Investment Functions with q in the Presence of Unobserved Persistent Shocks

Kyoo il Kim, Suyong Song, Jiawei (Brooke) Wang*

Abstract

We study the classical relationship between a firm's investment and q, for which an unobserved persistent shock is an important factor in the investment decision. In our setting, besides the potential measurement problem of q, controlling for the unobserved shock becomes a new challenge. We develop an estimation method that addresses both econometric issues given timing and information set assumptions. Using 16,256 unique public firms in the U.S. from 1975 to 2021, we find that q remains a significant factor of investment even after controlling for the unobserved shock and measurement error.

*Kyoo il Kim, kyookim@msu.edu, Michigan State University Department of Economics. Suyong Song, suyong-song@uiowa.edu, University of Iowa, Tippie College of Business, Department of Economics & Finance. Jiawei (Brooke) Wang, wangj249@miamioh.edu, Miami University Farmer School of Business, Department of Finance.

I. Introduction

The neoclassical theory of investment has a long history. It has been developed, tested, and refined across many decades since the seminal work of Jorgenson (1963) and Tobin (1969). The neoclassical theory suggests that the rate of investment is a function of Tobin's q, measured by the ratio of the market value of new additional investment goods to their replacement cost.[1] The foundation of modern q theory in Lucas and Prescott (1971) and Mussa (1977) is the firm's optimization condition—the marginal adjustment and direct purchasing costs of investment being equal to the shadow value of capital.[2] However, some recent studies challenge the empirical applicability of the q theory, citing difficulties in accurately measuring q, and propose alternative approaches that predict firms' investment decisions.

In this paper, we investigate the relationship between a firm's investment and q, for which an unobserved persistent shock to the investment cost function, such as information or technology shock, is an important factor in the firm's investment decision, derived from the firm's optimization problem. In our dynamic investment model to motivate the empirical investment equation, both the capital and the unobserved persistent shock are dynamic state variables; risk-neutral firms choose investment each period seeking to maximize the expected present value of their continuing future profits. For example, in this setting, firms experiencing a positive technology shock may face lower investment adjustment costs. Technological advances make equipment less expensive, make the investment process more efficient, and lead to improvements

[1] The intuition behind this theory even goes back to Keynes (1936).

[2] See also Hayashi (1982)'s work showing the relationship between marginal q and average q. The latter is the usual empirical measure of q as the ratio of the valuation of the firm's existing capital stock to its replacement cost, prone to measurement issues.

in the real investment opportunity set (Greenwood, Hercowitz, and Krusell (1997); Stiroh (2002); Fisher (2006); Kogan and Papanikolaou (2014)). As a result, the q-measure may become endogenous in the firm's investment equation if the unobserved shock is not properly accounted for. Importantly, we argue that incorporating the unobserved shock into the optimal investment function is essential, as this shock may directly impact capital adjustment costs, rather than solely influencing the firm's production function. Nonetheless, we demonstrate that this channel of dependency is not precluded by the existing classical investment theory.

The empirical concern we address here is a new challenge to the potential measurement problem of marginal q. It has been studied relatively well in the literature (e.g., Hayashi (1982); Blanchard, Rhee, and Summers (1993); Erickson and Whited (2000)), compared to the potential omitted variable problem we focus on. Unfortunately, controlling for measurement error in marginal q alone has been proven to be a difficult problem in the literature, as different empirical approaches taken to measurement errors rendered various and even contradictory conclusions on the roles of marginal q and internal funds in investment decisions. Addressing both the unobserved persistent shock and the measurement error problem is even more challenging. If the shock is omitted, not only does q become endogenous, but other observed regressors, such as cash flow or leverage, may also become endogenous. This underscores the critical importance of controlling for the unobserved persistent shock to estimate the investment function consistently.

To this end, we develop an econometric method that handles both issues. Our approach allows for time-varying investment adjustment costs and direct investment costs in firm-level panel data. Our identification strategy is based on a set of timing and information set assumptions about changes in the unobserved shock and adjustment costs. Given these restrictions, we derive moment conditions under which we identify both investment function parameters and dynamic parameters of the unobserved shock, and propose a Generalized Method of Moments (GMM) estimator. Our approach is robust to the endogeneity concerns in estimating the investment functions where q is correlated with the unobserved persistent shock and subject to measurement error.

Methodologically, we utilize a panel data approach, building on a similar method proposed by Blundell and Bond (1998, 2000) and Bajari, Fruehwirth, Kim, and Timmins (2012). Our estimation approach also generalizes differencing approaches used to control for correlated time-varying confounders. However, our context and problems are substantially different from those of existing studies. This is because, in the context of investment functions, not only is q mismeasured, but also the unobserved persistent shock is potentially correlated with other factors of investment, such as q. In standard dynamic panel models, endogeneity arises because differencing to remove a firm fixed effect induces correlation between the lagged dependent variable as a regressor and the differenced error term. In our investment equation, this endogeneity is present regardless of a firm fixed effect.

To motivate our insights on the unobserved persistent shock, we begin by incorporating the firm's estimated total factor productivity (TFP) as an additional regressor and estimate the augmented investment equation. For this purpose, we utilize the TFP measure from İmrohoroğlu and Tüzel (2014) who estimate firm-level production functions using Olley and Pakes (1996).[3] In particular, we estimate the investment equation both with and without TFP included as an

[3] We constructed a figure of TFP for four industries from the Fama–French 12 classification: Manufacturing, Energy, Business Equipment, and Shops (Figure 1). The figure shows industry trends in TFP from 1975 to 2021, with Energy having the highest and most volatile productivity, peaking around 2010. In contrast, Business Equipment shows steady long-term growth, while Manufacturing and Shops remain relatively flat at lower TFP levels.

additional observed state variable to assess whether our approach can effectively account for TFP when it is unobserved. We first proceed with a GMM estimator by including TFP as an additional observed state variable in place of the unobserved persistent shock. The results show that both q and TFP are statistically significant. We next estimate the investment equation using our proposed method to account for the unobserved persistent shock. It suggests that the unobserved persistent shock is significant. Importantly, q continues to be a significant factor even after the unobserved persistent shock is being controlled for. Lastly, to examine whether TFP contains information beyond that captured by the unobserved persistent shock, we include both TFP and the persistent shock in the investment function. Interestingly, once the unobserved shock is accounted for, TFP is no longer statistically significant. This suggests that our proposed method effectively captures the influence of the unobserved persistent shock, such as cost or technology shock, on investment decisions.[4]

We also examine the investment equation both with and without controlling for measurement error in q. When the measurement error is ignored, the estimated coefficient on q is substantially smaller than when the error is properly accounted for. This highlights the importance and empirical relevance of addressing both the omitted persistent shock and measurement error.

[4]One might suggest that including TFP as an additional control variable in an OLS regression provides a viable way to address endogeneity concerns. However, this approach has limitations, as it does not fully account for other important issues—particularly the broader nature of the unobserved persistent shock, besides other empirical issues. First, as shown in the summary statistics in Table 1, obtaining the TFP measure by using the estimation of firm-level production functions significantly reduces the number of observations from 149,429 to 107,183 due to data availability. The sample loss amounts to approximately 28%. Second, the OLS regression does not address the important issue of mismeasured q. Thus, this regression requires an additional step to control for the measurement error in q. In contrast, our estimator does not suffer from either of these limitations.

We also consider a case where the measurement error follows a more persistent process than the one assumed in our benchmark model. The estimation results remain very similar under this generalization, suggesting that our baseline specification performs effectively in the empirical setting.

We then conduct extensive empirical analyses, utilizing various measures of investment and q (for both physical and intangible measures) used in recent literature, for example, Peters and Taylor (2017). Using 16,256 unique firms from 1975 to 2021, our empirical results indicate the importance of controlling for the unobserved persistent shock in estimating investment functions. Across all investment equations considered, we find that the unobserved persistent shock plays a significant role. Importantly, our finding indicates that q is still a significant factor in investment decisions, even after controlling for the unobserved persistent shock and measurement error in q.

We contribute to the literature on the empirics of corporate investment in several significant dimensions. First, to motivate our specification of the investment equation, we allow a firm's adjustment cost of capital stock to depend on its unobserved persistent shock. We demonstrate that the optimal investment model is not only determined by q and other state variables but also by the unobserved persistent shock.

Second, we develop an estimation strategy for investment functions accounting for both endogeneity concerns due to the unobserved persistent shock and possibly mismeasured q. Our identifying moment conditions are derived from timing and information set assumptions that align with the firm's optimal decision-making process and are well grounded in the principle of rational expectations. Moreover, our estimator is straightforward to implement using standard computing software. We offer a set of diagnostic tests for the moment conditions.

Third, our empirical analysis confirms that q remains an important factor of investment even when other state variables such as firm size, employment, and cash flow (or leverage) are controlled for. Furthermore, we find that investment becomes more sensitive to q after accounting for the unobserved persistent shock and the measurement error problems. This result holds for our sub-period analysis, alternative definitions of investment and q, and a variety of robustness checks.

The rest of the paper proceeds as follows. Section II presents an investment model extending the models in Lucas and Prescott (1971) and Mussa (1977), in which the unobserved persistent shock factors into a firm's investment. Section III develops estimation methods. Section IV describes the data and variable construction. Section V reports the estimation results, and Section VI concludes.

II. Investment Model

To develop an empirical framework for an endogenous q model of investment in which both capital and the unobserved persistent shock are dynamic state variables, we present a simple standard dynamic investment model where risk-neutral firms choose investments each period to maximize the expected present value of continuing future profits. We use this simple dynamic investment model to motivate estimable equations and to discuss the nature of endogeneity problems in our empirical framework.

A. *q* Theory of Optimal Investment with Unobserved Shocks

We build on the original setting of Lucas and Prescott (1971) and Mussa (1977) but, as an important point of departure, we allow for the unobserved persistent shock to enter the investment cost function. Here we modify the dynamic investment model in Erickson and Whited (2000), in which capital is the endogenous quasi-fixed factor and the unobserved persistent shock is another fixed factor that evolves exogenously following a dynamic process (e.g. a first-order Markov process). The value of firm i at time t, from which the firm derives its optimal decision on investment to maximize the expected present value of the discounted flow of future profits, is given by

$$(1) \quad V_{it} = E\left[\sum_{j=0}^{\infty}\left(\prod_{s=1}^{j} b_{i,t+s}\right)[\Pi_{t+j}(K_{i,t+j},\varsigma_{i,t+j}) - \psi(I_{i,t+j},K_{i,t+j},W_{i,t+j},\nu_{i,t+j})|\Omega_{it}\right].$$

Here, $E[\cdot|\Omega_{it}]$ is the conditional expectation operator and Ω_{it} denotes the information set available to firm *i* at time *t*; K_{it} is the capital stock available at the beginning of period t; I_{it} is the investment and b_{it} is the firm's discount factor at time t; $\Pi_t(K_{it},\varsigma_{it})$ is the per period profit function, increasing in K_{it}, with ς_{it} being the shock to the profit function; $\psi(I_{it},K_{it},W_{it},\nu_{it})$ is the investment cost function including both the cost of adjusting the stock of capital and the direct purchase or sale cost of investment, where ν_{it} is an exogenous shock to adjustment cost. W_{it} denotes the vector of state variables other than capitals, which may include technology shock, demand and cost shocks, and other aggregate shocks.

The cost function $\psi(I_{it},K_{it},W_{it},\nu_{it})$ is increasing in I_{it}, decreasing in K_{it} and convex in the first two arguments. The shocks $(\varsigma_{it},\nu_{it})$ and state variables $W_{i,t}$ are observed by the firm at time t but these shocks and some components of W_{it} may not be fully observed by the econometrician. Finally, note that any other variable factors of production in the profit function (e.g. labor or materials) have been already optimized following static optimization problems by the firm. Also, for ease of notation, other observed factors are implicit and suppressed. For estimation, we will decompose W_{it} into observed factors and the unobserved persistent shock. We will add these additional factors in our empirical investment equation specifications later.

