When Is the Price of Analysts' Disagreement Risk Positive?

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Abstract

We provide robust evidence that the price of analysts' disagreement risk in the cross-section of stock returns changes sign; it's positive (negative) in periods of high (low) disagreement. We construct a general equilibrium model in which analysts have heterogeneous beliefs about aggregate earnings growth. Each asset's risk premium depends on (i) the market portfolio, (ii) the macroeconomic factor, and, (iii) a "squared-beta" factor. (i) decreases and (iii) increases with disagreement as investors choose lower cash flow beta assets during periods of high disagreement. We find support for such a flight-to-safety in the data.

Key Words: heterogeneous beliefs; general equilibrium; N-asset production economy; price of risk; flight-to-safety; ICAPM; 2-Stage Regressions

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I. Introduction

Intuitively, the disagreement of views among analysts is a systematic factor affecting stock returns. However, the sign of the impact of disagreement in analysts' beliefs on the cross-section of stock returns is one of the most controversial issues in finance. Alternative theoretical and empirical frameworks have led to different signs on how disagreement affects stock returns.[1] In this paper, we argue that the conflicting results in the literature have arisen since the price of disagreement risk changes sign over time: we provide robust evidence that it is significantly positive in periods of high disagreement, significantly negative in periods of low disagreement, and insignificant otherwise. In addition, we provide a general equilibrium model that sheds light on why the sign of the price of disagreement risk changes over time, structurally estimate this model, and provide evidence that a key mechanism arising in our model (a flight-to-safety) is present in the data as well.

Our proposed risk factor, aggregate disagreement, is shown in Figure 1.

[Insert Figure 1 approximately here]

In short, it is the value-weighted average of stock-level disagreement. Stock-level disagreement is the standard deviation of analysts' forecasts of the long-term growth rate of earnings per share (EPS). This "bottom-up" measure has been used in several recent papers (e.g. Yu (2011) and

[1]On one hand, Varian (1985), Varian et al. (1989), Abel (1989), Qu, Starks and Yan (2003), Doukas, Kim and Pantzalis (2004), Anderson, Ghysels and Juergens (2005), David (2008), Anderson, Ghysels and Juergens (2009), and, Carlin, Longstaff and Matoba (2014) find a positive relation. On the other hand, Miller (1977) theorizes that the divergence of investors' beliefs in the presence of short-sale constraints leads to overvaluation and lower returns. In support of this hypothesis, Diether, Malloy and Scherbina (2002), Chen, Hong and Stein (2002) , Park (2005), Sadka and Scherbina (2007), Yu (2011), Bali and Kelly (2022), and Brennan and Zhang (2022), find a negative relation between disagreement and excess returns.

Hong and Sraer (2016)) and incorporates the views of a vastly greater number of analysts than "top-down" measures, which are comprised of forecasts of the aggregate earnings growth rate. High (low) disagreement months are defined as months when aggregate disagreement is higher (lower) than the average aggregate disagreement plus (minus) 1 standard deviation; medium disagreement months are those in the intermediate range. [2]

To study the effects of aggregate disagreement on the cross-section of stock returns, we start by estimating the market and aggregate disagreement risk loadings, β and δ, using three-year rolling regressions at the monthly frequency of individual stock returns on excess market returns and one-month lagged aggregate disagreement. These estimated risk loadings allow us to analyze the price of disagreement risk using three different methods described next.

First, we form ten portfolios using δ, the estimated aggregate disagreement loadings. Figure 2 highlights the role played by aggregate disagreement in assets' returns using the full sample, and subsamples comprising low, medium, and high disagreement months.

[Insert Figure 2 approximately here]

For the full sample, we find a U-shaped relation between disagreement and portfolio returns. Limiting our sample to low (high) disagreement months, we observe a clear negative (positive) relationship between disagreement in investors' beliefs and returns. During low (high) disagreement months, a portfolio of stocks in the highest decile of disagreement loading underperforms (outperforms) a portfolio of stocks in the lowest decile of disagreement loading. In months of medium disagreement, the relationship is unclear.

[2] With one standard deviation bands, there are 51 (87) months of low (high) disagreement. The results of our paper also hold for several other cutoffs, such as 0.75 or 1.25 standard deviations. In the former case, the number of low (high) disagreement months increases to 127 (97), which are substantially larger sample sizes.

Second, we examine whether firms' exposure to aggregate disagreement is related to their characteristics, such as size or value, which are well-established priced factors in the asset pricing literature. For example, one might conjecture that smaller firms have greater exposure to aggregate disagreement, so that, based on size, we might expect higher-disagreement firms to have higher returns. We address this concern by double-sorting on size and exposure to aggregate disagreement; if the compensation for bearing a greater exposure to disagreement holds across size quintiles, then we would have established that the disagreement effect is distinct from the size effect. We perform this double sorting exercise for firms' disagreement exposure with market beta, size, two alternative measures of value, momentum, a comprehensive measure of market sentiment, and the forecast mean, which is used in the recent literature on "subjective beliefs" to measure analysts' overreaction to recent fundamental news, and confirm that the price of disagreement risk changes sign in different subsamples of the data as described in the previous paragraph.

Third, to extend the robustness of our findings to a very diverse set of firm characteristics in a multivariate setting, we estimate the price of disagreement risk using all portfolios on Ken French's webpage with at least 10 assets. The portfolios are based on standard factors such as size and value, as well as other characteristics, including investment, repurchases, and others. We find, quite remarkably, that the estimates of the price of risk for 35 of the 36 portfolio sets are very similar to those from the set of 100 beta-delta portfolios that we construct. In particular, the price of risk is positive during high-disagreement months and negative during low-disagreement months; furthermore, it is very stable across the subsamples.

