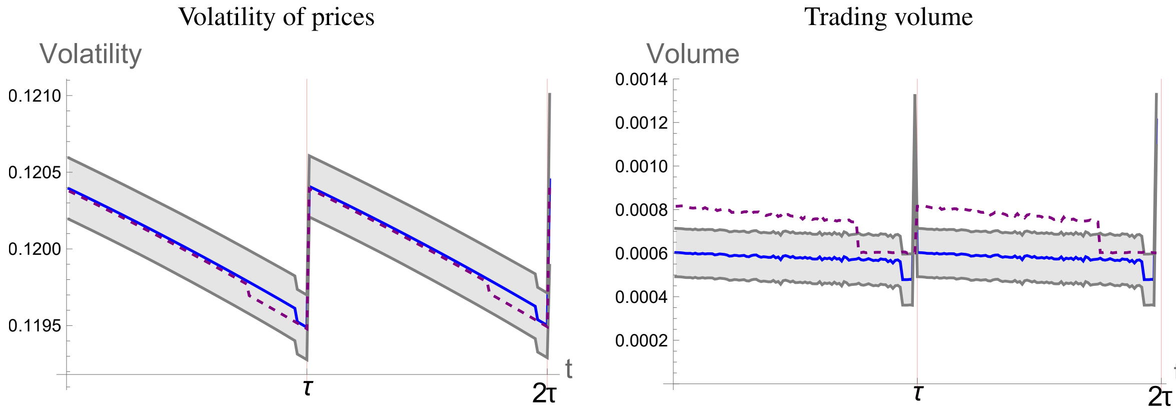
where the terms $\mu_{(\mathcal{RL})}$ and $\sigma_{(\mathcal{RL})}$ are present regardless of whether investors learn from the signal or not, and $\mathcal{F}(\cdot)$ is a function of $\hat{f}^A_t$, g_t, and a combination of the utility U^A_t, bond price B_t, and equity price P_t. The volume calculation uses $(d\theta_p)^2$, making the drift term irrelevant. As with volatility, the term related to the signal shock $dZ_{S,t}$ is present only when investors A learn from the signal. When the next announcement is imminent, investors stop learning from the signal, eliminating the hedging needs related to the signal shock and thus decreasing the trading volume.

For illustration, the volatility of prices and the average squared turnover of equity are shown in the left and right graphs of Figure 6, respectively. As an announcement approaches, the volatility decreases since the signal fluctuations no longer affect prices, which manifests as a

[21]These terms represent matrices of partial derivatives of the utility U^A_t, the equity price P_t, and the bond price B_t, respectively, with respect to the state variables D_t and η_t.

reduction in sentiment risk. Similarly, lower sentiment risk prompts investors to change their positions less frequently, resulting in lower trading volume.

Figure 6: Volatility and volume around announcements. The left graph shows the volatility and the right graph shows the trading volume over two announcement cycles. Two announcements are marked at times τ and 2τ with red vertical lines. The solid lines are for $t^* = 95\%$ and the dashed lines for $t^* = 75\%$.



B. Endogenous ϕ_t

Section III.A showed that when investors A stop learning from the signal ($\phi_t = 0$), a positive price drift emerges, coupled with low volatility and trading volume. While the timing t^* of this learning cessation was exogenous there, this section demonstrates that investors A endogenously choose to stop learning when an announcement is imminent.

This choice stems from the introduction of a fixed cost $\mathcal{C}$ per unit time for learning from the continuous signal in equation (4).[22] The benefit of learning is reduced variance in the growth rate filter of investors A, γ_t^A. However, if extracting information is costly, investors A must weigh the benefit of variance reduction against such an expense. In contrast, investors B, who deem the signal uninformative, never find it worthwhile to pay this cost.

[22]The cost is minimal, akin to the slight effort required to read an online report.

1. Properties of Learning

The variance reduction of investors A has the following properties:[23]

Proposition 3. (a) *The marginal variance reduction due to learning from the signal decreases monotonically over time.*

(b) *The maximum variance reduction is achieved if a non-learning period Δ is positioned as late as possible in an announcement cycle.*

Proposition 3(a) states that the signal's ability to reduce the posterior variance γ_t^A decreases over time, meaning that its benefit diminishes as the announcement nears. Proposition 3(b) follows as a consequence, indicating that investors prefer to learn earlier in the announcement cycle. If an investor were to ignore the signal for a period Δ to save on the cost $\Delta\mathcal{C}$, they would optimally place this break as close to the announcement as possible. In expectation, an investor would rather stop learning once than pause and restart later within the same announcement cycle.

Since the variance reduction diminishes over time while the cost $\mathcal{C}$ remains constant, investors A must compare the utility of extracting information ($\phi_t > 0$) with that of stopping altogether ($\phi_t = 0$). By such a comparison, the marginal investor A would be indifferent between these choices, ensuring no strategic interactions or deviations from the equilibrium learning. Using equation (12) and its closed-form solution in Proposition 1, the gain per unit time of extracting information is as follows:[24]

[23] The proof is provided in Appendix C.3.

[24] The proof is given in Appendix C.4.

Proposition 4. *The gain per unit of time of extracting information ϕ_t is*

$$G_t^A(Y_t,t) = \sum_{j=0}^{-\alpha} h(j)\Big(H_g(\phi_t,\cdot)\big[H_f(\phi_t,\cdot) - H_f(0,\cdot)\big] + H_f(0,\cdot)\big[H_g(\phi_t,\cdot) - H_g(0,\cdot)\big]\Big),$$

where Y_t represents the state variables $\{D_t, \hat{f}_t^A, g_t, \eta_t\}$, $h(j)$ is a deterministic function of the summation index j, and $H_f(\phi_t,\cdot)$ and $H_g(\phi_t,\cdot)$ are the exponential functions in Proposition 1, such that

(a) $H_f(\phi_t,\cdot) - H_f(0,\cdot)$ *is decreasing in t; and*

(b) $H_g(\phi_t,\cdot) - H_g(0,\cdot)$ *is hump-shaped in g and decreasing in g^2.*

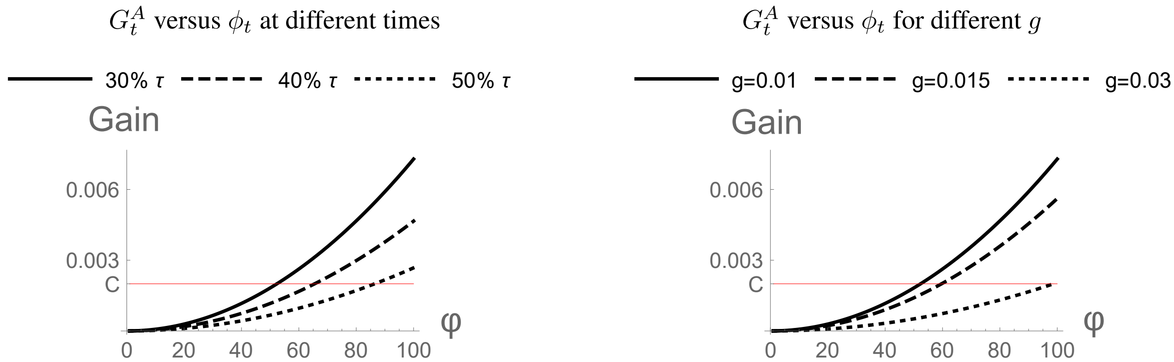
Proposition 4(a) formalizes the intuition that as an announcement approaches, investors have less incentive to extract information from the signal, a consequence of the diminished benefits of the signal over time (Proposition 3(a)). Proposition 4(b) states that higher sentiment risk (g^2) further discourages information extraction. This is because higher g^2 implies a riskier economy, which increases investors' sensitivity to uncertainty (Proposition 1), making them less willing to incur the cost of learning from the signal.

2. Properties of the Decision to Stop Learning

The decision of how much information to extract ultimately depends on the five-dimensional space of the state variables $\{D_t, \hat{f}_t^A, g_t, \eta_t\}$ and time. However, the effects of the dividend D_t, sentiment η_t, and expected growth rate $\hat{f}_t^A$ are straightforward (increasing for the first two, decreasing for the latter). Furthermore, from Proposition 2, stopping has consistent implications only for g_t^2 and γ_t^A, making them the key drivers in the decision to stop learning.

Figure 7 illustrates the gain G_t^A as a function of extracted information ϕ_t, showing its decline as time progresses (left graph) and as disagreement g rises (right graph).

Figure 7: Gain as a function of information extracted. The left graph shows the gain G_t^A as a function of extracted information ϕ_t at different times. The right graph shows G_t^A as a function of ϕ_t for different levels of disagreement g. The red line represents the cost $\mathcal{C}$.

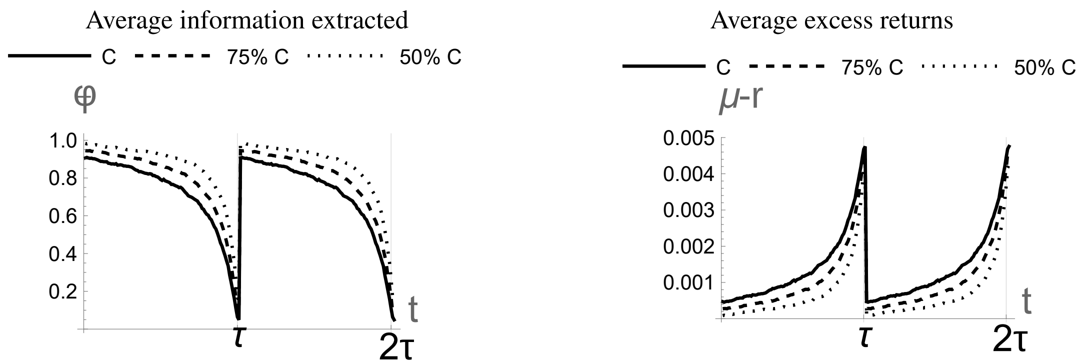


Since the gain function is convex in ϕ_t, an interior equilibrium for stopping learning is unattainable. Investors A fully extract the signal information $(\phi_t = 1)$ when the gain exceeds the cost but stop learning entirely ($\phi_t = 0$) otherwise. Thus, the decision is binary: investors A either extract information or ignore the signal.

