Optimal retirement saving and dissaving

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Abstract

Applying a rich model of individuals' life-cycle utility maximization, we comprehensively evaluate retirement saving plans. Across a range of individual characteristics, access to basic plans with constant expected payouts and no or full annuitization leads to individual utility gains of up to 5.07% of initial wealth and lifetime income ($43,800 in present value terms), and almost all individuals prefer full annuitization and a target-date fund investment strategy. With flexible plans allowing for partial annuitization and non-constant expected payouts, utility gains go up to 5.81%, and most individuals prefer a high degree of annuitization and expected payouts being increasing through retirement.

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I. Introduction

Retirement saving is gaining attention both in academia and among policy makers due to the inadequate savings of many retirees, the lack of funding of public pensions (e.g., Social Security in the U.S.), and the reforms of pension systems in numerous countries (e.g., replacing defined benefit programs with defined contribution programs). Much focus is on the accumulation phase, i.e., how individuals should be motivated or forced to build up sufficient retirement saving. This paper focuses on the decumulation phase, i.e., how the savings are paid back to the savers during retirement, but clearly the payout schedule affects individuals' contributions in the accumulation phase.

We set up a rich life-cycle model of the decisions many U.S. individuals face. The individual has Epstein-Zin utility of consumption and bequests, an unknown lifetime, starts adulthood with some wealth, and receives risky labor income until retirement and Social Security in retirement where she might have to pay significant out-of-pocket medical costs. The individual can save privately in a riskfree bond and the stock market index but can also save for retirement through 401(k)-like plans.[1] Each year, the individual contributes either a preset or a self-selected fraction of income to the plan, decides how much to save or dissave privately (consumption is residually determined), and chooses how to invest private savings. Accumulating savings in the retirement account is attractive due to a more lenient taxation on returns compared to private investments and the access to annuitization, but the retirement savings are illiquid and annuitization comes with a cost. For plans involving annuitization, individuals share lifetime risks

[1] According to 2025 estimates from the Investment Company Institute, around 70 million U.S. adults hold a total of $9 trillion in 401(k) plans.

through an annuity provider that makes lifelong payouts to the participants following a certain preset payout schedule, i.e., annuitization is an insurance against outliving your retirement savings. For non-annuitized plans, the saver can each year choose a payout equal to or exceeding a required minimum distribution.

First, we consider basic plans with either no or full annuitization and with flat expected payouts. With moderate preference parameters (e.g., a risk aversion of 4) and income and wealth in line with the median U.S. worker, an individual obtains a utility gain corresponding to 3.19% of initial wealth and lifetime income from having access to basic plans with self-selected contributions and a target-date investment strategy with the stock weight sloping from 90% to 30% between age 41 and 77. The utility gain translates into a present value gain of $27,600. Most of the gain is due to annuitization and only a small part due to the tax advantage. Plans with preset contribution schemes can also lead to significant utility gains, but they must avoid large required contributions for young individuals. When varying the preference parameters and the income level of the individual, we find utility gains of up to 5.07% (or $43,800) with self-selected contributions. As the base case individual, most individuals prefer the target-date investment strategy and full annuitization to no annuitization. As an exception, the individual with low risk aversion chooses no annuitization and all retirement savings invested in stocks. All the individuals optimally contribute a small part of income to the retirement account early in life and a much larger part in the years leading up to retirement, but contribution levels vary with individual characteristics.

Second, we introduce plan flexibility in terms of partial annuitization and non-flat expected payouts. Partial annuitization means that the individual can choose an annuitization ratio $I \in [0, 1]$ so that, upon death, the fraction $1 - I$ of the account balance is paid to the deceased's heirs, whereas the rest is distributed by the pension company to the accounts of surviving savers. The flexibility increases the utility gain of our base case individual from 3.19% to 3.57%, and the individual chooses an annuitization ratio of $I = 0.9$ and a payout schedule where expected payouts increase by 4% per year. Across the individuals considered, the flexibility increases the gain by up to 0.96 percentage points ($8,257 in PV-terms).[2] All individuals prefer a high degree of annuitization and increasing expected payouts through retirement, except the low risk aversion individual who sticks to no annuitization and self-selected payouts. With the plan flexibility, most individuals save less in the retirement plan and more privately. Interestingly, Beshears, Choi, Laibson, Madrian, and Zeldes (2014) report that many respondents in two surveys of hypothetical annuitization decisions prefer partial annuitization of retirement savings and an increasing payout pattern. Our theoretical study confirms that many individuals benefit from such retirement plan features and pinpoints the role of individual characteristics in the demand for these features.

Third, we investigate how our results depend on the U.S.-style out-of-pocket medical costs by repeating the analysis for the case with no such costs but a higher income tax rate, i.e., more in line with many European countries. We find that most individuals prefer the system with tax-financed medical expenses, except individuals with low risk aversion or high bequest weight. Generally, the utility gains from access to retirement plans are smaller when medical costs are tax financed, e.g., the gain drops from 3.57% to 1.68% for the base case individual with access to flexible pension plans. Moreover, with tax-financed medical costs, many individuals want to take more risk in the investment strategy of the pension plan, prefer more steeply increasing payouts, and a lower degree of annuitization.

[2] The Internet Appendix shows that adding the option to make premature withdrawals at a cost increases the gain of the base case individual further to 3.85%, but such an option might lead to higher annuity costs.

This paper seems to be the first to study a life-cycle consumption-investment choice model with both private, liquid savings and contributions to 401(k)-style plans with annuitization options and flexible payout schedules. Classical life-cycle papers ignore dedicated retirement saving schemes and annuitization, cf., e.g., Viceira (2001), Cocco, Gomes, and Maenhout (2005), and Gomes and Michaelides (2005). Dammon, Spatt, and Zhang (2004) add a tax-deferred, illiquid retirement account, assuming that the individual before retirement contributes a fixed fraction of income to the account and in retirement withdraws a pre-set fraction of the balance every year. For tractability, they specify annual income as a constant proportion of wealth which reduces the realism of the model. Gomes, Michaelides, and Polkovnichenko (2009) allow for non-spanned income and introduce a stock market entry cost for the private portfolio to distinguish between indirect stockholders (only holding stocks in the tax-deferred account) and direct stockholders (holding stocks in the private account).

By providing insurance against outliving your wealth, lifelong annuities can improve individuals' utility as first shown by Yaari (1965) in a highly stylized model. Our model has features that have been argued to reduce the attractiveness of annuities, namely bequest preferences, annuity costs, and Social Security benefits.[3] Nevertheless, we find that almost all individuals considered prefer full annuitization to no annuitization of retirement savings. When individuals can choose the annuitization ratio, most choose to annuitize at least 80% of retirement savings. In this sense our study confirms or even deepens the annuity puzzle, i.e., the observation

[3]Related papers discussing annuities include Mitchell, Poterba, Warshawsky, and Brown (1999), Davidoff, Brown, and Diamond (2005), Horneff, Maurer, and Stamos (2008), Horneff, Maurer, and Rogalla (2010a), Ameriks, Caplin, Laufer, and van Nieuwerburgh (2011), Beshears, Choi, Laibson, and Madrian (2011), Koijen, Nijman, and Werker (2011), Fitzpatrick (2015), Peijnenburg, Nijman, and Werker (2017), and Lockwood (2018).

that few individuals choose to annuitize despite theoretical large gains from doing so despite significant costs, implicit annuities through Social Security benefits, and bequest preferences. Our analysis suggests that annuity providers can make their products more attractive by offering plans with stock-heavy investment strategies and flexibility in terms of built-in partial annuitization and non-flat payouts. However, a key challenge is probably that most individuals fail to understand annuity products and the benefits from sharing lifetime risks, dislike giving up control over their savings, and find the costs too high; see Brown, Kapteyn, Luttmer, Mitchell, and Samek (2021) and the references therein. Mandatory or automatic (with possible opt-out) annuitization of pension savings can potentially reduce adverse selection problems and thus the costs of providing annuities, and such features are present in several countries with acclaimed pension systems (e.g., the Netherlands, Sweden, and Denmark). Just as automatic enrollment has increased participation in retirement saving plans, automatic annuitization features could help, but the exact design should be carefully considered, cf. the discussion in Iwry and Turner (2009).

Campbell, Cocco, Gomes, and Maenhout (2001), Dahlquist, Setty, and Vestman (2018), and Larsen and Munk (2023) discuss various models of mandatory retirement saving schemes and, among other things, determine the best common default investment strategy in a scheme covering heterogeneous savers. These papers feature separate accounts for retirement savings and include annuities. Our model overlaps with that of Larsen and Munk (2023), but their focus is on setting the optimal common contribution rate and default investment strategy in a mandatory saving scheme covering both rational individuals and individuals procrastinating on savings. In contrast, we consider rational individuals choosing how much to contribute to retirement savings and how savings are invested and paid out. Our results on optimal contribution rates through the accumulation phase and the optimal payout schedule in the decumulation phase are also relevant for the design of mandatory saving schemes.

Our paper adds to our understanding of how retirement savings are optimally decumulated. Most papers on annuities consider only a payout stream that is constant (in expectation) over time, but we find that many individuals prefer payouts to be growing through retirement. The payout profile is controlled by the expected return on the portfolio associated with the annuity and the annuity's so-called assumed interest rate (AIR). Balter and Werker (2020) derive and discuss the optimal AIR (and thus the payout schedule) of an annuity in a highly stylized setting, but we consider a more realistic setting. Horneff, Maurer, Mitchell, and Stamos (2010b) have a short discussion of the role of the AIR in a specific life-cycle model but do not explore what the optimal AIR is for different individuals. We also illustrate how some individuals benefit from "partial annuities" where the heirs receive a preset fraction of the deceased saver's retirement account balance.

The remainder of the paper is organized as follows. Section II describes various retirement saving plans and how their payouts are affected by the AIR and the annuitization ratio. Section III sets up the life-cycle model and explains the assumed parameter values. For our baseline set of parameter values, Section IV presents optimal decisions and pension plans and illustrates the impact of the access to a pension plan on life-cycle patterns in consumption, wealth, savings, and investments. Section V determines optimal pension plans across a range of individual characteristics. Section VI provides additional analyses of the role of uninsurable medical expenses, tax incentives and welfare considerations, heterogeneous mortality risk, and the option to make unscheduled withdrawals from pension savings. Finally, Section VII concludes.

II. Payout schedules of retirement saving plans

This section illustrates the range of payout schedules for retirement saving plans. We use annual time steps and assume individuals retire when turning $t_R = 67$ years old (the Social Security full-benefit retirement age when born 1960 or later) and may live on until the end of year $t_M = 100$. Being alive at age t, the probability of being alive at age $t+1$ is p_t with $p_{t_M} = 0$. We use mortality rates from the 2019 U.S. life table (Arias and Xu, 2022) where, e.g., an individual entering retirement expects to live for another 18 years.[4]

A. Plan characteristics

A wide range of the retirement saving plans can be characterized by an investment strategy, an annuitization ratio, a scheduled payout policy, and a cost structure.

The **investment strategy** specifies how retirement savings are invested in both the accumulation and the decumulation phase. We consider strategies involving a riskfree asset and a risky asset representing a stock market index. The strategy is defined by the share of investments—the portfolio weight—in the stock index at any time. We assume this weight is a pre-set function w_t of the individual's age, which encompasses fixed-weight strategies and target-date funds. The assumption disregards plans where the individual can actively change the portfolio, e.g., in response to income shocks, but the individual can still freely adjust the private portfolio. We focus on four strategies:[5]

[4]More information in the Internet Appendix.

[5]The '120-minus-age' strategy $w_t = 120 - 0.01 \times t$ generates similar results as the target-date fund strategy (IP4) but lower utility in all cases we consider.

IP1: $w_t = 0$, i.e., a riskfree investment all life (as seen in fixed annuities);

IP2: $w_t = 0.5$, i.e., 50% stocks all life (as seen in some variable annuities);

IP3: $w_t = 1$, i.e., a fully index-linked strategy;

IP4: $w_t = \min\{0.3, 0.9 - 0.6 \times \frac{(t-t_{sb})^+}{t_{se}-t_{sb}}\}$, i.e., 90% stocks until age $t_{sb} = t_R - 26$, slopes to 30% at age $t_{se} = t_R + 10$ and stays there, like Vanguard's target-date funds.[6]

We assume a constant annual riskfree log return of r, that the log return on the stock market over any period dt is normally distributed with expectation $(r + \mu_S - \frac{1}{2}\sigma_S^2)\,dt$ and standard deviation $\sigma_S\sqrt{dt}$, and that returns are independent in the time dimension. The expected annual rate of return on the stock is thus $\exp\{r + \mu_S\} - 1$, i.e., μ_S captures the excess expected stock return. We use the standard parameter values $r = 1\%$, $\mu_S = 4\%$, and $\sigma_S = 15.7\%$. By assuming that the portfolio is continuously rebalanced through year t to maintain a constant stock weight of $w_t \in [0, 1]$, the log return on the portfolio over the year is normally distributed with expectation $r + w_t\mu_S - \frac{1}{2}w_t^2\sigma_S^2$ and standard deviation $w_t\sigma_S$.[7] The gross after-tax return on retirement savings

[6]According to Vanguard's homepage accessed on January 3, 2024. Vanguard's description of the asset allocation glidepath involves U.S. stocks, international stocks, U.S. nominal bonds, international bonds, and short-term TIPS. We combine U.S. and international stocks to get the stock weight used in our model.

[7]Assume $dS_t = S_t[(r + \mu_S)\,dt + \sigma_S\,dz_t]$, where z is a standard Brownian motion. With a fraction w_t of wealth in the stock, the wealth dynamics are $dA_t = A_t[(r + w_t\mu_S)\,dt + w_t\sigma_S\,dz_t]$ which with a constant $w_t = w$ implies

$$A_{t+\Delta} = A_t \exp\left\{\left(r + w\mu_S - \frac{1}{2}w^2\sigma_S^2\right)\Delta + w\sigma_S\left(z_{t+\Delta} - z_t\right)\right\},$$

where $z_{t+\Delta} - z_t \sim N(0, \Delta)$. Hence, the pre-tax rate of return over a one-year period can be written as $\exp\{r + w\mu_S - \frac{1}{2}w^2\sigma_S^2 + w\sigma_S\varepsilon_S\} - 1$, where $\varepsilon_S \sim N(0, 1)$.

in year t is

$$(1) \qquad R_{At} = 1 + (1 - \tau_A) \left[\exp\left\{ r + w_t \mu_S - \frac{1}{2} w_t^2 \sigma_S^2 + w_t \sigma_S \varepsilon_{St} \right\} - 1 \right],$$

where $\varepsilon_{St} \sim N(0, 1)$ and τ_A is the tax rate with $\tau_A = 0$ as the baseline value.

The **annuitization ratio** defines what happens to the retirement saving balance upon death. In non-annuitized plans, savings go to the heirs of the account holder at death, after subtracting income tax since contributions are made before tax. In valuation terms, this is equivalent to the case where the product issuer after the death of the account holder continues to make the scheduled payments but to the heirs of the deceased. In contrast, lifelong annuities provide regular payments to the account holder until death, and the heirs receive nothing from the annuity provider who effectively distributes the remaining savings to surviving customers' accounts. We also consider "partial annuities" with an annuitization ratio $I \in [0, 1]$ so that the annuity provider retains the fraction I of savings upon death in retirement, while the remainder is paid to the heirs. For a large group of individuals of age $t \geq t_R$ with similar survival probabilities and account balances, the balance of each member surviving until age $t + 1$ can be added a fraction $d_t = I(1 - p_t)/p_t$ of the balance at the end of year t due to transfers from deceased customers.

Scheduled payouts are made from age t_F to t_L, and we focus on the case $t_R = t_F$ and $t_L = t_M$ and assume contributions to the account are made only before time t_F.[8] The scheduled payout policy is captured by an age-dependent function m_t stating the fraction of the retirement

[8]According to our extensive experimentation, none of the optimal plans we report for various individuals in later sections can be beaten by plans with shorter payout periods.

saving balance paid out at age t. If A_t is the opening balance in year t, the scheduled monetary payout in year t is $m_t A_t$. Next year's balance is

$$A_{t+1} = (1 - m_t) A_t R_{At} (1 + d_t). \tag{2}$$

We set $m_{t_L} = 1$ so the remaining balance A_{t_L} is paid out at age t_L.

The payout profiles we consider are controlled by a parameter x, the so-called *excess assumed interest rate (AIR)*, together with the expected investment returns and mortality risk. The payout rates are specified recursively as

$$m_t = \left(1 + \left\{m_{t+1} \mathrm{E}_t[R_{At}](1 + d_t) e^x\right\}^{-1}\right)^{-1}. \tag{3}$$

Since $e^{-x} = \mathrm{E}_t[m_{t+1} A_{t+1}]/(m_t A_t)$, the excess AIR x reflects the drop in expected payouts per year. For $x = 0$, expected payouts are flat which is a common feature of pension plans (for payments in real terms, constant payments mean CPI-adjusted payments). Details can be found in the Internet Appendix.

As a simple example, assume zero returns and taxes. A person turning 67 then needs un-annuitized savings of $340,000 to ensure $10,000 of consumption per year in case the person should live until the end of year 100 but only fully annuitized savings of $185,405 to ensure $10,000 per year until the end of life given the mortality rates of the U.S. population.

The **costs** associated with a retirement saving plan are also crucial. Our model assumes that the individual can privately invest without any trading or participation costs in a riskfree asset and in a stock index, so the modeled returns are really net of any such costs (e.g., the low cost on index ETFs). We assume that managing a retirement saving plan is not more costly than managing private, non-retirement funds. Any administrative fees charged by the plan provider (maybe $100-200 per year) are assumed to be similar to the costs (time consumption etc.) of handling private investments.