To better understand the changing sign of the price of disagreement risk, we construct a production economy N-asset general equilibrium model in the spirit of Merton's ICAPM, in which two analysts of different types have heterogeneous beliefs and provide different forecasts of a macroeconomic factor (aggregate earnings growth). Our model has a one-factor structure: all assets' cash flows linearly load on a single macroeconomic factor (aggregate earnings), but each asset also has significant idiosyncratic risk. We do not make a distinction between consumers and analysts, i.e., we assume that each analyst has the same beliefs as some group of consumers in the economy. Consumers invest their wealth in different assets in the economy and speculate with one another on the relative accuracy of their models' predictions. The beliefs of each analyst and their relative weight in the economy become state variables in the ICAPM framework whose shifts consumers hedge against. We derive an equilibrium cross-sectional pricing equation for our model, which has three endogenously determined factors. The consumers' beliefs and weighting process do not appear explicitly, but they affect the factors through the portfolio weights of the consumers' alternative assets.

The first factor compensates the investor for bearing market risk, similar to the CAPM. The only difference from the standard CAPM is that exposure to market risk is measured by the asset's cash flow beta relative to the market's cash flow beta, rather than the stock return beta. The second factor measures the risk premium for bearing the aggregate macroeconomic risk, as well as any undiversified idiosyncratic risk. The third factor is the premium for hedging the risk of shifts in the opportunity set in the ICAPM framework of Merton (1973). In our setup, this term depends on the product of the consumer's risk aversion and the weighted average of the squared cash-flow beta of each asset and the square of the weight the consumer invests in each asset. We call this term the "squared-beta" factor. The negative sign implies that in periods when the term is large, the risk premium for each asset is smaller. The negative sign arises because the consumer's wealth grows faster when she invests in higher-cash-flow beta assets; that is, her opportunity set is more attractive from a growth perspective. Consumers' portfolios display a ”flight-to-safety”: during periods of higher disagreement, they choose lower-cash-flow-beta assets. The model implies that risk premiums on assets from the hedging term will be higher when the consumer invests heavily in safer assets.

We structurally estimate our model using data on aggregate earnings growth and the series of aggregate analysts' disagreements. We find two sets of parameters (one for each type of analyst) that maximize the sum of the likelihoods of each agent type observing the fundamentals, as well as matching the model's time series of aggregate disagreement to that in the data. We show that analysts' differences in beliefs about regular high- and low-growth-rate states, as well as a disaster state, capture well the time variation in aggregate earnings disagreement.[3]

Our model has striking implications for the pricing of risk across the cross-section of stock returns. The model provides a positive correlation between disagreement in analysts' forecasts and the price of disagreement risk. The model's price of disagreement risk has a correlation of 0.45 with the price of disagreement risk from two-stage regressions in the data, even though the price of disagreement risk is not a targeted moment in our estimation. As discussed above, a crucial part of the model's mechanism is that wealth allocated to lower-cash-flow-beta assets increases in periods of higher disagreement, thereby lowering the squared-beta hedging term and

[3]Without modeling a disaster state, our model is unable to provide a good fit of fundamentals in the recessions of the current century, nor is it able to generate the high observed level of analysts' disagreement in these recessions. Our estimated model implies that disaster probabilities were low in the 1980s and 1990s, and hence aggregate disagreement was lower in these decades. This is our explanation for why studies on the price of disagreement risk that used data until 2000 typically find a negative price of disagreement risk.

increasing the risk premium on those assets. We provide support for such a flight-to-safety phenomenon in the data, which is again not a targeted moment of our estimation procedure.

The final exercise that we conduct in the paper is an out-of-sample one. Our structural model is estimated using data until Sept-2016.[4] We then verify that the prices of risk exhibit similar signs as developed in the in-sample period, based, in particular, on keeping the same intervals of high and low disagreement. From October 2016 to the end of 2021, the aggregate disagreement was in the high range for 43 of 63 months and in the medium range for the remaining months. As predicted by the in-sample results, the price of disagreement risk is positive and statistically significant in the high-disagreement months and insignificant otherwise. We also update our model-based disagreement based on realized earnings in the out-of-sample period, and it shows that analyst disagreement would be mostly high in this period, especially after the catastrophic drop in earnings during the pandemic. It is worth noting that the Fama-French three factors, as well as momentum, have statistically insignificant prices of risk in this period, which makes the disagreement factor even more compelling.

An extensive literature in asset pricing has examined the risk-return trade-off. The Capital Asset Pricing Model of the Sharpe (1964), Lintner (1965) and Black (1972) states that the security excess return is proportional to the sensitivity of its return to the market return, denoted by CAPM beta. Jensen, Black and Scholes (1972) point out that ”high beta” assets earn lower returns on average. More recently, Frazzini and Pedersen (2014) documents that a portfolio that holds low-beta assets and shorts high-beta earns a positive average return. Hong and Sraer (2016) relaxes the CAPM homogeneous expectation assumption and shows that when aggregate

[4] The date was chosen based on the availability of data at the time of the first version of this paper.

disagreement is low, expected return increases with beta as in the CAPM. However, when it is large, expected return can decrease with beta due to short-sale constraints. It is helpful to note that in Hong and Sraer (2016), investors' opportunity set is constant over time, and the only priced risk factor is the market index. In our setup, aggregate disagreement (AD) varies over time and endogenously affects aggregate investment opportunities. As a result, our paper is in the ICAPM framework of Merton (1973), rather than the CAPM framework, and hence AD is a systematically priced factor, which is the primary focus of our paper.