Simulated across cycles, the average extracted information $\bar{\phi}$ declines as an announcement nears; averaging is required since the binary switch from learning to non-learning is random depending on the state variables. Investors find it optimal, on average, to stop learning when they foresee an imminent announcement. The pattern of $\bar{\phi}$ depends on the cost of information $\mathcal{C}$; a higher cost leads to less information extraction across announcement cycles.

For illustration, the left graph in Figure 8 shows the average $\bar{\phi}$ over two announcement cycles simulated for different levels of $\mathcal{C}$. The right graph in Figure 8 shows the corresponding consequence for excess returns $\mu - r$, where the pre-announcement drift as per Definition 1 is evident. Intuitively, with a lower cost of information $\mathcal{C}$, investors A choose to learn for a longer time, i.e., the average amount of information $\bar{\phi}$ extracted is higher, which coincides with an earlier increase in excess returns.

Figure 8: Average information extracted and excess returns. The left graph displays the average amount of information extracted, $\bar{\phi}$, across announcement cycles for various levels of the cost $\mathcal{C}$. The right graph presents the corresponding average excess returns, $\mu - r$, across announcement cycles.



The speed of the pre-announcement drift is directly tied to the cost $\mathcal{C}$. All figures are calibrated to align the drift's timing with the pre-FOMC drift documented by Lucca and Moench (2015), which is extremely fast, occurring within 24 hours before an announcement. Matching this pace requires a minimal cost of $\mathcal{C} = 4.25 \times 10^{-6}$ (solid line in Figure 8). In an economy generating \$80 trillion in dividends, this translates to an information cost of roughly \$100 a day.[25]

C. Further Analysis

This section further examines pre-FOMC drift characteristics that the proposed model reproduces beyond the initial evidence on drift, volatility, and trading volume that motivated this paper. This strengthens the paper's contribution by demonstrating its ability to explain additional empirical evidence that existing theories often fail to capture.

[25] This estimate illustrates that a unit dividend corresponds to an information cost $\mathcal{C}$ that is several orders of magnitude smaller.

1. Evidence of VIX Explaining the Pre-Announcement Drift

The CBOE Volatility Index (VIX) generates a 30-day forward projection of volatility derived from S&P 500 index options. Empirically, Lucca and Moench (2015) documented that a higher VIX measured on a Friday before a FOMC meeting coincides with a higher pre-FOMC drift in the 24-hour period preceding the meeting. The proposed model can match these empirical findings by approximating the VIX with realized volatility around the meeting.[26]

In this model, stopping learning earlier in an announcement cycle has two effects: (i) a larger pre-announcement drift and (ii) higher realized volatility post-announcement. Stopping learning earlier allows more time for risk to decrease gradually, increasing the pre-announcement drift. Simultaneously, it leads to greater imprecision in investors A's estimate of the growth rate (higher posterior variance), causing them to react more strongly to the announcement information. This heightened reaction increases post-announcement volatility. Therefore, the model predicts a positive relationship between the pre-announcement drift and realized volatility after the announcement.

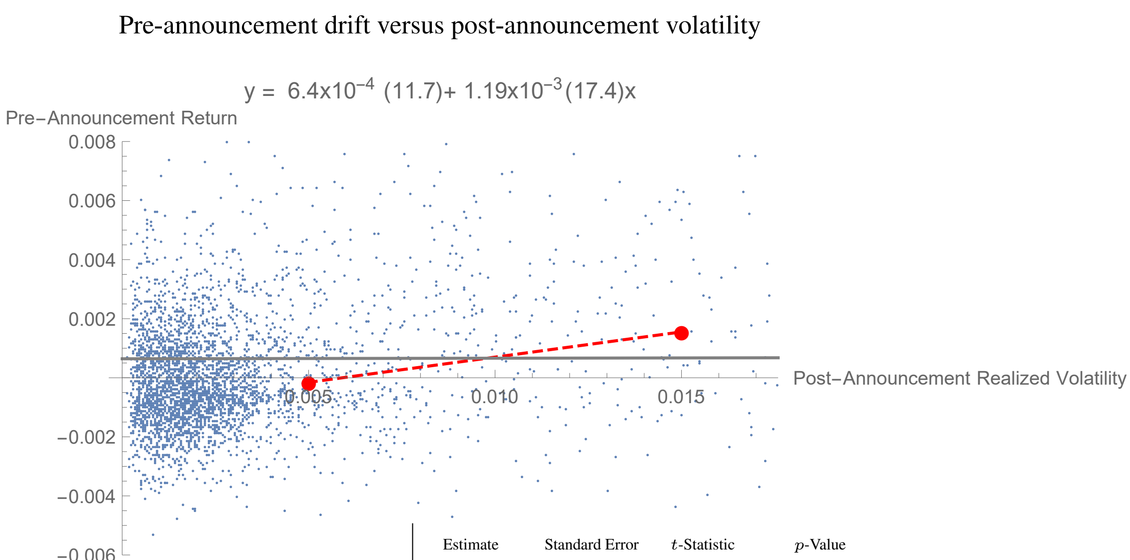
Proposition 2 supports the positive relationship between pre-announcement drift and post-announcement volatility. Part (a) shows that stopping earlier increases γ_{t-}^{A}, leading to higher volatility after the announcement. Part (c) explains that stopping earlier allows more time for the disagreement fluctuations to decrease gradually, resulting in a larger pre-announcement drift.

Simulated data from the model confirms this relationship. Plotting realized volatility two days post-announcement (as a proxy for the VIX) against the pre-announcement drift observed two days before reveals a positive correlation, as shown in Figure 9, along with the line of best fit

[26] The explicit calculation of a VIX equivalent is too technical to include in the current modeling framework.

and model summary. A larger pre-announcement drift is generally followed by higher realized volatility.

Figure 9: Scatter plot of pre-announcement drift versus realized post-announcement volatility based on simulation. Returns two days before an announcement plotted against realized volatility two days after the announcement, including the line of best fit (solid gray line), with corresponding t-statistics in parentheses, the mean of the low- and high-volatility groups (dashed red line), and the model summary as a table.



	Estimate	Standard Error	t-Statistic	p-Value
1	0.000643058	0.0000545292	11.7929	1.40861×10^{31}
x	0.0019204	0.000110035	17.4526	8.53881×10^{66}

2. Apparent Lack of Pre-Announcement Drift in the Treasuries Market

A puzzling aspect of the pre-FOMC drift, as noted by Lucca and Moench (2015), is the absence of a corresponding drift in treasury markets. This could stem from time-varying stock-bond correlations, which were notably positive in the 1990s and negative since the 2000s, with an average close to zero (see Campbell, Sunderam, and Viceira (2017)). In fact, Laarits (2019) leveraged this empirical observation, using a rolling-CAPM beta, and found a positive drift during periods with a positive beta and a negative drift during periods with a negative beta.

This paper's model features only a positive stock-bond correlation, which arises from both asset classes sharing exposure to sentiment risk. As shown in equation (15), when investors stop learning from the signal and sentiment risk decreases (lower g^2), the risk-free rate also drops. This decline in the risk-free rate manifests as a positive pre-announcement drift in treasury prices, aligning with the empirical evidence of a positive stock-bond beta. Du (2021) presents a theoretical model that includes disagreement about not only economic growth but also inflation, capturing the complex dynamics of time-varying stock-bond correlations that this paper does not address.

3. Empirical Evidence for Disagreement Generating a Pre-Announcement Drift

There are two main economic models that explain how prices, volatility, and volume respond to information. On the one hand, noisy rational-expectation equilibrium models assume that investors agree on how to interpret news, though their information sets differ. On the other hand, differences-of-opinion models, which this paper features, assume that investors observe the same information but interpret it differently. In noisy rational-expectation models, the relationship between volume and volatility is linear: order flow (volume) directly causes price changes (volatility). In contrast, in differences-of-opinion models, the relationship is looser because disagreement can generate additional trading volume without volatility.[27] In extreme cases of disagreement, trading volume may rise without any price change.

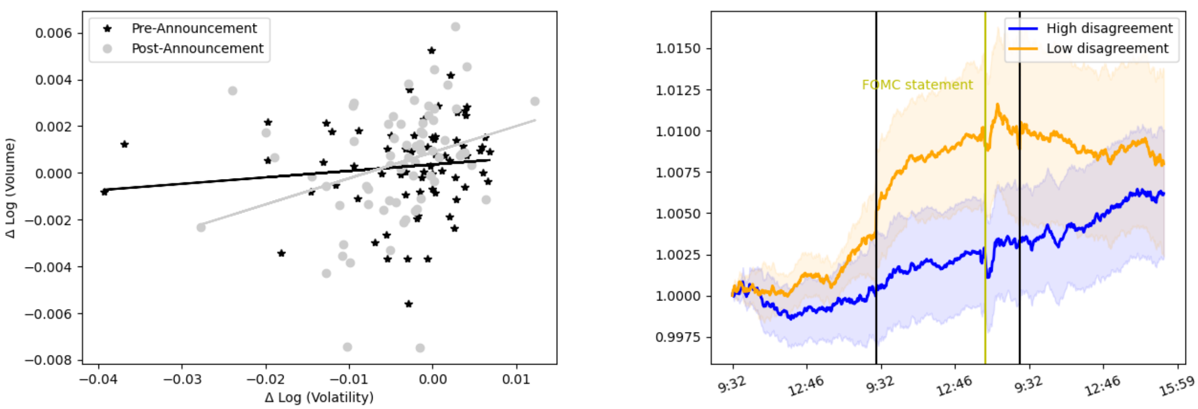
Testing how financial markets process information requires a careful empirical procedure using high-frequency intraday data around scheduled announcements. Bollerslev, Li, and Xue (2018) pioneered this approach, finding a weak relationship between trading volume and volatility

[27] Banerjee (2011) explores the contrasting relationships between volatility and volume in noisy rational-expectation models and in differences-of-opinion models.

around events such as FOMC announcements. Abnormal trading volume cannot be explained by abnormal price changes, suggesting that investors have additional motives, such as disagreement. Bollerslev et al. (2018) concluded that disagreement is the main driver, as volume-volatility elasticity estimates approach unity when known disagreement proxies are controlled for.