Annuities are often considered expensive. Mitchell et al. (1999) report that for a 65-year old annuitant the money's worth ratio of U.S. lifelong annuities is around 85% depending on the annuitant's gender and the discount rate used, and assuming population mortality rates. Hence, we assume a 15% cost on an annuity ($I = 1$) relative to a personal account ($I = 0$). Other studies report similar costs with variation across countries and products (Kaschützke and Maurer, 2011). These costs may stem from assessing and managing how the lifetime uncertainty of annuity buyers differs from that of the population and relates to adverse selection issues that individuals with a longer-than-average life expectancy benefit from a contract based on the population's life expectancy. A 15% charge roughly covers the extra payments if the annuitant's life expectancy at retirement is $(1/0.85) - 1 \approx 17.6\%$ longer than for the entire population. In the 2019 U.S. life table, an individual turning 67 can expect to live another 18 years, so for an annuity holder the same life expectancy can be up to 21.2 years without eliminating profits for the annuity issuer. For partial annuities with $0 < I < 1$, the cost is assumed proportional to I, so the money's worth ratio is

$$W(I) = 1 - KI, \quad K = 0.15, \tag{4}$$

i.e., a dollar contributed to retirement savings increases the account balance by $W(I)$ dollars.

B. Plans with constant expected payouts throughout retirement

Table 1 shows payouts for plans with constant expected payouts ($x = 0$) combining the four investment strategies specified above and three values (0, 0.5, and 1) of the annuitization ratio I. The individual is assumed in Panel A to spend \$100 on each product when retiring at age 67 and in Panel B to contribute an amount a at the beginning of every year from age 25 to 66. To facilitate comparisons across the panels, we fix $a = 100\, e^{-r}(1 - e^r)/(1 - e^{r(T_R - 25)}) \approx 1.9603$ so that a riskfree investment strategy with $I = 0$ generates a wealth of \$100 at retirement. In both cases, the product issuer subtracts costs so that only the fraction $W(I)$ of the contribution is invested to generate future payouts to the customer. For each case, the table shows the expected annual payout and the 10th and 90th percentiles in the distribution of possible payouts at ages 70, 80, 90, and 99. For plans involving stock investments, all numbers are based on 100,000 simulations of the annual stock returns.

[TABLE 1 about here.]

Table 1 illustrates two points. *First*, stock investments lead to notably higher expected payouts, in particular when savings are gradually built up as in Panel B. Focus on non-annuitized plans. The riskfree generates an annual payout of \$3.45, whereas one fully invested in stocks lead to an expected annual payout of \$5.97 if initiated at retirement and \$16.75 if build up over working life. The higher expected payout comes with a high upside potential and some downside risk. With the target-date fund (TDF), the expected annual payout is more modest but both the upside potential and the downside risk is, of course, lower than with a full stock investment. With gradual savings, the annual payout with the TDF strategy has an expectation of \$8.77 and thus 154$\%$ larger than with the riskfree strategy, and the TDF payout beats the riskfree payout at any age with a probability of more than 90% (cf. the 10th percentiles ranging from \$4.12 to \$3.62, so the downside risk is limited compared to the sizeable upside potential. Individuals might not be so concerned about the significant downside payout risk of stock-heavy plans at high ages (the 10th percentiles decrease in age) since they have only a small chance of surviving that long.

Second, the payouts increase significantly with the annuitization ratio I despite larger costs. With $I = 0$ the payments must be stretched until the maximum age, whereas with $I = 1$ the payments are only due until death. With a \$100 investment at retirement, the (expected) annual payout with $I = 1$ is 48% larger than for $I = 0$ (\$5.09 compared to \$3.45) when $w = 0$ and 22% larger (\$7.30 compared to \$5.97) when $w = 1$. With the gradual saving strategy, the (expected) annual payout is 70% larger with $I = 1$ than $I = 0$ when $w = 0$ and 44% larger when $w = 1$, and the 10th and 90th percentiles increase similarly.

The downside of a higher I is a lower bequest upon death. Figure 1 shows how the expected account value of selected plans varies with age.[9] The value of plans being build up from age 25 (light-colored curves) increases until retirement, and then savings decumulate in retirement—also for plans initiated at retirement (dark curves)—to zero at the maximum age. The solid curves represent non-annuitized plans and depict the pre-tax account value bequeathed in case of death at each age. The dashed curves represent the fully annuitized plans. The orange curves are for plans with $w = 1$ (index-linked) and the blue curves for plans with $w = 0$ (riskfree). The orange curves with savings from age 25 show that the account value is lower with $I = 1$ than $I = 0$, except for a few final years. At first, this is due to costs being subtracted. Later, transfers from deceased plan holders are added to the account when $I = 1$, reducing the distance

[9] Readers of the printed journal are referred to the online version for colors.

between the curves towards zero at retirement. In retirement, the value of the annuitized plan first declines more steeply due to larger payouts but, when the mortality rate picks up, increasing transfers from deceased portfolio holders bring the two curves closer. An individual with a fully annuitized instead of a non-annuitized version of the riskfree plan receives an extra annual payout of $\$5.09 - \$3.45 = \$1.64$ every year until death but, on the other hand, at death the heirs do not receive the pre-tax annuity value which is \$65.72 at age 80 and \$36.14 at age 90. Obviously, an individual's choice of plan depends on preferences for bequests versus consumption and on life expectancy.

[FIGURE 1 about here.]

C. Plans with other payout schedules

Table 2 provides examples of plans with age-dependent expected payouts controlled by the excess AIR x. With $x < 0$ and thus payouts increasing with age, more returns are made on savings, which leads to larger average payouts. A negative x generates a more steeply increasing payout when $I = 1$ than when $I = 0$. This follows from the mortality risk being increasing in age and the fact that payouts with $I = 1$ are conditional on survival. Hence, large conditional payouts late in life can be promised without reducing earlier payouts much. For example, with $w = 0$ and $x = -8\%$, expected payouts increase from $\$0.94$ at age 70 to \$9.57 at age 99 when $I = 0$ but from \$2.37 to \$24.15 when $I = 1$. For $x > 0$ and thus declining payouts, the difference in payments between $I = 0$ and $I = 1$ is smaller since the scheduled payments are low when mortality risk is high. The table also shows the 10th percentiles of annual payouts for the index-linked plans ($w = 1$). With a positive x, not only are expected payouts declining with age, there is also a large probability of ending up with very low payouts when living long. Note that plans with $w = 1$ and $x = -4\%$ have 10th percentiles being roughly flat through retirement together with increasing expected payouts.

[TABLE 2 about here.]

D. Required minimum distributions

U.S. legislation stipulates required minimum distributions (RMDs) that individuals must withdraw from their retirement accounts each year starting at age 73 (as of 2023); any required amounts not withdrawn are subject to a 50% tax. The RMD in dollar terms for a given age $t \geq 73$ is the account balance at the end of the previous year divided by a so-called distribution period associated with that age, which is published by the Internal Revenue Service (IRS), related to the remaining life expectancy, and thus decreasing with age. This regulation effectively defines a lower bound $\underline{m}_t$ on the payout rate m_t introduced above with $\underline{m}_t$ being equal to the reciprocal of the distribution period for that age.[10]

[FIGURE 2 about here.]

All plans considered above with a zero excess AIR satisfy the RMD bound, no matter what the annuitization ratio is. The solid curves in Figure 2 show the payout rates for some of these plans (on a log scale), and they are all above the dashed black curve that represents the lower bound. Panel A considers target-date fund strategies (IP4). The upper solid curve is for full annuitization ($I = 1$), the lower for no annuitization ($I = 0$). As discussed earlier, payout rates

[10]See IRS homepage and the link given there to Publication 590-B. We apply the distribution periods in Table III that pertains to all unmarried owners and most married owners.

are increasing in I. Panel B considers the strategies with either 0%, 50%, or 100% in stocks (IP1-3) and no annuitization. As payout rates are increasing in expected returns, the upper solid curve is for the most aggressive strategy. The curve for the least aggressive strategy barely meets the requirement; for example, the payout rate at age 73 is 4.07% while the minimum is 3.77%.

Payout rates are increasing in the excess AIR. Consequently, plans with a sufficiently negative AIR—and thus very steeply increasing expected payments—violate the lower bound. The dotted curves in Figure 2 represent strategies with an excess AIR of -10%, and they all fall below the lower bound over some age interval starting at age 73. In particular, for plans with low expected returns and no annuitization, the excess AIR can only be mildly negative, i.e., expected payouts can only be slowly increasing with age. Plans with high expected returns and full annuitization can have more steeply increasing expected payouts. For the TDF strategy, the lowest acceptable excess AIR is -8% with full annuitization and increases to 0% with no annuitization. For the all-in-stocks strategy, the lowest acceptable excess AIR is -10% with full annuitization and increases to -4% with no annuitization.[11]

[11] What about plans with a shorter payout period? For a fixed t_L, the payout rates are invariant to the starting age t_F of the payouts so just delaying the initiation of payouts plays no role for whether the RMD bound requirement is satisfied (of course, payouts have to begin at age 73 as the latest). In contrast, a plan inadmissible with payouts to age 100 may be admissible if payouts are terminated at an earlier age. For example, the orange dotted curve in Panel A is below the minimum for age 73-76. If payouts are terminated at age 98 instead of 100, the plan would be admissible.

III. A life-cycle model

We set up a life-cycle model to evaluate different retirement saving plans and to find the optimal retirement saving decisions of an individual consumer-investor. The model adapts the mandatory pension saving model of Larsen and Munk (2023), and we refer to their paper for details and for additional motivation of the baseline parameter values listed in Table 3. We model income and wealth in real terms (current dollars), and all returns are also in real terms. The model has annual time steps and represents the decision problem of an individual who has just turned $t_1 = 25$ years old, retires when turning $t_R = 67$ years old, and may live on until the end of her year $t_M = 100$. As above, p_t denotes the probability of being alive at age $t+1$ conditional on being alive at age t.

A. Income, Social Security, and medical costs

The individual receives pre-tax income Y_t at the beginning of year t from labor, a state pension, or other sources. The income dynamics are

$$Y_{t+1} = Y_t R_{Yt}, \tag{5}$$

where

$$R_{Yt} = \begin{cases} \exp\{\mu_{Yt} - \frac{1}{2}\sigma_Y^2 + \sigma_Y \varepsilon_{Yt}\} & \text{for } t = t_1, \dots, t_R - 2, \\ \zeta & \text{for } t = t_R - 1, \\ (1 - \phi_t h)(1 - \Phi_t H) & \text{for } t = t_R, \dots, t_M - 1, \end{cases} \tag{6}$$

with $\varepsilon_{Yt} \sim N(0,1)$ and independent over time. The income starts at $Y_{t_1} = \$40,000$, has a volatility of $\sigma_Y = 10\%$, and the expected growth rate μ_{Yt} follows a third-order polynomial with coefficients determined so that expected income peaks at age 55 at a value 50% above initial income and then drops 10% until retirement. The income is assumed uncorrelated with the stock index.[12] $\zeta = 0.45$ is the ratio of the annual state pension to pre-retirement income and leads to an expected after-tax annual state pension of \$17,328.

[TABLE 3 about here.]

Out-of-pocket medical costs are a major concern for U.S. retirees and can significantly impact saving and risk taking (De Nardi, French, and Jones, 2010). We let $\phi_t, \Phi_t \in \{0,1\}$ indicate whether an uninsured health shock with a small cost $h = 3\%$ (e.g., for prescription medicine), respectively large cost $H = 85\%$ (e.g., for nursing home spending), occurs at age t. The small-shock probability is the constant $q = \text{Prob}(\phi_t = 1) = 18\%$, whereas the large-shock probability $Q_t = \text{Prob}(\Phi_t = 1) = \min\{0.03 \times \frac{t-t_R}{t_M-t_R} + \left(\frac{(t-t_R-15)^+}{t_M-t_R-15}\right)^2, 0.5\}$ grows linearly until 15 years into retirement where it accelerates until reaching 50%. The medical costs are assumed to be tax deductible so h and H reflect the percentage reduction in income both before and after tax. Model simulations show that medical costs are expected to be 3.4%, 11.1%, 23.8%, and 77.9% of Social Security pay at ages 72, 79, 86, and 93, broadly in line with Koijen, van Nieuwerburgh, and Yogo (2016) and De Nardi, French, Jones, and McCauley (2016). For simplicity, life expectancy is assumed unchanged after a medical shock, and the shocks are assumed permanent (transitory shocks have little impact anyway).

[12] A zero correlation is in line with typical assumptions and estimates. The Internet Appendix shows similar results for the case of a correlation of 0.2 and provides additional discussion.

B. Investments and wealth dynamics

Both the pension fund and the individual can invest in a riskfree asset and a stock index. As in Section II, the riskfree log return is r per year, and the index has normally distributed log returns over any period dt with expectation $(r + \mu_S - \frac{1}{2}\sigma_S^2)\,dt$ and standard deviation $\sigma_S\sqrt{dt}$ and with returns being independent across time.[13]

We let F_t denote the individual's private wealth (outside the retirement account) and assume $F_{t_1} = \$5,000$.[14] In the beginning of each year, the individual (1) receives income and (in retirement) scheduled or self-selected payouts from her retirement saving account, (2) makes a scheduled or self-selected contribution to the retirement account, (3) pays income taxes, and (4) decides how much to consume and how to invest the remaining private wealth over the year. The contribution to the retirement account is a fraction $\alpha_t \in [0, \bar{\alpha})$ of pre-tax income with $\alpha_t = 0$ for $t \geq T_R$. Annual contributions to a 401(k) plan in the U.S. are currently capped at \$23,000 (2024-level, annually revised). If the cap is constant in real terms, it also applies close to retirement when income is expected to be up to around 50% higher than the initial \$40,000, and

[13]Some studies find (statistically weak) mean reversion or predictability in stock returns, motivating a glidepath strategy even without income, but the quantitative effects on investments of adding mean reversion to a life-cycle model seem modest, see Michaelides and Zhang (2017). Moreover, including mean reversion or predictability would add another state variable and, thus, severely prolong numerical computation time. See the Internet Appendix for additional discussion.

[14]Initial income and wealth are broadly matching the median 25-year old U.S. worker, cf. the 2019 Survey of Consumer Finances (Bhutta, Bricker, Chang, Dettling, Goodman, Hsu, Moore, Reber, Volz, and Windle, 2020, Table 2).

this is when high contribution rates are attractive in some cases. Hence, we set the upper bound to $\bar{\alpha} = 0.4$.

The year t payout from the retirement account is $m_t A_t$, where A_t is the account balance entering year t. We require $m_t = 0$ for $t < T_R$. For plans involving any annuitization ($I > 0$), the individual is part of a joint risk-sharing arrangement with other individuals and must stick to the scheduled payouts $m_t A_t$ described in Section II, and only plans satisfying the RMD are allowed. For a non-annuitized plan ($I = 0$), the individual's retirement savings are separate from other individuals so the individual can chooses the payout ratio m_t each year under the RMD constraint $m_t \geq \underline{m}_t$. The balance A_t of the retirement account starts at $A_{t_1} = 0$. Adding contributions with annuity-linked costs to (2), the retirement account balance at the beginning of year $t+1$, provided the individual survives year t, is

$$A_{t+1} = \left[(1 - m_t)A_t + W\alpha_t Y_t\right] R_{At}(1 + d_t). \tag{7}$$

The income after contributions to—or payouts from—the retirement account is subject to a proportional tax rate of $\tau_Y = 30\%$. The disposable private wealth in year t is

$$\tilde{F}_t = F_t + (1 - \tau_Y)\left[(1 - \alpha_t)Y_t + m_t A_t\right] \tag{8}$$

of which the individual consumes a fraction $c_t \in (0, 1]$. The remainder is invested and, with R_{Ft} denoting the gross after-tax return, next year's private wealth becomes

$$F_{t+1} = (1 - c_t)\tilde{F}_t R_{Ft}. \tag{9}$$

We assume the private portfolio is continuously rebalanced to keep a constant fraction π_t of wealth in the stock index throughout year t. All private returns—realized or not—are taxed year-end at a rate of $\tau_F = 20\%$. Similarly to Eq. (1), the after-tax gross return is then

$$R_{Ft} = 1 + (1 - \tau_F)\left[\exp\left\{r + \pi_t\mu_S - \frac{1}{2}\pi_t^2\sigma_S^2 + \pi_t\sigma_S\varepsilon_{St}\right\} - 1\right].$$

C. Preferences and decisions

The retirement saving plans we consider are characterized by t_F, t_L, I, x, and $w = (w_t)$. For a plan with self-selected contributions, the individual chooses c_t, π_t, and α_t for $t = t_1, t_1 + 1, \ldots, t_M$ (and m_t if $I = 0$) to maximize lifetime utility. We assume Epstein-Zin utility with indirect utility J_t satisfying the recursion

$$J_t = \max_{c_t, \pi_t, \alpha_t}\left\{\left(c_t\tilde{F}_t\right)^{1-\frac{1}{\psi}} + \beta\,\mathrm{CE}_t^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{10}$$

where appropriate bounds are imposed on the controls c_t, π_t, α_t, and where

$$\mathrm{CE}_t = \left(p_t \mathrm{E}_t\left[J_{t+1}^{1-\gamma}\right] + (1 - p_t)\mathrm{E}_t\left[\bar{U}_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \tag{11}$$

is the certainty equivalent of next period's utility which is J_{t+1} if surviving and the bequest utility $\bar{U}_{t+1} = \xi^{\frac{1}{\psi-1}} B_{t+1}$ if not. The bequest if dying at the end of year t is the sum of the private wealth and the fraction $1 - I$ of after-tax retirement wealth,

$$B_{t+1} = F_{t+1} + (1 - I)(1 - \tau_Y)[(1 - m_t)A_t + W\alpha_t Y_t]R_{At}.$$

Should the individual reach the maximum age, the pension account has already been paid out, so $B_{t_M+1} = F_{t_M+1}$. Without access to a retirement saving plan, the choice variable α and the state variable A are identical to zero.