On the theoretical front, our paper contributes to the growing literature on general equilibrium models with heterogeneous beliefs. Building on the seminal papers of Detemple and Murthy (1994) and Basak (2000), David (2008) studies the implications of heterogeneous beliefs for the risk premium on the market index within this framework, while Dumas, Kurshev and Uppal (2009) studies the implications for "excess volatility" of the market index. Gallmeyer and Hollifield (2008) study the implications for asset prices in such a model with an added short-sales constraint, while Burashi, Trojani and Vedolin (2014) extend the framework to multiple stocks in an exchange economy setting with multiple trees. Baker, Hollifield and Osambela (2016) study investment in a single production technology where the representative agent has Epstein-Zin preferences. This paper obtains a tractable equilibrium characterization by working within the Cox-Ingersoll-Ross (1985) framework with multiple firms, each with access to a linear production technology. In particular, the scale of each firm is endogenous, and we explicitly study the riskiness of the market portfolio with changing disagreement of beliefs.[5]

[5] In addition to the papers mentioned in this paragraph, the following papers in this literature are in the same general framework: Jouini and Napp (2007), Bhamra and Uppal (2014), Buraschi, Piatti and Whelan (2018),Borovicka (2020).

In a new and related set of papers, Malmendier and Nagel (2016) and Das, Kuhnen and Nagel (2020) examine the origins of the different underlying consumer models in the economy. They provide empirical evidence that different life experiences and various characteristics, such as education, affect beliefs. We would conjecture that analysts' forecasting models are affected by similar characteristics.

The remainder of this paper is organized as follows. In Section II, we describe the data used and provide results of predictability regressions on aggregate disagreement in the time-series. In Section III, we provide our main empirical results on the time variation of the price of disagreement risk in the cross-section of stock returns using two-stage regressions. In Section IV, we provide a theoretical model that prices disagreement risk in the cross-section, and in Section V, we show that its pricing implications are in line with our empirical findings. Section VI concludes. All proofs of propositions are in the appendix. An online appendix provides additional results on univariate sorting and out-of-sample evidence for the model's predictions.

II. Data and Variables

The data in this study are the intersection of the Institutional Brokers Estimate System (I/B/E/S) and the Center of Research in Securities Prices database (CRSP) between December 1981 and September 2016. From I/B/E/S, we obtain the analyst forecasts data while from CRSP, we obtain the monthly returns. We include all stocks listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and Nasdaq with share code 10 or 11 (common stocks). In order to ensure that the illiquid stocks are not considered in our analysis, we exclude penny stocks (price $< \$5$) and micro-caps (stocks in the bottom 2 deciles of the monthly size distribution). A firm is kept in our sample if it has more than 12 consecutive monthly observations. We end up with a sample of 6,428 firms.

We start by displaying the main variable of interest, which is aggregate disagreement. Similar to Hong and Sraer (2016), we measure aggregate disagreement as the value-weighted average of the standard deviation of analysts' forecasts of the 5-year growth in earnings per share. Figure 1 plots the time series of the aggregate disagreement measure, as well as the low and high disagreement thresholds shown by the two horizontal lines. We define low (high) disagreement months as those in which the previous month's aggregate disagreement is lower (higher) than the average aggregate disagreement minus (plus) one standard deviation. Of our sample, 87 months are high disagreement months while 55 months are low disagreement months. The remaining 276 months are medium disagreement months. Low disagreement months are concentrated prior to 2000, while high disagreement months occurred mostly after 2000. As can be seen in Figure 1, high levels of disagreement occur both during recession and growth periods. Table 1 presents the summary statistics of the aggregate disagreement measure. On average during the full sample period, the aggregate disagreement equals 3.51.

[Insert Table 1approximately here]

The average ranges from 2.8 in low disagreement months to 4.48 in high disagreement months. The correlations between aggregate disagreement and standard factors are provided in the online appendix.

III. The Price of Disagreement Risk in the Cross-Section

To obtain time-varying risk loadings, we run 3-year rolling monthly time-series regressions of stock returns on excess market returns as well as one-month lagged aggregate disagreement.

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \delta_{i,t} \times Agg\_disp_{t-1} + \epsilon_{i,t} \quad (1)$$

The estimated risk loadings, $\beta_{i,t}$ and $\delta_{i,t}$, are called the pre-ranking coefficients. Sorting on these betas and deltas, we form portfolios of stocks. Then we can repeat the regression using the portfolio returns and estimate $\beta_{p,t}$, and $\delta_{p,t}$ for each portfolio, which are the post-ranking betas and deltas. We analyze the price of disagreement risk across the cross-section in two ways. First, we study returns on portfolios based on univariate sorts on firms' δs, their disagreement risk exposure. Next, to determine whether our results on δ sorts are due to relationships between firms' characteristics and their deltas, we form portfolios based on double sorts of disagreement loadings and various standard firm characteristics. Finally, to estimate the price of disagreement risk across subsamples, we use standard cross-sectional two-stage regressions.