Figure 10: Empirical evidence of disagreement. The left graph shows a scatter plot of the average change in minute volume versus the average change in minute volatility for the pre- and post-announcement periods, along with their respective trend lines. The right graph shows the pre-FOMC drift classed into “high disagreement” and “low disagreement” cases based on the slope of the univariate regression of volume explained by volatility. The vertical black lines represent days, and the yellow line represents the time at which the FOMC statement was released. The sample period is from January 2001 through March 2011.

Trading volume versus volatility | Pre-FOMC drift: high- and low-disagreement cases



Their methodology motivates an exploration of the explanatory power of volatility for volume in pre- and post-FOMC periods using the proposed model. We define the pre-announcement period as the interval from the market opening on the day before the FOMC meeting until the release, and the post-announcement period as the interval from the release until the market closing on the following day. The left graph in Figure 10 depicts the findings: the relationship between trading volume and volatility is looser, indicated by a flatter slope, in the pre-announcement period than in the post-announcement period. This suggests more disagreement, on average, among investors ahead of announcements.[28] Next, if announcements are divided into those preceded by high disagreement and those preceded by low disagreement based on the slope of the volume-volatility relationship in the pre-announcement period (with high disagreement corresponding to a flatter slope and low disagreement to a steeper slope), then, as shown in the right graph of Figure 10, the pre-announcement drift is significantly positive only when low disagreement precedes the FOMC announcement. According to our proposed model, the cause of the pre-announcement drift is a reduction in sentiment risk, i.e., disagreement-squared, a conclusion that is supported by empirical evidence of a higher pre-FOMC drift in instances of lower disagreement.

IV. Reduced Models

This section demonstrates that without information costs (Section IV.A) or without disagreement (Section IV.B), the model cannot fully account for the observed patterns in prices, volume, and volatility before announcements. It analyzes two reduced models: one with a costless signal to isolate the role of disagreement from its interaction with intermittent learning, and another without disagreement or a costly signal, used to show that scheduled announcements alone do not generate a pre-announcement drift.

[28] Define $\Delta\log(\sigma_t) = \log(\sigma_t) - \log(\sigma_{t-1})$ and $\Delta\log(m_t) = \log(m_t) - \log(m_{t-1})$, where σ_t is the root of the squared return per minute and m_t is the sum of all trades sizes per minute.

A. Reduced Model: No Information Cost

This section incorporates scheduled announcements into a disagreement model à la Dumas et al. (2009). The economy follows the setup in Section II.A, with the same information structure as in Section II.B, except that the signal's informativeness (equation (4)) is fixed at a constant ϕ. To interpret the model's implications, it is useful to examine the behavior of the state variables before and after the announcement.

1. Pre-Announcement Behavior

Ahead of an announcement, the only variables that follow a persistent pattern are the posterior variances γ_t^A and γ_t^B. As in the main model, γ_t^A and γ_t^B increase throughout an announcement cycle and drop when an announcement occurs (see Figure 1). Furthermore, the difference $\gamma_t^A - \gamma_t^B$ decreases throughout an announcement cycle because investors A extract information from the signal while investors B do not, so that γ_t^A increases more slowly than γ_t^B. From equation (17), both of these behaviors contribute to decreasing the fundamental volatility of returns:

$$\underbrace{\bar{\sigma}_R}_{\downarrow} = \sigma_D + \underbrace{\frac{\gamma_t^A}{\sigma_D}}_{\uparrow} \underbrace{\frac{\partial P}{\partial \hat{f}_t}}_{<0} \frac{1}{P} + \underbrace{\frac{(\gamma_t^A - \gamma_t^B)}{\sigma_D}}_{\downarrow} \underbrace{\frac{\partial P}{\partial g_t}}_{>0} \frac{1}{P}.$$

Since the fundamental volatility of returns is reduced leading up to an announcement, excess returns are reduced ahead of an announcement (see equation (18)). Therefore, a positive pre-announcement drift, as per Definition 1, is impossible in this model.

2. Post-Announcement Behavior

Immediately after an announcement, several changes occur simultaneously:

(i) The posterior variances γ_t^A and γ_t^B drop, as in the main model.

(ii) The difference between the posterior variances, $\gamma_t^A - \gamma_t^B$, increases because investors B extract more information from the announcement than investors A, who are relatively "agnostic" about the announcement.

(iii) The disagreement-squared, g_t^2, increases due to the differing interpretations of the announcement information by the two groups of investors ($\psi \neq 1$). The signed disagreement, g_t, can either increase or decrease depending on the realization of the announcement I_t and the prior estimates of the growth rates, $\hat{f}_t^{A-}$ and $\hat{f}_t^{B-}$.

(iv) The sentiment variable, η_t, rises under the objective measure due to the assumption that the truth is represented by the beliefs of investors B about the announcement, which reduces the consumption share of investors A.

The implications for the excess returns in equation (18) are that

$$\mu - r = \underbrace{K_0}_{\uparrow} + K_1 g_t + \underbrace{K_2}_{\downarrow <0} \underbrace{g_t^2}_{\uparrow},$$

where K_0 increases due to the rise in fundamental volatility from (i) and (ii); K_1 cannot be signed due to the conflicting effects of (i) and (ii) versus (iv); and K_2 decreases (becomes more negative) due to (iv), while g^2 rises from (iii). Intuitively, two opposing forces influence excess returns at an announcement: on the one hand, the information released reduces the posterior variance of the growth rate, increasing excess returns; on the other hand, the differing opinions on the announcement's content increase the disagreement-squared (variance of sentiment), which decreases excess returns. These effects directly counterbalance each other. Simulations with the parameters in Table 1 show that the reduction in posterior variance slightly outweighs the increase in disagreement-squared at the announcement. Thus, while prices may jump, they do not consistently drift up or down, as there is no directional change in risk perception.

Although the occurrence of an announcement has no definite effect on returns, its effects on the volatility and trading volume (which go hand in hand) can be precisely signed. From equation (17), it is evident that (i) and (ii) increase the fundamental volatility, even though the sentiment component depends on the direction of disagreement. Furthermore, the increase in volatility upon an announcement increases hedging needs and trading volume. Hence, trading volume, calculated as the squared turnover over time, increases when an announcement occurs. The previous three phenomena that occur after an announcement—no average effect on returns, high volatility, and high trading volume—match well-established empirical evidence around scheduled announcements and justify the introduction of disagreement about announcements, i.e., $\psi \neq 1$, even though it does not affect the pre-announcement drift.[29]

Figure 11 shows simulation results for excess returns, volatility, and volume before and after an announcement, together with confidence intervals. The simulation makes evident the absence of a pre-announcement drift, as in Definition 1, while accurately capturing the post-announcement empirical patterns for volatility and trading volume.

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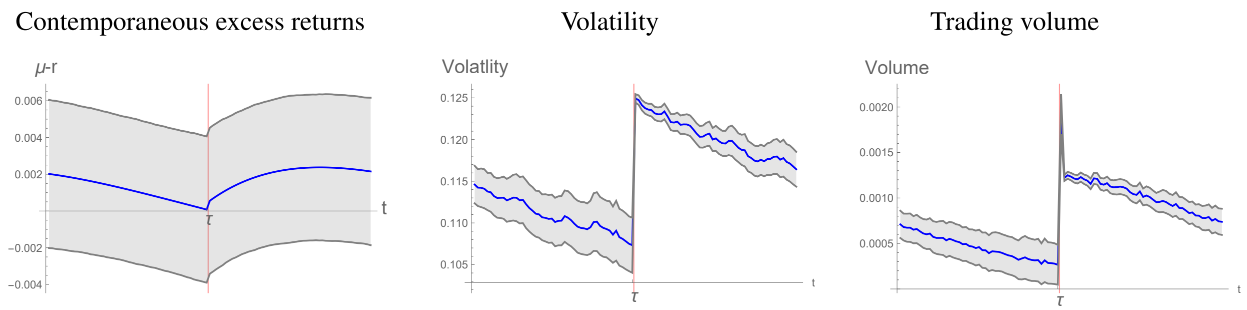
[29] See the empirical evidence in Bollerslev et al. (2018) for FOMC announcements, in Kandel and Pearson (1995) for corporate earnings announcements, and in Li, Maug, and Schwartz-Ziv (2022) for shareholder meetings.

Figure 11: Simulated excess returns, volatility, and trading volume around an announcement. The left graph shows the contemporaneous (expected) excess returns, the middle graph shows the volatility, and the right graph shows the trading volume around an announcement at time τ, marked in red.



B. Baseline Model: No Disagreement

This section presents a baseline model that focuses solely on scheduled announcements while disregarding elements such as disagreement or information costs. The model explores whether the mere existence of scheduled announcements in an economy defined as in Section II.A can generate a pre-announcement drift. We first describe the economy's equilibrium assuming that only investors A populate the economy. Specifically, $\psi = 0$ in equation (3), and ϕ_t in equation (4) is fixed at all times. This section concludes with the finding that a positive pre-announcement drift is unattainable within such a model.

1. Equilibrium and Excess Returns

The problem that investors wish to solve is reduced to

$$\sup_{c_t} E \int_0^\infty e^{-\rho t} \frac{1}{\alpha} (c_t)^\alpha \, dt \quad \text{subject to} \quad E \int_0^\infty \xi_t c_t \, dt = E \int_0^\infty \xi_t \theta D_t \, dt,$$

where θ represents the initial portfolio allocations. The solution of this optimization gives the state price density ξ_t as

$$\xi_t = e^{-\rho t} \left[\left(\frac{1}{\lambda} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} D_t^{\alpha-1}, \tag{20}$$

where λ is the Lagrange multiplier. The pricing of assets depends on the expectation of the distribution of D_u, where u is a future time relative to t, to certain powers, as summarized in the following proposition.