Base case preferences are characterized by the relative risk aversion (RRA) $\gamma = 4$, the elasticity of intertemporal substitution (EIS) $\psi = 0.25$, the subjective discount factor $\beta = 0.96$, and the strength of the bequest motive $\xi = 1$, but we also consider alternative values.[15]

Given our set-up, the indirect utility is a function $J_t(F_t, Y_t, A_t)$ of age, private wealth, income, and retirement savings. The dimension of the state space can be reduced by one by exploiting a homogeneity property (see the Internet Appendix):

$$J_t = \left(F_t + [1-\tau_Y]A_t\right) G_t\left(y_t, a_t\right), \tag{12}$$

where

$$y_t = \frac{[1-\tau_Y]Y_t}{F_t + [1-\tau_Y]A_t}, \qquad a_t = \frac{[1-\tau_Y]A_t}{F_t + [1-\tau_Y]A_t}. \tag{13}$$

Here a_t is bounded by 0 and 1, whereas y_t is bounded from below by zero but unbounded from above. Both A_t and F_t depend on the optimal controls so that y_t typically starts out very high as annual income tends to be large relative to financial wealth for young individuals. As wealth accumulates over life, y_t typically drops considerably approaching retirement and then jumps down at retirement after which it varies again due to medical costs and wealth decumulation.[16]

[15]Cases with $\gamma = 1$ or $\psi = 1$ must be studied separately with appropriate adjustments of (10) and (11).

[16]Such variations are problematic when implementing the dynamic programming approach on an equidistant and relatively sparse grid. The Internet Appendix explains how to handle this.

We solve for G and optimal decisions by backwards dynamic programming on a grid. We simulate 10,000 paths forward and report averages at each age to indicate an expected life-cycle pattern. Of course, without access to a retirement saving plan the variable a is identical to zero and the dimension is thus reduced further.

D. Utility gain measure

Let $J_{t_1}(F, Y; \text{P})$ denote the initial (age 25) indirect utility when the individual follows the retirement saving plan P defined by a specific combination of t_F, t_L, I, x, and $w = (w_t)$. The initial retirement account value is zero, so A is dropped from the notation. As a common benchmark, we compare with the indirect lifetime utility $J_{t_1}(F, Y; \text{no})$ when the individual does not have access to any retirement saving plan. If the individual is not forced to save in the retirement saving product, she is at least as well off with access to such a product as without. We quantify the utility gain to the individual of having plan P by the fraction λ of additional life-time labor income and initial wealth that the individual without any plan would need to receive to obtain the same lifetime utility as with access to the plan. With the additional income and wealth, the indirect utility without retirement saving products is $J_{t_1}([1+\lambda]F, [1+\lambda]Y; \text{no}) = (1+\lambda)J_{t_1}(F, Y; \text{no})$. Equating this with $J_{t_1}(F, Y; \text{P})$, we find

$$\lambda = \frac{J_{t_1}(F, Y; \text{P})}{J_{t_1}(F, Y; \text{no})} - 1. \tag{14}$$

Following Larsen and Munk (2023), we transform the utility gain into a dollar amount by multiplying λ by the sum of the initial financial wealth and the present value (PV) of lifetime after-tax income (from labor and Social Security less medical expenses). Since the income is not spanned by traded assets, there is no unique way to fix the discount rate for future expected income. We apply a discount rate of 3.55%, the sum of the riskfree rate 1% and a premium of $4\% \times 10\%/15.7\%$ calculated as a volatility-scaling of the equity premium. The PV of lifetime income is then $859,242 and adding the financial wealth of $5,000, a utility gain λ of 1% corresponds to $8,642.

IV. Optimal decisions and plans: base case preferences

This section focuses on individuals with base case preference parameters. First, we illustrate optimal decisions with and without access to pension plans. Next, we show how the utility and outcomes of the individual are affected by a range of basic plans, i.e., plans with either no or full annuitization and with flat expected payouts. Finally, we add plan flexibility in the form of partial annuitization and non-flat expected payouts.

A. Optimal decisions with and without a pension plan

As shown in the next subsection, the optimal basic pension plan features full annuitization, a TDF investment strategy, and self-selected contributions. Figure 3 illustrates the optimal controls with and without this pension plan. The controls are shown at three different age levels and always as functions of the ratio y of income to total wealth; note that the relevant range of y and thus the horizontal axis in these graphs change with age. With access to a pension plan, the controls also depend on the ratio a of pension wealth to total wealth, and we show the controls for $a = 0.3$ (labeled 'low') and $a = 0.6$ ('high').

[FIGURE 3 about here.]

The upper panels of Figure 3 show that the consumption-wealth ratio c is increasing in y with or without the pension plan and in a with the plan. The consumption-wealth ratio is large for a young individual with little wealth, decreases until retirement where total wealth peaks, after which it often increases as wealth is decumulated. The consumption-wealth ratio is larger with access to the pension plan (dotted curves) than without (solid curves).

The panels in the middle of Figure 3 show the plan's stock weight (flat blue line) and the optimal private stock weight π with (dotted curves) or without the plan (solid dark curves). Due to the bond-like human capital, the private stock weight is at the maximum 100% for the young individual and tends to decrease with age when the income-wealth ratio drops, as is known from standard life-cycle models (e.g., Cocco et al., 2005). At age 60, the stock weight is still 100% if the income-wealth ratio is high but below 100% if the income-wealth ratio is low. At age 90, the human capital is small (only the PV of Social Security in the remaining years) and has little impact on the stock weight which is therefore lower and approaching the no-income Merton weight of $\mu_S/(\gamma\sigma_S^2) \approx 0.406$. An individual with a pension plan adjusts the private stock weight so that the stock weight in her total portfolio comes close to what is optimal in the absence of a pension plan. At age 90, the plan's 30% stock weight is too low relative to the benchmark individual's optimal total stock weight, so she invests a larger fraction of private wealth in stocks, and more so if a is high so that a lot of wealth is tied up in the pension plan. Due to the more lenient return taxation, the individual prefers more stocks in the retirement account than in the private account.

The lower panels of Figure 3 show the optimal contribution rate α to the preferred pension plan at age 30, 45, and 60. The optimal contribution rate α is low—in most cases zero—at age 30. A young individual who has preferences for consumption smoothing and expects a significant income growth does not want to save much in an illiquid pension account but builds up a small private buffer for "rainy days." At age 45, the incentives for pension savings are bigger: much of the income growth is already realized, a private buffer has been built, and the retirement period is getting closer. The optimal contribution rate is lower if a is large, i.e., a relatively large pension wealth has already been accumulated. The contribution rate is hump-shaped in y, so if income is either low or high relative to total wealth, a smaller fraction of income is contributed to the pension account compared to the case with a medium income-wealth ratio. At age 60, the individual contributes in many cases the maximum 40% of income. At this age, pension savings are preferred to private savings because of the more lenient return taxation, and the individual is not concerned with the illiquidity of the pension savings as retirement is near and pension payouts are thus starting soon.

While the optimal control diagrams in Figure 3 are informative, they must be coupled with the probabilities of ending up in different values of y and a at each age-level to see typical life-cycle patterns. Hence, we simulate 10,000 possible life-time paths. Along each path, we draw random numbers to represent the annual shocks to labor income, stock prices, and health costs. Starting from the stated initial values and using the optimal controls derived with our numerical optimization approach, this generates life-cycle paths of income, consumption, portfolio weights, contribution rates, private wealth, pension wealth, etc. The age-specific averages across the 10,000 paths indicate expected life-cycle patterns.

The upper-left panel of Figure 4 shows that expected consumption (solid curves) has the hump-shaped life-cycle pattern seen in the data (Thurow, 1969; Gourinchas and Parker, 2002). The dotted blue curve shows expected income (including Social Security) after tax and health costs. Expected consumption is larger at every age when the individual has access to the pension plan. The dashed curves indicate the 5th percentile of consumption which is also larger with the pension plan than without, so the plan reduces the risk of low consumption at any age and mostly so in retirement.

[FIGURE 4 about here.]

The upper-right panel illustrates that, without a pension plan (dark curve), the fraction of income saved starts at around 15%, is almost flat until age 45, and then declines significantly and turns negative at age 60. With a pension plan, the individual saves both privately (orange curve) and via the pension plan (yellow curve). In the first years, the individual prefers private savings to build up a liquid wealth buffer, but pension contributions increase from about 5% at age 30 to 10% around age 50 and then steeply to 30% in the final years before retirement where wealth is moved from the private account to the pension account.[17]

The lower-left panel shows that, without the pension plan (dark curve), the individual invests all savings in stocks until around age 55, where the stock weight starts to decline and ends up around 50%. The yellow kinked line depicts the glidepath strategy of the pension plan which this individual finds too conservative so the private stock weight (orange curve) is kept at 100% longer and subsequently kept larger than in the case without a plan.

The lower-right panel displays the typical tent-shaped wealth pattern. With the pension plan, most wealth is accumulated in the plan (yellow curve) with an expected after-tax value of

[17]The orange and yellow curves are non-smooth since it is difficult to precisely identify the optimal contribution rate at any specific age. The individual's utility is almost unaffected if she today saves a little less in the pension account and a little more in the private account and then does the opposite next year. Hence, we focus on the broad age-trend in the contribution rate and on how the accumulated pension wealth evolves.

$333,000 at retirement. Private wealth (orange curve) peaks at age 57 at $122,000, is reduced to $50,000 towards retirement (financing high contributions), and then grows to $73,000 in the late 80s, mainly due to the risk of large medical expenses. Total expected wealth is lower with a plan (green curve) than without (dark curve), e.g., $371,000 vs. $491,000 at age 65. The plan's annuitization insures the individual against "outliving her wealth" so lower savings is needed which facilitates higher consumption throughout life.

B. Basic pension plans

The top row of Table 4 refers to the benchmark case with no pension plan, where the optimal decisions lead to an expected ratio of private wealth to after-tax income of 1.9 at age 35, 6.1 at age 50, and 14.3 at age 65. In retirement, the ratio is calculated using the Social Security benefits as income (without subtracting medical costs), so the ratio increases at retirement where income drops. Subsequently, the ratio declines when wealth is decumulated to finance consumption expenditures exceeding Social Security but, as indicated by the wealth-income ratio of 21.1 at age 85, the decumulation is slow since the individual faces the risk of substantial medical costs as well as the risk of living long in which case wealth must cover consumption for many years.

[TABLE 4 about here.]

Table 4 shows utility gains, relative to the no-plan case, and private and pension wealth relative to income for a range of basic pensions plans following one of the investment strategies IP1-4. When choosing each year the contribution to a non-annuitized plan with a preset flat payout schedule, the individual prefers a plan with full stock investments, cf. Panel A of Table 4. This generates a utility gain of 0.66% or $5,700 in PV terms. The accumulated pension wealth relative to income is expected to be 0.7 at age 35, 3.0 at age 50, and 9.9 at age 65, jumps at retirement and then declines as payouts are made. With the plan, less private wealth is accumulated. At age 65, the aggregate expected wealth-income ratio is 15.1 (sum of 5.2 and 9.9), compared to 14.3 in the no-plan case. The utility gain is lower for the other investment strategies, e.g., 0.52% with a TDF strategy. By definition, utility gains are lower with a predetermined contribution schedule instead of self-selected contributions. As explained above, the individual dislikes plans with large contributions in early adulthood, and the best fixed contribution rate is 5% when contributions are made from age 25, 7% from age 30, 11% from age 35, and 15% from age 40. For these plans, the gain reduction relative to the plan with self-selected contributions is modest as the individual can undo much of the unwanted pension savings by adjusting private savings.

Also the additional utility gain from selecting the payout ratio each year instead of following a preset schedule is modest, cf. Panel B of Table 4. For example, with self-selected contributions and the all-stocks strategy, the utility gain is 0.77% with self-selected payouts compared to 0.66% with flat payouts. The modest difference is partly due to the RMD lower bound that limits the individual's flexibility regarding payout ratios. With self-selected payouts, the pension savings are accessible throughout retirement, so the individual can substitute private savings by pension savings and thereby reduce capital gain taxes.

Panel C of Table 4 shows that utility gains are substantially larger with the fully annuitized plans than the non-annuitized plans, despite the assumed 15% cost of annuities. Non-annuitized plans benefit individuals only through lower capital gains taxes, whereas fully annuitized plans also allow individuals to share lifetime risks and thus provide insurance against living long. On the other hand, with annuitized plans, individuals must stick to a predetermined payout schedule which, in this section, is required to deliver flat expected payouts. With fully annuitized plans, our baseline individual prefers that pension savings are invested following the TDF strategy. With self-selected contributions, this leads to a utility gain of 3.19% (or $27,600) instead of 0.77% [0.66%] for the non-annuitized plan with self-selected [flat] payouts. While the expected pension wealth-income ratio is similar in the two cases, the private wealth-income ratio is considerably lower with the fully annuitized plan than with the similar non-annuitized plan with flat payouts, e.g., 2.0 instead of 4.3 at age 65 with self-selected contributions and the TDF strategy.

Among fully annuitized plans with a constant contribution rate, a 7% contribution is optimal if contributions must be made from age 25, but the individual is better off with a larger contribution rate if contributions start later, e.g., a plan with 19% contributions from age 40 leads to a utility gain of 2.71%, not far from the 3.19% gain with self-selected contributions. If well designed, plans leaving few choices to individuals (and thus being less exposed to behavioral mistakes) can still lead to substantial utility gains.

C. Flexible pension plans

Table 5 explores how much the base case individual appreciates payout flexibility in the form of partial annuitization (I less than 1) and non-flat expected payouts (x different from zero). For all plans considered, pension savings are invested according to the TDF strategy, which is the optimal strategy of the base case individual for most plans, including those generating the largest utility gains. For tractability, we consider only I in multiples of 0.1 and x in multiples of 2% and, given the RMD lower payout bound, -8% is the lowest admissible value for the fully annuitized plan with a TDF strategy. Among plans with self-selected contributions, Panel A shows that when maintaining flat expected payouts, the optimal annuitization ratio is 0.9 which leads to an increase in the utility gain of only 0.07 percentage points compared to full annuitization. Panel B shows that when maintaining full annuitization, a plan with an excess AIR of -4% (i.e., expected payouts increasing about 4% each year) adds 0.33 percentage points to the utility gain compared to flat payouts. Panel C shows that when varying both, the optimal annuitization ratio is still 0.9 and the excess AIR still -4% with a combined incremental utility gain of 0.38 percentage points or around $3,300 in PV terms.

[TABLE 5 about here.]

The other columns of Table 5 show similar results for plans with a fixed contribution rate from a certain age. The lower the contribution rate and the later the starting age, the less valuable is partial annuitization and the more valuable is a more steeply increasing payout schedule. When both I and x are flexible, an annuitization ratio I of 0.9 is optimal for all plans, and most plans are best with a negative excess AIR x and thus increasing expected payouts through retirement. The additional utility gain is between 0.33 and 0.63 percentage points ($2,800-5,500 in PV terms) compared to the basic version of the plan.

In our main specification, the individual selects the payout parameters I and x when initiating the pension plan at age 25. The Internet Appendix considers the alternative specification where I and x are chosen at retirement depending on the individual's pre-tax income Y, private wealth F, and pension wealth A entering retirement. Across a range of combinations of Y, F, and A, the optimal annuitization ratio is 0.9 for the base case individual and thus identical to the optimal choice at age 25 for all the contribution schemes covered by Table 5. When pension wealth is relatively small and private wealth large, the optimal excess AIR x at retirement is -8%

so that the small pension wealth is decumulated slowly in the beginning of retirement. When pension wealth is relatively large and private wealth small, the optimal x at retirement is -4% or even -2% so that the large pension wealth is decumulated more rapidly although still with increasing payouts through time. These findings are consistent with Table 5 where a more negative x is chosen at age 25 for plans with lower contributions. Letting the individual choose these parameters at retirement invalidates the homogeneity property (12) before retirement and thus drastically increases computational complexity (see the Internet Appendix), so we do not consider optimal pre-retirement behavior and initial utility gains for this specification. However, the results indicate some additional utility gain from delaying the choice of excess AIR until retirement.