The firm maximizes the expected present value of the future profits V_{it}, subject to the capital stock accumulation identity

$$K_{i,t+1} = (1 - d_i)K_{it} + I_{it}, \tag{2}$$

where d_i is the firm *i*'s capital depreciation. We then obtain the "marginal" q_{it} from $\frac{\partial V_{it}}{\partial K_{it}}$, which measures the benefit of adding an incremental unit of capital to the firm. The first-order condition for maximizing the value of the firm in Equation (1) subject to Equation (2) then yields

$$\frac{\partial \psi(\cdot)}{\partial I_{it}} \equiv \psi_I(I_{it}, K_{it}, W_{it}, \nu_{it}) = q_{it}. \tag{3}$$

In the original setting of Lucas and Prescott (1971) and Mussa (1977) (see also Erickson and Whited (2000)), which is common in the literature, a firm's unobserved persistent shock does not enter the investment cost function. Our main innovation is to incorporate an additional source of unobserved firm heterogeneity into the firm's investment decision problem. We develop an empirical investment equation that aligns with the theoretical model and propose a consistent estimation procedure that accounts for this unobserved factor. Importantly, our framework allows the unobserved persistent shock to influence the optimal investment decision not only through the production function but also by affecting the cost of investment. We argue that this feature remains consistent with the neoclassical theory of investment. The first-order condition above highlights that incorporating the unobserved shock into the investment cost function $\psi(\cdot)$ is essential for the dependence of the optimal investment on the unobserved shock given the marginal q_{it}. This is because the direct impact of this shock on the firm's profit function $\Phi_t(\cdot)$ through its production function is already subsumed in q_{it} and the unobserved shock only shows up in the optimal investment through $\psi(\cdot)$ as in Equation (3).

B. Empirical Model of Investment Equation

To develop an empirical framework, we now present an investment equation consistent with the firm's optimal investment decision problem in Equation (1). Write $W_{it} \equiv (Z_{it}, \omega_{it})$ where Z_{it} represents the observed state variables, which may proxy for firm heterogeneity and demand factors, and ω_{it} denotes the unobserved persistent shock. We consider a class of investment cost functions, including the cost of adjusting the stock of capital, as

$$\psi(I_{it}, K_{it}, W_{it}, \nu_{it}) = K_{it}\left[\tilde{f}_0(Z_{it}, \nu_{it}, \omega_{it}) + \tilde{f}_1(Z_{it}, \nu_{it}, \omega_{it})\frac{I_{it}}{K_{it}} + \frac{\gamma_{it}}{2}\left(\frac{I_{it}}{K_{it}}\right)^2\right], \tag{4}$$

where, in particular, $\tilde{f}_1(Z_{it}, \nu_{it}, \omega_{it})$ denotes the linear adjustment cost.

From Equation (4), it is clear that the feature of the model that renders the investment function to depend on ω_{it} is specifically due to the linear adjustment cost $\tilde{f}_1(Z_{it}, \nu_{it}, \omega_{it})$, which is a function of the shock ω_{it}, not merely due to the investment cost function $\psi(I_{it}, K_{it}, W_{it}, \nu_{it})$ depending on ω_{it}. For example, if we have $\tilde{f}_1(Z_{it}, \nu_{it}, \omega_{it}) = \tilde{f}_1(Z_{it}, \nu_{it})$, the cost function still depends on ω_{it} because of $\tilde{f}_0(Z_{it}, \nu_{it}, \omega_{it})$, but this additive adjustment cost does not enter the investment equation as we can see below. In the literature, it is also typically assumed that the adjustment cost parameter γ_{it} is constant across firms as γ (but it may vary by the time t). Combining these, we obtain

(5) $$\frac{\partial \psi(\cdot)}{\partial I_{it}} \equiv \psi_I(I_{it}, K_{it}, W_{it}, \nu_{it}) = \tilde{f}_1(Z_{it}, \nu_{it}, \omega_{it}) + \gamma \frac{I_{it}}{K_{it}} = q_{it}.$$

This equation clearly indicates that q_{it} is dependent on the unobserved persistent shock ω_{it}, unless the linear adjustment cost $\tilde{f}_1(Z_{it}, \nu_{it}, \omega_{it})$ is independent of ω_{it}. Finally, the above equation can be rewritten, as in the literature, yielding the investment equation for which now both q_{it} and ω_{it} enter as factors of investment:

$$y_{it} = \beta q_{it} - f_1(Z_{it}, \nu_{it}, \omega_{it}),$$

where $y_{it} = \frac{I_{it}}{K_{it}}$, $\beta = 1/\gamma$, and $f_1(Z_{it}, \nu_{it}, \omega_{it}) = \tilde{f}_1(Z_{it}, \nu_{it}, \omega_{it})/\gamma$. To develop a simple regression equation in line with the literature, we can let (e.g.) $\tilde{f}_1(Z_{it}, \nu_{it}, \omega_{it}) = -Z_{it}\tilde{\theta} - \tilde{\alpha}\omega_{it} + \nu_{it}$.[5]

We then obtain the familiar regression equation as an extension of Erickson and Whited (2000) (Equation (6) below becomes their equation (6) if we set $\theta = \alpha = 0$):

(6) $$y_{it} = \beta q_{it} + Z_{it}\theta + \alpha\omega_{it} + u_{it},$$

where $\theta = \tilde{\theta}/\gamma$, $\alpha = \tilde{\alpha}/\gamma$, and $u_{it} = -\nu_{it}/\gamma$. An important implication of this investment equation is that, if omitted in the regression, the unobserved persistent shock ω_{it} is a potential

[5]We abstract away from whether the adjustment cost is decreasing or increasing in the unobserved persistent shock ω_{it}, our main point is that ω_{it} is an omitted factor of a firm's investment decision, and our identifying restrictions and the estimation procedure do not rely on any sign condition.

source of endogeneity. It can be correlated with q_{it}, while u_{it} is the usual exogenous shock. Note that, for ease of notation, other observed factors in both the profit and cost functions are included in Z_{it}. These variables can be added to the empirical investment equation and may not create additional endogeneity problems once the omitted shock ω_{it} is controlled for.

C. Interpretation of the Persistent Shocks

In this subsection, we set out our interpretation of the persistent shock ω in the context of the firm investment, adjustment costs, and Tobin's q literature. Investment adjustment costs are central to dynamic models of capital accumulation, as they determine the speed and efficiency with which firms respond to changes in economic conditions. Hayashi (1982) formalized the link between q and investment under convex adjustment costs, showing that marginal q governs optimal investment decisions in the presence of installation frictions. These costs arise because capital goods cannot be instantaneously installed without incurring inefficiencies, such as production disruptions or resource misallocation.

In dynamic investment models, unobserved persistent shocks, such as technological advancements or improvements in information efficiency, play a critical role in shaping firms' investment behavior by influencing adjustment costs (Greenwood et al. (1997); Stiroh (2002); Fisher (2006); Kogan and Papanikolaou (2014)). These shocks affect the marginal cost of capital adjustment, thereby altering optimal investment trajectories and the speed of capital accumulation. Enhanced information efficiency, for instance, reduces informational frictions and uncertainty, enabling firms to make more accurate and timely investment decisions. This improvement mitigates costs associated with misallocation, delays, and errors, ultimately fostering a more efficient allocation of resources and, in turn, firm productivity.

Similarly, positive technology shocks can lower the costs and time required to upgrade capital equipment or adopt new production technologies. For example, the diffusion of cloud computing and automation technologies has enabled firms to scale operations rapidly without incurring the high fixed costs traditionally associated with IT infrastructure upgrades. In manufacturing, the integration of advanced robotics has streamlined production processes, reducing downtime and adjustment costs during technology transitions. In the renewable energy sector, technological improvements in battery storage and solar panel efficiency have accelerated investment cycles, making capital upgrades less costly and more frequent.

Our interpretation of q aligns with the existing literature on measurement error in q. As shown in Equation (5), q is on the right-hand side of the first-order condition and, in theory, it perfectly measures the marginal benefit of adding an incremental unit of capital to the firm. However, in practice, since it is unobserved and replaced with the average q, it is subject to measurement error and may not fully reflect, for example, a firm's productivity variation stemming from intangibles, information asymmetries, or market inefficiencies.

Regarding how to handle this measurement issue in q, we depart from the existing literature by introducing unobserved persistent shocks ω that affect adjustment costs on the left-hand side and can also capture (e.g.,) firm productivity variation if it is not fully reflected in q. In our model, the unobserved persistent shocks effectively streamline the investment process and improve overall productivity. We incorporate them in the structural models for the firm's optimization problem, but we test the theory based on the reduced-form model in Equation (5). As shown below, the empirical findings confirm that the unobserved persistent shocks are statistically significant and economically important in corporate investment decisions. The results stay robust even for the investment model with the total q that includes both physical and intangible capital. Failure to account for these unobserved persistent shocks may lead to biased estimates of investment dynamics and misinformed policy prescriptions. Incorporating such factors into investment models is therefore essential for accurately capturing the interplay between technological progress, firm behavior, and economic outcomes.

III. Estimation Strategy

The endogeneity of q_{it} due to the unobserved persistent shock ω_{it} is another important potential confounder in the regression of the investment function, in addition to the well-noted problem of the measurement issue of the marginal q. The mismeasurement of q relevant to the neoclassical theory of optimal investment can arise from several sources (see Hayashi (1982) and Erickson and Whited (2000)). Marginal q is not usually equal to average q in realistic market settings, as originally noted by Hayashi (1982), unless constant returns to scale and perfect competition conditions are all satisfied. Another source of measurement error is the divergence of average q from marginal q due to inefficiencies in financial markets, as discussed by Blanchard et al. (1993). Besides these issues, there remain several other empirical challenges to correctly measuring q.

Here we develop an estimation strategy that can handle both concerns of endogeneity in estimating the investment function (6), where q_{it} is measured with error and is potentially correlated with ω_{it}. Our purpose is two-fold. First, we test whether the unobserved ω_{it} is a relevant factor of investment (in addition to the usual suspects, such as cash flow or leverage, firm size, etc., as considered in the literature). Second, we develop an estimation of the investment function, which is robust to the potential measurement error in q.

Our estimation strategy is based on a set of timing and information set assumptions about changes in the unobserved persistent shock and adjustment cost. Given these assumptions, we derive moment conditions under which we identify both investment function parameters and a dynamic parameter of the persistent shock ω_{it}. Our approach to tackling both concerns of endogeneity in estimating the investment function (6) is robust, whether q is correlated with the unobserved ω_{it} and is measured with error.

We adopt a panel data approach, extending the methods proposed by Blundell and Bond (1998, 2000) in production functions and Bajari et al. (2012) in hedonic models. Our estimation approach is similar in spirit to these generalized differencing approaches used for controlling for correlated time-varying confounders. An important difference is that unobserved ω_{it} in the investment function context is potentially correlated with other factors of investment, such as q, and this q itself is also mismeasured.

A. Modeling the Unobserved Persistent Shock

We consider the empirical investment equation that generalizes Equation (6) as

$$y_{it} = \alpha_i + \beta q_{it} + Z_{it}\theta + \omega_{it} + u_{it}, \tag{7}$$

where y_{it} is the investment ratio and α_i is the firm fixed effect. Compared to the investment equation (6), without loss of generality, we normalize the coefficient on ω_{it} to be one because this persistent shock is an unobserved factor.

The true q may not be directly observable and can only be measured with error as $q_{it} = q^*_{it} + e_{it}$, where q^*_{it} and e_{it} denote the true q and possible measurement error, respectively. The vector of state variables, Z_{it}, includes other potential observable factors of investment, such as cash flow (or leverage) and firm size, which proxy for firm heterogeneity and demand shocks. These variables can be incorporated into the investment equation, and their inclusion in the estimation does not introduce additional endogeneity problems once the unobserved ω_{it} is controlled for. However, if ω_{it} is omitted, these additional observed factors, including cash flow, can become endogenous regressors as well. This highlights the importance of controlling for the unobserved persistent shock in the investment equation to consistently estimate coefficients of both q and other observed factors.

The investment equation contains two unobserved shocks, ω_{it} and u_{it}. Motivated by Equation (5), here we allow q_{it} to be correlated with ω_{it}. Both q_{it} and Z_{it} are not correlated with the exogenous shock u_{it}. Following a standard setting in the literature to deal with the persistent error (Olley and Pakes, 1996; Blundell and Bond, 2000), we assume ω_{it} follows a Markov process, such as a simple autoregressive process of order one (AR(1)).[6]

Assumption III.1 *Let ω_{it} be an unobserved persistent shock, a factor of the investment cost in Equation (1). We assume that*

$$\omega_{it} = \rho\omega_{i,t-1} + \xi_{it}, \tag{8}$$

where ξ_{it} denotes the innovation term in the process, and the dynamic parameter ρ satisfies $|\rho| < 1$.

[6]Extending this to more general specifications of the autoregressive process is possible with additional notation.

The investment equation can be estimated with or without the firm fixed effect α_i. We primarily focus on the case with the fixed effect in our approach; the estimation without the fixed effect can proceed without the first-order differences to remove the fixed effect, as below. For the empirical implementation of the estimator, in Section V, we provide more details on the model with and/or without the firm fixed effect.