Proposition 5. *The conditional expected value of the future discount factor is*

$$E\left[\left(\frac{D_u}{D_t} \right)^{\epsilon} \middle| \hat{f}_t \right] = H_f(\hat{f}_t, u, t, \epsilon)$$
$$\text{where} \quad \frac{\partial H_f(\hat{f}_t, u, t, \epsilon)}{\partial \hat{f}_t} = \frac{\epsilon(1 - e^{-\zeta(u-t)})}{\zeta} H_f(\hat{f}_t, u, t, \epsilon)$$

has the sign of ϵ. The function $H_f(\hat{f}_t, u, t, \epsilon)$ is given in closed form in Appendix C.5.

The pricing measure in equation (20) encompasses both the equilibrium's risk-free rate and market prices of risk. Applying Itô's lemma yields explicit expressions for these. The contemporaneous (expected) excess returns are the product of the market prices of risk and the volatility of stock returns, resulting in

$$\mu - r = (1-\alpha)\sigma_D^2 + \gamma_t (1-\alpha) \underbrace{\frac{\partial P}{\partial \hat{f}_t} \frac{1}{P}}_{<0}. \tag{21}$$

Contemporaneous (expected) excess returns increase with the dividend's volatility, σ_D, but decrease with the uncertainty about (i.e., posterior variance of) the growth rate of the dividend, γ_t. The latter relationship is due to $\frac{\partial P}{\partial \hat{f}_t} < 0$, as stated in Proposition 5 (the derivative has the sign of ϵ, which is negative) and stems from the anticipation that an increase in today's growth rate will lead to higher consumption tomorrow. Consequently, investors reduce their savings in the asset, thereby driving prices down.[30]

It is worth delving deeper into the contrasting effects of σ_D and γ_t on excess returns. Both quantities reflect certain aspects of "risk," but while a higher volatility σ_D increases the risk premium, a higher posterior variance γ_t decreases it by making the asset more attractive for hedging. Recall that the growth rate is unobservable, but dividend realizations shed light on it. A negative dividend realization prompts investors to revise their expectations of the future growth rate downward, leading them to anticipate lower future consumption. In response, they increase savings in the asset, driving up its price. As this high price coincides with low consumption (following a negative dividend realization), the resulting negative correlation induces a negative risk premium. With higher γ_t, investors' hedging demand becomes more sensitive to dividend realizations, further diminishing the risk premium as the asset becomes indispensable for hedging. In contrast, when $\gamma_t = 0$, investors know the growth rate with certainty, and there is no revision regardless of the dividend realization; thus, their hedging demand for the asset remains unaffected.[31]

2. Results

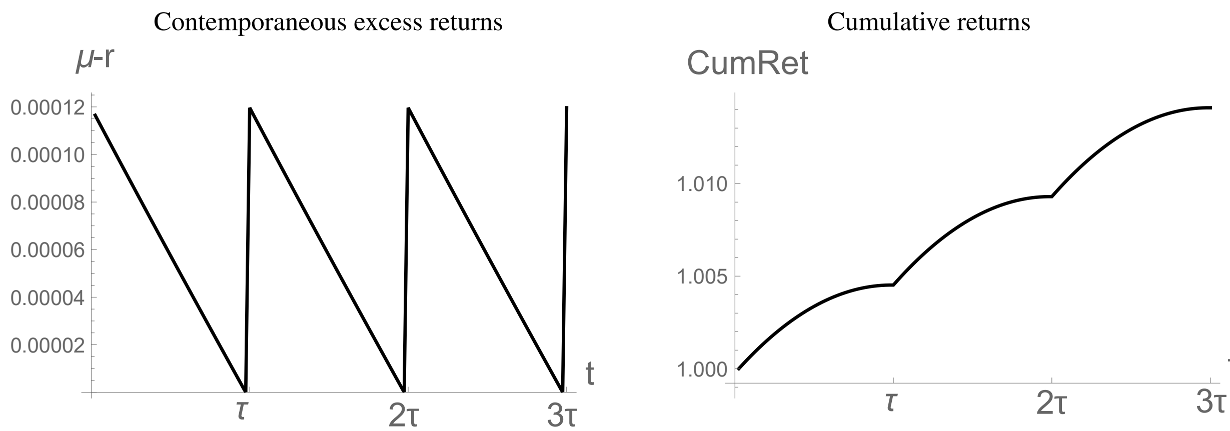
In this model, the only variable that follows a periodic pattern is the posterior variance γ_t, which increases during an announcement cycle and drops immediately after the announcement (as shown in Figure 1). From Proposition 5 and equation (21), an increase in γ_t leads to a decrease

[30] With risk aversion greater than 1, the income effect dominates the effect of increased future dividends; see Veronesi (2000).

[31] See Veronesi (2000) for further discussion on the lack of a risk premium for noisy signals.

in expected returns. Figure 12 illustrates the excess returns (left graph) and cumulative returns (right graph) over three announcement cycles. As an announcement nears, expected returns decrease due to the rising posterior variance γ_t. Consequently, this model does not allow for a positive pre-announcement drift, as per Definition 1, in anticipation of future announcements.

Figure 12: Excess returns and cumulative returns. The left graph shows the contemporaneous (expected) excess returns, $\mu - r$, for three announcement cycles. The right graph shows the cumulative returns for the same three announcement cycles.



V. Conclusion

This paper proposes an economic mechanism that can explain, within a unified framework, the *pre-announcement drift*—a puzzling positive price drift in the aggregate equity market, accompanied by low volatility and low trading volume, ahead of scheduled announcements. A differences-of-opinion model à la Dumas et al. (2009) is developed and shown to reproduce the empirical phenomena surrounding the pre-announcement drift, in contrast to alternative asset pricing explanations.

In the proposed general equilibrium model, investors interpret a costly signal differently. This heterogeneous reaction to information creates sentiment risk, which manifests itself in the movement of prices. The scheduled nature of the announcements and the cost of learning from the continuous signal generate intermittent learning: investors who pay to learn from the continuous signal optimally choose to stop learning as an announcement approaches. The decision to stop learning decreases sentiment risk just before an announcement and hence causes a positive realized price drift. The decrease in sentiment risk simultaneously results in low volatility and low trading volume when an announcement is imminent.

The proposed modeling framework can be helpful not only for explaining the pre-announcement drift but also as a basis for analyzing the behavior of other heterogeneous sets of players. For example, it can be adapted to study investors that follow fundamental (idiosyncratic) versus macro (systematic) news, or the new wave of high-frequency trading compared with trading based on information received at lower frequencies. These are settings in which investors with different beliefs coexist in financial markets, and the model developed in this paper may be helpful for gaining a better understanding of their interactions.

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A. Appendix: Leakage of Information

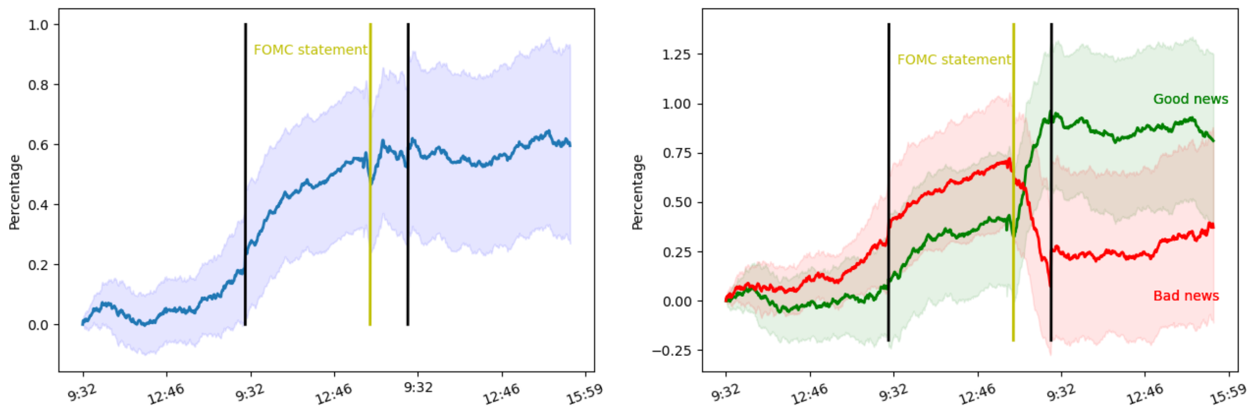
This section summarizes evidence against information leakage as the cause of the pre-announcement drift. The leakage hypothesis is unlikely since the drift is always positive, implying selective leakage of good news, though it remains plausible under short-sell constraints. Lucca and Moench (2015) rejects the leakage explanation for two reasons: (1) the drift is uncorrelated with post-announcement price movements, and (2) the Fed enforces a one-week blackout period before each FOMC meeting, preventing public statements and making leakage improbable.

We test the information leakage hypothesis by splitting the S&P 500's cumulative returns into "good news" and "bad news" in a three-day window around FOMC meetings. The classification is based on market reaction (defined as the price change from the announcement, at 2:30 p.m., to the market close, at 3:59 p.m.) rather than on the statement content. The right graph in Figure 13 shows the result of this decomposition.

Figure 13: Cumulative returns on the S&P 500 index in three-day windows. The left graph shows the the pre-FOMC drift as the average cumulative return on the S&P 500 index between 9:30 a.m. EST on the day before a scheduled FOMC announcement and 4:00 p.m. EST on the day after the announcement. The vertical black lines represent days, and the yellow line represents the moment when the FOMC statement is released. The right graph shows the pre-FOMC drift decomposed into "good news" (green) and "bad news" (red) based on the market reaction. The shaded areas are pointwise 95% confidence bands around the average returns. The sample period is from September 1994 through March 2011.

Average cumulative return, showing pre-FOMC drift

Pre-FOMC drift split into "good news" and "bad news"



The pre-announcement drift occurs before both "good news" and "bad news" without significant statistical differences, despite the latter having a higher mean. This undermines the information leakage hypothesis: if investors expected negative reactions, short-selling would have caused a price drop. Even with partial leakage, the "bad news" drift should have been smaller than the "good news" one. While leakage may reduce risk, as suggested by Ying (2020b), this effect is secondary to the price response to the announcement.[32]

[32]The risk reduction would dominate if it came from massive information leakage. However, such a massive leakage would imply a reaction of close to zero price after the announcement, which contradicts the empirical evidence, too.