V. Preferred plans of different individuals

This section presents results for individuals with different characteristics. Compared to the baseline case, we consider the effects of changing one parameter at a time: an RRA γ of 2 or 6 (instead of 4), an EIS ψ of 0.1 or 0.5 (instead of 0.25), a bequest weight ξ of 0.2 or 5 (instead of 1), a subjective discount factor β of 0.93 or 0.99 (instead of 0.96), and an initial pre-tax annual income Y_{t_1} of \$30,000 or \$50,000 (instead of \$40,000).[18]

Panel A of Table 6 shows the best basic plans for the different individuals, i.e., plans with (i) full annuitization ($I = 1$) and preset flat expected payouts ($x = 0$) or (ii) no annuitization with

[18]When varying the initial income, we keep the state pension unchanged in dollar terms by simultaneously changing ζ from 0.45 to 0.6 and 0.36, respectively, and we keep the same maximum annual dollar contribution by changing $\bar{\alpha}$ from 0.4 to 0.533 and 0.32, respectively.

self-selected payouts respecting the RMD lower bound. The utility gains range from 0.81% to 5.07% or from $6,996 to $43,811 in PV-terms. The RRA 2 individual has the smallest gain and, as the only individual considered, prefers a non-annuitized plan that invests all savings in stocks. The utility gain is completely due to the tax advantage for this individual, who is willing to keep a low liquid wealth and thus build up significant retirement savings with high expected and untaxed returns. All other individuals prefer a fully annuitized plan, despite the 15% cost and Social Security benefits, with savings invested according to the TDF strategy. The largest gains are obtained by the individuals with a high discount factor or high RRA who are most concerned with ensuring a decent consumption level if they should live long. The individual with a high bequest incentive builds up more liquid wealth (which can be bequeathed) than pension wealth, whereas the others are willing to tie up most of their wealth in the pension account. For all individuals, a big part of the pension wealth at retirement stems from large contributions in the years just before retirement, as was illustrated for the base case preferences in Figure 4. Especially for individuals with a high RRA, a low bequest weight, or a high discount factor, the gains are much larger for the fully annuitized plans than for the corresponding non-annuitized plans. For example, with RRA 6, the utility gain is only 0.84% for the non-annuitized plan with preset flat expected payouts compared to 4.97% with the fully annuitized plan.

[TABLE 6 about here.]

Panel B of Table 6 allows partial annuitization and non-flat payouts. While the RRA 2 individual still prefers the non-annuitized plan, the other individuals appreciate the extra flexibility with an additional utility gain of up to 0.96 percentage points or $8,257. Most individuals prefer an annuitization ratio around 0.9 but, as expected, the individual with a high bequest incentive selects a lower value. Most individuals choose a plan with expected payouts increasing around 4% each year. The flexibility in scheduled payouts affect the saving patterns of the individuals. The base case individual accumulates less wealth in the pension plan and more private wealth when having access to the preferred flexible plan than the preferred basic plan: at age 65, the ratio of pension wealth to annual income is 7.5 instead of 8.8 and the ratio of private wealth to annual after-tax income is 3.5 instead of 2.0. A few individuals change their saving behavior in the opposite direction. For example, for the individual with high bequest weight, the age-65 ratio of pension wealth to annual income is 8.0 with the flexible plan instead of 6.8 with the basic plan and the ratio of private wealth to annual after-tax income is 6.1 instead of 7.3 but, due to a simultaneous decrease in the annuitization ratio from 1 to 0.5, the desired high bequest upon death is maintained as 50% of the pension account is now also bequeathed.

VI. Additional analyses

A. The role of uninsurable late-life medical expenses

Our main model features significant out-of-pocket medical expenses in retirement which reflects the situation of many U.S. households. This section considers the case with tax-financed medical expenses as in many European countries. With the assumed dynamics and initial level, the present value of lifetime income after medical expenses and a 30% income tax is $859,242. Eliminating the medical expenses increases the present value to $879,049 if the tax rate is maintained at 30%, but by increasing the tax rate to 31.5773% the present value of income is back at $859,242 so this is the tax rate assumed in the following calculations.

Panel A of Table 7 shows that, without access to pension plans, 9 out of the 11 individuals considered prefer tax-financed medical costs with a utility gain of up to 1.85% or almost $16,000 in PV-terms. Only the low-RRA individual prefers self-paid medical expenses in exchange for a lower income tax rate, whereas the high-bequest individual is essentially indifferent. With self-paid medical expenses individuals build up savings at age 65 that are up to 39% higher (for the low-income individual) than with tax-financed expenses.

Panel B of Table 7 displays the best basic and best flexible pension plan with self-selected contributions when medical expenses are tax financed. Utility gains are sizeable, although significantly smaller than in the main setting with out-of-pocket expenses, see Table 6, due to the reduced need for large retirement savings. For the base case individual, the utility gain is 1.47% instead of 3.19% for basic plans and 1.68% instead of 3.57% for flexible plans. A well-designed pension plan is more valuable to individuals in a U.S.-like system with out-of-pocket medical expenses than in a European-style system with tax-financed medical expenses. Facing less disposable income risk, many of the individuals take more investment risk and prefer plans investing 100% in stocks. When restricted to basic plans, the individual with a high bequest weight now prefers a non-annuitized plan with self-selected payouts to a fully annuitized plan, but this individual marginally prefers a flexible plan with a modest annuitization ratio. With plan flexibility, most individuals still choose a relatively large—although sometimes slightly lower—annuitization ratio and upward-sloping payouts. The flexibility increases the utility gain by up to 0.64 percentage points or $5,514 in PV-terms. Also when equipped with the best pension plan, total retirement savings tend to be somewhat lower with tax-financed medical expenses than without.

[TABLE 7 about here.]

B. Tax incentives and welfare considerations

The access to annuitization and tax-free returns on pension savings makes pension plans attractive, whereas the illiquidity of pension savings is the main downside. With pension plans, individuals accumulate less total savings and much less savings in taxed private accounts, so total tax revenue decreases. From a social welfare perspective the individuals' utility gains are balanced by the lower tax revenue. We calculate the PV of life-time taxes on income and returns as the sum of expected tax payments at each age (average across simulated paths) discounted using the discount rate applied to income, cf. Section D.

For the base case individual, the PV of taxes is $433,600 without pension plans. With the optimal basic [flexible] plan, the individual's utility gain in PV-terms is $27,600 [$30,900] while the PV of taxes drops by $37,600 [$36,500], cf. the first row in each panel in Table 8. These numbers question whether such plans improve social welfare, but a full-scale social welfare analysis must include other hard-to-quantify effects excluded from our model. For example, with a pension plan, the individual increases consumption throughout life which boosts sales tax revenues and employment with ripple effects on corporate and income tax revenue. Also, with sizeable and annuitized savings, retirees draw less on public support.

[TABLE 8 about here.]

The tax advantage is not the main source of the utility gain. Table 8 shows that with a 20% tax rate on all returns, the utility gain of the best basic plan would still be 2.45% (instead of 3.19%) or $21,200 in PV-terms, and in this case tax revenues are only $16,300 lower than in the no-plan setting. For the best flexible plan, the gain is \$25,800 compared to a reduced tax revenue of \$15,300. The utility gains now exceed the drop in taxes. While the overall savings are roughly unchanged, removing the tax exemption leads to lower savings in the pension account and full annuitization of those lower savings becomes optimal (I^* goes from 0.9 to 1) with a more steeply increasing payout profile (x^* goes from -4% to -6%).

The largest source of the utility gain is the access to annuitization, despite the assumed 15% cost. Reducing this to 5%, the gains increase by up to another percentage point or more than \$8,000 in PV-terms, and the optimal annuitization ratio and pension savings go up (offset by lower private savings), cf. Table 8. Even the RRA 2 individual would then annuitize, with an annuitization ratio of 0.5 and an excess AIR of 2% that generates a 0.95% utility gain, 0.14 percentage points better than the non-annuitized plan with self-selected payouts. Initiatives lowering actual or perceived costs of annuitization could generate substantial utility gains to retirement savers.

C. Heterogeneous mortality risk and annuity demand

Our main analysis applies population mortality rates when calculating annuity payouts and assumes that the individual's mortality risk is identical to that of the population. What happens if the individual has a different life expectancy than the average individual in the population? We consider a "strong" individual who, at each age until 100, has a probability of dying which is 50% below that of the population and a "weak" ["very weak"] individual with a 50% [150%] higher mortality risk at each age. The top panel of Table 9 shows the implications for the expected lifetime at different ages. When entering retirement at age 67, the average individual expects to live until age 85.0, whereas the very weak, the weak, and the strong individuals expect to live until 78.7, 82.1, and 90.1, respectively.[19]

[TABLE 9 about here.]

Table 9 confirms that the strong individual with a higher-than-average expected lifetime benefits more from annuities. For flexible plans, the utility gain is 4.96% for the strong individual compared to 3.57% for an individual with an average mortality. With a gain of 1.23%, even the very weak individual benefits from annuitization. Despite the assumed 15% costs, full or partial annuitization is attractive even to individuals with a considerably lower expected lifetime than assumed in the calculation of annuity payouts. The weaker individuals build up less total retirement savings and keep a smaller fraction of overall savings in the pension account. All the individuals choose an annuitization ratio of 0.9, whereas the desired payout profile is more steeply increasing for individuals with longer expected lifetimes.

D. Option to make unscheduled withdrawals

As an extension, we now allow the individual at the beginning of each year t to choose a fraction $M_t \geq 0$ of the pension account A_t to be paid out in addition to any scheduled payouts. To obtain an unscheduled cash payout of $M_t A_t$, the value of the annuity portfolio is reduced by

[19]We assume the individual applies correct mortality rates when making decisions. Heimer, Myrseth, and Schoenle (2019) find that young [old] individuals underestimate [overestimate] expected lifetime and study the impact on saving behavior in a life-cycle model without pension plans. O'Dea and Sturrock (2023) show how underestimation causes lower annuity participation. To isolate the effects of varying the mortality risk, we fix the probability and consequences of medical shocks but, obviously, these risks might be related.

$(1 + k_t)M_tA_t$, where $k_t \geq 0$ reflects a cost or penalty. For 401(k)s in the U.S., a 10% tax penalty is paid on withdrawals before the age of 59.5 years.[20] In addition, we assume a cost equal to 1% of any unscheduled payout to represent any actual and psychological burdens related to obtaining such payouts. Then we also avoid any simultaneous contributions and withdrawals from age 60 to retirement. Hence, we let $k_t = 0.11$ for $t \leq 60$ and $k_t = 0.01$ for $t > 60$. See Section IA.5 of the Internet Appendix for additional information.

For the base case individual, the withdrawal option increases the utility gain associated with the best basic pension plan marginally from 3.19% to 3.28%. Early withdrawals occur only when the individual has a relatively high pension wealth (large a) and low income (small y). At each age below 60, no early withdrawals are made in more than 95% of the simulations. After age 60, early withdrawals are more common but typically small; in more than 95% of the simulations, the maximum unscheduled withdrawal is less than $300.

The withdrawal option has a more notable impact on flexible plans. With this option, the base case individual prefers a steeper payout profile with an excess AIR of -8% instead of -4%. The scheduled payouts can be more backloaded since the individual can make early unscheduled payouts when needed, thus making the total payouts state dependent. The optimal annuitization ratio remains unchanged at 0.9. Adding the withdrawal option increases the utility gain for the best flexible plan from 3.57% to 3.85%, an increase corresponding to around $2,377 in PV-terms. On the other hand, the withdrawal option complicates the risk management procedures of the

[20] Exceptions to the 10% tax exist and include some higher-education expenses, first home purchase, uninsured medical expenses, terminal illness, and death. Unemployed individuals can also make some non-penalized withdrawals according to specific rules. Our model disregards these exceptions (medical expenses are included only for retired individuals where the penalty does not apply in any case).

annuity provider and may thus lead to a larger annuity cost. If the withdrawal option causes the cost to increase from 15% to 19% or more, the utility gain is indeed lower with the option than without.

VII. Conclusion

In a rich life-cycle model, we show that the access to well-designed retirement saving plans greatly improves the utility of rational workers across a range of individual characteristics. The utility gains are due to the access to annuitization (despite significant costs) and a return tax advantage, and the gains are significantly increased if the plans allow for partial annuitization, non-flat payout schedules, and maybe also early withdrawal options if this flexibility does not considerably increase annuity costs. Most of the individuals considered prefer a target-date fund investment strategy, pension payouts that are expected to increase through retirement, as well as a large degree of (but less than full) annuitization of the retirement savings. Individuals' optimal contributions to the retirement saving plans are typically small early in working life and then increases slowly with age until the final years before retirement where optimal contributions tend to be large. Whether the possibly large medical expenses in retirement are tax-financed or paid out of the individual's own pockets matters for which retirement saving plan is best and how big the associated utility gain is.

Various extensions of our model seem interesting to explore. First, while the Vanguard-style target-date fund strategy included in our study is preferred by many of the individuals considered, alternative specifications of the strategy might be even better, e.g., with a larger stock weight in the final part of the strategy or a different period over which the stock weight is reduced. Second, additional individual characteristics (such as home ownership and thus the possible saving through home equity accumulation) and alternative preferences (such as habit formation) could be investigated, although at significant computational costs.

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FIGURE 1

Account values

The individual's age is depicted along the horizontal axis. The blue lines show the value of a riskfree plan at the beginning of each year, and the orange-red lines the expected value of an index-linked plan. The solid lines represent personal products $(I = 0)$ and the dashed lines lifelong annuities ($I = 1$). The dark-colored lines are for plans initiated at retirement with a \$100 investment, whereas the light-colored lines are for a plan with gradual savings of \$1.9063 every year from age 25 to retirement at age 67 (indicated by (G) in the legend). Additional information can be found in the main text. Please see online version for correct colors.

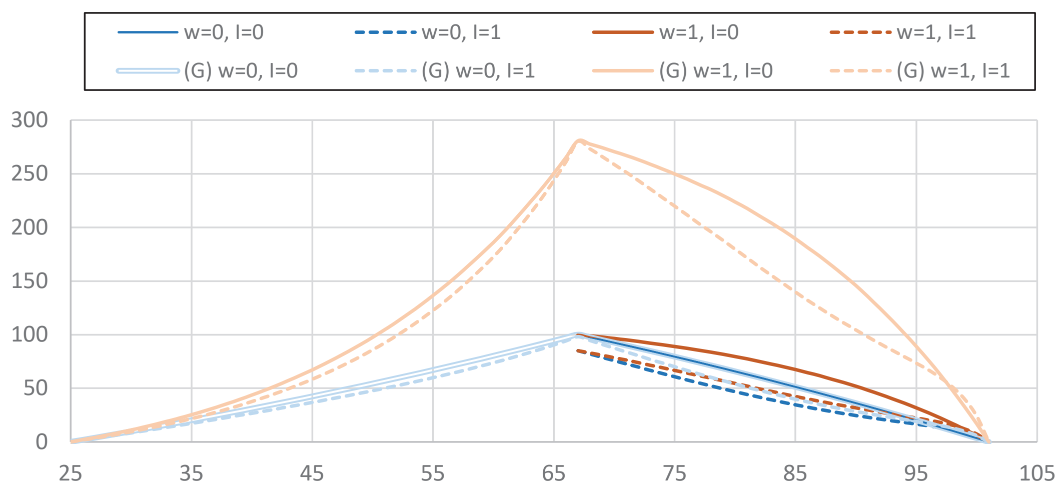

FIGURE 2

Payout rates

The figure shows payout rates m_t as a function of age t for different retirement saving plans. Panel A considers plans where investments follow the target-date fund strategy with solid curves representing plans with an excess AIR of $x = 0$ and dotted curves plans with $x = -10\%$; the orange curves are for plans with full annuitization ($I = 1$) and the gray curves for plans with no annuitization ($I = 0$). The black dashed curve depicts the required minimum distribution. Panel B considers non-annuitized plans with a constant stock weight of either $w = 0$ (orange curves), $w = 0.5$ (gray curves), or $w = 1$ (blue curves); the solid curves are for plans with an excess AIR of $x = 0$ and the dotted curves for plans with $x = -10\%$. Note that the vertical axes have a logarithmic scale. Please see online version for correct colors.

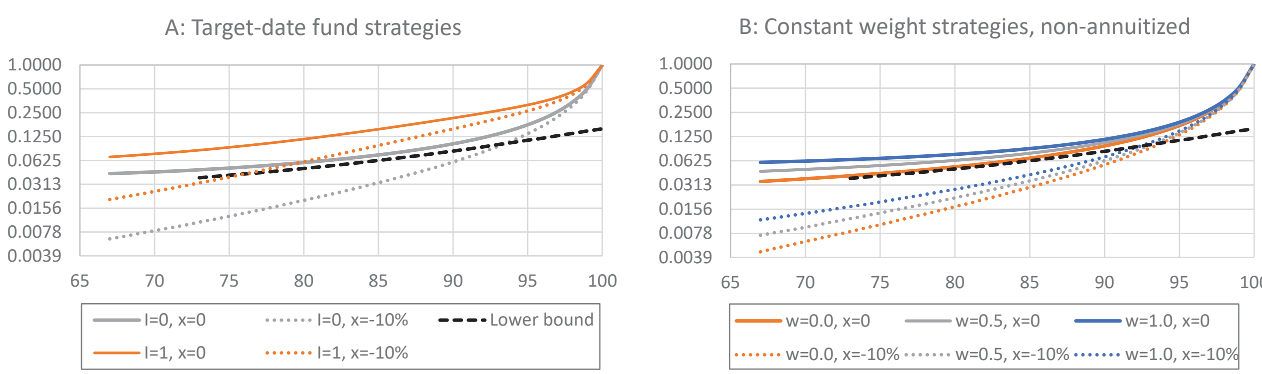

FIGURE 3

Optimal controls

The figure shows, at different age levels, how optimal controls vary with the income-wealth ratio y. The upper panels show c, i.e., the optimal consumption as a fraction of disposable wealth. The middle panels show π, i.e., the fraction of private wealth invested in stocks. The lower panels show the contribution rate α, i.e., the fraction of income saved in the pension account. The solid gray curves are for the case without a pension plan. With a pension plan, the optimal controls also depend on a, the fraction of pension wealth to total wealth. Here, the dashed orange curves are for $a = 0.3$ and the dotted green lines are for $a = 0.6$. The baseline parameter values from Table 3 are assumed. The pension plan applied is the optimal basic pension plan with full annuitization and the target-date investment strategy. Please see online version for correct colors.