Estimation without measurement error

We first consider the model where q_{it} is measured without error and we set $q_{it} = q^*_{it}$ where q^*_{it} denotes the true q. Applying generalized differencing to Equation (7), we obtain

$$y_{it} = (1-\rho)\alpha_i + \rho y_{i,t-1} + \beta(q_{it} - \rho q_{i,t-1}) + (Z_{it} - \rho Z_{i,t-1})\theta + u_{i,t} - \rho u_{i,t-1} + \xi_{it}.$$

By taking the first-order differences to remove the firm fixed effect, we then obtain

$$\Delta y_{it} = \rho \Delta y_{i,t-1} + \beta(\Delta q_{it} - \rho \Delta q_{i,t-1}) + (\Delta Z_{it} - \rho \Delta Z_{i,t-1})\theta + \Delta u_{it} - \rho \Delta u_{i,t-1} + \Delta \xi_{it}.$$

We now utilize a set of timing and information set assumptions as our identifying conditions. We make the following assumptions.

Assumption III.2 *Let u_{it} be an idiosyncratic shock in the investment equation (7) and ξ_{it} be the innovation term in the persistent shock process (8). Let Z_{it} denote other observed factors of investment, which are conditionally mean independent of the innovation ξ_{it}. The shocks satisfy*

$$E[u_{it}|J_{it}] = 0, E[u_{it'}|Z_{it}] = 0$$

for all t and t' and

$$E[\xi_{it}|J_{i,t-1}, Z_{it}] = 0, or\ E[\Delta\xi_{it}|J_{i,t-2}, \Delta Z_{it}, \Delta Z_{i,t-1}] = 0$$

where J_{it} denotes the information available to the firm i at a point in time t when the firm makes the investment decision.

Note that, by construction, J_{it} includes all current observables at the time of the investment decision and their lags. For example, J_{it} may include q_{it}, Z_{it}, and $y_{i,t-1}$ (and their respective lags). However, for estimation, the valid instruments may consist of only a subset of J_{it} or may include additional available variables, depending on the moment conditions, as detailed in our data-driven IV selection.

Assumption III.2 states that (i) u_{it}, the exogenous shock to adjustment cost, is not systematically over- or under-predicted, given the information available at time *t*, and this shock is also strictly exogenous with respect to Z_{it}. It also imposes that (ii) the innovation of the persistent shock process is uncorrelated with any information available at time $t - 1$ or other observed factors Z_{it}; this is reasonable since ω_{it} follows an exogenous Markov process. Note that this assumption allows other observed factors Z_{it} to be correlated with ω_{it} but not with the innovation term ξ_{it}.

In the dynamic panel literature, Assumption III.2 is often referred to as $J_{i,t-1}$ including "predetermined" variables. Our identifying conditions are also motivated by rational expectations in the sense that, given available information, a firm does not over- or under-invest on average. In other words, from the available information, we cannot predict systematic over- or under-investment by firms. Ackerberg (2023) further provides details on how these assumptions can be strengthened or relaxed. An important point he elaborates is that what matters is not only the timing of when firms choose the “predetermined” variables but also what they know at that time. In this sense, these restrictions are referred to as the timing and information set assumptions, not only timing assumptions.

These assumptions allow that q_{it} is potentially endogenous, even being free of measurement error. Under Assumptions III.1 and III.2, we then obtain the moment condition

$$E[\Delta u_{it} - \rho\Delta u_{i,t-1} + \Delta\xi_{it}|J_{i,t-2}, \Delta Z_{it}, \Delta Z_{i,t-1}] = 0,$$

from which we obtain the moment condition for GMM estimation

$$E[\Delta y_{it} - \{\rho\Delta y_{i,t-1} + \beta(\Delta q_{it} - \rho\Delta q_{i,t-1}) + (\Delta Z_{it} - \rho\Delta Z_{i,t-1})\theta\}|J_{i,t-2}, \Delta Z_{it}, \Delta Z_{i,t-1}] = 0. \tag{9}$$

Estimation with measurement error

Next we consider the measurement error in $q_{it} = q^*_{it} + e_{it}$. The regression equation derived from Equation (7) becomes

$$\begin{aligned}\Delta y_{it} = \rho\Delta y_{i,t-1} + \beta(\Delta q_{it} - \rho\Delta q_{i,t-1}) + (\Delta Z_{it} - \rho\Delta Z_{i,t-1})\theta \\ +\Delta u_{it} - \rho\Delta u_{i,t-1} + \Delta\xi_{it} - \beta(\Delta e_{it} - \rho\Delta e_{i,t-1}).\end{aligned}$$

We further assume that the measurement error is not persistent in the sense that it is not correlated with lagged information $J_{i,t-1}$, and is also not correlated with other observable factors of the investment Z_{it}, as follows:

Assumption III.3 *Let $q_{it} = q^*_{it} + e_{it}$, where q^*_{it} denotes the true q and e_{it} denotes its measurement error. The measurement error satisfies for all t and t',*

$$E[e_{it}|J_{i,t-1}] = 0 \quad \textit{and} \quad E[e_{it'}|Z_{it}] = 0.$$

This assumption about the measurement error commonly appears in the literature, which rules out q being systematically mismeasured. This is a reasonable condition since the market's perception of the firm's true q is continuously updated by rationally incorporating information available at the market up to the current date. The assumption of the measurement error being uncorrelated with the first-order lagged information, $J_{i,t-1}$, is plausible, given the annual frequency of the data that is empirically used to estimate the investment equation. For instance, if the measurement error follows a moving average process of order one (MA(1)), the assumption is satisfied. It follows from this assumption that

$$E[\Delta e_{it}|J_{i,t-2}, Z_{it}] = 0 \quad \text{and} \quad E[\Delta e_{i,t-1}|J_{i,t-3}, Z_{it}] = 0.$$

By combining these conditional moment conditions, we then obtain

$$E[\Delta e_{it} - \rho\Delta e_{i,t-1}|J_{i,t-3}, \Delta Z_{it}, \Delta Z_{i,t-1}] = 0.$$

From this result, it is clear that, given Assumption (III.3), the moment condition (9) can be made robust to the measurement error of q_{it} by using the following moment condition:[7]

$$(10)\quad E[\Delta y_{it} - \{\rho\Delta y_{i,t-1} + \beta(\Delta q_{it} - \rho\Delta q_{i,t-1}) + (\Delta Z_{it} - \rho\Delta Z_{i,t-1})\theta\} | J_{i,t-3}, \Delta Z_{it}, \Delta Z_{i,t-1}] = 0.$$

It is worth mentioning that most studies assume the classical measurement error in q and do not allow for a persistent measurement error. Nevertheless, Assumption III.3 can be modified to allow for a more persistent measurement error; this would require changing the conditioning variables in the moment condition. In our empirical applications (Section 5), we examine this scenario using a more persistent process and find that the measurement error process outlined in Assumption III.3 aligns more appropriately with the empirical settings.

With the use of additional notation, we can also allow the coefficients β and γ to vary by time or period. In Section 5, we provide further details on how to choose instruments to implement GMM estimation, based on these timing and information set assumptions.

B. Implementation of Estimation

We discuss here how to implement the GMM estimation for the investment equation (7), with measurement error in the measured $q_{it} = q^*_{it} + e_{it}$, where q^*_{it} denotes the true q. We consider two cases: the model without and with the firm fixed effect, respectively:

[7]A simple modification of Equation (9) reveals that this moment condition (10) is robust to an alternative timing and information set assumption that u_{it} is only conditionally mean independent with the lagged information $J_{i,t-1}$, such as for all t and t'

$$E[u_{it}|J_{i,t-1}] = 0 \quad \text{and} \quad E[u_{it'}|Z_{it}] = 0.$$

$$\iota_{it} = \alpha + \beta q_{it} + Z_{it}\theta + \omega_{it} + u_{it} - \beta e_{it},$$

and

$$\iota_{it} = \alpha_i + \beta q_{it} + Z_{it}\theta + \omega_{it} + u_{it} - \beta e_{it},$$

where we now use the notation ι_{it}, instead of y_{it}, to denote various investment measures in our analyses.[8] Here α_i denotes the firm fixed effect; ω_{it} denotes the unobserved persistent shock; u_{it} is an exogenous shock to the adjustment cost; e_{it} is the measurement error in q^*_{it}.

The model without the firm fixed effect, after applying the generalized differencing due to the AR(1) process of the persistent shock (8), yields

$$\begin{aligned}\iota_{it} = \alpha(1-\rho) + \rho\iota_{i,t-1} + \beta(q_{it} - \rho q_{i,t-1}) + (Z_{it} - \rho Z_{i,t-1})\theta \\ +u_{it} - \rho u_{i,t-1} + \xi_{it} - \beta(e_{it} - \rho e_{i,t-1}).\end{aligned}$$

We note that the variables $\{Z_{it}, Z_{i,t-1}\}$ satisfy the moment condition and serve as instruments for themselves, while the variables $\{\iota_{i,t-1}, q_{it}, q_{i,t-1}\}$ do not. Therefore, we can use the following set of further lagged variables as excluded instrumental variables (IVs), because they are not correlated with the error terms $[u_{it} - \rho u_{i,t-1} + \xi_{it} - \beta(e_{it} - \rho e_{i,t-1})]$:

$$\left\{\begin{array}{c} \iota_{i,t-2}, \iota_{i,t-3}, \iota_{i,t-4}, \cdots \\ q_{i,t-2}, q_{i,t-3}, q_{i,t-4}, \cdots \\ Z_{i,t-2}, Z_{i,t-3}, Z_{i,t-4}, \ldots \end{array}\right\}$$

[8] In addition to standard investment rates in our main analysis, we examine physical, intangible, and total investment.

We justify these IVs based on the assumptions about timing and information set, as discussed in the previous section, and we adopt data-driven criteria to select IVs among this set of variables, as we detail in Subsection C below.

Define the vector of parameters $\vartheta \equiv (\alpha, \beta, \theta', \rho)'$. Let H_{it} be a $K \times 1$ vector that stacks the IVs we select, given firm *i* and time *t*. Let $\widehat{W}$ be a $K \times K$ weighting matrix satisfying the $\widehat{W} \rightarrow_p W$ condition, with a symmetric positive definite matrix *W*. The GMM estimator of ϑ is defined as

$$\hat{\vartheta} \equiv \text{argmin}_{\theta}\, g_n(\vartheta)'\widehat{W} g_n(\vartheta), \tag{11}$$

with

$$g_n(\vartheta) \equiv \frac{1}{n} \sum_{i=1}^{n} \sum_{t=t_0}^{T_i} H_{it} \cdot (\iota_{it} - \alpha(1-\rho) - \rho\iota_{i,t-1} - \beta(q_{it} - \rho q_{i,t-1}) - (Z_{it} - \rho Z_{i,t-1})\theta)\,,$$

where t_0 is determined by the availability of lagged variables in the instruments H_{it}, depending on the choice of lags in the instruments. Under standard regularity conditions for GMM, the estimator achieves consistency and asymptotic normality:

$$\sqrt{n}(\hat{\vartheta} - \vartheta) \rightarrow_d N(0, V_{\vartheta}),$$

with the asymptotic variance-covariance matrix, V_{ϑ}.

Similarly, for the model with the firm fixed effect, after applying the generalized differencing to the first-differenced equation, we obtain

$$\Delta\iota_{it} = \rho\Delta\iota_{i,t-1} + \beta(\Delta q_{it} - \rho\Delta q_{i,t-1}) + (\Delta Z_{it} - \rho\Delta Z_{i,t-1})\theta$$
$$+\Delta u_{it} - \rho\Delta u_{i,t-1} + \Delta\xi_{it} - \beta(\Delta e_{it} - \rho\Delta e_{i,t-1}).$$

In this case, the variables $\{\Delta Z_{it}, \Delta Z_{i,t-1}\}$ satisfy the moment condition, while the variables $\{\Delta\iota_{i,t-1}, \Delta q_{it}, \Delta q_{i,t-1}\}$ do not. Then the following set of further lagged variables can be used as excluded IVs, because they are orthogonal to the error terms $[\Delta u_{it} - \rho\Delta u_{i,t-1} + \Delta\xi_{it} - \beta(\Delta e_{it} - \rho\Delta e_{i,t-1})]$:

$$\left\{ \begin{array}{c} \iota_{i,t-3}, \iota_{i,t-4}, \iota_{i,t-5}, \ldots \\ q_{i,t-3}, q_{i,t-4}, q_{i,t-5}, \ldots \\ Z_{i,t-3}, Z_{i,t-4}, Z_{i,t-5}, \ldots \end{array} \right\} \tag{12}$$

We note that the set of instruments consisting of the differenced version of the IVs can be also used in place of the IVs above:

$$\left\{ \begin{array}{c} \Delta\iota_{i,t-3}, \Delta\iota_{i,t-4}, \Delta\iota_{i,t-5}, \ldots \\ \Delta q_{i,t-3}, \Delta q_{i,t-4}, \Delta q_{i,t-5}, \ldots \\ \Delta Z_{i,t-3}, \Delta Z_{i,t-4}, \Delta Z_{i,t-5}, \ldots \end{array} \right\} \tag{13}$$

Define the vector of parameters $\tilde{\vartheta} \equiv (\beta, \theta', \rho)'$. Then, the GMM estimator of $\tilde{\vartheta}$ is defined as in equation (11):

$$\hat{\tilde{\vartheta}} \equiv \text{argmin}_{\tilde{\vartheta}}\, \tilde{g}_n(\tilde{\vartheta})'\widehat{W}\tilde{g}_n(\tilde{\vartheta}), \tag{14}$$

with

$$\tilde{g}_n(\tilde{\vartheta}) \equiv \frac{1}{n}\sum_{i=1}^{n}\sum_{t=t_1}^{T_i} \widetilde{H}_{it} \cdot \left(\Delta\iota_{it} - \rho\Delta\iota_{i,t-1} - \beta(\Delta q_{it} - \rho\Delta q_{i,t-1}) - (\Delta Z_{it} - \rho\Delta Z_{i,t-1})\theta\right).$$

$\widetilde{H}_{it}$ denotes the vector that stacks the instruments equation (12) or equation (13), and t_1 is determined by the availability of lagged variables in the instruments $\widetilde{H}_{it}$, depending on the choice of lags in the instruments. We discuss our criteria for selecting instruments and provide some practical guidelines in Subsection C.