A. Univariate Portfolio Sorts

We use the estimated aggregate disagreement exposure, δ, from Equation (1), to test if aggregate disagreement is priced in the cross-section of stock returns for the full sample, as well as subsamples comprising low, medium, and high disagreement months. To do so, we form portfolios each month by sorting firms on their aggregate disagreement loadings. Each month, we sort stocks into 10 δ-deciles with the first decile having the lowest pre-ranking δ and the tenth decile having the highest pre-ranking δ. We calculate the monthly portfolio return, R_t^P, as the value-weighted average of the returns of all stocks in the P^{th} δ-sorted portfolio. Figure 2 highlights the role played by aggregate disagreement in an asset's returns. For the full sample, we find a U-shaped cross-sectional relation between disagreement and returns (Panel a). Quite strikingly, the relation is negative for low disagreement months (Panel b); inconclusive for medium disagreement months (Panel c), and positive for high disagreement months (Panel d). These results suggest a negative (positive) compensation for exposure to aggregate disagreement during periods of low (high) disagreement, which we analyze further in the following subsections.

B. Bivariate Portfolio Sorts

In this section, we investigate the robustness of our finding on the changing sign for compensation for aggregate disagreement exposure. One possibility is that firms' exposure to aggregate disagreement is related to their characteristics, such as size or value, which are well-established priced factors in the asset pricing literature. For example, one might conjecture that smaller firms have greater exposure to aggregate disagreement, so that, based on size, we might expect higher-disagreement firms to have higher returns. One way to address such a concern would be to double-sort on size and exposure to aggregate disagreement; if the compensation for bearing a greater exposure to disagreement holds across size quintiles, then we would have established that the disagreement effect is distinct from the size effect. In this section, in Table 3, we present the results of the double-sorting exercise for the following seven characteristics/risk loadings, all well known in the literature: market beta, size, two alternative measures of value, momentum, subjective beliefs, and sentiment. We verify if the disagreement effect holds across the quintiles of the characteristic/risk measure under consideration.

[Insert able 3 approximately here]

We note that Table 3 does not show the compensation for bearing the risk in the alternative characteristics/risk-loadings. The full results of the double-sort and comments on the results are in Section B of the online appendix.

Using the firm's beta as an example, we first sort stocks into five quintiles based on their market beta calculated in the previous month, which is calculated using (1). Then, within each quintile, we sort stocks into five quintiles based on their aggregate disagreement loadings estimated at the end of the previous month. We overall obtain 25 portfolios value-weighted portfolios based on firms' betas and their deltas, and study their mean returns. We repeat this exercise for the other characteristics/risk loadings. For each characteristic/risk loading, we provide results for all months, high-disagreement months, and low-disagreement months. Along the rows, we have firms in different quintiles of the characteristic/risk-loading. We provide the mean monthly value-weighted returns from a strategy that longs the highest-disagreement quintile firms and shorts the lowest-disagreement quintile firms (we refer to this as the mean return (5-1)) and its t-statistic. We provide a summary of the results below.

1. We first examine whether our results on the changing sign of the price of disagreement risk are due to differences in firms' market risk exposures. For all months, we see along each row that, in all but one of the beta quintiles, the return differential, 5-1, which is the reward for disagreement risk exposure, is not significantly different from zero; nor is it significant in the All firms row. In the high (low) disagreement months, all return differentials are

positive (negative) across all beta quintiles and all firms. Therefore, the compensation for bearing disagreement risk is unrelated to the firm's beta, but changes sign across high- and low-disagreement samples.

2. Size is the market capitalization of the company. The pattern of compensation for disagreement risk is the same as for beta across all size quintiles. For all months, the compensation is insignificant, while it is positive (negative) for high (low) disagreement months.

3. Firms' book-to-market ratios are calculated following Asness and Frazzini (2013) and Asness, Frazzini, Israel and Moskowitz (2015); We calculate the book-to-market ratio using the current market value, but we update the book equity value to the previous fiscal year's value every June. The pattern of compensation for disagreement risk is the same as for beta across all value quintiles, with a few exceptions. For all months, the disagreement compensation is positive for the second and 5th value quintiles, whereas in low-disagreement months, it is negative but insignificant for the third and 5th value quintiles.

4. Recently, Eisfeldt, Kim and Papankolaou (2022) have attributed the inconsistent performance of the standard B/M ratio in portfolio selection to the omission of intangible assets in the definition of the book equity value. In particular, technology stocks have a larger proportion of intangible assets to total assets than other stocks, and in recent decades, technology stocks have a larger weight in market indices. To jointly assess compensation for value and disagreement loading, we therefore repeat the double-sorting exercise using the intangible-augmented book-to-market ratio, denoted B/M^{INT}. As in the construction

of the standard B/M, every June-end, BE_t^{INT} is updated using the previous fiscal year's book equity value [6]. The market value is the current market capitalization. Without exception, the pattern of risk compensation for disagreement is the same across alternative subsamples and all quintiles of intangible value.

5. The momentum of stock i is calculated as the six-month lagged six-month return: $\text{ret}_i(t-6) + \text{ret}_i(t-7) + \cdots + \text{ret}_i(t-11)$. The pattern of compensation for disagreement risk is the same as for the other variables, with two exceptions. For all months, the compensation is positive for the 2nd and 3rd quintiles of firms sorted by momentum.