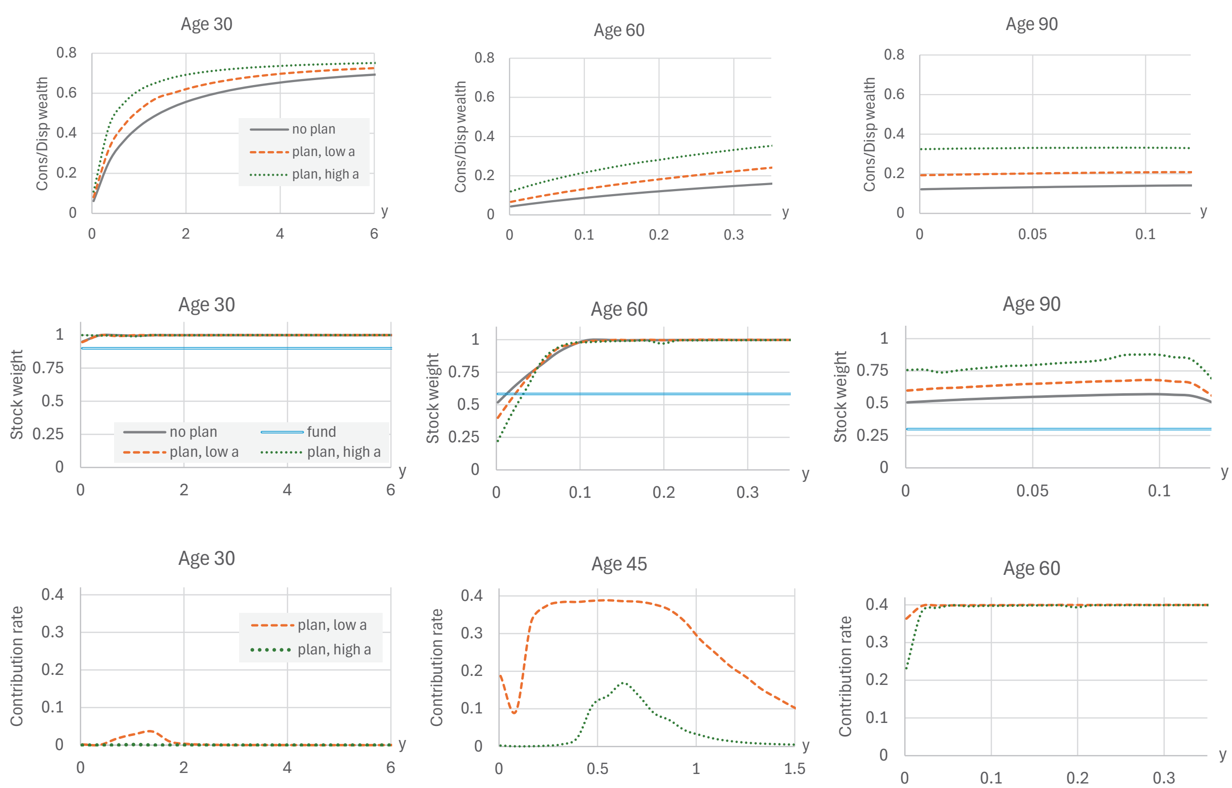

FIGURE 4

Life-cycle patterns with and without a pension plan

The diagrams show expected consumption, saving rates, private stock weight, and wealth as a function of age, both without a pension plan (solid dark curves) and with a pension plan characterized by a TDF investment strategy, full annuitization, and flat expected payouts. The upper-left diagram shows consumption with a plan (orange) and expected income after tax and medical expenses (dotted blue), as well as the 5th percentiles of consumption at each age without (dashed gray) and with (dashed orange) the pension plan. For the case with access to the plan, the panels with saving rates, stock weight, and wealth show both the private component (orange) and the pension component (yellow). The lower-right panel also depicts total wealth with a pension plan (green). Parameter values are taken from Table 3. The income and plan wealth shown are after income tax. Please see online version for correct colors.

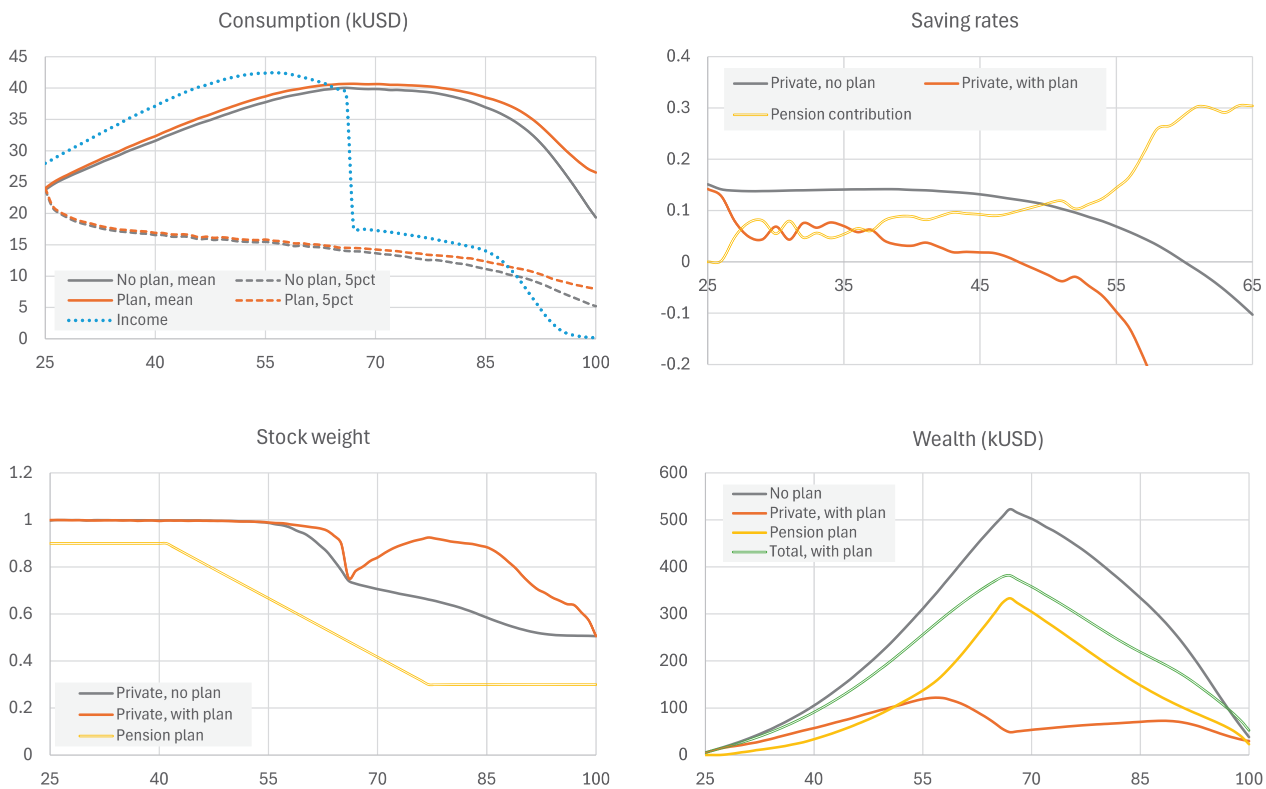

TABLE 1

Annual payouts from plans with constant expected payouts

The table illustrates annual payouts from various retirement saving plans with constant expected payouts from retirement until death. Each row corresponds to a given plan defined by the stock weight w and the annuitization ratio I; 'TDF' in the w-column refers to the glidepath strategy IP4 defined in the text. In Panel A, the individual invests \$100 in the plan just after turning 67 years old. In Panel B, the individual invests \$1.9063 in the plan every year from age 25 to age 66 which for the plan with $w = I = 0$ leads to \$100 of savings at retirement. The plan provider charges a fee of $I \times 15\%$ of any investment and invests the rest in the given portfolio. The third column shows the expected annual payout and the right part of the panel shows the 10th and the 90th percentiles in the distribution of payouts at age 70, 80, 90, and 99. The expectation and percentiles are based on 100,000 simulated paths.

w	I	Expected	10th percentile at age				90th percentile at age			
		payout	70	80	90	99	70	80	90	99
Panel A: \$100 invested at age 67										
0	0	3.45	3.45	3.45	3.45	3.45	3.45	3.45	3.45	3.45
	0.5	4.49	4.49	4.49	4.49	4.49	4.49	4.49	4.49	4.49
	1	5.09	5.09	5.09	5.09	5.09	5.09	5.09	5.09	5.09
0.5	0	4.62	3.85	3.09	2.65	2.37	5.45	6.38	6.99	7.40
	0.5	5.64	4.69	3.77	3.24	2.89	6.65	7.78	8.52	9.03
	1	6.16	5.12	4.12	3.53	3.15	7.26	8.49	9.31	9.86
1	0	5.97	4.05	2.46	1.71	1.28	8.15	10.49	11.83	12.56
	0.5	6.89	4.68	2.84	1.97	1.48	9.41	12.11	13.67	14.51
	1	7.30	4.96	3.01	2.09	1.57	9.97	12.84	14.48	15.37
TDF	0	4.26	3.61	3.18	2.95	2.79	4.95	5.47	5.75	5.96
	0.5	5.32	4.51	3.96	3.69	3.48	6.18	6.83	7.17	7.44
	1	5.88	4.98	4.38	4.07	3.85	6.82	7.54	7.93	8.22
Panel B: Constant contribution age 25-66										
0	0	3.45	3.45	3.45	3.45	3.45	3.45	3.45	3.45	3.45
	0.5	4.83	4.83	4.83	4.83	4.83	4.83	4.83	4.83	4.83
	1	5.86	5.86	5.86	5.86	5.86	5.86	5.86	5.86	5.86
0.5	0	7.54	4.43	3.89	3.47	3.14	11.29	12.07	12.72	13.22
	0.5	9.92	5.81	5.10	4.55	4.12	14.90	15.91	16.77	17.41
	1	11.69	6.82	5.99	5.34	4.84	17.59	18.77	19.78	20.54
1	0	16.75	4.80	3.48	2.61	2.03	33.27	35.41	36.79	37.48
	0.5	20.95	5.95	4.32	3.24	2.52	41.71	44.36	46.07	46.93
	1	24.05	6.78	4.93	3.69	2.88	47.97	50.99	52.91	53.85
TDF	0	8.77	4.12	3.90	3.74	3.62	14.82	15.11	15.31	15.52
	0.5	11.84	5.52	5.23	5.02	4.85	20.06	20.46	20.72	20.99
	1	14.16	6.55	6.21	5.96	5.76	24.04	24.51	24.82	25.14

TABLE 2

Products with non-constant expected payouts

The table covers various retirement saving plans with payouts throughout the retirement period but with potentially non-constant expected payouts. Each row corresponds to a plan defined by the stock weight w, the annuitization ratio I, and the excess AIR x. The individual invests $100 in the retirement savings product just after turning 67 years old. The product issuer charges a fee of $\$15 \times I$ and invests the rest in a portfolio. The table shows the average of the expected payouts at each age from 67 to 99, as well as the expectation and the 10th percentile of the payout at age 70, 80, 90, and 99. For investment strategies involving stocks the results are based on 100,000 simulated paths.

			Average	Expected payout at age				10th percentile at age			
w	I	x	expected	70	80	90	99	70	80	90	99
0	0	-8%	3.70	0.94	2.09	4.66	9.57	0.94	2.09	4.66	9.57
		-4%	3.58	1.94	2.89	4.31	6.18	1.94	2.89	4.31	6.18
		0%	3.45	3.45	3.45	3.45	3.45	3.45	3.45	3.45	3.45
		4%	3.33	5.29	3.55	2.38	1.66	5.29	3.55	2.38	1.66
		8%	3.23	7.10	3.19	1.43	0.70	7.10	3.19	1.43	0.70
0	1	-8%	9.35	2.37	5.28	11.76	24.15	2.37	5.28	11.76	24.15
		-4%	6.77	3.66	5.45	8.13	11.66	3.66	5.45	8.13	11.66
		0%	5.09	5.09	5.09	5.09	5.09	5.09	5.09	5.09	5.09
		4%	4.07	6.48	4.34	2.91	2.03	6.48	4.34	2.91	2.03
		8%	3.49	7.68	3.45	1.55	0.75	7.68	3.45	1.55	0.75
1	0	-8%	8.60	2.18	4.86	10.83	22.21	1.48	2.01	3.09	4.78
		-4%	7.20	3.89	5.81	8.68	12.41	2.64	2.40	2.48	2.67
		0%	5.97	5.97	5.97	5.98	5.97	4.05	2.46	1.71	1.28
		4%	5.03	8.01	5.37	3.60	2.51	5.44	2.22	1.03	0.54
		8%	4.41	9.70	4.36	1.96	0.95	6.59	1.80	0.56	0.21
1	1	-8%	16.23	4.12	9.18	20.45	41.93	2.80	3.78	5.84	9.02
		-4%	10.61	5.73	8.56	12.78	18.29	3.90	3.53	3.65	3.93
		0%	7.30	7.30	7.30	7.30	7.30	4.96	3.01	2.09	1.57
		4%	5.44	8.66	5.81	3.90	2.71	5.88	2.40	1.11	0.58
		8%	4.41	9.71	4.37	1.96	0.95	6.60	1.80	0.56	0.21

TABLE 3

Baseline parameter values

See the main text for the motivation of the assumed parameter values.

Parameter	Description	Value
	Financial assets	
r	Riskfree interest rate	0.01
μ_S	Expected excess stock return	0.04
σ_S	Stock volatility	0.157
	Horizon, preferences, and initial wealth	
t_1	Initial age in years	25
t_R	Retirement age in years	67
t_M	Maximum age in years	100
γ	Relative risk aversion	4
ψ	Elasticity of intertemporal substitution	0.25
β	Subjective discount factor	0.96
ξ	Bequest strength parameter	1
F_{t_1}	Initial financial wealth (thousand USD)	5
A_{t_1}	Initial pension wealth (thousand USD)	0
	Income	
Y_{t_1}	Initial annual income (thousand USD)	40
σ_Y	Income volatility	0.1
ρ_{YS}	Income-stock correlation	0
ζ	Social Security relative to final salary	0.45
	Tax rates and costs	
τ_Y	Income tax rate	0.3
τ_F	Tax rate on private returns	0.2
τ_A	Tax rate on retirement returns	0.0
K	Proportional annuity costs	0.15

TABLE 4

Base case preferences: basic plans

The top row refers to the case without a pension plan, Panel A to pension plans with no annuitization ($I = 0$) and a preset flat expected payout schedule ($x = 0$), Panel B to plans with no annuitization and self-selected payouts, and Panel C to fully annuitized ($I = 1$) plans with flat expected payouts ($x = 0$). The first two columns indicate how plan contributions and plan investments are made; e.g., '5 from 25' means a 5% contribution rate from age 25 to retirement. The middle columns show the expectation of the ratio of financial wealth to after-tax income at the ages 35, 50, 65, 70, and 85 years. The right-most columns show the same for the ratio of after-tax pension wealth to after-tax income. The expectations are based on averages over 10,000 simulated outcomes. In retirement, the income is represented by the after-tax Social Security benefits without subtracting any medical expenses. The base case parameters from Table 3 are used.