In practice, the proposed estimator is easy to implement in standard computing software. For illustrative purposes, we utilize the Stata command, `gmm`, to implement the proposed estimator in the empirical estimation. We use the option, robust, for the weighting matrix and cluster the standard errors of the parameter estimates at the firm level.

C. Selection of Instrumental Variables

We adopt data-driven criteria to select IVs that satisfy legitimate instrumental variable conditions. Since the number of IVs can be more than the number of endogenous variables as long as the moment condition is satisfied, in principle, the model can be over-identified. So, our first criterion is Hansen's J-test for over-identification. To ensure consistency of the estimator and its desirable finite sample performance, IVs should not suffer from weak instrument problems. The second criterion is strong instrument tests. In particular, we consider Sanderson and Windmeijer's F-tests for weak identification and under-identification since multiple endogenous variables exist at the moment condition for the investment equation. Third, we select the specification that minimizes the residual of the GMM objective function (11) or (14) as small as possible. This guarantees that the estimates are the global minimizers of the optimization problem. Lastly, we conduct AR(1) and AR(2) diagnostic tests on the regression residuals obtained from our estimation (using e.g., Stata's `arima` command). These tests examine whether the residuals exhibit first- or second-order serial correlation. The absence of significant higher-order autocorrelation provides additional support that our moment conditions are not misspecified and that the GMM framework is appropriately designed. These practical criteria guarantee that the selected instruments are valid and relevant for the moment conditions.

IV. Data and Construction of Variables

In this section, we describe the construction of the sample and the main variables. We construct the key variables of interest, following Peters and Taylor (2017), Erickson and Whited (2000), Hadlock and Pierce (2010), and Gala, Gomes, and Liu (2020).

Our sample ranges from 1975 to 2021. The sample contains all Compustat North American firms, except for utility firms (SIC codes 4900-4999), financial firms (6000-6999), and firms identified as public service, international affairs, or non-operating establishments (9000+). Using the standard procedure from the literature, we only include firms with non-missing or non-negative book values of assets or sales and firms with at least $5 million in physical capital. The sample has 16,256 unique firms and 149,429 observations. We winsorize all regression variables at the 1% level to reduce the impact of outliers.

In the following, we describe our construction of the variables—investment, q, cash flow, firm size, employment-to-capital ratio, leverage, and sales—for our analysis. Standard investment is defined as capital expenditures (Compustat item *capex*) scaled by the replacement cost of physical capital (Compustat item *ppegt*). q is constructed as the firm's market value scaled by the replacement of physical capital. The market value of a firm is defined as the market value of outstanding equity (Compustat items *prcc_c* times *csho*), plus the book value of debt (Compustat items *dltt+dlc*), minus the firm's current assets (Compustat item *act*) which include cash, marketable securities, and inventory. Cash flow is the sum of income before extraordinary items (*ib*) and depreciation expense (*dp*) scaled by the replacement cost of physical capital. Firm size is the natural logarithm of physical capital stock, and employment-to-capital ratio is the natural logarithm of the number of employees scaled by the physical capital stock. We define leverage as the sum of long-term and short-term debt scaled by total assets, and net leverage as the total debt minus cash and short-term investments, scaled by total assets. We construct sales as sales normalized by physical capital. Table 1 reports summary statistics of the key variables. Detailed definitions of the firm's physical, intangible, and total investment rates are provided in Appendix F.

V. Empirical Results

This section outlines our approach to estimating the empirical investment function and presents the results. Our primary objective is to investigate the well-established relationship between a firm's investment and q, accounting for firm heterogeneity and the unobserved persistent shock. Specifically, we focus on standard investment, defined as capital expenditure scaled by physical capital, and later extend the analysis to other types of investment, including total, physical, and intangible investments.

A. Motivating Preliminary Analyses

We first estimate the investment equation with the higher-order polynomial OLS model, where a non-linear function of the state variables, such as cash flow (CF), firm size (lnK), and employment (lnN_K), is considered.[9] The results are reported in Appendix A of the online appendix. We start by estimating a simple model with polynomial approximation in the state variables and confirm that the state variables in place of q explain the investment. However, we find that q remains significant in the estimation equations even after controlling for higher-order polynomials of these variables and firm fixed effects. We then add the estimated TFP to a linear investment equation and find that TFP is statistically significant at the 1% level. Estimating a more flexible model with higher-order terms does not alter the result. These estimation results motivate us to treat q and the unobserved persistent shock as the main factors of the investment function. Since the coefficients of the higher-order terms have small magnitudes and little economic significance, our main analyses primarily focus on the linear model with a firm fixed effect, while sub-period analysis and non-linear models serve as robustness checks.

We next formally implement our GMM estimation approach to account for measurement error in q and the potential unobserved persistent shock to investment policy. The results are presented in Table 2. We estimate our model both with and without TFP included as an additional observed state variable to assess whether our approach can effectively account for TFP when it is unobserved.

As reported in Column (1), we estimate the GMM regression with TFP, q, cash flow, size,

[9]Gala et al. (2020) argue that observed state variables explain corporate investment better than q and propose a flexible polynomial regression approach to avoid model misspecification. Song and Wee (2025) find evidence of heterogeneity in the investment-q relation, using nonlinear investment equations.

employment, and a firm fixed effect in the investment function, while we do not account for the unobserved shock ω. We find that both q and TFP are statistically significant for this specification. The coefficient of q is 0.009 and significant at the 1% level, and the coefficient of TFP is 0.016 and also significant at the 1% level. In Column (2), we account for the unobserved shock ω using our approach instead of the TFP proxy. However, we estimate the model using firms for which TFP is available to facilitate comparison. The result shows that q remains significant at a 1% level with a magnitude of 0.016, which is improved from 0.009 in Column (1) when we include TFP as an additional state variable.

Next, in Column (3), we include TFP as a control variable and simultaneously account for ω in the GMM estimation. Here, interestingly, TFP becomes statistically insignificant, while the AR(1) parameter of ω remains significant at the 1% level. The coefficient of q is statistically significant at the 1% level with a magnitude of 0.025. Notably, TFP is redundant in this model specification. This indicates that accounting for the persistent unobserved shock using our approach can effectively subsume this TFP shock, which is not fully captured by the observed q, even if it is omitted.

Overall, the results in Table 2 highlight the advantages of accounting for the persistent shock in estimating the investment equation, thereby supporting the efficacy of our proposed approach.[10]

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[10]We recognize that introducing the TFP proxy reduces the sample size, which might induce sample selection bias. To examine the possibility of bias, we re-estimate the model using the full sample and remove the requirement of TFP availability that is imposed in the estimation of Column (2) in Table 2, which is in fact the same as our main specification reported in Column (4) of Table 3. The test result for the coefficient difference confirms that the estimated q coefficients are qualitatively similar. The difference between the estimated q coefficients in Table 2

B. Main Results

We now consider the investment equations for the full sample period from 1975 to 2021, for which we include q along with other state variables in the investment function and account for the unobserved persistent shock using our proposed approach. The estimation results are reported in Table 3 Panel A.

We first estimate the OLS regression with q, cash flow, size, and employment in Column (1). The results show that when the OLS does not account for the persistent shock and the measurement error in q, the coefficient of q is significant at the 1% level, but its magnitude is as small as 0.006, suggesting a downward bias. In Columns (2)-(6), we implement our proposed approach in different model specifications to examine the robustness of the results. Columns (2) and (4) are based on the specifications of the measurement error as in Assumption III.3 (e.g., MA(1)). Column (5) assumes no measurement error in q and Column (6) assumes more persistent measurement error in q (e.g., MA(2)). We use the criteria for choosing IVs discussed in the previous section and only report the estimation results that satisfy the selection criteria to avoid redundant tables. Throughout the section, the employed IVs are reported at the top of each table. The residual (e(Q)) represents the difference between the actual observed values of the dependent variable and those predicted by the model. In the context of GMM regression, a low residual means that the predicted values of investment generated by the GMM estimates are close to their observed values. The P-value of Hansen's J-test (P value of Hansen J) is reported to check the validity of the IVs. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak

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Column (2) with the TFP availability restriction and Table 3 Column (4) with the full sample is statistically insignificant (t-statistic = 0.23), which ensures that the q coefficient is not affected by the restricted sample of TFP.

identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. In addition, we check diagnostic tests of the autoregressive process on the estimation residuals and report them in Appendix E of the online appendix. These test results confirm that the selected IVs are valid and strong.[11]

We start with the MA(1) measurement error specification in Columns (2)-(4). Column (2) includes q, cash flow, and size and employment as other state variables. Column (3) eliminates size and employment and only includes q, cash flow, and the unobserved persistent shock ω. First, the result in Column (2) confirms that the state variables are significant in explaining corporate investment. Furthermore, we find both q and the dynamic parameter (AR(1)) of ω are statistically significant at the 1% level. When we include the unobserved persistent shock in Column (3), it is highly significant at a 1% level, suggesting that it is a significant factor of the investment function. In Column (4), we include the state variables in addition to the unobserved persistent shock. We can infer that the unobserved persistent shock is still a significant factor of the investment function, and q remains statistically and economically significant after controlling for cash flow, size, and employment. We also observe an increased magnitude of the coefficient of q from 0.006 in Column (1) the OLS estimation to 0.017 in Column (4) the GMM estimation. This discrepancy arises from both attenuation bias and omitted variable bias inherent in OLS estimation. The magnitude of the coefficient of q converted to dollar value is \$1.72 million increase in capital expenditures with a one-unit increase in q for the median firms in the sample. For the top and

[11]We perform a couple of robustness checks and report them in Appendix C of the online appendix. First, we check robustness to the timing of ω and report the results in Table C1. The findings confirm that our identification strategy is robust to modest shifts in the assumed timing of ω. Second, we estimate the specification using IVs with 3-4 lags and the results remain very similar to the main specification (Table C2).

bottom quartile firms in the sample, the values are \$9.67 million and \$0.44 million.[12] Overall, the evidence highlights the advantage of our empirical approach, which accounts for both the measurement error in q and the unobserved persistent shock to investment.

Next, to gauge the empirical relevance of the measurement error in q and the unobserved persistent shock in our estimation separately, we examine the specification that does not account for either the measurement error or the persistent shock. The results are reported in Columns (4)-(5). First, under the specification without measurement error in Column (5), the persistent shock is statistically significant, and the coefficient of q is significantly larger than the one in Column (1). This implies that the OLS estimator is downward-biased if the persistent shock is omitted. Second, comparing the results in Column (4) under the MA(1) specification with those in Column (5) without measurement error, we find that measurement error in q attenuates its estimated coefficient. After accounting for measurement error, the coefficient of q in Column (4) is significantly larger than that in Column (5). Specifically, the difference in the coefficients of q is statistically significant, with a t-statistic of 2.57.