B. Appendix: Simulation Parameters

The average dividend growth rate $\bar{f}$, the dividend volatility σ_D, and the growth rate volatility σ_f are set to the values in Dumas et al. (2009), which are similar to the estimation results reported in Brennan and Xia (2001). The announcement volatility, σ_I, is arbitrarily set to an annual value of 10%; equivalent results can be obtained using a wide range of values, the sole restriction being that $\sigma_I > \sigma_f$.[33]

Table 1: Parameter values used for figures.

Parameter	Symbol	Value
Long-term average growth rate of the dividend process	$\bar{f}$	0.015
Reversion parameter of the dividend process	ζ	0.2
Volatility of dividends	σ_D	0.13
Volatility of announcements	σ_I	0.15
Volatility of dividend shock	σ_f	0.03
Investors' relative risk aversion	$1-\alpha$	3
Rate of impatience	ρ	0.1
Informativeness of continuous signal	ϕ_t	0.95
Informativeness of announcements	ψ	0.5

[33]Time-separable preferences in this CRRA framework provide tractable formulas but pose challenges for quantitative analysis. The model focuses on the qualitative economic mechanism; quantitative calibration is beyond the scope of this paper.

C. Appendix: Proofs

Throughout this appendix, the subscript t in ϕ_t is omitted for convenience.

1. Proof of Proposition 1

Proof. The solution depends on the joint conditional distribution of η_u and D_u given the state variables $\{g_t, \eta_t, \hat{f}_t^A, D_t\}$ and time t:

$$E^A_{\hat{f}^A,g,t}\left[D_u^{\varepsilon}\eta_u^{\chi}\right] = H(D_t, \eta_t, \hat{f}^A, g, t, u; \varepsilon, \chi).$$

The solution H should satisfy

$$0 = \mathcal{L}H(D_t, \eta_t, \hat{f}^A, g, t, u; \varepsilon, \chi) + \frac{\partial H(D_t, \eta_t, \hat{f}^A, g, t, u; \varepsilon, \chi)}{\partial t}$$

where $\mathcal{L}$ is the differential generator of $(D_t, \eta_t, \hat{f}_t^A, g_t)$ under the probability measure of investors A. Written out in full while omitting the subscripts t of the state variables, the preceding equation is

$$\begin{aligned} 0 = {} & \frac{\partial H}{\partial D} D\hat{f}^A - \frac{\partial H}{\partial \hat{f}^A}\zeta(\hat{f}^A - \bar{f}) - \frac{\partial H}{\partial g} g\left(\zeta + \frac{\gamma^B}{\sigma_D^2}\right) + \frac{1}{2}\frac{\partial^2 H}{\partial D^2}D^2\sigma_D^2 + \frac{1}{2}\frac{\partial^2 H}{\partial \eta^2}\frac{\eta^2 g^2}{\sigma_D^2} \\ & + \frac{1}{2}\frac{\partial^2 H}{\partial g^2}\left(\frac{(\gamma^A - \gamma^B)^2}{\sigma_D^2} + \phi^2\sigma_f^2\right) + \frac{1}{2}\frac{\partial^2 H}{\partial (\hat{f}^A)^2}\left(\frac{(\gamma^A)^2}{\sigma_D^2} + \phi^2\sigma_f^2\right) - \frac{\partial^2 H}{\partial D \partial \eta} D\eta g \\ & + \frac{\partial^2 H}{\partial D \partial g} D(\gamma^A - \gamma^B) + \frac{\partial^2 H}{\partial D \partial \hat{f}^A} D\gamma^A - \frac{\partial^2 H}{\partial \eta \partial g}\left(\frac{g\eta(\gamma^A - \gamma^B)}{\sigma_D^2}\right) \\ & - \frac{\partial^2 H}{\partial \eta \partial \hat{f}^A}\frac{\eta g \gamma^A}{\sigma_D^2} + \frac{\partial^2 H}{\partial g \partial \hat{f}^A}\gamma^A\left(\frac{\gamma^A(\gamma^A - \gamma^B)}{\sigma_D^2} + \phi^2\sigma_f^2\right) + \frac{\partial H}{\partial t}, \end{aligned}$$

and the solution is of the form

$$H(D_t, \eta_t, \hat{f}_t^A, g_t, t, u; \varepsilon, \chi) = D_t^{\varepsilon}\eta_t^{\chi} H_f(\hat{f}_t^A, t, u; \varepsilon) H_g(g_t, t, u; \varepsilon, \chi).$$

Substituting the expressions for the partial derivatives yields

$$
\begin{aligned}
0 &= \varepsilon \hat{f}^A - \frac{\partial H_f}{\partial \hat{f}^A}\frac{1}{H_f}\zeta(\hat{f}^A - \bar{f}) - \frac{\partial H_g}{\partial g}\frac{1}{H_g} g\left(\zeta + \frac{\gamma^B}{\sigma_D^2}\right) + \frac{1}{2}\varepsilon(\varepsilon - 1)\sigma_D^2 \\
&+ \frac{1}{2}\chi(\chi-1)\frac{g^2}{\sigma_D^2} + \frac{1}{2}\frac{\partial^2 H_g}{\partial g^2}\frac{1}{H_g}\left(\frac{(\gamma^A - \gamma^B)^2}{\sigma_D^2} + \phi^2\sigma_f^2\right) + \frac{1}{2}\frac{\partial^2 H_f}{\partial(\hat{f}^A)^2}\frac{1}{H_f}\left(\frac{(\gamma^A)^2}{\sigma_D^2} + \phi^2\sigma_f^2\right) \\
&- \varepsilon\chi g + \frac{\partial H_g}{\partial g}\frac{\varepsilon}{H_g}(\gamma^A - \gamma^B) + \frac{\partial H_f}{\partial \hat{f}^A}\frac{\varepsilon\gamma^A}{H_f} - \frac{\partial H_g}{\partial g}\frac{\chi}{H_g}\left(\frac{g(\gamma^A - \gamma^B)}{\sigma_D^2}\right) \\
&- \frac{\partial H_f}{\partial \hat{f}^A}\frac{\chi}{H_f}\frac{g\gamma^A}{\sigma_D^2} + \frac{\partial H_f}{\partial \hat{f}^A}\frac{\partial H_g}{\partial g}\frac{1}{H_g H_f}\left(\frac{\gamma^A(\gamma^A - \gamma^B)}{\sigma_D^2} + \phi^2\sigma_f^2\right)) + \frac{\partial H_f}{\partial t}\frac{1}{H_f} + \frac{\partial H_g}{\partial t}\frac{1}{H_g},
\end{aligned}
$$

where again the subscripts t of $\hat{f}_t^A, \gamma_t^A, \gamma_t^B$, and g_t have been omitted. Grouping together the terms involving only $\hat{f}^A$ and functions of time such as γ^A and γ^B, and collecting the terms with g and η separately, results in two equations. One equation defines the partial differential equation that H_f should satisfy,

$$
\begin{aligned}
0 &= \varepsilon \hat{f}^A - \frac{\partial H_f}{\partial \hat{f}^A}\frac{1}{H_f}\zeta(\hat{f}^A - \bar{f}) + \frac{1}{2}\varepsilon(\varepsilon - 1)\sigma_D^2 + \frac{1}{2}\frac{\partial^2 H_f}{\partial(\hat{f}^A)^2}\frac{1}{H_f}\left(\frac{(\gamma^A)^2}{\sigma_D^2} + \phi^2\sigma_f^2\right) \\
&+ \frac{\partial H_f}{\partial \hat{f}^A}\frac{\varepsilon\gamma^A}{H_f} + \frac{\partial H_f}{\partial t}\frac{1}{H_f},
\end{aligned}
$$

which, using $H_f(\hat{f}_t^A, t, t; \varepsilon) = 1$ as the boundary condition, is solved by

$$
\begin{aligned}
H_f(\hat{f}_t, u, t) = \exp\Bigg[&\epsilon\left(\frac{(\hat{f}_t - \bar{f})(1 - e^{-\zeta(u-t)})}{\zeta} + \bar{f}(u-t)\right) + \frac{(\gamma\epsilon^2)\left(\zeta(u-t) - (1 - e^{-\zeta(u-t)})\right)}{\zeta^2} \\
&+ \frac{\epsilon^2\left(4e^{-\zeta(u-t)} - e^{-2\zeta(u-t)} + 2\zeta(u-t) - 3\right)\left(\gamma^2 + \sigma_D^2\sigma_f^2\phi^2\right)}{4\zeta^3\sigma_D^2} + \frac{1}{2}\sigma_D^2(u-t)(\epsilon - 1)\epsilon\Bigg].
\end{aligned}
$$

The other equation, which contains g and η and cross terms, is

$$
\begin{aligned}
0 = &- \frac{\partial H_g}{\partial g}\frac{1}{H_g} g\left(\zeta + \frac{\gamma^B}{\sigma_D^2}\right) + \frac{1}{2}\chi(\chi-1)\frac{g^2}{\sigma_D^2} + \frac{1}{2}\frac{\partial^2 H_g}{\partial g^2}\frac{1}{H_g}\left(\frac{(\gamma^A - \gamma^B)^2}{\sigma_D^2} + \phi^2\sigma_f^2\right) - \varepsilon\chi g \\
&+ \frac{\partial H_g}{\partial g}\frac{\varepsilon}{H_g}(\gamma^A - \gamma^B) - \frac{\partial H_g}{\partial g}\frac{\chi}{H_g}\left(\frac{g(\gamma^A - \gamma^B)}{\sigma_D^2}\right) - \frac{\partial H_f}{\partial \hat{f}^A}\frac{\chi}{H_f}\frac{g\gamma^A}{\sigma_D^2} \\
&+ \frac{\partial H_f}{\partial \hat{f}^A}\frac{\partial H_g}{\partial g}\frac{1}{H_g H_f}\left(\frac{\gamma^A(\gamma^A - \gamma^B)}{\sigma_D^2} + \phi^2\sigma_f^2\right) + \frac{\partial H_g}{\partial t}\frac{1}{H_g}.
\end{aligned}
$$