Contrib	Invest	Utility gain		Expected private wealth/inc					Expected pension wealth/inc				
		Pct	kUSD	35	50	65	70	85	35	50	65	70	85
No plan		0.00	0.0	1.9	6.1	14.3	32.7	21.1	0.0	0.0	0.0	0.0	0.0
				Panel A: Non-annuitized plans, flat payouts									
Self-select	w=0	0.08	0.7	1.9	6.1	11.5	25.8	17.3	0.0	0.0	2.7	6.5	3.6
	w=0.5	0.26	2.3	1.9	5.2	4.7	9.5	6.8	0.0	0.8	9.1	21.7	13.6
	w=1	0.66	5.7	1.2	3.2	5.2	11.2	9.0	0.7	3.0	9.9	24.2	16.7
	TDF	0.52	4.5	1.3	2.9	4.3	9.2	6.5	0.6	3.2	9.7	22.6	13.7
5 from 25	w=1	0.44	3.8	1.3	3.9	7.4	16.3	11.4	0.6	2.4	7.5	18.4	12.7
	TDF	0.36	3.1	1.3	3.9	8.4	19.0	12.8	0.6	2.2	5.7	13.4	8.1
7 from 30	w=1	0.45	3.9	1.5	3.8	6.9	15.0	10.7	0.4	2.4	8.0	19.7	13.6
11 from 35	w=1	0.42	3.6	1.9	3.7	5.7	12.2	9.5	0.0	2.5	9.3	23.1	15.9
15 from 40	w=1	0.37	3.2	1.9	4.1	5.7	12.0	9.3	0.0	2.0	9.1	22.8	15.7
				Panel B: Non-annuitized plans, self-selected payouts									
Self-select	w=0	0.10	0.8	1.9	6.0	11.4	24.3	16.1	0.0	0.0	2.8	7.9	4.7
	w=0.5	0.31	2.7	1.9	5.3	5.1	6.8	1.1	0.0	0.7	8.6	24.6	19.1
	w=1	0.77	6.6	1.2	3.0	3.7	14.8	12.0	0.7	3.2	11.4	20.4	10.9
	TDF	0.57	4.9	1.2	2.9	2.8	5.4	6.4	0.7	3.2	11.1	26.2	13.7
5 from 25	w=1	0.52	4.5	1.3	3.8	7.3	17.3	12.0	0.6	2.4	7.5	16.8	10.3
	TDF	0.40	3.4	1.3	3.9	8.4	17.2	9.9	0.6	2.2	5.7	15.1	10.9
8 from 30	w=1	0.53	4.6	1.4	3.5	5.9	15.8	11.9	0.5	2.7	9.2	18.9	10.7
12 from 35	w=1	0.51	4.4	1.9	3.5	4.9	15.0	11.9	0.0	2.7	10.2	19.9	10.9
17 from 40	w=1	0.46	4.0	1.9	3.8	4.5	14.6	11.8	0.0	2.3	10.4	20.0	10.8
				Panel C: Fully annuitized plans, flat payouts									
Self-select	w=0	1.96	17.0	1.8	5.5	6.3	13.4	9.4	0.0	0.2	4.9	10.9	5.0
	w=0.5	2.73	23.6	1.7	4.2	2.0	3.4	4.2	0.0	1.1	8.6	19.4	9.6
	w=1	2.25	19.4	1.2	2.7	2.7	6.1	8.0	0.6	2.7	9.9	23.3	12.4
	TDF	3.19	27.6	1.2	2.7	2.0	3.3	4.1	0.5	2.4	8.8	20.0	9.7
7 from 25	w=1	1.52	13.2	1.1	2.9	4.4	10.0	9.5	0.7	2.8	9.0	21.0	11.2
	TDF	2.24	19.3	1.0	2.7	4.7	10.2	7.8	0.7	2.6	6.8	15.3	7.4
10 from 30	TDF	2.54	22.0	1.3	2.6	3.8	7.8	6.4	0.5	2.7	7.5	17.0	8.9
14 from 35	TDF	2.68	23.1	1.8	2.7	3.1	6.1	5.4	0.0	2.5	8.0	18.2	9.1
19 from 40	TDF	2.71	23.4	1.7	3.1	2.9	5.5	5.0	0.0	2.1	8.0	18.3	8.9
27 from 45	TDF	2.68	23.2	1.7	4.0	2.9	5.1	4.8	0.0	1.3	8.0	18.6	9.1

TABLE 5

Base case preferences: partial annuitization and non-flat payouts

Panels A and B show percentage utility gains for plans with the contributions stated in the column heading where, e.g., '7 from 25' means a 7% contribution rate from age 25 to retirement. Panel A varies the annuitization ratio I while keeping a flat expected payout schedule ($x = 0$). Panel B varies the excess AIR x for fully annuitized plans ($I = 1$). Numbers in boldface show the largest gain in each column. For the case where both I and x can be varied, Panel C shows the best combination and the associated percentage utility gain as well as the increase in the utility gain relative to the best basic plan, i.e., the plan with $I = 1$ and $x = 0$. In all cases, the pension plan follows the target-date fund investment strategy. The base case parameters from Table 3 are used.

I	x in pct	Self-select	7 from 25	10 from 25	13 from 25	10 from 30	15 from 30	20 from 30	14 from 35	19 from 40
Panel A: Pct utility gains for plans with partial annuitization and flat expected payouts										
0	0	0.52	0.23	-0.53	-1.99	0.32	-0.59	-2.75	0.35	0.33
0.1	0	1.06	0.61	-0.09	-1.57	0.75	-0.13	-2.36	0.81	0.80
0.2	0	1.57	0.95	0.28	-1.21	1.12	0.26	-2.04	1.21	1.20
0.3	0	2.00	1.24	0.60	-0.92	1.45	0.59	-1.78	1.55	1.56
0.4	0	2.37	1.49	0.87	-0.68	1.72	0.86	-1.56	1.84	1.86
0.5	0	2.68	1.70	1.09	-0.47	1.96	1.09	-1.38	2.09	2.12
0.6	0	2.93	1.88	1.27	-0.31	2.16	1.28	-1.24	2.30	2.33
0.7	0	3.11	2.03	1.42	-0.18	2.32	1.43	-1.12	2.47	2.50
0.8	0	3.24	2.14	1.53	-0.10	2.45	1.54	-1.04	2.60	2.63
0.9	0	**3.26**	2.22	**1.59**	**-0.06**	2.53	**1.59**	**-1.01**	**2.68**	**2.71**
1	0	3.19	**2.24**	1.51	-0.24	**2.54**	1.45	-1.27	2.68	2.71
Panel B: Pct utility gains for plans with full annuitization and non-flat expected payouts										
1	-8	3.42	2.58	1.07	-1.08	2.62	0.57	-2.42	2.53	2.51
1	-6	3.50	2.77	1.46	-0.61	2.90	1.07	-1.88	2.88	2.86
1	-4	**3.52**	**2.80**	1.70	-0.29	**3.01**	1.42	-1.49	**3.06**	**3.06**
1	-2	3.45	2.62	**1.72**	**-0.15**	2.90	**1.56**	-1.28	3.00	3.03
1	0	3.19	2.24	1.51	-0.24	2.54	1.45	**-1.27**	2.68	2.71
1	2	2.69	1.74	1.09	-0.53	2.02	1.11	-1.46	2.14	2.16
1	4	2.07	1.25	0.62	-0.92	1.48	0.66	-1.76	1.57	1.57
Panel C: Best plan with partial annuitization and non-flat expected payouts										
Utility gain, pct		3.57	2.87	1.89	0.10	3.09	1.78	-0.94	3.17	3.19
Extra gain, pct		0.38	0.63	0.38	0.34	0.55	0.33	0.34	0.49	0.48
Optimal I		0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
Optimal x in pct		-4	-6	-4	-2	-4	-2	0	-4	-4

TABLE 6

Heterogeneous individuals: Best plans with self-selected contributions.

Each column represents an individual, first the individual with the base case parameter values, followed by individuals where one parameter is changed as indicated by the column heading. Individuals select contributions to the pension plans optimally depending on age and state variables. Panel A shows the best basic plans, i.e., plans with (i) full annuitization ($I = 1$) and flat expected payouts ($x = 0$) or (ii) no annuitization with self-selected payouts respecting the RMD bound. Panel B allows partial annuitization and non-flat expected payouts. In each case, the percentage utility gain and the optimal plan characteristics are shown, together with the ratio of the pension balance to annual income (i.e., A/Y) and the ratio of private wealth to after-tax-income (i.e., F/Y' where $Y' = [1 - \tau_Y]Y$) both at age 65; the numbers shown are averages across the 10,000 simulated paths.

	Base	RRA (4)		EIS (0.25)		Bequest (1)		Disc fac (0.96)		Income (40)	
	case	$\gamma=2$	$\gamma=6$	$\psi=0.1$	$\psi=0.5$	$\xi=0.2$	$\xi=5$	$\beta=0.93$	$\beta=0.99$	$Y=30$	$Y=50$
Panel A: Best basic plans											
Utility gain, pct	3.19	0.81	4.97	2.79	3.54	3.55	1.11	1.67	5.07	2.77	3.46
Optimal I	1	0	1	1	1	1	1	1	1	1	1
Invest, w	tdf	w=1	tdf	tdf	tdf	tdf	tdf	tdf	tdf	tdf	tdf
$\overline{A/Y}$, age 65	8.8	7.8	11.2	8.3	9.4	9.1	6.8	6.1	12.3	7.8	9.4
$\overline{F/Y'}$, age 65	2.0	0.7	1.7	3.1	1.0	1.2	7.3	1.8	2.3	2.0	2.0
Panel B: Best flexible plans											
Utility gain, pct	3.57	0.81	5.80	3.02	4.49	3.87	1.60	1.87	5.81	3.21	3.81
Extra gain, pct	0.38	0.00	0.83	0.23	0.96	0.32	0.49	0.20	0.74	0.44	0.35
Optimal I	0.9	0	0.9	0.8	1	1	0.5	0.9	0.9	0.9	0.9
Optimal x in pct	-4	0	-4	-2	-8	-4	-4	-4	-4	-6	-4
Invest, w	tdf	w=1	tdf	tdf	tdf	tdf	tdf	tdf	tdf	tdf	tdf
$\overline{A/Y}$, age 65	7.5	7.8	10.0	8.5	6.4	7.0	8.0	5.1	10.8	5.8	7.9
$\overline{F/Y'}$, age 65	3.5	0.7	2.7	3.0	4.1	3.5	6.1	2.8	3.9	4.1	3.8

TABLE 7

Utility gains and best plans with tax-financed medical expenses.

Without access to pension plans, Panel A shows the utility gain of having no out-of-pocket medical expenses and a higher income tax rate instead of significant out-of-pocket expenses and a lower income tax rate, as well as the expected ratio of wealth to after-tax income at age 65 in both cases. Panel B presents the best basic and best flexible pension plan with self-selected contributions in the situation with no out-of-pocket medical expenses and a higher income tax rate. For basic plans, the annuitization ratio is restricted to 0 or 1; with $I = 0$, the individual self-selects payouts from the pension plan. With $I > 0$, the payout plan is preset with the restriction $x = 0$ for basic plans.

	Base	RRA (4)		EIS (0.25)		Bequest (1)		Disc fac (0.96)		Income (40)	
	case	$\gamma=2$	$\gamma=6$	$\psi=0.1$	$\psi=0.5$	$\xi=0.2$	$\xi=5$	$\beta=0.93$	$\beta=0.99$	$Y=30$	$Y=50$
Panel A: No plan; gain relative to case with low tax but medical expenses											
Utility gain in pct	1.21	-1.02	1.64	1.03	1.23	1.30	-0.03	0.34	1.85	1.77	0.76
$\overline{F/Y'}$, age 65											
– w/ medical exp.	14.3	8.1	17.0	15.2	13.1	14.2	16.1	10.1	19.7	12.9	15.3
– w/o medical exp.	11.4	6.7	14.5	12.2	10.4	11.1	13.8	7.4	16.8	9.2	12.8
Panel B: No medical expenses; gain relative to case with no plan and no medical expenses											
Best basic plan											
Utility gain, pct	1.47	0.73	3.09	1.12	1.89	1.76	0.53	0.61	2.79	1.09	1.73
Optimal I	1	0	1	1	1	1	0	1	1	1	1
Invest, w	w=1	w=1	tdf	w=1	w=1	w=1	w=1	w=1	tdf	w=1	w=1
$\overline{A/Y}$, age 65	7.9	5.9	9.3	7.1	8.8	8.3	10.5	4.5	10.3	6.2	8.8
$\overline{F/Y'}$, age 65	2.6	0.5	2.3	3.9	1.3	1.7	2.6	2.6	3.3	2.7	2.7
Best flexible plan											
Utility gain, pct	1.68	0.73	3.29	1.26	2.53	1.96	0.54	0.69	3.18	1.25	2.02
Extra gain, pct	0.21	0.00	0.20	0.14	0.64	0.19	0.02	0.08	0.39	0.16	0.29
Optimal I	0.8	0	0.9	0.7	1	1	0.4	0.7	0.9	0.8	0.9
Optimal x in pct	-4	0	-2	-4	-8	-6	-6	-2	-6	-4	-6
Invest, w	w=1	w=1	tdf	w=1	w=1	w=1	w=1	w=1	w=1	w=1	w=1
$\overline{A/Y}$, age 65	7.5	5.9	9.0	7.0	6.8	6.1	7.5	5.0	9.8	6.1	7.0
$\overline{F/Y'}$, age 65	3.1	0.5	2.7	4.3	3.3	4.0	6.4	2.3	5.1	2.9	4.6

TABLE 8

Base case preferences: the role of taxes and annuity costs

The table shows the effects of varying the tax rate τ_A and the annuity cost K from their base values of 0 and 15%. Panel A focuses on basic plans where the annuitization ratio I is 0 or 1 and flat expected payouts ($x = 0$) if $I = 1$, whereas Panel B allows flexibility in I and x. For all plans, the individual self-selects contributions. Except for τ_A and K, the base case parameters of Table 3 are used.

τ_A	K	I	x	Utility gain		$\Delta \text{PV}_{\text{taxes}}$	Wealth-income ratio, age 65		
				Pct	kUSD	kUSD	Pension	Private	Total
				Panel A: Best basic plan					
0	15%	1	0	3.19	27.6	-37.6	8.8	2.0	10.8
0	5%	1	0	4.16	36.0	-37.6	9.3	1.9	11.2
0.2	15%	1	0	2.45	21.2	-16.3	8.0	2.7	10.7
0.2	5%	1	0	3.45	29.8	-16.4	8.7	2.2	11.0
				Panel B: Best flexible plan					
0	15%	0.9	-4	3.57	30.9	-36.5	7.5	3.5	11.0
0	5%	0.9	-2	4.36	37.7	-39.3	9.5	1.6	11.1
0.2	15%	1	-6	2.99	25.8	-15.3	5.1	6.0	11.2
0.2	5%	1	-4	3.69	31.9	-16.0	6.8	4.4	11.1

TABLE 9

Best pension plans with alternative mortality risk

We assume that the individual optimally chooses the contributions to pension plans. With the non-annuitized plan, the individual also chooses plan payouts under the RMD constraints. For all plans, the individual self-selects contributions. Except for the mortality risk, the base case assumptions and parameter values in Table 3 are used.

	Population	Very weak	Weak	Strong
Expected age at death				
When age 25	79.8	69.3	75.3	87.0
When age 67	85.0	78.7	82.1	90.1
When age 80	89.3	85.1	87.2	92.9
No plan				
$\overline{F/Y'}$, age 65	14.3	10.9	13.0	16.0
Non-annuitized plan				
Utility gain, pct	0.77	0.59	0.70	0.84
$\overline{A/Y}$, age 65	11.4	8.8	10.5	12.5
$\overline{F/Y'}$, age 65	3.7	2.8	3.3	4.3
Invest, w	1	1	1	1
Best basic plan				
Utility gain, pct	3.19	1.16	2.35	4.12
Optimal I	1	1	1	1
$\overline{A/Y}$, age 65	8.8	5.6	7.6	10.1
$\overline{F/Y'}$, age 65	2.0	3.5	2.5	1.6
Invest, w	tdf	tdf	tdf	tdf
Best flexible plan				
Utility gain, pct	3.57	1.23	2.53	4.96
Optimal I	0.9	0.9	0.9	0.9
Optimal x in pct	-4	-2	-4	-6
$\overline{A/Y}$, age 65	7.5	5.3	6.3	7.5
$\overline{F/Y'}$, age 65	3.5	3.9	4.1	4.2
Invest, w	tdf	tdf	tdf	tdf

Internet Appendix to
"Optimal retirement saving and dissaving"

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Claus Munk

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This Internet Appendix supplements the main paper in the following way. Section IA.1 provides additional information about and illustrations of the assumed mortality risk. Section IA.2 clarifies some details regarding the scheduled payout policy. Section IA.3 explains how the utility maximization problem is solved by a dynamic programming approach. Section IA.4 covers the case where the individual can choose the annuitization ratio and the excess assumed interest rate (AIR) at retirement. Section IA.5 discusses some alternative assumptions about stock returns. Finally, Section IA.6 considers the option to make unscheduled withdrawals from the pension account.

IA.1 Lifetime uncertainty

We apply mortality rates from the 2019 U.S. life table (Arias and Xu, 2022) with an imposed maximum age of 100. The left panel of Figure IA.1 shows the probability of being alive at a given age conditional on being alive at age 25 (blue line) or age 67 (green line), as well as the probability of dying at every age (red line). The right panel shows the expected age at death given that you have survived to the current age shown on the horizontal axis. The dark solid curve is based on the population mortality rates in the 2019 life table. The other curves represent the alternative mortality assumptions considered in Section 6.3: the strong individual has, at each age until 100, a probability of dying which is 50% below that of the population, whereas the weak [very weak] individual has a 50% [150%] higher mortality risk at each age. The expected age at death for a 67-year old is written near each curve.

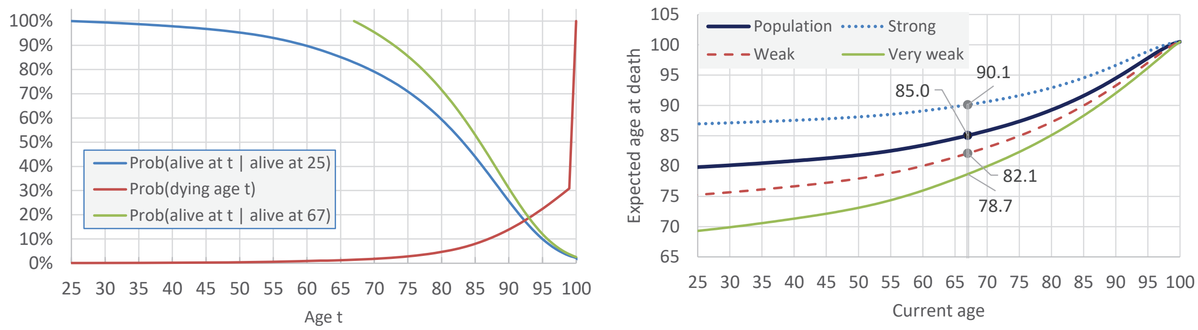

Figure IA.1: **Lifetime uncertainty.** The individual's age is depicted along the horizontal axis. The red line shows the probability of dying in each year. The blue [green] line shows the probability of being alive at the beginning of a given year conditional on being alive when turning 25 [67]. The lines are based on the 2019 life table for the U.S. population (Arias and Xu, 2022) with an imposed maximum age of 100.