We also investigate the case where the measurement error of q is more persistent, such as an MA(2) process, and report the results in Column (6). In this case, we modify the conditioning variables in the moment condition from $J_{i,t-1}$ to $J_{i,t-2}$. Under the MA(2) specification, the coefficients on q and the persistent shock remain very close to those under the MA(1) specification in Column (4), and the differences are not statistically significant, indicating that our results are robust to allowing for more persistent measurement error. Specifically, the t-statistics

[12]The median of the sample firms' *ppegt* is \$101.46 million. The top quartile and bottom quartile of *ppegt* are \$568.89 million and \$25.69 million, respectively. We also compare our estimates with those from Erickson and Whited (2000) and Peters and Taylor (2017) in Appendix D of the online appendix.

of the coefficient differences are 0.219 for q and 0.979 for ω, respectively. Thus, the more parsimonious model in Column (4) can be selected. As a result, the estimation confirms that the investment equation, which includes the persistent shock ω and measurement error (Column (4)), is the most preferred model specification. In particular, these results highlight the importance and empirical relevance of addressing both the omitted persistent shock and measurement error.

Model Comparison

We compare the coefficients of q across different specifications in Table 3 Panel B. For each model, we report the magnitudes and standard errors of the coefficients, as well as the *t*-statistics of the pairwise differences. First, comparing the OLS and GMM specifications with MA(1) measurement error (Columns (1) and (2)), the difference in the magnitudes of q is not significant (*t*-statistic = 1.35). In contrast, the difference between OLS and the GMM specification with both MA(1) measurement error and the unobserved shock ω is highly significant (*t*-statistic = 8.60). The coefficient of q improves from 0.006 in the OLS to 0.017 in GMM with both MA(1) measurement error and ω.

Next, comparing the GMM specifications that include MA(1) measurement error but differ in whether ω is controlled for (Columns (2) and (4)), the coefficient on q increases substantially, nearly doubling from 0.009 to 0.017, and the difference is statistically significant (*t*-statistic = 3.14). Finally, comparing the specifications of GMM with ω alone with the GMM model with both MA(1) measurement error and ω (Columns (4) and (5)), we find that the coefficient of q improves from 0.013 to 0.017. The difference in q is statistically significant with a *t*-statistic of 2.57. Overall, our preferred model, the GMM specification incorporating both MA(1) measurement error and ω, yields a q coefficient that is significantly different from those obtained in the alternative specifications and is the largest in magnitude.

Taken together, the findings suggest that although measurement error must be addressed, controlling for unobserved persistent shocks is even more pivotal, suggesting that a comprehensive approach that tackles both sources of endogeneity is essential; addressing measurement error alone does not seem sufficient to fully resolve the endogeneity of the observed q, thereby strengthening the motivation for the proposed approach.

Cash flow sensitivity

In the OLS estimation reported in Column (1) of Table 3, cash flow appears significant. However, once ω and measurement error in q are controlled for in the GMM estimation in Column (4), the cash flow effect becomes insignificant. Whether cash flow should be included in the investment equation has long been debated. Prior studies (e.g., Erickson and Whited (2000); Gomes (2001); Kaplan and Zingales (1997)) emphasize that the empirical significance of cash flow often reflects measurement error in q or omitted variable bias. Erickson and Whited (2000) examine investment–cash flow sensitivity after purging for measurement error in q and find that the estimated cash flow coefficients are small and statistically insignificant, even for financially constrained firms. This evidence calls into question the interpretation of cash flow effects as direct evidence of financing frictions. Similarly, Gomes (2001) argues that cash flow can appear predictive in investment equations because of correlations with underlying technology shocks, even in the absence of financial frictions. Moreover, Kaplan and Zingales (1997) show that less financially constrained firms can exhibit greater investment–cash flow sensitivity than more constrained firms, suggesting that cash flow often proxies for unobserved investment opportunities not captured by Tobin's q.

Our results indicate that after explicitly controlling for the unobserved persistent shock ω and measurement error in q, the coefficient of cash flow becomes insignificant. This suggests that the significance of the coefficient on cash flow observed in Column (1) likely reflects the correlation between cash flow and unobserved factors. In Table B1 of Appendix B, we further split the sample into financially constrained and financially unconstrained firms, using size, the WW index by Whited and Wu (2006), and the KZ index by Kaplan and Zingales (1997). The results show that the coefficient of cash flow remains insignificant for both financially constrained and unconstrained firms, further supporting our interpretation.

C. Estimation with Leverage

Existing studies, such as Whited (1992) and Hennessy, Levy, and Whited (2007), suggest that financial liabilities should be a state variable for the optimal investment policy. In this section, we use leverage as a relevant state variable. We employ two variables: leverage (the sum of long-term and short-term debt, scaled by total assets) and net leverage (the total debt netting cash and short-term investments, scaled by total assets). We estimate both GMM and OLS models by including either leverage or net leverage in place of cash flow. The results are presented in Panels A and B of Table 4, respectively.

In Panel A, Columns (1) and (2), we find that q is statistically significant in the GMM estimations, while leverage and net leverage are negatively associated with optimal investment. In Columns (3) and (4), we include the sales-to-capital ratio, but the results remain robust. The negative coefficients on leverage and net leverage are consistent with the findings in Gala et al. (2020) that higher financial liabilities constrain investment. Panel B presents the OLS results. Comparing the magnitude of the coefficients on q, the GMM estimates (ranging from 0.037 to 0.040 in Panel A) are substantially larger than the OLS estimates (ranging from 0.005 to 0.006 in Panel B), indicating that controlling for endogeneity strengthens the sensitivity of investment to q.

D. Sub-period Analysis

To examine possible structural change due to the financial crisis in the investment equation over time, we split the sample into two sub-periods (1975-2009 and 2010-2021) and estimate the equations for each sub-period. The OLS and GMM estimation results are reported in Table 5. As before, we estimate investment equations with a firm fixed effect. The OLS results show that the coefficients of q are statistically significant in both sub-periods, with the magnitude of the coefficient being smaller for the sub-period of 2010-2021. The magnitude of q for the estimation before 2010 is 0.007 and decreases to 0.004 for the estimation period of 2010 to 2021. The GMM results in Columns (3) and (4) show that q and ω are significant in both sub-periods. The coefficient on ω increases from 0.122 (for the period before 2010) to 0.238 (after 2010). Thus, accounting for the unobserved persistent shock has become increasingly important in the estimation of investment equations in recent periods. We can also infer that q has exhibited increasing importance since 2010, in contrast to the results from the OLS. It is worth noting that in both sub-periods, the magnitudes of the coefficient of q in the GMM estimation are larger than those of the OLS estimation. It is consistent with the main results that the investment-q sensitivity becomes larger after controlling for measurement error and the unobserved persistent shock using our approach.

E. Estimation of Other Investment Types

The results presented so far are for standard investment, defined as capital expenditures scaled by physical capital. In this section, we examine the robustness of our GMM estimation approach to different types of investment, i.e., total, physical, and intangible investments. We follow Peters and Taylor (2017) to construct these variables.[13] The results are reported in Table 6. Panels A and B present GMM and OLS estimation results, respectively. In the GMM estimations in Columns (1) to (3) for total, physical, and intangible investment, respectively, we find that total q is statistically significant at the 1% level across different types of investment, after controlling for the state variables of cash flow, size, and employment. In fact, the total q includes both physical and intangible capital. The unobserved persistent shock ω remains a significant factor in the investment-q estimation. This reinforces our argument that ω may capture other unobserved investment factors, such as technology or information efficiency shocks, and is still important to explain investment even after controlling for observed intangible measures proposed in the existing literature.

Moreover, comparing with the OLS estimates, we continue to observe the increased magnitudes of q in the GMM estimates, removing the downward bias of the OLS estimations. Specifically, in the GMM estimations, the magnitudes of the coefficient of q are 0.091, 0.060, and 0.034 for total, physical, and intangible investments, respectively. On the contrary, in the OLS

[13]Definitions of these investments are provided in Appendix F of the online appendix. We also adopt total q and total cash flow when we estimate the GMM model for total, physical, and intangible investments.

regressions, the magnitudes of the coefficient of q are 0.028, 0.015, and 0.011, respectively. Therefore, our proposed GMM approach corrects the downward bias present in OLS estimation. From these results, we confirm that the results from our approach are robust across various types of investment.

F. Non-linear Model Estimations

In this subsection, we examine the possibility that investment may respond non-linearly to state variables. To explore this, we perform non-linear regression analyses to evaluate the effectiveness of our proposed estimation approach for non-linear models. We include higher-order terms such as lnK^2, lnK^3, and lnN_K^2. The estimation results are reported in Table 7. In Column (1), we incorporate both lnK^2 and lnN_K^2 into the regression. On the one hand, the coefficient of lnK^2 is significant at the 1% level, although the magnitude of 0.005 is relatively small. On the other hand, the coefficient of lnN_K^2 is not statistically significant. We find that ω and q remain statistically significant at the 1% level. In Column (2), we further check the non-linearity of lnK by including its squared and cubic terms in the regression. While the coefficients of lnK^2 and lnK^3 have small magnitudes of -0.004 and 0.001, respectively, they are statistically significant at the 1% level. Regardless of the functional form of the investment equation, ω and q are still statistically significant and economically meaningful. The coefficients of ω and q are comparable to those in Table 3.

Therefore, even after accounting for the non-linearity of investment in state variables, the dynamic coefficient of the persistent shock remains statistically significant. Overall, the empirical evidence supports the importance of controlling for the unobserved persistent shock and measurement error in q when estimating investment functions.

VI. Conclusion

We extend the classical theory of optimal investment and contend that an unobserved persistent shock is a relevant factor in a firm's investment decisions, arising from the firm's optimization problem. The key condition for this result is that the persistent shock affects both the profit function and the investment cost function. We demonstrate that this framework is not only consistent with the neoclassical q theory of investment but also empirically relevant.

Given our theoretical framework, the presence of the unobserved persistent shock in the investment equation can be regarded as an omitted variable problem, which hence creates another source of endogeneity. To resolve the empirical challenges of the unobserved persistent shock and the potential measurement problem of marginal q, we propose a panel GMM estimation approach, grounded in a set of timing and information set assumptions. Our identifying conditions are based on rational expectations, where firms, given the available information, do not over- or under-invest on average.

We show that the persistent shock in the empirical application to *Compustat* firms is significant in all specifications of the investment equations we consider. We also examine the investment equations with and without accounting for measurement error in q. Ignoring this error leads to a substantially smaller estimated coefficient, underscoring the importance of addressing both measurement error and the omitted persistent shock. Our results remain robust across various alternative definitions of investment, q, cash flow, and through different sub-period analyses.

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FIGURE 1

TFP by Industry

This figure shows TFP by industry for manufacturing, energy, business equipment, and shops. The industries are classified using Fama-French 12 industry classification. In this figure, we use the raw TFP data, while the TFP data in the regression below is log-transformed.

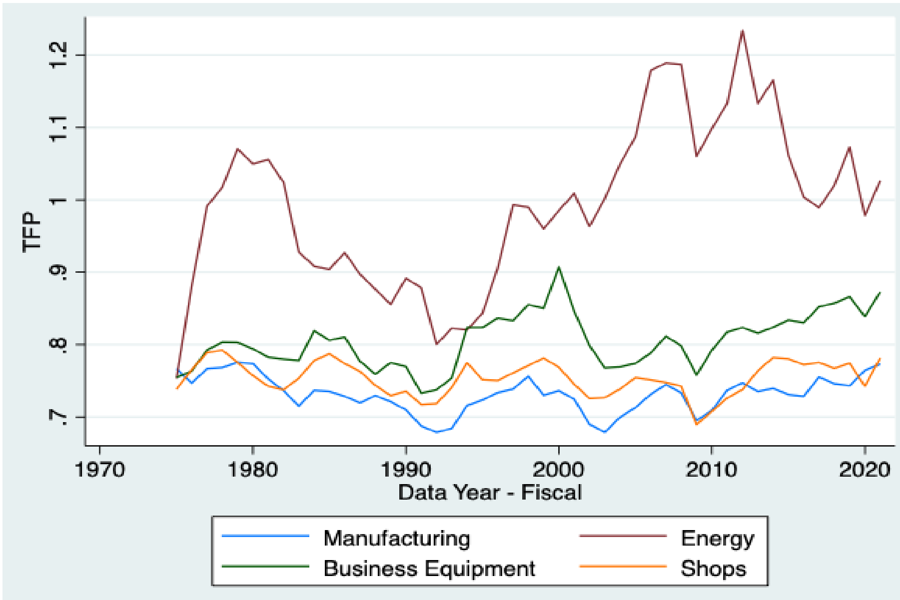

TABLE 1

Summary Statistics

This table presents the summary statistics of the main variables. All variables are winsorized at the 1% and 99% levels to reduce the impact of outliers. We report the summary statistics of TFP and other key variables in the sample from 1975 to 2021, with 16,256 unique firms. The TFP data (natural logarithm transformed) is obtained from İmrohoroğlu and Tüzel (2014). The summary statistics indicate that TFP data is only available for about two-thirds of the sample firms.