The solution for H_g is quadratic in g and of the form

$$H_g(g,t,u;\varepsilon,\chi) = \exp\big\{A_1(\chi;u-t) + \varepsilon^2 A_2(\chi;u-t) + \varepsilon g B(\chi;u-t) + g^2 C(\chi;u-t)\big\}.$$

Define the following constants at time t:

$$a = 2\left(\frac{(\gamma^A-\gamma^B)^2}{\sigma_D^2} + \phi^2\sigma_f^2\right), \qquad b = \zeta + \frac{\gamma^B + \chi(\gamma^A-\gamma^B)}{\sigma_D^2}, \qquad c = \frac{\chi(\chi-1)}{2\sigma_D^2},$$

$$q = \sqrt{b^2-ac}, \qquad k = -\chi\left(1+\frac{\gamma^A}{\sigma_D^2\zeta}\right), \qquad l = \frac{\chi\gamma^A}{\sigma_D^2\zeta},$$

$$m = (\gamma^A-\gamma^B)\left(1+\frac{\gamma^A}{\sigma_D^2\zeta}\right) + \frac{\phi^2\sigma_f^2}{\zeta}, \qquad n = -\left(\frac{\gamma^A}{\zeta}\frac{(\gamma^A-\gamma^B)}{\sigma_D^2} + \frac{\phi^2\sigma_f^2}{\zeta}\right)$$

and

$$\upsilon_1 = 0, \qquad \upsilon_2 = 2q, \qquad \upsilon_3 = \zeta, \qquad \upsilon_4 = 2q+\zeta, \qquad \upsilon_5 = q$$

$$\vartheta_1 = \frac{k\,(b+q)+2cm}{q}, \qquad \vartheta_2 = \frac{k\,(b-q)+2cm}{q}, \qquad \vartheta_3 = \frac{l\,(b+q)+2cn}{q-\zeta},$$

$$\vartheta_4 = \frac{l\,(b-q)+2cn}{q+\zeta}, \qquad \vartheta_5 = -(\vartheta_1+\vartheta_2+\vartheta_3+\vartheta_4).$$

The differential equations and initial conditions that determine H_g are

$$C'(u-t) = aC^2(u-t) - 2bC(u-t) + c, \qquad C(0)=0,$$

$$B'(u-t) = B(u-t)[aC(u-t)-b] + 2C(u-t)(m+ne^{-\zeta(u-t)}) + k + le^{-\zeta(u-t)}, \qquad B(0)=0,$$

$$A_1'(u-t) = \frac{a}{2}C(u-t), \qquad A_1(0)=0,$$

$$A_2'(u-t) = B(u-t)\Big(\frac{a}{4}B(u-t) + d + ne^{-\zeta(u-t)}\Big), \qquad A_2(0)=0.$$

The solutions to these equations are

$$\begin{aligned}
C(u-t) &= \frac{c\left(1-e^{-2q(u-t)}\right)}{(q-b)e^{-2q(u-t)}+b+q},\\
B(u-t) &= \frac{\sum_{i=1}^{5}\vartheta_i e^{-\upsilon_i(u-t)}}{(q-b)e^{-2q(u-t)}+b+q},\\
A_1(u-t) &= \frac{1}{2}\Big((u-t)(b-q)-\log\big((q-b)e^{-2q(u-t)}+b+q\big)+\log(2q)\Big),\\
A_2(u-t) &= \int_t^u B(h-t)\Big(\frac{a}{4}B(h-t)+d+ne^{-\zeta(h-t)}\Big)\,dh,
\end{aligned}$$

where the integral that defines $A_2(u-t)$ can be obtained in closed form.

This solution for H_g is well-defined when $b^2-ac>0$, i.e., when q is real. Note that b^2-ac can be written as a quadratic equation in χ:

$$\begin{aligned}
f(\chi) &= b^2-ac = k_1+k_2\chi+k_3\chi^2\\
&= \frac{(\gamma^B)^2}{\sigma_D^4}+\chi\left(\frac{(\gamma^B-\gamma^A)^2}{\sigma_D^4}+\frac{(\phi\gamma^A)^2}{\sigma_D^2}-\frac{2\gamma^B(\gamma^B-\gamma^A)}{\sigma_D^4}\right)-\chi^2\frac{(\phi\gamma^A)^2}{\sigma_D^2}.
\end{aligned}$$

Because k_3 is negative, the function $f(\chi)$ defines a parabola that opens downward. Since $f(0)=k_1>0$ and $f(1)=\frac{(\gamma^A)^2}{\sigma_D^2}>0$, the parabola defined by $f(\chi)$ is always positive in the interval $\chi\in[0,1]$.

The next step is to show that $C(\chi;u-t)\le 0$ so that $H_g(\cdot)$ is decreasing in g^2. Note that the sentiment variable η_t is a change of measure, hence a martingale, and must satisfy

$$E_t^A\left[\frac{\eta_u}{\eta_t}\right]=1.$$

By using the solution for $H_g(\cdot)$, the expectations of η_t are given by

$$E_t^A\left[\left(\frac{\eta_u}{\eta_t}\right)^{\chi}\right]=H_g(g,t,u;0,\chi)=e^{A_1(\chi;u-t)+g^2C(\chi;u-t)}.$$

Since $\chi \in [0,1]$, applying a concave transformation by Jensen's inequality yields

$$E_t^A\left[\left(\frac{\eta_u}{\eta_t}\right)^{\chi}\right] \leq 1,$$

which implies that $A_1(\chi; u-t) \leq 0$ and $C(\chi; u-t) \leq 0$. □

2. Proof of Proposition 2

Proof. **(a)** Begin with the solution to the law of motion of the posterior variance γ_t^A in equation (6):

$$\gamma_t^A = \frac{\gamma_0^A \kappa^A(e^{2t\kappa^A}+1) + \big(\sigma_f^2(1-\phi^2) - \zeta\gamma_0^A\big)(e^{2t\kappa^A}-1)}{\kappa^A(e^{2t\kappa^A}+1) + \big(\frac{\gamma_0^A}{\sigma_D^2}+\zeta\big)(e^{2t\kappa^A}-1)}, \tag{22}$$

where $\kappa^A \equiv \sqrt{\zeta^2 + \frac{\sigma_f^2}{\sigma_D^2}(1-\phi^2)}$. It follows that

$$\frac{\partial \gamma_t^A}{\partial \phi} = -\frac{\sigma_f^2\phi}{\kappa^A\Big(\kappa^A(e^{2t\kappa^A}+1) + \big(\frac{\gamma_0^A}{\sigma_D^2}+\zeta\big)(e^{2t\kappa^A}-1)\Big)^2}\left[\frac{\gamma_0^A}{\sigma_D^2}\left(2\zeta + \frac{\gamma_0^A}{\sigma_D^2}\right)\underbrace{\left(-4t\kappa^A e^{2t\kappa^A} + e^{4t\kappa^A} - 1\right)}_{>0}\right.$$
$$\left. + \left(2\zeta^2 + \frac{\sigma_f^2}{\sigma_D^2}(1-\phi^2)\right)(e^{4t\kappa^A}-1) + \kappa^A\left(2\zeta + 2\frac{\gamma_0^A}{\sigma_D^2}\right)(e^{2t\kappa^A}-1)^2 + \frac{\sigma_f^2}{\sigma_D^2}(1-\phi^2)\big(4t\kappa^A e^{2t\kappa^A}\big)\right].$$

This derivative is negative because the underbraced expression is of the form $f(x) = -4xe^{2x} + e^{4x} - 1$, which is increasing and non-negative for all $x > 0$, since $f(0) = 0$ and $f'(x) = 4e^{2x}(e^{2x} - 2x - 1)$. Therefore, γ_t^A decreases in ϕ, which implies that γ_t^A increases as ϕ decreases to zero.

(b) The increase of $\gamma_t^A - \gamma_t^B$ follows from the fact that γ_t^B does not depend on ϕ, so that $\frac{\partial(\gamma_t^A - \gamma_t^B)}{\partial\phi} = \frac{\partial\gamma_t^A}{\partial\phi}$, which is negative as shown in part (a). Therefore, $\gamma_t^A - \gamma_t^B$ increases as ϕ decreases to zero.

For the absolute value of the difference, $|\gamma_t^A - \gamma_t^B|$, it is important to note that when investors A stop learning the signal, i.e., $\phi = 0$, the law of motion of their posterior variance is the same as that for the posterior variance of investors B. Specifically, the solution of the posterior variance of investors A becomes

$$\gamma_t^A = \frac{\gamma_0^A \kappa^A (e^{2t\kappa^A} + 1) + (\sigma_f^2 - \zeta\gamma_0^A)(e^{2t\kappa^A} - 1)}{\kappa^A (e^{2t\kappa^A} + 1) + \left(\frac{\gamma_0^A}{\sigma_D^2} + \zeta\right)(e^{2t\kappa^A} - 1)}$$

where $\kappa^A \equiv \sqrt{\zeta^2 + \frac{\sigma_f^2}{\sigma_D^2}}$. This solution is a monotonic function of time since its derivative

$$\frac{\partial \gamma_t^A}{\partial t} = \frac{4e^{2t\kappa^A} (\kappa^A)^2 \left(\sigma_f - \frac{(\gamma_0^A)^2}{\sigma_D^2} - 2\zeta\gamma_0^A\right)}{\left(\kappa^A (e^{2t\kappa^A} + 1) + \left(\frac{\gamma_0^A}{\sigma_D^2} + \zeta\right)(e^{2t\kappa^A} - 1)\right)^2}$$

has a consistent sign over the entire domain $t \geq 0$. Furthermore, as time increases, this function converges to a unique value. Specifically, from equation (6), it converges to

$$\lim_{t\to\infty} \gamma_t^B = \lim_{t\to\infty} \gamma_t^A = \bar{\gamma} = \sigma_D^2 \left(\sqrt{\zeta^2 + \frac{\sigma_f^2}{\sigma_D^2}} - \zeta \right).$$

Although the starting points of the law of motion for γ_t^A and γ_t^B are different, both converge monotonically to the same constant over time. Therefore, their absolute difference $|\gamma_t^A - \gamma_t^B|$ must decrease when investors A stop learning from the signal.