IA.2 Details on scheduled payout policy

With a scheduled payout policy given by the age-dependent function m_t, next year's opening account balance is

$$A_{t+1} = (1 - m_t) A_t R_{At} (1 + d_t). \tag{IA.1}$$

The payout pattern is controlled by the so-called *assumed interest rate* (AIR) schedule $\tilde{r}_{t_F}, \ldots, \tilde{r}_{t_L-1}$ together with the mortality-implied write-ups captured by $d_t = I(1-p_t)/p_t$. Specifically, the associated payout rates are defined recursively by

$$m_t = \left(1 + m_{t+1}^{-1} e^{-\tilde{r}_t} (1 + d_t)^{-1}\right)^{-1}, \quad t = t_F, t_F + 1, \ldots, t_{L-1}. \tag{IA.2}$$

The recursion (IA.2) with the terminal value $m_{t_L} = 1$ is solved by

$$\begin{aligned} m_t = \Big(1 &+ e^{-\tilde{r}_t}(1+d_t)^{-1} + e^{-(\tilde{r}_t + \tilde{r}_{t+1})}(1+d_t)^{-1}(1+d_{t+1})^{-1} + \ldots \\ &+ e^{-(\tilde{r}_t + \tilde{r}_{t+1} + \cdots + \tilde{r}_{t_L-1})}(1+d_t)^{-1}(1+d_{t+1})^{-1} \ldots (1+d_{t_L-1})^{-1}\Big)^{-1}. \end{aligned} \tag{IA.3}$$

The recursion implies that

$$m_{t+1} = e^{-\tilde{r}_t}(1+d_t)^{-1} \frac{m_t}{1 - m_t}. \tag{IA.4}$$

Combining the dynamics of the pension account balance in (IA.1) with the above expression for m_{t+1}, we get

$$m_{t+1}A_{t+1} = e^{-\tilde{r}_t}(1+d_t)^{-1}\frac{m_t}{1-m_t}(1-m_t)A_t R_{At}(1+d_t) = m_t A_t e^{-\tilde{r}_t} R_{At}. \tag{IA.5}$$

The excess AIR is

$$x = \tilde{r}_t - \ln \mathrm{E}_t[R_{At}] \tag{IA.6}$$

which we assume constant, and we see that

$$m_{t+1}A_{t+1} = m_t A_t e^{-x}. \tag{IA.7}$$

If $x = 0$ and thus

$$\tilde{r}_t = \ln \mathrm{E}_t[R_{At}] \equiv \ln\left(\tau_A + (1-\tau_A)\exp\{r + w_t \mu_S\}\right), \tag{IA.8}$$

then the scheduled payouts are constant in expectation through retirement. The payout is increasing [decreasing] if the realized log after-tax return $\ln R_{At}$ is greater [smaller] than the assumed interest rate $\tilde{r}_t$ at age t. After substitution of (IA.6), we can rewrite the payout rate recursion (IA.2) as

$$m_t = \left(1 + \left\{m_{t+1}\mathrm{E}_t[R_{At}](1+d_t)e^x\right\}^{-1}\right)^{-1}. \tag{IA.9}$$

IA.3 Solving the utility maximization problem

We assume here that the annuitization ratio I, the excess assumed interest rate x, the pension payout period $[t_F, t_L]$, and the pension investment strategy w_t are given. The scheduled payout pattern represented by m_t is then known. The dynamic programming problem when contributions are chosen by the individual is

$$J_t = \max_{c_t,\pi_t,\alpha_t}\left\{\left(c_t\tilde{F}_t\right)^{1-\frac{1}{\psi}} + \beta\, \mathrm{CE}_t^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{IA.10}$$

where appropriate bounds are imposed on the controls c_t, π_t, α_t and

$$\mathrm{CE}_t = \left(p_t \mathrm{E}_t\left[J_{t+1}^{1-\gamma}\right] + (1-p_t)\mathrm{E}_t\left[\bar{U}_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}. \tag{IA.11}$$

With straightforward minor adjustments, the problems with either predetermined contributions or with self-chosen, non-annuitized payouts (subject to the RMD lower bound) are solved in the same way as the problem (IA.10).

We let $\bar{Y}_t = (1-\tau_Y)Y_t$ and $\bar{A}_t = (1-\tau_Y)A_t$. Following Section IA2.1 in the Internet Appendix of Larsen and Munk (2023), we now verify that

$$J_t = \left(F_t + \bar{A}_t\right) G_t\left(y_t, a_t\right), \tag{IA.12}$$

where

$$y_t = \frac{\bar{Y}_t}{F_t + \bar{A}_t}, \qquad a_t = \frac{\bar{A}_t}{F_t + \bar{A}_t}, \tag{IA.13}$$

and we determine a recursive relation for G.

Final year, $t = t_M$. The individual is sure to die at the end of the period ($p_{t_M} = 0$), leaving a bequest of

$$B_{t_M+1} = F_{t_M+1} = (1 - c_{t_M})\left(F_{t_M} + \bar{Y}_{t_M} + \bar{A}_{t_M}\right) R_{F,t_M}. \tag{IA.14}$$

Here we use that, if the pension account is still positive at t_M, everything is paid out. The certainty equivalent is therefore

$$\begin{aligned} \mathrm{CE}_{t_M} &= \left(\mathrm{E}_{t_M}\left[\xi^{\frac{1-\gamma}{\psi-1}} B_{t_M+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \\ &= \xi^{\frac{1}{\psi-1}}(1-c_{t_M})\left(F_{t_M} + \bar{A}_{t_M}\right)(1+y_{t_M})\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}. \end{aligned} \tag{IA.15}$$

In the final year, there is no contribution to or non-scheduled payment from the pension account, so the individual only has to choose consumption and the private portfolio weight. The indirect utility is thus

$$\begin{aligned} J_{t_M} &= \max_{c_{t_M}, \pi_{t_M}} \left\{ \left(c_{t_M}[F_{t_M} + \bar{Y}_{t_M} + \bar{A}_{t_M}]\right)^{1-\frac{1}{\psi}} + \beta\, \mathrm{CE}_{t_M}^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &= \left(F_{t_M} + \bar{A}_{t_M}\right) G_{t_M}(y_{t_M}, a_{t_M}), \end{aligned} \tag{IA.16}$$

where

$$G_{t_M}(y_{t_M}, a_{t_M}) = (1 + y_{t_M}) \max_{c_{t_M}, \pi_{t_M}} \left\{ c_{t_M}^{1-\frac{1}{\psi}} + \beta \xi^{\frac{1}{\psi}} (1 - c_{t_M})^{1-\frac{1}{\psi}} \left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{IA.17}$$

which in fact is independent of a_{t_M}. The optimal portfolio weight $\pi^*_{t_M}$ is determined numerically by maximizing $\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$. The optimal consumption rate is

$$c^*_{t_M} = \left(1 + \xi\beta^{\psi}\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{\psi-1}{1-\gamma}}\right)^{-1}. \tag{IA.18}$$

Note that $c^*_{t_M} \approx 1/(1+\xi)$ so the fraction of disposable wealth not consumed, and thus used for bequest, is approximately $\xi/(1+\xi)$. Hence, the bequest is approximately ξ times the amount consumed in the final year.

Non-final year, $t = t_1, \ldots, t_M - 1$. If dying at the end of year t, the bequest is

$$B_{t+1} = F_{t+1} + (1-\tau_Y)(1-I)[(1-m_t)A_t + W\alpha_t Y_t]R_{At} = F_{t+1} + \frac{(1-I)\bar{A}_{t+1}}{1+d_t}. \tag{IA.19}$$

To apply an induction argument, assume that $J_{t+1} = (F_{t+1} + \bar{A}_{t+1})G_{t+1}(y_{t+1}, a_{t+1})$. Then the certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_t &= \Big(p_t \mathrm{E}_t\left[(F_{t+1} + \bar{A}_{t+1})^{1-\gamma} G_{t+1}(y_{t+1}, a_{t+1})^{1-\gamma}\right] \\
&\qquad + (1-p_t)\mathrm{E}_t\left[\xi^{\frac{1-\gamma}{\psi-1}}(F_{t+1} + \frac{1-I}{1+d_t}\bar{A}_{t+1})^{1-\gamma}\right]\Big)^{\frac{1}{1-\gamma}} \\
&= (F_t + \bar{A}_t)\Bigg(p_t \mathrm{E}_t\bigg[\left(\tfrac{F_{t+1}+\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma} G_{t+1}(y_{t+1}, a_{t+1})^{1-\gamma}\bigg] \\
&\qquad + (1-p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\bigg[\left(\tfrac{F_{t+1}+\frac{1-I}{1+d_t}\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma}\bigg]\Bigg)^{\frac{1}{1-\gamma}} \\
&\equiv (F_t + \bar{A}_t)\mathcal{C}_t(y_t, a_t).
\end{aligned} \tag{IA.20}$$

Here, the expectation is over the stock price shock ε_{St} and the income shock ε_{Yt} before retirement as well as health shocks ϕ_t and Φ_t in retirement. We use that (for $\omega = 1$ or $\omega = (1-I)/(1+d_t)$)

$$\begin{aligned}
\frac{F_{t+1} + \omega\bar{A}_{t+1}}{F_t + \bar{A}_t} &= (1-c_t)\left(1 + (1-\alpha_t)y_t - (1-m_t)a_t\right)R_{Ft} \\
&\qquad + \omega\left((1-m_t)a_t + W\alpha_t y_t\right)R_{At}(1+d_t),
\end{aligned} \tag{IA.21}$$

and

$$y_{t+1} = \frac{y_t R_{Yt}}{(1-c_t)\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)R_{Ft}+\left((1-m_t)a_t+W\alpha_t y_t\right)R_{At}(1+d_t)}, \tag{IA.22}$$

$$a_{t+1} = \frac{\left((1-m_t)a_t+W\alpha_t y_t\right)R_{At}(1+d_t)}{(1-c_t)\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)R_{Ft}+\left((1-m_t)a_t+W\alpha_t y_t\right)R_{At}(1+d_t)}. \tag{IA.23}$$

The utility recursion implies that

$$\begin{aligned}
J_t &= \max_{c_t,\pi_t,\alpha_t}\left\{c_t^{1-\frac{1}{\psi}}\left(F_t+(1-\alpha_t)\bar{Y}_t+m_t\bar{A}_t\right)^{1-\frac{1}{\psi}}+\beta\left(F_t+\bar{A}_t\right)^{1-\frac{1}{\psi}}\mathcal{C}_t(y_t,a_t)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}\\
&= \left(F_t+\bar{A}_t\right)\max_{c_t,\pi_t,\alpha_t}\left\{c_t^{1-\frac{1}{\psi}}\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)^{1-\frac{1}{\psi}}+\beta\mathcal{C}_t(y_t,a_t)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}\\
&\equiv \left(F_t+\bar{A}_t\right)G_t\left(y_t,a_t\right).
\end{aligned} \tag{IA.24}$$

Since the expectation in $\mathcal{C}_t$ involves G_{t+1}, we get the recursion

$$\begin{aligned}
G_t\left(y_t,a_t\right) = \max_{c_t,\pi_t,\alpha_t}\Bigg\{ & c_t^{1-\frac{1}{\psi}}\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)^{1-\frac{1}{\psi}}\\
&+\beta\Bigg(p_t\mathrm{E}_t\left[\left(\tfrac{F_{t+1}+\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma}G_{t+1}(y_{t+1},a_{t+1})^{1-\gamma}\right]\\
&+(1-p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\left[\left(\frac{F_{t+1}+\frac{1-I}{1+d_t}\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma}\right]\Bigg)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\Bigg\}^{\frac{1}{1-\frac{1}{\psi}}}
\end{aligned} \tag{IA.25}$$

We solve this backwards starting with $t = t_M - 1$ where G_{t_M} is known from (IA.17).

The numerical implementation largely follows Larsen and Munk (2023) so we only provide a short description here. We derive G and the optimal strategies for (c,π,α) by backwards dynamic programming on a 21×21 equidistant grid of points $(\hat{y}_i, a_j)$ where $\hat{y}_t$ is a deterministic transformation of y_t to ensure a more stable state variable over time. Following Larsen and Munk (2023), we define

$$\hat{y}_t = y_t\exp\{-k_y(t_R-1-t)^+ + k_y^{\mathrm{ret}}(t-t_R)^+\}/\left(1-(1-\zeta)\mathbf{1}_{\{t\geq t_R\}}\right) \tag{IA.26}$$

and form the grid using $\hat{y}$ instead of y since $\hat{y}$ is more stable over time than y with an appropriate choice of k_y and k_y^{ret}, which depends on parameters and initial conditions. The expectations are approximated by Gauss-Hermite quadrature. To obtain life-cycle patterns, we simulate many possible paths forward drawing random shocks to stock prices, labor income, and medical expenses using interpolation and extrapolation when simulated values of $\hat{y}$ and a are off the grid. We report averages at each age to indicate an expected life-cycle pattern.

IA.4 Annuitization ratio and excess AIR chosen at retirement

Suppose that the individual can choose the annuitization ratio I and the excess AIR x at time t_{R-}, i.e., immediately before retiring. Let $Y_{t_{R-}}$, $F_{t_{R-}}$, and $A_{t_{R-}}$ denote the labor income, the private wealth, and the retirement savings immediately before retirement. The first retirement income (Social Security benefit) is $Y_{t_R} = \zeta Y_{t_{R-}}$. In this case, any annuitization costs are paid at retirement with the balance of the pension account being reduced to $A_{t_R} = W(I) A_{t_{R-}}$. The private wealth at retirement does not jump. The scaled state variables y and a immediately after retirement are thus

$$y_{t_R} = \frac{(1-\tau_Y)\zeta Y_{t_{R-}}}{F_{t_{R-}} + (1-\tau_Y)W(I)A_{t_{R-}}}, \quad a_{t_R} = \frac{(1-\tau_Y)W(I)A_{t_{R-}}}{F_{t_{R-}} + (1-\tau_Y)W(I)A_{t_{R-}}}. \tag{IA.27}$$

In retirement, for fixed (I, x), we go backwards period by period from t_M to t_R as explained in Section IA.3 above, maximizing over c_t and π_t in each period. Hence, the indirect utility for a given choice of (I, x) is

$$J_{t_{R-}}(F_{t_{R-}}, Y_{t_{R-}}, A_{t_{R-}}; I, x) = (F_{t_R} + W(I)\bar{A}_{t_R})\, G_{t_R}(y_{t_R}, a_{t_R}; I, x) \tag{IA.28}$$

going backwards period by period from t_M as explained in the above subsection, maximizing over c_t and π_t in each period. We can then determine the optimal choice of (I, x) by maximizing $J_{t_{R-}}(F_{t_{R-}}, Y_{t_{R-}}, A_{t_{R-}}; I, x)$ over all feasible combinations (I, x), acknowledging the required minimum distributions.

Note that the optimal plan characteristics I^* and x^* depend on all three variables F, Y, A, not just the scaled variables y, a. Hence, in this case, the indirect utility J_t in the pre-retirement phase, $t < t_R$, cannot be separated as $(F + \bar{A})G_t(y, a)$. This significantly

	$A = 300$		$A = 400$		$A = 500$	
	I^*	x^*, pct	I^*	x^*, pct	I^*	x^*, pct
Income $Y = 40$						
$F = 60$	0.9	-4	0.9	-4	0.9	-2
$F = 100$	0.9	-6	0.9	-4	0.9	-4
$F = 140$	0.9	-6	0.9	-6	0.9	-4
Income $Y = 60$						
$F = 60$	0.9	-6	0.9	-4	0.9	-4
$F = 100$	0.9	-6	0.9	-6	0.9	-4
$F = 140$	0.9	-8	0.9	-6	0.9	-4
Income $Y = 80$						
$F = 60$	0.9	-6	0.9	-4	0.9	-4
$F = 100$	0.9	-8	0.9	-6	0.9	-4
$F = 140$	0.9	-8	0.9	-6	0.9	-6

Table IA.1: **Base case preferences: Choosing payout parameters at retirement.** The table shows the base case individual's optimal choice of annuitization ratio I and excess AIR x at retirement for different combinations of pre-tax income Y, private wealth F, and pension wealth A entering retirement.

increases the computational complexity and, therefore, we have not solved such problems. Obviously, a pension plan that allows you to select I and x at retirement (depending on your situation at that date) is more valuable than a pension plan where I and x is chosen and fixed when you start paying into the plan.

Table IA.1 lists the base case individual's optimal choice of I and x at retirement for a range of combinations of pre-tax income Y, private wealth F, and pension wealth A entering retirement. In all cases the optimal annuitization ratio is 0.9, which is identical to the optimal choice at age 25 for all the contribution schemes covered by Table 5. In contrast, the optimal x varies across the income and wealth combinations. When pension wealth is small and private wealth large, the optimal excess AIR x at retirement is -8%, which leads the small pension wealth to be decumulated slowly in the beginning of retirement. When pension wealth is large and private wealth small, the optimal x at retirement is -4% or even -2%, which leads the large pension wealth to be decumulated more rapidly, although still with increasing payouts through time.

IA.5 Alternative assumptions about stock returns

Our main specification assumes that stock returns are IID and uncorrelated with labor income. Both assumptions are typical in life-cycle models.

Our baseline assumption of a zero stock-income correlation is consistent with various empirical studies that report a near-zero correlation for most population groups, even over longer horizons, see, e.g., Davis and Willen (2000), Cocco, Gomes, and Maenhout (2005), Pennacchi and Rastad (2011), and Fagereng, Gottlieb, and Guiso (2017). However, some studies report a moderately positive correlation at least for workers in some industries or lines of occupation. Notably, a non-zero stock-income correlation does not significantly complicate our numerical solution method.