6. Subjective beliefs are measured as the mean of analysts' long-term growth forecasts. While most of the measures are standard and do not require explanation, we make the following comments on subjective beliefs and sentiment. Subjective beliefs uncover the role of biased beliefs in forecasting stock returns (see e.g. Shleifer and Gennaioli (2018) and Nagel and Xu (2023)). In these papers, investors extrapolate recent firms' performance, so that their forecasts are overly (under)optimistic following good (bad) earnings news. Since these erroneous beliefs eventually correct, high mean forecasts of agents predict low returns. It is worth noting that these papers forecast at horizons of one year or more, a horizon long enough for beliefs to correct, while we examine returns over the following month. That said, the compensation for disagreement risk follows the same patterns as the other characteristics, with two exceptions. For all months, the 2nd and 4th quintiles, sorted by the mean forecast, show significant positive compensation for disagreement risk.

7. A growing literature finds that market sentiment is a significant explanatory factor in the

[6] We obtain the book equity BE^{INT} values from Eisfeldt et al. (2020).

cross-section of stock returns.[7] Building on the work of Baker and Wurgler (2006) and Baker and Wurgler (2007), Huang, Jiang, Tu and Zhou (2015) constructs a sentiment index that negatively predicts stock returns, which we use to construct sentiment loadings of firms.[8] Before we present our results, it is helpful to note that empirically, aggregate disagreement and sentiment are not highly correlated (see Figure 4). While a full investigation of sentiment is beyond the scope of this paper, it is important to keep in mind that the sentiment series is based on various liquidity and financing conditions. In contrast, aggregate disagreement is based solely on firms' forecasted earnings growth.

At the first stage, the sentiment loading, $\theta_{i,t}$, is estimated each month using a rolling three-year window of lagged individual stock returns through the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times \boldsymbol{R}_{m,t} + \theta_{i,t} \times \text{Aggregate Sentiment}_{t-1} + \epsilon_{i,t}.$$

We then double-sort firms on their disagreement and sentiment loadings in the second stage. The final risk/loading of Table 3 shows the compensation for disagreement risk loading for alternative quintiles of sentiment loading. With two exceptions, the price of disagreement risk follows the same pattern as the other characteristics. The two exceptions

[7]Ben-Rephael, Kandel and Wohl (2012) finds that investor sentiment affects mutual fund flows and stock returns. Chen, Han and Pan (2022) finds that exposure to sentiment fluctuations is a source of excess returns for hedge funds. Chen, Liu, Wang, Wang and Yu (2025) find that returns to standard characteristic-based portfolios are different in high and low sentiment regimes.

[8]The index combines information in six series: (i) closed-end discount, (ii) first day IPO returns, (iii) number of IPOs, (iv) share turnover, (v) dividend premium, and (vi) equity share of new issues. The series is available on Guofu Zhou's webpage. Ung, Gebka and Anderson (2024) also provides a composite sentiment index.

are the 2nd and 3rd quintiles of sentiment during high disagreement months, for which the compensation for disagreement is not significant, even though it is positive. Therefore, changes in the sign of the price of disagreement risk are robust to alternative levels of sentiment. Finally, as shown in Table A9 in the online appendix, the compensation for sentiment risk also changes sign in low- and high-disagreement months. This and other properties of sentiment risk pricing are topics for future research.

C. Two-stage Regressions

In this section, we analyze whether aggregate disagreement is a priced risk factor using our value-weighted β-δ portfolios. To construct the β-δ portfolios, we use the estimated parameters, β and δ, obtained from Equation (1). Each month, we sort stocks into 10 β-deciles based on their pre-ranking β. For every β-decile, we sort stocks based on the pre-ranking δ. We thus obtain 100 portfolios formed monthly. We calculate the value-weighted monthly returns R_t^P on these 100 portfolios. We report the summary statistics in Table A2 for monthly β-δ portfolio return and pre-ranking risk loadings. The numbers reported represent time-series averages of the monthly cross-sectional mean, standard deviation, and the 10^{th} to 90^{th} percentiles.

[Insert Table A2 approximately here]

We use the Fama-Macbeth (FM) two-stage procedure as in Fama and MacBeth (1973) to estimate the prices of disagreement risk. In fact, we estimate the price of risk using four alternative specifications, which differ in the set of control variables. Specification 1 includes market excess returns, which correspond to the conditional CAPM. Specification 2 adds the one-month-lagged aggregate disagreement measure. Specification three includes the Fama and French (1993) factors and the momentum factor (Jegadeesh and Titman (1993)). Finally, Specification four includes the four factors and aggregate disagreement. We closely follow the steps outlined in Section 12.3 of Cochrane (2001).

We obtain monthly returns on the factors (R_t^m, SMB, HML, and UMD) from the data web page of Ken French. Each month, post-ranking risk loadings are estimated using the current and previous 35 monthly returns. For example, for the fourth specification, we estimate the first-stage equation:

$$\text{(2)} \quad R_{t-s}^P = \alpha_{P,t} + \beta_{P,t}^{MKT} \times R_{t-s}^m + \delta_{P,t} \times Agg\_disp_{t-s-1} + \beta_{P,t}^{SMB} \times SMB_{t-s} + \beta_{P,t}^{HML} \times HML_{t-s} + \beta_{P,t}^{UMD} \times UMD_{t-s} + \epsilon_{P,t-s},$$

for $P = 1, ..., 100$, and $s = 0, \cdots, 35$. R_t^P is the value-weighted monthly return of the P^{th} $\beta - \delta$-sorted portfolio at t. Next, for the second stage, each month we estimate the prices of risk at date t using the equation