(c) From equation (7),

$$dg_t^2 = \frac{(\gamma_t^B - \gamma_t^A)^2}{\sigma_D^2}\, dt + \phi^2\, dt.$$

It follows from part (b) that $(\gamma_t^B - \gamma_t^A)^2$ decreases with time when $\phi = 0$ and hence the riskiness of the sentiment variable (i.e., g_t^2) decreases. Nevertheless, when $\phi = 0$, the process of g_t remains an Orstein–Uhlenbeck process:

$$dg_t = \vartheta_t g_t\, dt + \sigma_{D,t}^g\, dZ_{D,t}^A,$$

$$\vartheta_t = -\left(\zeta + \frac{\gamma_t^B}{\sigma_D^2}\right), \qquad \sigma_{D,t}^g = \frac{\gamma_t^A - \gamma_t^B}{\sigma_D}.$$

□

3. Proof of Proposition 3

Proof. **(a)** Define the marginal variance reduction per unit time due to learning from the signal as $\frac{\partial \gamma_t^A}{\partial t}$. Using the solution in equation (22), for any ϕ,

$$\frac{\partial \gamma_t^A}{\partial t} = \frac{4e^{2t\kappa^A}(\kappa^A)^2\left(\sigma_f(1-\phi^2) - \frac{(\gamma_0^A)^2}{\sigma_D^2} - 2\zeta\gamma_0^A\right)}{\left(\kappa^A(e^{2t\kappa^A}+1) + \left(\frac{\gamma_0^A}{\sigma_D^2}+\zeta\right)(e^{2t\kappa^A}-1)\right)^2}.$$

Note that $\frac{\partial \gamma_t^A}{\partial t}$ is a monotonic function of time since

$$\frac{\partial^2 \gamma_t^A}{\partial t^2} = -\frac{8e^{2t\kappa^A}(\kappa^A)^3\gamma_0^A\left(\kappa^A(e^{2t\kappa^A}-1) + \left(\frac{\gamma_0^A}{\sigma_D^2}+\zeta\right)(e^{2t\kappa^A}+1)\right)}{\left(\kappa^A(e^{2t\kappa^A}+1) + \left(\frac{\gamma_0^A}{\sigma_D^2}+\zeta\right)(e^{2t\kappa^A}-1)\right)^3}\left(\sigma_f(1-\phi^2) - \frac{(\gamma_0^A)^2}{\sigma_D^2} - 2\zeta\gamma_0^A\right)$$

is strictly positive for $\sigma_f(1-\phi^2) - \frac{(\gamma_0^A)^2}{\sigma_D^2} - 2\zeta\gamma_0^A < 0$ or strictly negative for $\sigma_f(1-\phi^2) - \frac{(\gamma_0^A)^2}{\sigma_D^2} - 2\zeta\gamma_0^A > 0$, so $\frac{\partial \gamma_t^A}{\partial t}$ is monotone regardless of the choice of parameters.

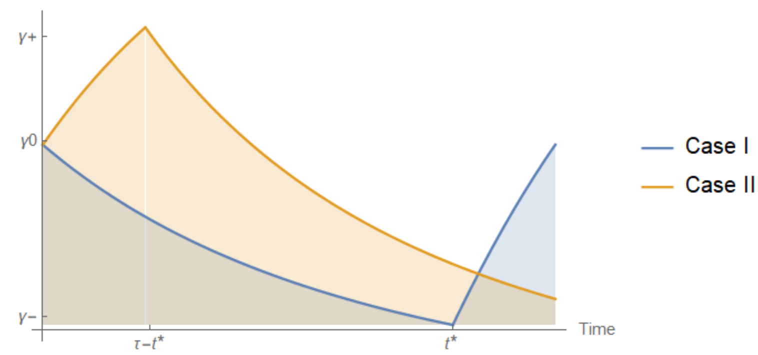
It follows that, since $\kappa^A > 0$, $\frac{\partial \gamma_t}{\partial t} \to 0$ as $t \to \infty$ by L'Hospital rule:

$$\begin{aligned}
&\lim_{t\to\infty} \frac{4e^{2t\kappa^A}(\kappa^A)^2\left(\sigma_f(1-\phi^2) - \frac{(\gamma_0^A)^2}{\sigma_D^2} - 2\zeta\gamma_0^A\right)}{\left(\kappa^A(e^{2t\kappa^A}+1) + \left(\frac{\gamma_0^A}{\sigma_D^2}+\zeta\right)(e^{2t\kappa^A}-1)\right)^2} \\
&= \lim_{t\to\infty} \frac{2(\kappa^A)^2\gamma_0^A\left(\sigma_f(1-\phi^2) - \frac{(\gamma_0^A)^2}{\sigma_D^2} - 2\zeta\gamma_0^A\right)}{\left(\frac{\gamma_0^A}{\sigma_D^2}+\zeta+\kappa^A\right)\left(\kappa^A(e^{2t\kappa^A}+1) + \left(\frac{\gamma_0^A}{\sigma_D^2}+\zeta\right)(e^{2t\kappa^A}-1)\right)} = 0.
\end{aligned}$$

Therefore, as time increases, the marginal variance reduction of the signal monotonically decreases.

(b) Consider two cases: In Case I, investors A extract information from $t = 0$ to $t = t^*$, with a non-learning interval of $\Delta = \tau - t^*$ until the next announcement at τ. In Case II, investors A learn from $t = 0$ to $t = t_1$, pause, and resume learning at $t = t_1 + \tau - t^*$, maintaining the same non-learning interval $\Delta = \tau - t^*$. Since both cases have identical information costs, for $t_1 < t^*$, Case II is effectively a time-shifted version of Case I. Thus, Case I represents learning followed by non-learning, while Case II represents the reverse; see Figure 14.

Figure 14: Posterior variance of investors A in two cases. The posterior variance γ_t^A if investors learn first and then stop learning at time t^* (Case I) or if they don't learn initially and then start learning at time $\tau - t^*$ (Case II).



In Case I, γ_t^A decreases from the original level γ_0 to $\gamma_- < \gamma_0$ by time t^*. In Case II, γ_t^A increases from γ_0 to $\gamma_+ > \gamma_0$ by time $\tau - t^*$. It is shown below that the total variance, i.e., the area under the curve in Figure 14, is larger in Case II (shaded orange) than in Case I (shaded blue).

From equation (22), the integral over time t of the variance process that starts at γ_0 is $\Gamma(\gamma, t, \phi)$. Using this notation, the following difference is positive:

$$\begin{aligned}
&\text{total variance in Case II} - \text{total variance in Case I} \\
&= \Gamma(\gamma_0, \tau - t^*, 0) + \Gamma(\gamma_+, t^*, \phi) - \big(\Gamma(\gamma_0, t^*, \phi) + \Gamma(\gamma_-, \tau - t^*, 0)\big) \\
&= g_L(\gamma_+) - g_L(\gamma_0) + g_N(\gamma_0) - g_N(\gamma_-) > 0,
\end{aligned}$$

where $g_L(\gamma)$ and $g_N(\gamma)$ are functions of γ given by

$$g_L(\gamma) = -\sigma_D^2(\zeta + \kappa^A)t^* \log\left[\frac{2\kappa^A}{\kappa^A(e^{2t^*\kappa^A}+1) + \left(\frac{\gamma}{\sigma_D^2}+\zeta\right)(e^{2t^*\kappa^A}-1)}\right],$$

$$g_N(\gamma) = -\sigma_D^2(\zeta + \kappa^B)(\tau - t^*) \log\left[\frac{2\kappa^B}{\kappa^B(e^{2(\tau-t^*)\kappa^B}+1) + \left(\frac{\gamma}{\sigma_D^2}+\zeta\right)(e^{2(\tau-t^*)\kappa^B}-1)}\right],$$

$$\text{with} \quad \kappa^A \equiv \sqrt{\zeta^2 + \frac{\sigma_f^2}{\sigma_D^2}(1-\phi^2)} \qquad \text{and} \qquad \kappa^B \equiv \sqrt{\zeta^2 + \frac{\sigma_f^2}{\sigma_D^2}}.$$

It suffices to show that these are increasing functions of γ, since their derivatives are positive:

$$\frac{\partial g_L(\gamma)}{\partial \gamma} = \frac{e^{2t^*\kappa^A}-1}{\kappa^A\left(e^{2t^*\kappa^A}+1\right) + \left(\frac{\gamma}{\sigma_D^2}+\zeta\right)\left(e^{2t^*\kappa^A}-1\right)} > 0,$$

$$\frac{\partial g_N(\gamma)}{\partial \gamma} = \frac{e^{2(\tau-t^*)\kappa^B}-1}{\kappa^B\left(e^{2(\tau-t^*)\kappa^B}+1\right) + \left(\frac{\gamma}{\sigma_D^2}+\zeta\right)\left(e^{2(\tau-t^*)\kappa^B}-1\right)} > 0.$$

Given that $\gamma_+ > \gamma_0$ and $\gamma_0 > \gamma_-$, it follows that $g_L(\gamma_+) - g_L(\gamma_0) > 0$ and $g_N(\gamma_0) - g_N(\gamma_-) > 0$. Therefore, the total variance in Case II (non-learning first) is higher than that in Case I (learning first). □

4. Proof of Proposition 4

Proof. Begin by writing the utility of investors A in terms of the solution in equation (14):

$$E^A \int_t^\tau e^{-\rho(u-t)} \frac{1}{\alpha}(c_u^A)^\alpha \, du = \int_t^\tau e^{-\rho(u-t)} \frac{1}{\alpha} E^A\left[(\omega(\eta_u)D_u)^\alpha\right] du.$$