Compared to our baseline assumption of a zero correlation, a positive correlation makes the human capital more stock-like so that less wealth should be invested directly in the stock market to maintain the risk of the investor's total wealth, i.e., the sum of financial wealth and human capital. A zero or low correlation generates the typical downward-sloping glidepath for the optimal stock weight over life that we also find in Figure 5 of the main paper, consistent with the common advice to hold "more stocks when young," at least in terms of the fraction of financial wealth invested in stocks. A high enough correlation can lead to an upward-sloping stock weight with "less stocks when young." In addition to the relatively straightforward effect on the stock weight, a positive stock-income correlation may have implications for consumption/savings decisions as well as for how the individual evaluates pension plans.

As a robustness check, we have derived results in our model for an individual with a stock-income correlation of $\rho_{YS} = 0.2$ but otherwise assuming the baseline parameter values from Table 3, including the preference parameters.

Figure IA.2 illustrates the life-cycle patterns of the stock weight in the left panel and wealth in the right panels. Solid lines refer to the base case with a zero correlation, whereas dotted lines are for a correlation of 0.2. The left panel confirms that without any pension plan the optimal stock weight is slightly lower with a positive correlation than with a zero correlation (black curves). With a positive correlation, the stock weight starts deviating from 100% at an earlier age and continues until retirement at a lower level. In retirement, there is no correlation in any case. In the situation with a pension plan, we assume pension savings are invested according to the same target-date fund strategy (yellow kinked line) no matter what the value of the stock-income correlation is. To ensure a lower total stock weight when the correlation is positive, a lower fraction of the private savings are invested in stocks compared to the zero-correlation case (orange curves). With the best basic pension plan, the investor saves a slightly lower fraction of total savings in the pension account when facing a positive correlation instead of a zero correlation (see

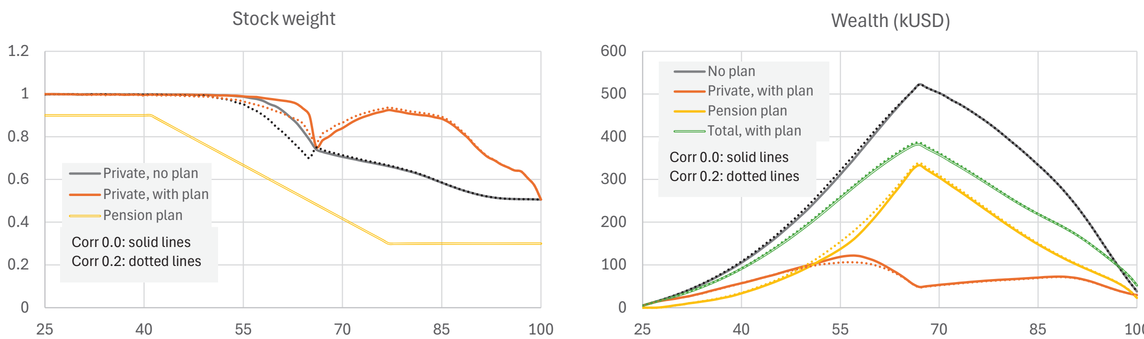

Figure IA.2: **Life-cycle patterns and stock-income correlation.** The diagrams show the base-case individual's stock weights and wealth as a function of age, both without a pension plan (dark curves) and with a pension plan characterized by a TDF investment strategy, full annuitization, and flat expected payouts. Solid lines are for the baseline case of a zero stock-income correlation ρ_{YS}, whereas dotted lines are for a correlation of 0.2. Other parameter values are taken from Table 3. For the case with access to the plan, the panels show both the private component (orange) and the pension component (yellow). The right panel also depicts total wealth with a pension plan (green). The plan wealth shown are after income tax. All curves represent averages across 10,000 simulated paths.

Table IA.2). Hence, the investor must keep a slightly higher fraction of private wealth in stocks to obtain the same total stock weight in retirement (where the stock weight in the pension plan is "too low") as can be seen by the dotted orange curve being above the solid orange curve at least in the early retirement years. The right panel of Figure IA.2 shows that changing the stock-income correlation from 0 to 0.2 has marginal effects on the expected savings over the life time.

Table IA.2 shows that with the best self-made pension plan, i.e. a non-annuitized plan with self-chosen contributions and payouts, the utility gain is 0.62% with a 0.2 correlation and 0.77% with a zero correlation. In contrast, the utility gain associated with the best basic annuitized plan ($I = 1$, $x = 0$) and best flexible plan ($I = 0.9$, $x = -4\%$) is larger for a positive correlation than for a zero correlation. With a positive stock-income correlation, the risk of ending up with both low income and low wealth (from low returns) increases, which makes a pension plan with annuitization more appreciated.

Benzoni, Collin-Dufresne, and Goldstein (2007) argue that long-run income-stock correlations can be substantial due to cointegration, which can potentially invalidate the argument for the glide path investment strategy. However, they do not present strong evidence for such a cointegration, and empirical estimates of long-run stock-income corre-

	Correlation $\rho_{YS} = 0.0$			Correlation $\rho_{YS} = 0.2$		
	Gain %	$\overline{F/Y'}$ at 65	$\overline{A/Y}$ at 65	Gain %	$\overline{F/Y'}$ at 65	$\overline{A/Y}$ at 65
No plan	0.00	14.3	0.0	0.00	13.9	0.0
Self-made plan, $w = 1$	0.77	3.7	11.4	0.62	3.9	9.3
Basic: $I = 1$, $x = 0$, tdf	3.19	2.0	8.8	3.41	1.9	8.7
Flex: $I = 0.9$, $x = -4$, tdf	3.57	3.5	7.5	3.80	3.2	7.4

Table IA.2: **The role of the stock-income correlation.** The table lists key information about the individual's utility and retirement saving for different pension plans where the stock-income correlation ρ_{YS} either has the baseline value of 0.0 or a value of 0.2. The rows correspond to the case with no pension plan, the case with a non-annuitized plan with self-selected contributions and payouts, the case with the best basic plan, and the case with the best flexible plan. For both values of the correlation, the best non-annuitized plan has 100% stock investments, the best basic plan has full annuitization and follows a target-date fund strategy, and the best flexible plan has an annuitization ratio of 0.9, an excess AIR of -4%, and follows a target-date fund strategy. 'Gain %' refers to the percentage utility gain relative to the case with no plan. The columns with the headings $\overline{F/Y'}$ and $\overline{A/Y}$ show the average ratio of private wealth to (after-tax) income and pension wealth to income (both either before or after tax) at the age of 65 based on 10,000 simulated paths.

lations do not seem significant (e.g. Pennacchi and Rastad, 2011).

Some empirical studies report statistically significant mean reversion in stock prices so that high recent returns predict low future returns and vice versa. Intuitively, the mean reversion reduces the long-run variance of stock returns, which makes stock investments more attractive for long-term investors. Various related studies find other statistically significant predictors of stock returns (such as the price-dividend or price-earnings ratio, the interest rate level, the term spread, and consumption growth) that lead to counter-cyclical variations in stock returns which also makes stocks more attractive to long-term investors compared to the IID case; see, e.g., Cochrane (2005, Chap. 20), Goyal and Welch (2008), and Goyal, Welch, and Zafirov (2024).[1] However, the statistical significance is often marginal and not always replicable out of sample. Moreover, while the return predictability leads to time-varying optimal portfolio weights, realistic specifications often show a modest impact on the average values of the optimal portfolio weight compared to an IID return specification.

To incorporate cointegration or predictability, the model needs an extra state variable which significantly complicates and slows down the numerical optimization algorithm, so

[1] Also the labor income growth rate may be predictable by business cycle indicators. A few papers study the quantitative effects of return and income predictability in life-cycle models that ignore pension savings (Munk and Sørensen, 2010; Lynch and Tan, 2011; Michaelides and Zhang, 2017; Kraft, Munk, and Weiss, 2019).

this is practically infeasible in our already rich setting. As argued above, the impact of realistic cointegration or predictability on average optimal decisions and expected life-cycle patterns is likely small anyway.

IA.6 Extension to non-scheduled payouts at a cost

As an extension to the setting of the main paper, we now allow the individual at the beginning of each year t to choose a fraction $M_t \geq 0$ of the pension account A_t to be paid out in addition to any scheduled payouts. Since all payouts are subject to income tax, the disposable wealth in year t is then

$$\tilde{F}_t = F_t + (1 - \tau_Y) \left[(1 - \alpha_t) Y_t + (m_t + M_t) A_t \right]. \tag{IA.29}$$

To obtain an unscheduled cash payout of $M_t A_t$, the value of the annuity portfolio is reduced by $(1 + k_t) M_t A_t$, where $k_t \geq 0$ reflects a cost or penalty. For 401(k)s in the U.S., a 10% tax penalty is paid on withdrawals before the age of 59.5 years.[2] In addition, we assume a cost equal to 1% of any unscheduled payout to represent any actual and psychological burdens related to obtaining such payouts. Hence, we let

$$k_t = \begin{cases} 0.11 & \text{for } t \leq 60, \\ 0.01 & \text{for } t > 60. \end{cases} \tag{IA.30}$$

The positive cost of unscheduled withdrawals has the nice implication that the individual will never make a simultaneous contribution to and an unscheduled withdrawal from the pension account. This is shown below. Without this cost, the optimal contributions and unscheduled withdrawals would be indeterminate.

If the individual survives year t, next year's initial value of the retirement saving account is

$$A_{t+1} = \left[(1 - m_t - (1 + k_t) M_t) A_t + (1 - KI) \alpha_t Y_t \right] R_{At} (1 + d_t). \tag{IA.31}$$

[2] Exceptions to the 10% tax exist and include some higher-education expenses, first home purchase, uninsured medical expenses, terminal illness, and death. Unemployed individuals can also make some non-penalized withdrawals according to specific rules. Our model disregards these exceptions (medical expenses are included only for retired individuals where the penalty does not apply in any case).

In this case the indirect utility J_t satisfies the recursion

$$J_t = \max_{c_t,\pi_t,\alpha_t,M_t} \left\{ \left(c_t \tilde{F}_t\right)^{1-\frac{1}{\psi}} + \beta\, \mathrm{CE}_t^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{IA.32}$$

where appropriate bounds are imposed on the controls $c_t, \pi_t, \alpha_t, M_t$, and where

$$\mathrm{CE}_t = \left(p_t \mathrm{E}_t \left[J_{t+1}^{1-\gamma} \right] + (1-p_t)\mathrm{E}_t \left[\bar{U}_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} \tag{IA.33}$$

is the certainty equivalent of next period's utility which is J_{t+1} if surviving and the bequest utility $\bar{U}_{t+1} = \xi^{\frac{1}{\psi-1}} B_{t+1}$ if not. Here the bequest if dying at the end of year t is

$$B_{t+1} = F_{t+1} + (1-I)(1-\tau_Y)[(1-m_t-(1+k_t)M_t)A_t + (1-KI)\alpha_t Y_t]R_{At}. \tag{IA.34}$$

The numerical solution procedure follows Section IA.3 with straightforward minor adjustments of the dynamics of the scaled state variables y and a.

With $k_t > 0$, a simultaneous contribution to and an unscheduled payout from the pension account is never optimal. To see this, suppose that you consider a simultaneous contribution at the rate $\alpha_t > 0$ and an unscheduled withdrawal characterized by $M_t > 0$. Since $\alpha_t > 0$ we have $m_t = 0$, i.e., no scheduled payouts. At the beginning of year t, this leads to an increase in disposable wealth of

$$\Delta F = (1-\tau_Y)[(1-\alpha_t)Y_t + M_t A_t] \tag{IA.35}$$

and an increase in the retirement account balance of

$$\Delta A = (1-KI)\alpha_t Y_t - (1+k_t)M_t A_t. \tag{IA.36}$$

First assume that $\alpha_t Y_t - M_t A_t \geq 0$. The alternative strategy of making no unscheduled payout and a contribution at the smaller rate $\widehat{\alpha}_t = \alpha_t - M_t A_t / Y_t \geq 0$ leads to the same change ΔF in disposable wealth, whereas the increase in the retirement account is

$$\Delta \widehat{A} = (1-KI)\widehat{\alpha}_t Y_t = (1-KI)\alpha_t Y_t - (1-KI)M_t A_t > \Delta A, \tag{IA.37}$$

and, hence, the alternative strategy is preferred. Suppose, instead, that $\alpha_t Y_t - M_t A_t < 0$. Then the alternative strategy of a zero contribution and the smaller unscheduled payout defined by $\widehat{M}_t = M_t - \alpha_t Y_t / A_t$ generates the same change ΔF in disposable wealth and an increase in the retirement account of

$$\Delta \widehat{A} = -(1 + k_t)\widehat{M}_t A_t = (1 + k_t)\alpha_t Y_t - (1 + k_t) M_t A_t > \Delta A, \qquad \text{(IA.38)}$$

so that the alternative strategy is preferred.

We have studied how the withdrawal option affects the decisions and utility gain of the base case individual under the assumption that the individual can select each year can select the contributions to the pension account. For the best basic plan with full annuitization, flat expected payouts, and a TDF investment strategy, the withdrawal option increases the utility gain only marginally from 3.19% to 3.28%. Early withdrawals are in this case quite rare and occur only when the individual has a relatively high pension wealth (large a) and low income (small y). At each age below 60, no early withdrawals are made in more than 95% of the simulations. After age 60, early withdrawals are more common but typically small; in more than 95% of the simulations, the maximum unscheduled withdrawal is less than \$300. The dotted curves in Figure IA.3 show for each age the median, the 90th percentile, and the 95th percentile of the unscheduled dollar withdrawal across 10,000 simulations. Since the numbers are so small, the curves are almost indistinguishable from the horizontal axis.

Turning to flexible pension plans, the optimal plan without the withdrawal option has an annuitization ratio of 0.9 and an excess AIR of -4%. If we add the withdrawal option to this plan, the utility gain goes up from 3.5705% to 3.7399%, and the individual will increase pension savings and reduce private savings. However, with the withdrawal option, the individual will optimally choose an excess AIR of -8% instead of -4% (still with a 0.9 annuitization ratio), which increases the utility gain further to 3.8456%. The total additional utility gain due to the withdrawal option is therefore $3.8456 - 3.5705 = 0.2751$ percentage points or \$2,377 in present value terms. Having the option to make unscheduled withdrawals if needed, the individual prefers the scheduled payouts to be low early in retirement and more steeply increasing through retirement. The solid curves in Figure IA.3 show for each age the median, the 90th percentile, and the 95th percentile of the unscheduled dollar withdrawal across 10,000 simulations when the pension plan has $I = 0.9$ and $x = -8\%$. The withdrawals are now more frequent and larger than for the best basic plan. The median withdrawal peaks at age 80 with a value of around \$5,700. In 5% of the simulations, the unscheduled withdrawal at age 80 is more than \$18,000.

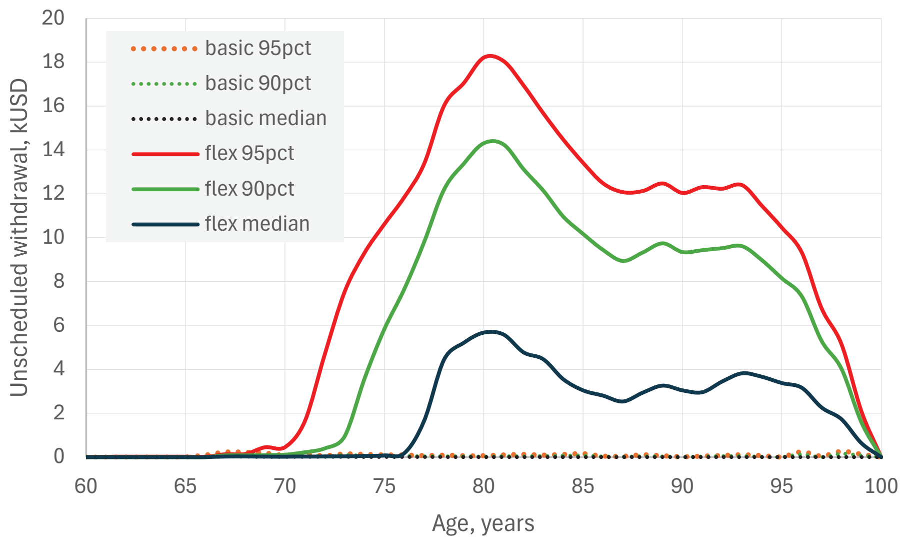

Figure IA.3: **Unscheduled withdrawals.** For any given pension plan, we calculate the optimal unscheduled withdrawal policy together with the optimal strategy for contributions, consumption, and the stock weight in the private portfolio. The dotted curves near the horizontal axis are for the basic plan with full annuitization, flat expected payouts, and a target-date fund investment strategy. The solid curves are for a flexible plan with an annuitization ratio of 0.9 and an excess AIR of -8%, still with a target-date fund investment strategy. Based on 10,000 simulations using the optimal decisions for each pension plan, the curves show, at each age, the median (blue), the 90th percentile (green), and the 95th percentile (red). The base case parameter values are used, also for preferences.

Withdrawals are still very rare before retirement and in the first few years in retirement.

On the other hand, the withdrawal option makes the payouts that the annuity provider has to make less predictable, which complicates the risk management procedures of the annuity provider and may thus lead to a larger annuity cost. If the withdrawal option causes the cost to increase from 15% to 19% or more, brings the utility gain associated with the best flexible plan (with $x = -8\%$ and $I = 0.9$) down to 3.55%, which is below the utility gain attainable without the withdrawal option. In general, the costs of introducing payout flexibility should be considered alongside the benefits of flexibility.

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