Variable	Firm-Year sample	Mean	STD	P25	Median	P75
Inv	149,429	0.17	0.20	0.06	0.11	0.19
q	149,429	3.54	7.78	0.35	1.02	3.05
CF	149,429	0.14	0.68	0.05	0.15	0.31
Inv_total	149,429	0.21	0.19	0.10	0.16	0.25
Inv_physical	149,429	0.09	0.12	0.02	0.05	0.10
Inv_intangible	149,429	0.12	0.12	0.04	0.09	0.16
q_total	149,429	1.04	1.61	0.20	0.57	1.20
CF_total	149,429	0.16	0.19	0.08	0.15	0.23
lnK (size)	149,429	4.97	2.09	3.28	4.67	6.40
lnN_K (employment)	149,429	-4.50	1.55	-5.18	-4.25	-3.47
Leverage	149,429	0.25	0.22	0.07	0.22	0.37
Net Leverage	149,423	0.11	0.32	-0.09	0.13	0.31
lnY_K (sales)	131,875	0.79	1.10	0.21	0.90	1.48
TFP	107,183	-0.33	0.45	-0.53	-0.31	-0.10

TABLE 2

Preliminary GMM Regression Results Using TFP Proxy

This table presents the results of GMM estimations with and without TFP. We estimate the investment equation under our identifying assumptions by implementing the `gmm` command in Stata. In Column (1), we include TFP as an exogenous variable and estimate the GMM model without the AR(1) process for the unobserved shock ω. The IVs are d.l3.q d.l4.q. TFP is measured at time t as ω. In Column (2), to facilitate comparison, we estimate the GMM model with ω and require TFP to be available. The IVs are l3.inv l4.inv l4.q l5.CF. In Column (3), in addition to ω, TFP is also included in the regression as an exogenous variable. The IVs are l4.CF l4.inv l4.q l5.q. Firm fixed effects are included. Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. t-statistics are reported in parentheses.

VARIABLES	(1)	(2)	(3)
q	0.009***	0.016***	0.025***
	(3.15)	(14.29)	(16.02)
CF	0.006**	0.003	-0.001
	(2.12)	(1.44)	(-0.24)
lnK (size)	-0.123***	-0.128***	-0.157***
	(-26.56)	(-22.63)	(-17.55)
lnN_K (employment)	0.066***	0.048***	0.048***
	(13.58)	(11.21)	(10.16)
ω		0.160***	0.435***
		(4.55)	(10.79)
TFP	0.016***		0.004
	(5.78)		(1.37)
Firm-Year Sample	107,183	107,183	107,183
e(Q)	0.00000	0.00294	0.00279
P value of Hansen J	0.78	0.24	0.55
F stats of inv(t-1)		160.58	33.55
F stats of q (t-1)	27.72	40.30	38.18
F stats of q (t-2)		69.65	67.78
SWP of inv(t-1)		0.00	0.00
SWP of q (t-1)	0.00	0.00	0.00
SWP of q (t-2)		0.00	0.00
SWκ^2P of inv(t-1)		0.00	0.00
SWκ^2P of q (t-1)	0.00	0.00	0.00
SWκ^2P of q (t-2)		0.00	0.00
Firm FE	Y	Y	Y

TABLE 3

Main Results with Full Sample Using the Proposed Approach

This table presents the OLS and GMM estimation results from 1975 to 2021. For Panel A, Column (1) presents the OLS results. In Column (2), we estimate the GMM model with size and employment in addition to q, cash flow, and MA (1) measurement error, but without ω. The IVs are d.l3.q and d.l4.q. Column (3) presents the GMM results with q, cash flow, the AR (1) process for ω and MA(1) measurement error. The IVs are l4.CF l4.q l5.q l3.inv. In Column (4), we estimate the GMM model with all the factors. The IVs are l3.inv l4.inv l5.CF l4.q. In Column (5), we estimate the GMM model without measurement error. The IVs are l3.inv l4.inv l3.q l3.CF. In Column (6), we estimate with MA(2) measurement error and the IVs are l3.inv l4.inv l5.q l5.CF. Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. t-statistics are reported in parentheses. Panel B summarizes the estimated q coefficients from Panel A Columns (1) OLS, (2) GMM-MA(1), and (4) GMM-MA(1) and ω, (5) GMM with ω only, respectively.

Panel A: Regression results

	OLS	GMM				
	(1)	(2)	(3)	(4)	(5)	(6)
q	0.006***	0.009***	0.024***	0.017***	0.013***	0.016***
	(27.72)	(4.14)	(13.79)	(13.26)	(13.76)	(13.59)
CF	0.010***	0.006**	0.006*	0.003	0.003*	0.002
	(5.91)	(2.57)	(1.89)	(1.21)	(1.72)	(0.83)
lnK (size)	-0.185***	-0.136***		-0.141***	-0.148***	-0.136***
	(-54.84)	(-27.64)		(-26.66)	(-31.05)	(-26.57)
lnN_K (employment)	0.053***	0.046***		0.032***	0.033***	0.035***
	(15.05)	(10.24)		(7.49)	(8.03)	(8.29)
ω			0.503***	0.205***	0.159***	0.161***
			(22.98)	(6.48)	(6.55)	(5.03)
Measurement error	No	MA(1)	MA(1)	MA(1)	No	MA(2)
Firm-Year Sample	149,429	149,429	149,429	149,429	149,429	149,429
R-squared	0.209					
e(Q)		0.00000	0.00009	0.00228	0.00332	0.00209
P value of Hansen J		0.61	0.59	0.99	0.81	0.79
F stats of inv(t-1)			676.25	219.14	275.37	222.53
F stats of q (t-1)		42.96	46.22	42.52	75.07	36.11
F stats of q (t-2)			144.88	91.98	157.39	79.48
SWP of inv(t-1)			0.00	0.01	0.00	0.01
SWP of q (t-1)		0.00	0.03	0.01	0.01	0.00
SWP of q (t-2)			0.03	0.01	0.01	0.00
SWκ^2P of inv(t-1)			0.00	0.01	0.00	0.01
SWκ^2P of q (t-1)		0.00	0.03	0.01	0.01	0.00
SWκ^2P of q (t-2)			0.03	0.01	0.01	0.00

Panel B: Coefficient comparison

	(1) OLS	(2) GMM-MA(1)	(4) GMM-MA(1) and ω	(5) GMM-ω
Coefficient of q	0.006	0.009	0.017	0.013
Standard error of q	0.00022	0.00221	0.00126	0.00091
t-statistics of the difference of q				
OLS vs. GMM-MA(1)	1.35			
GMM MA(1) vs. GMM MA(1) and ω	3.14			
OLS vs. GMM MA(1) and ω	8.60			
GMM-ω vs. GMM-MA(1) and ω	2.57			

TABLE 4

Estimation with Leverage

This table presents the results of GMM and OLS estimations from 1975 to 2021. We include leverage or net leverage and sales in the regressions. For Panel A, Column (1) presents the regression results with leverage. The IVs are l4.lnK l3.inv l4.*q* l5.lnN_K. In Column (2), we estimate the GMM model with net leverage. The IVs are d.l4.net leverge l3.inv l4.*q* l5.lnN_K. In columns (3) to (4), we estimate the GMM model with leverage or net leverage and sales. The IVs are the same for Columns (3) and (4), l4.lnK l4.d.*q* l5.lnY_K (sales) l3.inv. Firm fixed effects are included. Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. *t*-statistics are reported in parentheses. Panel B reports the corresponding results in each columns.

Panel A: GMM Estimation with Leverage

VARIABLES	(1)	(2)	(3)	(4)
q	0.040***	0.039***	0.038***	0.037***
	(15.07)	(13.31)	(16.06)	(15.98)
Leverage	-0.102***		-0.109***	
	(-9.68)		(-12.87)	
Net leverage		-0.084***		-0.097***
		(-9.16)		(-13.48)
lnK (size)	-0.119***	-0.112***	-0.092***	-0.088***
	(-16.58)	(-14.64)	(-17.32)	(-17.12)
lnN_K (employment)	0.012**	0.018***	0.014***	0.020***
	(2.18)	(3.24)	(2.85)	(4.16)
lnY_K (sales)			0.007**	0.007**
			(2.22)	(2.52)
ω	0.468***	0.462***	0.128***	0.111**
	(12.96)	(12.81)	(2.66)	(2.39)
Firm-Year Sample	149,429	149,423	131,875	131,869
e(Q)	0.00328	0.00341	0.00201	0.00193
P value of Hansen J	0.84	0.47	0.23	0.19
F stats of inv(t-1)	347.29	336.54	216.71	213.04
F stats of q (t-1)	127.52	119.64	72.79	71.96
F stats of q (t-2)	205.29	195.42	87.67	84.99
SWP of inv(t-1)	0.00	0.00	0.00	0.00
SWP of q (t-1)	0.00	0.00	0.00	0.00
SWP of q (t-2)	0.00	0.00	0.00	0.00
SWκ^2P of inv(t-1)	0.00	0.00	0.00	0.00
SWκ^2P of q (t-1)	0.00	0.00	0.00	0.00
SWκ^2P of q (t-2)	0.00	0.00	0.00	0.00
Firm FE	Y	Y	Y	Y

Panel B: OLS Estimation with Leverage

VARIABLES	(1)	(2)	(3)	(4)
q	0.006***	0.006***	0.005***	0.005***
	(27.96)	(27.05)	(22.51)	(21.87)
Leverage	-0.140***		-0.118***	
	(-22.79)		(-20.06)	
Net leverage		-0.151***		-0.130***
		(-31.46)		(-27.74)
lnK (size)	-0.178***	-0.165***	-0.169***	-0.158***
	(-53.32)	(-49.38)	(-48.56)	(-45.39)
lnN_K (employment)	0.052***	0.056***	0.042***	0.046***
	(15.02)	(16.18)	(11.17)	(12.22)
lnY_K (sales)			0.027***	0.028***
			(13.51)	(13.88)
Firm-Year Sample	149,429	149,423	131,875	131,869
R-squared	0.217	0.227	0.197	0.206
Firm FE	Y	Y	Y	Y

TABLE 5

Sub-period Analyses

This table presents the OLS and GMM estimation results for the two sub-periods, i.e., 1975 to 2009 and 2010 to 2021. Columns (1) and (2) present the OLS regression results. Columns (3) and (4) estimate the GMM model. The instrumental variables in Column (3) are l4.CF l5.CF l4.q, and l3.inv. The instrumental variables in Column (4) are d.l4.CF d.l5.CF l4.q, and l3.inv. Firm fixed effects are included. Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. t-statistics are reported in parentheses.

	OLS		GMM	
VARIABLES	(1)	(2)	(3)	(4)
	<2010	2010-2021	<2010	2010-2021
q	0.007***	0.004***	0.013***	0.017***
	(24.94)	(13.27)	(12.82)	(6.81)
CF	0.014***	0.002	0.010***	-0.005
	(6.42)	(0.71)	(3.50)	(-1.51)
lnK (size)	-0.195***	-0.158***	-0.146***	-0.133***
	(-47.58)	(-27.26)	(-23.19)	(-14.44)
lnN_K (employment)	0.062***	0.036***	0.044***	0.020***
	(13.55)	(7.52)	(8.40)	(3.61)
ω			0.122***	0.238***
			(3.25)	(4.30)
Firm-Year Sample	108,862	40,567	108,862	40,567
R-squared	0.224	0.179		
e(Q)			0.00377	0.00073
P value of Hansen J			0.18	0.85
F stats of inv(t-1)			150.58	64.46
F stats of q (t-1)			89.90	12.27
F stats of q (t-2)			80.22	19.75
SWP of inv(t-1)			0.00	0.01
SWP of q (t-1)			0.00	0.04
SWP of q (t-2)			0.00	0.01
SWκ^2P of inv(t-1)			0.00	0.01
SWκ^2P of q (t-1)			0.00	0.04
SWκ^2P of q (t-2)			0.00	0.01
Firm FE	Y	Y	Y	Y

TABLE 6

Total, Physical, and Intangible Investment

This table presents the results of GMM and OLS estimations from 1975 to 2021. Columns (1) to (3) in Panel A (B) present the GMM (OLS) regression results for total, physical, and intangible investments, respectively. In this table, we use total q and total cash flow as defined in Peters and Taylor (2017) (see Appendix F). The IVs are (1) l4.total inv l3.total inv l5.total CF l4.total q, and (2) l4.lnK l4.total q l5.total q l3.physical inv, and (3) d.l4.lnN_K l4.total q l5.total q l3.intangible inv. Firm fixed effects are included. Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. t-statistics are reported in parentheses.