The expectation term in the integrand can be written as

$$E^A_{\hat{f}_t^A, g_t}\left[(\omega(\eta_u)D_u)^\alpha\right] = E^A_{\hat{f}_t^A, g_t}\left[\left(\frac{\left(\frac{1}{\lambda^A}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{1}{\lambda^A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\eta_u}{\lambda^B}\right)^{\frac{1}{1-\alpha}}}\right)^\alpha D_u^\alpha\right]$$

$$= E^A_{\hat{f}_t^A, g_t}\left[\left(1 + \left(\frac{\eta_u \lambda^A}{\lambda^B}\right)^{\frac{1}{1-\alpha}}\right)^{-\alpha} D_u^\alpha\right].$$

By focusing on positive integer values of the risk aversion $1-\alpha$, so that $-\alpha \in \mathbb{Z}$, the binomial formula can be used to expand the first term of the expectation as

$$
\begin{aligned}
E^A_{\hat{f}^A_t, g_t}\left[\left(1+\left(\frac{\eta_u \lambda^A}{\lambda^B}\right)^{\frac{1}{1-\alpha}}\right)^{-\alpha} D^\alpha_u\right] &= E^A_{\hat{f}^A_t, g_t}\left[D^\alpha_u \sum_{j=0}^{-\alpha} \frac{(-\alpha)!}{j!(-\alpha-j)!}\left(\frac{\eta_u \lambda^A}{\lambda^B}\right)^{\frac{j}{1-\alpha}}\right] \\
&= \sum_{j=0}^{-\alpha} \frac{(-\alpha)!}{j!(-\alpha-j)!}\left(\frac{\lambda^A}{\lambda^B}\right)^{\frac{j}{1-\alpha}} E^A_{\hat{f}^A_t, g_t}\big[D^\alpha_u \left(\eta_u\right)^{\frac{j}{1-\alpha}}\big] \\
&= \sum_{j=0}^{-\alpha} \frac{(-\alpha)!}{j!(-\alpha-j)!}\left(\frac{\lambda^A}{\lambda^B}\right)^{\frac{j}{1-\alpha}} D^\alpha_t \left(\eta_t\right)^{\frac{j}{1-\alpha}} H_f(\hat{f}^A, t, u; \alpha) \times H_g\left(g, t, u; \alpha, \frac{j}{1-\alpha}\right).
\end{aligned}
$$

Therefore, the gain function can be expressed as the expectation when $\phi > 0$ minus the expectation when $\phi = 0$, which can be written as

$$
\begin{aligned}
G^A_t(Y_t, t) = \sum_{j=0}^{-\alpha} \frac{(-\alpha)!}{j!(-\alpha-j)!}\left(\frac{\lambda^A}{\lambda^B}\right)^{\frac{j}{1-\alpha}} D^\alpha_t \left(\eta_t\right)^{\frac{j}{1-\alpha}} \Bigg[& H^{\phi_t \neq 0}_f(\hat{f}^A, t, u; \alpha) \times H^{\phi_t \neq 0}_g\left(g, t, u; \alpha, \frac{j}{1-\alpha}\right) \\
& - H^{\phi_t = 0}_f(\hat{f}^A, t, u; \alpha) \times H^{\phi_t = 0}_g\left(g, t, u; \alpha, \frac{j}{1-\alpha}\right)\Bigg].
\end{aligned}
$$

(a) From the solution of $H_f(\cdot)$ in equation (14), it follows that

$$
\frac{\partial H_f(\cdot)}{\partial \phi} = \frac{\sigma^2_f \epsilon^2 \phi}{2\zeta^3} H_f(\cdot)\left[2\zeta(u-t) + 4e^{-\zeta(u-t)} - e^{-2\zeta(u-t)} - 3\right] \geq 0.
$$

This is non-negative because the function in brackets is of the form $h(x) = 2x + 4e^{-x} - e^{-2x} - 3$, which is non-negative in the range $x \geq 0$ since $h(0) = 0$ and $h'(x) = 2e^{-2x}\left(1-e^x\right)^2 \geq 0$. The derivative with respect to t is negative:

$$
\begin{aligned}
\frac{\partial^2 H_f(\cdot)}{\partial \phi \partial t} = -\frac{\sigma^2_f \epsilon^2 \phi}{2\zeta^3} H_f(\cdot) \Bigg[& \zeta\big(2e^{-2\zeta(u-t)} - 4e^{-\zeta(u-t)} + 2\big) + \epsilon\big(2\zeta(u-t) + 4e^{-\zeta(u-t)} - e^{-2\zeta(u-t)} - 3\big) \\
& \times \Bigg(\bar{f} + (\hat{f}^A - \bar{f})e^{-\zeta(u-t)} + \frac{\epsilon(1-e^{-\zeta(u-t)})^2\left((\gamma^A)^2 + \sigma^2_D \sigma^2_f \phi^2\right)}{2\zeta^2 \sigma^2_D} \\
& + \frac{\gamma^A \epsilon(1-e^{-\zeta(u-t)})}{\zeta} + \frac{\sigma^2_D(\epsilon-1)}{2}\Bigg)\Bigg] < 0,
\end{aligned}
$$

where the first function in parentheses is of the form $h(x) = 2e^{-2x} - 4e^{-x} + 2$, which is non-negative in the range $x \geq 0$ since $h(0) = 3$, a minimum occurs at $x = \ln(4)$, and the function increases thereafter as $h'(x) = e^{-2x}\left(e^{x} - 4\right)^2 > 0$ for all $x > \ln(4)$.

(b) Recall that the function $H_g(\cdot)$ is of the form

$$H_g(g, t, u; \varepsilon, \chi) = \exp\left\{A_1(\chi; u-t) + \varepsilon^2 A_2(\chi; u-t) + \varepsilon g B(\chi; u-t) + g^2 C(\chi; u-t)\right\},$$

which is quadratic and decreasing in g^2, since $C(\chi; u-t) \leq 0$ (see the proof of Proposition 1). For small values of g, one can approximate the difference $H_g(\phi, \cdot) - H_g(0, \cdot)$ by the ratio $H_g(\phi, \cdot)/H_g(0, \cdot)$. Therefore, for the function to be decreasing in g^2, the sign of $C(\phi, \chi; u-t) - C(0, \chi; u-t)$ must be positive, which is the case since $\frac{\partial C(\chi; u-t)}{\partial \phi} > 0$ from

$$\frac{\partial C(\chi; u-t)}{\partial \phi} = -\frac{4c\sigma_f^2 \phi}{2\sqrt{b^2 - ac}} \frac{c\left(e^{4q(u-t)} - 4q(u-t)e^{2q(u-t)} - 1\right)}{\left(b\left(e^{2q(u-t)} - 1\right) + q\left(e^{2q(u-t)} + 1\right)\right)^2},$$

where the function $h(x) = e^{4x} - 4xe^{2x} - 1$ in the numerator is non-negative in the range $x \geq 0$ because $h(0) = 0$ and $h'(x) = 4e^{2x}\left(e^{2x} - 1 - 2x\right)^2 \geq 0$. $\square$

5. Proof of Proposition 5

Proof. The solution depends on the current state variable $\hat{f}_t$ in the following way:

$$E\left[(D_u)^{\epsilon} \middle| \hat{f}_t\right] = H(\hat{f}_t, u, t) = D_t^{\epsilon} H_f(\hat{f}_t, u, t).$$

In equilibrium it should satisfy

$$\mathcal{L}H(\hat{f}_t, u, t) + \frac{\partial H(\hat{f}_t, u, t)}{\partial t} = 0,$$

where $\mathcal{L}$ is the differential generator of $(D_t, \hat{f}_t)$. Writing this out in full gives

$$0 = \frac{\partial H}{\partial D} D_t \hat{f}_t - \frac{\partial H}{\partial \hat{f}_t}\zeta(\hat{f}_t - \bar{f}) + \frac{1}{2}\frac{\partial^2 H}{\partial D_t^2} D_t^2 \sigma_D^2 + \frac{1}{2}\frac{\partial^2 H}{\partial (\hat{f}_t)^2}\left(\frac{(\gamma)^2}{\sigma_D^2} + \phi^2 \sigma_f^2\right) + \frac{\partial^2 H}{\partial D_t \partial \hat{f}_t} D_t \gamma + \frac{\partial H}{\partial t}.$$

Replacing by $H(\cdot) = D_t^{\epsilon} H_f(\cdot)$ results in

$$0 = \epsilon \hat{f}_t - \frac{\partial H_f(\cdot)}{\partial \hat{f}_t} \frac{1}{H_f(\cdot)} \zeta(\hat{f}_t - \bar{f}) + \frac{1}{2}\epsilon(\epsilon - 1)\sigma_D^2 + \frac{1}{2}\frac{\partial^2 H_f(\cdot)}{\partial(\hat{f}_t)^2} \frac{1}{H_f(\cdot)} \left(\frac{(\gamma)^2}{\sigma_D^2} + \phi^2 \sigma_f^2 \right)$$
$$+ \frac{\partial H_f(\cdot)}{\partial \hat{f}_t} \frac{1}{H_f(\cdot)} \epsilon\gamma + \frac{\partial H_f(\cdot)}{\partial t} \frac{1}{H_f(\cdot)},$$

which is solved by the function

$$H_f(\hat{f}, u, t) = \exp\Bigg[\epsilon \left(\frac{(\hat{f}_t - \bar{f})(1 - e^{-\zeta(u-t)})}{\zeta} + \bar{f}(u - t) \right) + \frac{(\gamma\epsilon^2)\big(\zeta(u-t) - (1 - e^{-\zeta(u-t)})\big)}{\zeta^2}$$
$$+ \frac{\epsilon^2 \big(4e^{-\zeta(u-t)} - e^{-2\zeta(u-t)} + 2\zeta(u-t) - 3\big)\big(\gamma^2 + \sigma_D^2 \sigma_f^2 \phi^2\big)}{4\zeta^3 \sigma_D^2} + \frac{1}{2}\sigma_D^2 (u-t)(\epsilon - 1)\epsilon \Bigg].$$

□