$$\text{(3)} \quad R_t^P = \kappa_t + \pi_t \times \beta_{P,t}^{MKT} + \omega_t \times \delta_{P,t} + \phi_t^{SMB} \times \beta_{P,t}^{SMB} + \phi_t^{HML} \times \beta_{P,t}^{HML} + \phi_t^{UMD} \times \beta_{P,t}^{UMD} + \epsilon_{t,P},$$

for $P = 1, ..., 100$. Finally, we calculate the time-series mean of the estimated prices of risk and the estimated variance (our equations illustrate the case of the price of disagreement risk), as

$$\text{(4)} \quad \hat{\omega} = \frac{1}{T} \sum_{t=1}^{T} \hat{\omega}_t$$

$$\text{(5)} \quad \sigma^2(\hat{\omega}) = \frac{1}{T^2} \sum_{t=1}^{T} (\hat{\omega}_t - \hat{\omega})^2,$$

respectively. The prices of risk of the other factors are estimated analogously.

In Table 4, we report the average prices of risk from the second-stage regression for each factor across the four specifications introduced above.

[Insert Table 4 approximately here]

For each specification, we show the time-series mean and T-statistics of the mean estimate. We also show the time-series average of the $\bar{R}^2$ for each regression specification. The first column shows the results for specification (1), which has the market as the only factor. The mean estimate of the price of market risk in isolation is 0.67. However, as indicated by its t-statistic, this estimate is not statistically different from zero. Interestingly, the time-series average of the $\bar{R}^2$ is 0.142. The reader might wonder why the average $\bar{R}^2$ is so high despite the only price of risk being statistically insignificant. The answer lies in the high volatility of the risk estimate's price. Even though the period-by-period $\bar{R}^2$ is relatively large, the estimate $\hat{\pi}_t$ of the price of risk varies widely, leading to an insignificant t-statistic for the sample mean. The second column shows the results for specification (2), which include the market and aggregate disagreement as the two factors. In this specification, the average price of market risk is similar to specification (1) at 0.77, while the average price of disagreement risk is 0.046; however, due to the high variability of the prices of risk over time, the means of each of the prices of risk are statistically insignificantly different from zero. The average $\bar{R}^2$ of the two-factor model is 0.25, which is 11 percentage points higher than in specification (1), which implies that in each period, the aggregate disagreement factor explains a significant amount of variation in the second-stage regression.

The following two specifications add the Fama-French factors and momentum (collectively the four-factor model) to the market and aggregate disagreement factors. Column three shows that the average $\bar{R}^2$ of the four-factor model is 0.31; however, due to the time-series volatility in the factors' prices of risk, the t-statistics for each factor are insignificant. In column four, we have specification (4), which includes the four-factor model and aggregate disagreement. Here, the average $\bar{R}^2$ increases to 0.475 (16.5 percentage points higher relative to specification (3)), even though the estimates of all the prices of risk are statistically insignificant.

The main point of our paper is that the price of disagreement risk is variable, but we can understand its variation to an extent. We shed light on the variation in the price of disagreement risk in Figure 3.

[Insert Figure 3 approximately here]

As seen, the price of disagreement risk was stable and low during the 1985 to 1995 period, and has been higher and more volatile in the 2000s. We also show the aggregate disagreement series in the same plot. Most remarkably, the two series show a positive correlation (correlation coefficient of 0.22), i.e., disagreement risk is higher during periods of higher disagreement. This finding motivates the next part of our analysis: we examine a two-stage regression across alternative subsamples based on the level of aggregate disagreement.

In Table 5, we present the second-stage regression results for three sub-samples: low, medium, and high disagreement months.

[Insert Table 5 approximately here]

We again consider the four specifications in Table 4. We make two striking observations: First, adding aggregate disagreement to specifications (1) and (3) increases the $\bar{R}^2$ by similar magnitudes to the full sample analysis in Table A2. However, the mean estimates of the prices of risk of the four factors are each insignificantly different from zero. Second, the price of disagreement risk is significantly negative during periods of low disagreement (when aggregate disagreement is one standard deviation below its mean) and significantly positive in periods of high disagreement (when aggregate disagreement is one standard deviation above its mean). It is statistically insignificant in periods of medium disagreement (when aggregate disagreement is within one standard deviation of its mean). The main reason that the price of disagreement risk is significant in the low and high sub-samples is that it is relatively stable in these sub-samples. It is also worth noting that the magnitude of the price of disagreement risk is more pronounced in high-disagreement months (0.38) than in low-disagreement months (-0.08), and we will address this asymmetry when we discuss our theoretical model.

D. Additional Test Portfolios

In this section, we study whether the relation between the price of disagreement risk and aggregate disagreement is robust, that is, it holds in alternative sets of test portfolios. By examining multiple sets of test portfolios, we reduce the likelihood of finding 'false' factors in any one set, as noted by Lewellen, Nagel and Shanken (2010). In particular, we repeat the two-stage Fame-Macbeth regressions for each of the sets of portfolios on Ken French's data page that have at least 6 portfolios. Overall, we have 36 sets of test portfolios, which we list in Table 2.

[Insert Table 2 approximately here]

It is noteworthy that the test portfolios are created from a very diverse set of firm characteristics, such as size, market-to-book, investment, repurchases, and others, and hence the portfolios are unlikely to be based on similar sets of firms.