Panel A: GMM Estimations

VARIABLES	Total (1)	Physical (2)	Intangible (3)
q_total	0.091***	0.060***	0.034***
	(20.50)	(18.89)	(18.19)
CF_total	0.031***	0.004	0.023***
	(4.20)	(0.81)	(6.46)
lnK (size)	-0.125***	-0.075***	-0.046***
	(-22.83)	(-19.63)	(-23.39)
lnN_K (employment)	-0.001	0.005*	-0.008***
	(-0.40)	(1.65)	(-6.26)
ω	0.504***	0.532***	0.597***
	(18.27)	(22.29)	(29.91)
Firm-Year Sample	149,429	149,429	149,429
e(Q)	0.00100	0.00186	0.00084
P value of Hansen J	0.80	0.47	0.59
F stats of inv(t-1)	332.80	359.37	324.36
F stats of q (t-1)	88.08	158.52	96.21
F stats of q (t-2)	232.23	345.23	220.46
SWP of inv(t-1)	0.00	0.00	0.00
SWP of q (t-1)	0.00	0.00	0.02
SWP of q (t-2)	0.00	0.00	0.03
SWκ^2P of inv(t-1)	0.00	0.00	0.00
SWκ^2P of q (t-1)	0.00	0.00	0.02
SWκ^2P of q (t-2)	0.00	0.00	0.03
Firm FE	Y	Y	Y

Panel B: OLS Estimations

VARIABLES	Total (1)	Physical (2)	Intangible (3)
q_total	0.028***	0.015***	0.011***
	(39.35)	(29.90)	(32.62)
CF_total	0.105***	0.058***	0.046***
	(18.49)	(14.78)	(17.00)
lnK (size)	-0.126***	-0.081***	-0.037***
	(-46.01)	(-37.61)	(-37.21)
lnN_K (employment)	0.024***	0.017***	0.008***
	(9.09)	(8.52)	(9.11)
Firm-Year Sample	149,429	149,429	149,429
Adjusted R-squared	0.249	0.158	0.192
Firm FE	Y	Y	Y

TABLE 7

Non-linear Estimations

This table presents the results of non-linear GMM estimation for total investment from 1975 to 2021. In this table, we use total q and total cash flow as defined in Peters and Taylor (2017) (see Appendix F). The IVs are the same for Columns (1) to (2), l5.lnN_K d.l4.lnK l4.q^2 l5.q^2 l4.q l3.inv. Firm fixed effects are included. Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. *t*-statistics are reported in parentheses.

VARIABLES	(1)	(2)
q	0.023***	0.023***
	(16.87)	(17.26)
CF	0.000	0.000
	(0.19)	(0.02)
lnK (size)	-0.187***	-0.154***
	(-12.79)	(-12.34)
lnK^2 (size)	0.005***	-0.004***
	(4.28)	(-3.76)
lnK^3 (size)		0.001***
		(8.47)
lnN_K (employment)	0.033***	0.022***
	(2.78)	(5.05)
lnN_K^2 (employment)	0.001	
	(0.99)	
ω	0.317***	0.323***
	(10.16)	(10.36)
Firm-Year Sample	149,429	149,429
e(Q)	0.00306	0.00315
P value of Hansen J	0.44	0.45
F stats of inv(t-1)	220.06	219.59
F stats of q(t-1)	82.12	82.07
F stats of q(t-2)	165.93	167.24
SWP of inv(t-1)	0.00	0.00
SWP of q(t-1)	0.00	0.00
SWP of q(t-2)	0.00	0.00
SWChi2P of inv(t-1)	0.00	0.00
SWκ^2P of q(t-1)	0.00	0.00
SWκ^2P of q(t-2)	0.00	0.00
Firm FE	Y	Y

I. Internet Appendix

A. Estimation with Higher-order Polynomial OLS

In this section, we consider OLS estimation of the investment equation as a non-linear function of state variables, including cash flow, firm size (lnK), and the employment to capital ratio (lnN_K), while controlling for firm fixed effects. We employ several polynomial terms of these state variables to approximate the non-linear function, as reported in Table A1. The result in Column (1) shows that these state variables are statistically significant, and the correct specification of the functional form is non-linear.

Next, to examine whether q is redundant, we include q and the squared and cubic terms of state variables in the investment function, taking advantage of the flexible terms of these state variables in Column (2). We find that q is statistically significant at a 1% level with a magnitude of 0.014. However, the higher orders of the variables, e.g., q^2, q^3, lnK^3, and lnN_K^3, have a small magnitude close to zero, albeit statistically significant. Adding these polynomial terms does not significantly improve the fit as seen from the adjusted R^2, comparing the R^2 of 0.216 in Column (2) with that of 0.189 in column (1). Gala et al. (2020) claim that q becomes irrelevant to the firm's investment after taking into account state variables such as firm size, sales, and cash flows. In contrast, our estimation result confirms that q remains a significant factor of the investment, even after controlling for the flexible terms of other state variables.

In Column (3), we use the firm-level TFP measure from İmrohoroğlu and Tüzel (2014) to illustrate the empirical significance of accounting for the unobserved shock. We regress investment on TFP, q, cash flow, size, and employment. The result shows that q remains a significant factor while TFP is also statistically significant at a 1% level. This highlights the importance of controlling for persistent shocks—potentially including TFP—in the investment function. In Column (4), we examine whether q remains statistically significant by adding the squared and cubic terms of the state variables and controlling for firm fixed effects. We find that q remains relevant both in terms of the magnitude of the estimated coefficient and its statistical significance, while the higher-order terms of the variables, e.g., q^2, q^3, lnK^3, and lnN_K^3, have small magnitudes, albeit statistically significant. Compared with Column (4), we additionally control for a year fixed effect in Column (5). The result suggests that the coefficient of q remains robust as in Column (4).

TABLE A1

Higher-order OLS Estimation

This table presents the results of OLS estimations of different polynomial models. In Column (1), we estimate the OLS model with CF, lnK (size), lnN_K (employment), and the square of these variables. We add *q* to the regressions and include the square and cube terms of the variables in Column (2). In Column (3), we regress investment on the first-order TFP, *q*, CF, lnK (size), and lnN_K (employment). Column (4), we include TFP and the first, square, and cube terms of *q*, CF, lnK (size), and lnN_K (employment). We additionally control for a year fix effect in Column (5). Because of the availability of TFP, to facilitate comparison, we require TFP to be available when estimating the regressions in Columns (1) and (2). Standard errors are clustered at the firm level. *t*-statistics are reported in parentheses.

VARIABLES	(1)	(2)	(3)	(4)	(5)
TFP			0.024***	0.017***	0.010***
			(10.44)	(7.37)	(4.28)
q		0.014***	0.005***	0.013***	0.012***
		(20.02)	(21.57)	(19.60)	(16.02)
CF	0.030***	0.023***	0.012***	0.019***	0.017***
	(13.87)	(10.99)	(6.09)	(8.36)	(7.45)
lnK (size)	-0.231***	-0.182***	-0.158***	-0.181***	-0.251***
	(-25.90)	(-22.94)	(-47.13)	(-22.79)	(-24.26)
lnN_K (employment)	0.137***	0.078***	0.079***	0.078***	0.100***
	(12.03)	(4.44)	(22.27)	(4.44)	(4.78)
q^2		-0.000***		-0.000***	-0.000***
		(-9.21)		(-8.98)	(-7.33)
CF^2	0.015***	0.011***		0.010***	0.009***
	(13.28)	(9.86)		(9.15)	(8.04)
lnK^2 (size)	0.008***	-0.003***		-0.003***	-0.003***
	(9.98)	(-3.36)		(-3.34)	(-3.20)
lnN_K^2 (employment)	0.006***	-0.005		-0.005	-0.001
	(4.83)	(-1.51)		(-1.48)	(-0.32)
q^3		0.000***		0.000***	0.000***
		(6.57)		(6.41)	(5.25)
lnK^3 (size)		0.001***		0.001***	0.001***
		(12.78)		(12.68)	(15.14)
lnN_K^3 (employment)		-0.001***		-0.001***	-0.001**
		(-3.34)		(-3.32)	(-2.09)
Firm-Year Sample	107,183	107,183	107,183	107,183	107,183
R-squared	0.189	0.216	0.202	0.218	0.231
Firm FE	Y	Y	Y	Y	Y
Year FE	N	N	N	N	Y

B. Estimation for Financially Constrained and Unconstrained Firms

TABLE B1

Financially Constrained vs. Unconstrained Firms

This table presents the GMM estimation results from 1975 to 2021. Columns (1)-(3) present the results for financially constrained firms, proxied by size, Whited-Wu (WW), and Kaplan-Zingales (KZ) indexes, respectively. Columns (4)-(6) present the results for financially unconstrained firms. The IVs are d.l3.inv d.l4.inv l4.TQ l4.lnK for the estimation. Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. t-statistics are reported in parentheses.

	Financially constrained firms			Financially unconstrained firms		
	(1) Size	(2) WW index	(3) KZ index	(4) Size	(5) WW index	(6) KZ index
q	0.020***	0.018***	0.019***	0.028***	0.032***	0.026***
	(9.75)	(10.17)	(11.02)	(13.04)	(12.16)	(9.94)
CF	-0.003	-0.0005	-0.001	0.004	0.002	0.002
	(-0.96)	(-0.18)	(-0.20)	(0.84)	(0.32)	(0.53)
lnK (size)	-0.158***	-0.159***	-0.168***	-0.121***	-0.124***	-0.165***
	(-12.09)	(-13.38)	(-15.08)	(-13.55)	(-11.91)	(-12.71)
lnN_K (employment)	0.030***	0.034***	0.031***	0.011	0.012	0.009
	(5.38)	(5.92)	(5.51)	(1.42)	(1.38)	(1.16)
ω	0.281***	0.259***	0.244***	0.427***	0.486***	0.807***
	(3.86)	(3.68)	(3.81)	(7.43)	(7.94)	(20.15)
Firm-Year Sample	72,622	65,200	62,664	76,807	66,600	63,043
e(Q)	0.00109	0.00141	0.00161	0.00400	0.00391	0.00553
P value of Hansen J	0.83	0.39	0.32	0.25	0.44	0.38
F stats of inv(t-1)	53.47	57.32	65.15	82.69	76.53	59.97
F stats of q (t-1)	36.37	36.31	68.61	72.60	62.18	48.26
F stats of q (t-2)	75.46	83.82	83.80	107.31	78.26	91.95
SWP of inv(t-1)	0.00	0.00	0.00	0.00	0.00	0.02
SWP of q (t-1)	0.00	0.00	0.00	0.00	0.00	0.02
SWP of q (t-2)	0.00	0.00	0.00	0.00	0.00	0.02
SWκ^2P of inv(t-1)	0.00	0.00	0.00	0.00	0.00	0.02
SWκ^2P of q (t-1)	0.00	0.00	0.00	0.00	0.00	0.01
SWκ^2P of q (t-2)	0.00	0.00	0.00	0.00	0.00	0.02

C. Robustness Checks

Estimation When ω is Observed at $t-1$

In our original setting, we assume that ω is observed at t. In this section, we perform robustness checks including (i) re-estimating the model under an alternative assumption that ω is observed at $t-1$ rather than t, and (ii) using lagged TFP as instruments in a placebo test. The results are reported in Table C1 below. In Column (1), we replicate our main regression in Table 3 Column (4) for comparison. Column (2) reports the results of the estimation with the alternative assumption that ω is observed at $t-1$ rather than t. We find that the resulting coefficient estimates are very close to the baseline in Column (1): the coefficients of q and ω are still positive and significant. The difference in the q coefficient relative to the baseline is statistically insignificant (t-statistic = 1.39).

Column (3) reports the estimation results using lagged TFP as instruments in a placebo test. The IV set passes the first-stage F-tests, Hansen's J-test, and Sanderson–Windmeijer diagnostics. However, it produces a higher estimation residual e(Q) than that in Column (1). Therefore, the baseline model in Column (1) is preferable. Moreover, the coefficients of q and ω are qualitatively similar to those in the baseline model in Column (1).

Thus, this slight violation of the specification in the model assumptions does not alter the main findings that the unobserved persistent shock ω is an important factor in the firm's investment decision and that q remains significant even after controlling for the unobserved shock. These robustness checks indicate that our identification strategy is robust to modest shifts in the assumed timing of ω in practice.