We repeat our estimation of the prices of risk of specification (4), which includes the four-factor model and aggregate disagreement, in Section C. In particular, we use (2) and (3) to estimate the conditional prices of risk at each date, and then, (4) and (5) to calculate the mean estimate and the standard error of the estimate of the price of disagreement risk. Our estimates for all 36 portfolios are concisely presented in Figure 5.

[Insert Table 1 approximately here]

On the horizontal axis of each panel, we have the portfolio set number, as indexed in Figure 5. The top panel shows the time-series average of the price of disagreement risk in a portfolio set. We have three sets of results: for all time periods and for high- and low-disagreement periods (as discussed in Section C). Our results for each set of portfolios are very similar to those for the 100 beta-delta portfolios in Section C: for each portfolio set, the mean estimates are close to zero using data from all periods, are positive (around 0.35) during high disagreement periods, and negative (around -0.08) during low disagreement periods. There is minimal variation in these estimates across portfolio sets (the sole exception being portfolio set 14). The bottom panel shows the t-statistics, and the results are quite remarkable. The estimates using data for all months are insignificantly different from zero, whereas for the high-disagreement months, the estimates are significantly positive (t-statistics greater than 2). In low-disagreement months, the estimates are negative for 34 of the 36 portfolio sets. The estimates of the price of disagreement risk for the full sample are insignificant because of the large time-series variation for each set. However, just as in Section C, the price of disagreement risk is mostly positive during high disagreement periods, and negative during low disagreement periods. In addition, within these periods, the prices of risk are stable, and hence the estimates are statistically significant.

IV. The Model

In this section, we provide a general equilibrium model that sheds light on why the price of disagreement risk can change sign over time depending on the level of aggregate disagreement. The model is based on the N asset (technology) production economy framework of Cox, Ingersoll and Ross (1985) (CIR) with three important specification choices. First, there are heterogeneous investor types, while CIR has a single investor. Second, the state variables that we choose are the beliefs of each investor in the economy, as well as a state variable that measures the importance of each investor in the economy. Third, we restrict the weights in each asset to be positive in aggregate.

A. The Macroeconomic Factor

A macroeconomic factor follows the process:

$$\frac{dY_t}{Y_t} = \nu_t dt + \sigma_Y d\tilde{W}_{Yt}. \tag{6}$$

The drift ν follows an S-state Markov chain that shifts between the states $\{\nu_1, \cdots, \nu_S\}$ with generator matrix Λ.[9]

There are two types of consumers indexed by $m = 1, 2$. Each type m assumes that the process for ν has the correct specification, but differs in the estimates of the states and the generator. Let $\nu^{(m)}$ denote the estimated drift vector estimated by agent of type m, and let $\Lambda^{(m)}$ be

[9]Over the infinitesimal time interval of length dt, $\Lambda_{ij}dt = \text{prob}\,(\nu_{t+dt} = \nu_j | \nu_t = \nu_i)$, for $i \neq j$, and $\Lambda_{ii} = -\Lambda_{ij}$. The transition matrix over any finite interval of time, s, is $\exp(\Lambda s)$.

their estimated generator matrix. Neither type of agent can observe the realizations of ν, although each does observe the entire history of Y. Based on their assumed models, analysts of type m form the posterior probability $\pi_{st}^{(m)} = \text{prob}(\nu_t = \nu_s^{(m)} | \mathcal{F}_t^{(m)})$ of ν being in state s at time t for each $s \in \{1, \cdots, S\}$. We denote conditional means with bars, for example, $\overline{\nu}_t^{(m)} = \sum_{s=1}^{S} \pi_{st}^{(m)} \nu_s^{(m)}$. Given an initial belief $0 \leq \pi_{s0}^{(m)} \leq 1$, the probabilities $\pi_{st}^{(m)}$ follow the stochastic differential equations

$$d\pi_{st}^{(m)} = \mu_{st}^{(m)} dt + \sigma_{st}^{(m)} d\tilde{W}_{Yt}^{(m)}, \tag{7}$$

where

$$\mu_{st}^{(m)} = [\pi_t \Lambda^{(m)}]_s; \qquad \sigma_{st}^{(m)} = \frac{\pi_{st}^{(m)} (\nu_s^{(m)} - \bar{\nu}_t^{(m)})}{\sigma_Y}, \tag{8}$$

$$d\tilde{W}_{Yt}^{(m)} = \frac{1}{\sigma_Y} \left(\frac{dY_t}{Y_t} - E_t^{(m)} [\frac{dY_t}{Y_t}] \right) = \frac{(\nu_t - \overline{\nu}^{(m)})}{\sigma_Y} dt \ + \ d\tilde{W}_{Yt}. \tag{9}$$

In particular, the belief $\pi_{st}^{(m)}$ mean-reverts to its unconditional mean, and its volatility is proportional to agent m's uncertainty about the underlying state (see David (1997) for further details of the updating processes).