TABLE C1

Robustness Checks When ω is Observed at $t-1$

This table presents the GMM estimation results with MA(1) measurement error from 1975 to 2021. Column (1) presents the original regression results in Column (4) of Table 3 in the manuscript. We estimate the GMM model with size and employment in addition to q, cash flow, and the unobserved persistent shock. The IVs are l3.inv l4.inv l5.CF l4.q. In Column (2), we estimate with MA(1) measurement error under an alternative assumption that ω is observed at $t-1$ and the IVs are d.l4.CF l4.q d.l4.inv d.l5.CF. In Column (3), we include lagged TFP as instrumental variables and the IVs are l4.CF l4.q l4.inv l3.TFP. Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. t-statistics are reported in parentheses.

VARIABLES	(1)	(2)	(3)
q	0.017***	0.020***	0.021***
	(13.26)	(11.44)	(15.30)
CF	0.003	0.001	0.003
	(1.21)	(0.40)	(1.13)
lnK (size)	-0.141***	-0.167***	-0.176***
	(-26.66)	(-15.59)	(-17.96)
lnN_K (employment)	0.032***	0.029***	0.033***
	(7.49)	(5.92)	(6.41)
ω	0.205***	0.379***	0.481***
	(6.48)	(7.30)	(11.18)
Firm-Year Sample	149,429	149,429	149,429
e(Q)	0.00228	0.00150	0.00306
P value of Hansen J	0.99	0.87	0.96
F stats of inv(t-1)	219.14	28.24	117.19
F stats of q (t-1)	42.52	15.97	70.93
F stats of q (t-2)	91.98	65.22	123.18
SWP of inv(t-1)	0.01	0.01	0.00
SWP of q (t-1)	0.01	0.07	0.00
SWP of q (t-2)	0.01	0.05	0.00
SWκ^2P of inv(t-1)	0.01	0.01	0.00
SWκ^2P of q (t-1)	0.01	0.07	0.00
SWκ^2P of q (t-2)	0.01	0.05	0.00

Instrument Robustness

We conducted robustness checks using 3–4 periods lagged instruments, with results reported in Columns (2)-(3) of Table C2. For comparison, Column (1) reproduces the main specification results using the original IV set in Table 3, Column (4). The estimated coefficient on q remains statistically significant and closely aligned with our main estimate; the differences between the new estimates and the baseline coefficient of q are not statistically significant. The corresponding t-statistics for testing these differences are 0.29 and 0.80 for the two alternative instrument sets in Columns (2) and (3), respectively. Meanwhile, the coefficients on ω remain highly significant.

TABLE C2

Robustness Checks with Alternative IVs

This table presents the GMM estimation results from 1975 to 2021. For comparison, Column (1) reproduces the main specification in Table 3 Column (4) of the manuscript. The IVs are l3.inv l4.inv l5.CF l4.q. Columns (2)-(3) presents the results of the estimations that restrict IV lags to 3-4 periods. The IVs are l3.inv l4.inv l4.q d.l4.lnN_K for Column (2) and l3.inv l4.inv d.l4.CF l4.q for Column (3). Standard errors are clustered at the firm level. e(Q) represents the residual from the GMM estimation. SW P-value (SWP) is Sanderson and Windmeijer's F-test for the weak identification problem. SW κ^2 P-value (SWκ^2P) is Sanderson and Windmeijer's F-test for the under-identification problem. *t*-statistics are reported in parentheses.

	(1)	(2)	(3)
q	0.017***	0.016***	0.016***
	(13.26)	(13.17)	(12.96)
CF	0.003	0.003	0.004
	(1.21)	(1.50)	(1.59)
lnK (size)	-0.141***	-0.140***	-0.141***
	(-26.66)	(-26.42)	(-27.58)
lnN_K (employment)	0.032***	0.034***	0.034***
	(7.49)	(7.90)	(8.07)
ω	0.205***	0.193***	0.174***
	(6.48)	(5.81)	(5.51)
Measurement error	MA(1)	MA(1)	MA(1)
Firm-Year Sample	149,429	149,429	149,429
e(Q)	0.00228	0.00217	0.00212
P value of Hansen J	0.99	0.11	0.50
F stats of inv(t-1)	219.14	209.04	213.24
F stats of q (t-1)	42.52	42.14	42.25
F stats of q (t-2)	91.98	93.53	91.32
SWP of inv(t-1)	0.01	0.06	0.03
SWP of q (t-1)	0.01	0.04	0.04
SWP of q (t-2)	0.01	0.04	0.04
SWκ^2P of inv(t-1)	0.01	0.06	0.03
SWκ^2P of q (t-1)	0.01	0.04	0.04
SWκ^2P of q (t-2)	0.01	0.04	0.04

D. Comparing Magnitude of *q*

In this section, we compare *q* coefficients of the OLS and GMM estimations in Erickson and Whited (2000), Peters and Taylor (2017), and our manuscript. First, the sample period of Erickson and Whited (2000) is 1992-1995, and their sample focuses on 737 manufacturing firms. The sample period of Peters and Taylor (2017) is 1975-2011, while our sample period is 1975-2021. Second, Erickson and Whited (2000) do not control for firm fixed effects, whereas Peters and Taylor (2017) and our study do. Third, we control for firm size and employment in the models, while these two studies do not. Because of these differences, we report the relative magnitudes of *q* coefficients of the OLS and GMM estimations in Table D1. In Erickson and Whited (2000), the GMM estimates with high-order moment equations (GMM3, GMM4, and GMM5) for each year are from 1.6 to 5.9 times larger than the OLS estimate from the same year. The GMM3-MD, GMM4-MD, and GMM5-MD estimates, which combine the GMM estimates from different years by a Minimum Distance (MD) estimator, are from 2.4 to 3.2 times larger than the OLS-MD estimate. In Peters and Taylor (2017), the GMM estimate is 2.1 times larger than the OLS estimate. After controlling for the unobserved persistent shock and measurement error in *q*, our GMM estimate is 2.8 times larger than the OLS estimate.

TABLE D1

Coefficients of *q* Comparison

This table presents a comparison of the *q* coefficients of the OLS and GMM estimations and their relative ratios in Erickson and Whited (2000), Peters and Taylor (2017), and our manuscript.

q coefficients	OLS	GMM3-MD	GMM4-MD	GMM5-MD	GMM	Ratio of $\frac{GMM}{OLS}$
Erikson and Whited (2000)	0.014	0.045	0.034	0.033		(2.4-3.2)
Peters and Taylor (2017)	0.017				0.035	2.1
Our study	0.006				0.017	2.8

E. Diagnostic Tests of Autoregressive Process

This section presents the results of the diagnostic tests on the underlying assumptions of our model. In our GMM framework, we assume the persistent shock (ω) follows an AR(1) process. Since it is treated as an unobserved state variable and is not directly estimated, we indirectly test the assumption by using TFP as a proxy variable. The results reported in Table E1 Column (1) show that TFP follows an AR(1) process: the AR(1) coefficient is 0.909 (*t*-statistic = 7.01), while the AR(2) coefficient is –0.250 (*t*-statistic = –1.36, insignificant), which lends support to our AR(1) assumption on ω.

Next, in line with the original Arellano-Bond AR(2) tests, we conduct AR(1) and AR(2) diagnostics directly on the regression residuals from our main estimation in Table 3 Column (4), the GMM estimation with MA(1) measurement error. Columns (2)-(3) in Table E1 report these results: the AR(2) test statistic is insignificant, and similarly, the AR(1) test also fails to reject the null of no first-order autocorrelation. This confirms that our moment conditions are valid under the GMM framework.

TABLE E1

Diagnostic AR Tests

Column (1) presents the AR(2) test on the TFP, which confirms that the TFP follows an AR(1) process, indirectly supporting our assumption of AR(1) process on ω. Columns (2) and (3) report the AR tests on the residual of the main regression in Table 3 Column (4). This confirms that both AR(1) and AR(2) are rejected. σ is the standard deviation of the error term in the AR process. Standard errors are reported in parentheses.

	TFP	Table 3 (4) Residual	Table 3 (4) Residual
AR(1)	0.909***	-0.148	-0.121
	(0.130)	(0.134)	(0.139)
AR(2)	-0.250	-0.171	
	(0.183)	(0.132)	
Constant	-0.330***	0.004***	0.004***
	(0.010)	(0.001)	(0.001)
σ	0.019***	0.011	0.109***
	(0.002)	(0.002)	(0.001)
Obs.	47	44	44
Wald $\chi^2(1)$	62.01	2.51	0.77
Log likelihood	119.07	136.86	136.22
Prob $> \chi^2$	0.00	0.29	0.38

F. Other Types of Investments

Following Peters and Taylor (2017), we define the firm's physical, intangible, and total investment rates as

$$\iota_{it}^{phy} = \frac{Inv_{it}^{phy}}{K_{i,t-1}^{tot}} \tag{15}$$

$$\iota_{it}^{int} = \frac{Inv_{it}^{int}}{K_{i,t-1}^{tot}} \tag{16}$$

$$\iota_{it}^{tot} = \frac{Inv_{it}^{tot}}{K_{i,t-1}^{tot}}. \tag{17}$$

Physical investment Inv^{phy} is measured as capital expenditures (Compustat item capx). We measure intangible investment Inv^{int} based on R&D and selling, general and administrative (SG&A) expenses as $R\&D + (0.3 \times SG\&A)$. As in the definition, we assume that 30% of SG&A is an investment. Total investment Inv^{tot} is the sum of Inv^{phy} and Inv^{int}.

The denominator total capital K^{tot} is the sum of physical capital K^{phy} and intangible capital K^{int}. The replacement cost of physical capital is K^{phy}, defined as the book value of property, plant and equipment (Compustat item ppegt). The data on the replacement cost of intangible capital, K^{int}, is obtained directly through Wharton Research Data Services (WRDS) that is available by Peters and Taylor (2017). They define K^{int} to be the sum of the firm's externally purchased and internally created intangible capital.

Externally purchased intangible capital is defined as intangible assets from the balance sheet (Compustat item intan). It is set to be zero if missing. However, since the internally created intangible capital does not appear on the balance sheet, it is hard to measure. Peters and Taylor (2017) construct a proxy by accumulating past intangible investments, and they define the stock of internal intangible capital as the sum of knowledge capital and organization capital. Firms develop knowledge capital through input on R&D. Therefore, this can be constructed by accumulating past R&D spending using the perpetual inventory method: $G_{it} = (1 - \delta_{R\&D})G_{i,t-1} + R\&D_{it}$, where G_{it} is the end-of-period stock of knowledge capital, $\delta_{R\&D}$ is its depreciation rate, and $R\&D_{it}$ is real expenditures on R&D spending (Compustat item xrd) during the year. Peters and Taylor (2017) use BEA's industry-specific R&D depreciation rates.

Next, the stock of organization capital is measured by accumulating a fraction of past SG&A spending using the perpetual inventory method. Peters and Taylor (2017) argue at least part of SG&A represents an investment in organization capital through advertising, spending on distribution systems, employee training, and payments to strategy consultants. They count 30% of SG&A spending as an investment in intangible capital and use a depreciation rate of $\delta_{SG\&A} = 20\%$.

We use two methods to measure q, the standard measure of Tobin's q as in Erickson and Whited (2000), and total q from Peters and Taylor (2017). We measure total q by scaling firm value by the sum of physical and intangible capital:

$$q_{it}^{tot} = \frac{M_{it}}{K_{it}^{phy} + K_{it}^{int}},$$

where M is the firm's market value. This equals the market value of outstanding equity (Compustat items prcc_c times csho), plus the book value of debt (Compustat items dltt+dlc), minus the firm's current assets (Compustat item act), which includes cash, marketable securities, and inventory. The denominator, total capital, is the sum of physical capital K^{phy} and intangible capital K^{int}. According to Peters and Taylor (2017), it is reasonable to assume physical and intangible capital share the same marginal q. This marginal q also becomes equivalent to average q, that is the ratio of firm value to its total capital stock, if the assumptions of the constant returns to scale, perfect competition, and perfect substitutes in production and depreciation, are all satisfied. Therefore, given that these assumptions are reasonably justified, we may measure Tobin's q as q^{tot}, firm value divided by K^{tot}, the sum of physical and intangible capital. Peters and Taylor (2017) predict that the firm's optimal physical and intangible investment rates vary with q^{tot}.

Peters and Taylor (2017) propose an alternative measure of cash flow by adding intangible investments into the free cash flow, to measure the profits available for total investment as

$$CF_{it}^{tot} = \frac{IB_{it} + DP_{it} + Inv_{it}^{int}(1-\kappa)}{K_{i,t-1}^{phy} + K_{i,t-1}^{int}}.$$

Here, κ is the marginal tax rate, which is either simulated marginal tax rate from Graham (1996) when available, or it is assumed to be 30%. The effective cost of a dollar of intangible capital is $(1-\kappa)$ because accounting rules allow firms to expense intangible investments.