The two types of agents perceive the history of fundamentals differently. The process $\{\tilde{W}_{Yt}^{(m)}\}$ is the "innovations" process of analysts of type m, and is the shock process to the macroeconomic factor as perceived by agents of type m. According to analyst m, the macroeconomic factor dynamics are:

$$\frac{dY_t}{Y_t} = \bar{\nu}_t^{(m)} dt + \sigma_Y d\tilde{W}_{Yt}^{(m)}. \tag{10}$$

Taking the difference between the innovations of the two analysts, we have

(11) $$d\tilde{W}_{Yt}^{(2)} = d\tilde{W}_{Yt}^{(1)} + \sigma_{\eta t} dt,$$

where

(12) $$\sigma_{\eta t} = \frac{(\bar{\nu}_t{}^{(1)} - \bar{\nu}_t{}^{(2)})}{\sigma_Y}.$$

Let $\mathcal{P}_t^{(m)}$ be analyst m's probability measure over the path of Y_s, $s \in [0, t]$. Appealing to the results in David (2008) (see Corollary 1), the Radon-Nikodym derivative of $\mathcal{P}_t^{(2)}$ with respect to $\mathcal{P}_t^{(1)}$ is given by the process η_t, which follows;

(13) $$\frac{d\eta_t}{\eta_t} = \sigma_{\eta t} d\tilde{W}_{Yt}^{(1)},$$

which is a martingale with respect to $\mathcal{P}_t^{(1)}$. By relating the two innovation processes, we can write the beliefs of analyst 2 from the perspective of analyst 1 as

(14) $$d\pi_{1t}^{(2)} = (\mu_{1t}^{(2)} + \sigma_{1t}^{(2)} \sigma_{\eta_t}) dt + \sigma_{1t}^{(2)} d\tilde{W}_{Yt}^{(1)},$$

and consequently solve the equilibrium of the model under analyst 1's probability measure. Alternatively, we could also solve it under Analyst 2's probability measure, or even the objective probability measure.

B. Firms in the Economy

We model firms in a production economy with stochastically linear technologies as in Cox et al. (1985). We assume that there are N production technologies, which we shall refer to as assets. The transformation of an investment of an amount X_i of the single good in the economy in the ith asset is given by the

$$\frac{dX_{it}}{X_{it}} = b_i \frac{dY_t}{Y_t} + \sigma_i d\tilde{W}_i. \tag{15}$$

Therefore, the return on each technology is driven by the macroeconomic factor and an idiosyncratic firm-specific shock. The coefficient b_i is the "cash flow beta" of the ith technology. We assume that analysts agree on the value b_i because it can be estimated by regressing each asset's cash flow growth on the macroeconomic factor's growth. Investment in assets is made through competitive, value-maximizing firms. With free entry of firms and stochastic constant returns to scale, there is no incentive for firms to enter or leave industry i if and only if the returns on each firm's shares are identical to the technologically determined physical returns on that asset. The equilibrium scale of each firm is determined by the supply of investment to that firm.

C. Equilibrium in the Economy

We do not make a distinction between consumers and analysts, i.e., we assume that each analyst has the same beliefs as some group of consumers in the economy. More specifically, we assume that there are two types of consumers, with consumer m holding the beliefs of analyst

m.[10] According to the model of analyst m, the drift of the ith technology is $\alpha_i^{(m)} = b_i\,\nu^{(m)}$. Each consumer m in the economy, $m = 1, 2$, has the standard CRRA utility function

$$U^{(m)}(c) = E^{(m)}[\int_0^{\infty} \exp(-\rho s) \cdot u(c_s)ds], \tag{16}$$

with time discount factor ρ and felicity $u(c_t) = c_t^{\gamma}/\gamma$. The felicity function $u(.)$ has a constant coefficient of relative risk aversion $1-\gamma$, and satisfies the Inada conditions $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$.

To facilitate the analysis of the equilibrium, we follow the approach of Cuoco and Hè (1994) to solve for the equilibrium in the A heterogeneous agent model by formulating stochastic weights for the representative agent. The solution method was extended to models with heterogeneous beliefs by Basak (2000) and Basak and Cuoco (1998). While the above models are in exchange economy settings, Buss, Uppal and Vilkov (2016) and Baker et al. (2016) use the same weights to decentralize The social planner's solution in a production economy, which we have in this model. More specifically, for given weights λ_{1t} and λ_{2t} for the two agents, the representative agent's utility function solves:

$$U(c_t; \lambda_{1t}, \lambda_{2t}) = e^{-\rho t} \max_{c_{1t}+c_{2t}=c_t} \lambda_{1t}\frac{c_{1t}^{\gamma}}{\gamma} + \lambda_{2t}\frac{c_{2t}^{\gamma}}{\gamma}. \tag{17}$$

[10]In an earlier version of this paper, we assumed that was a single consumer who aggregated the beliefs of different agents, using Bayesian Model Averaging (BMA). The equilibrium results with the alternative assumptions are very similar. The results with BMA are available upon request from the authors.

Solving this problem gives the individual consumption rates

$$c_{1t} = \frac{(\lambda_{2t}/\lambda_{1t})^{\frac{1}{\gamma-1}}}{1+(\lambda_{2t}/\lambda_{1t})^{\frac{1}{\gamma-1}}}\, c_t, \quad \text{and} \tag{18}$$

$$c_{2t} = \frac{1}{1+(\lambda_{2t}/\lambda_{1t})^{\frac{1}{\gamma-1}}}\, c_t, \tag{19}$$

and the representative agent's utility as

$$U(c_t;\lambda_{1t},\lambda_{2t}) = e^{-\rho t}\,\frac{c_t^{\gamma}}{\gamma}\lambda_{1t}\left(1+\left(\frac{\lambda_{2t}}{\lambda_{1t}}\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma}. \tag{20}$$

Following the analysis in Basak (2000), we formulate the equilibrium with the weights $\lambda_{1t} = 1/y_1$ and $\lambda_{2t} = \eta_t/y_2$, where η_t is the change of measure process in (13), and y_1 and y_2 are the Lagrange multipliers associated with the budget constraints of the two agents at time 0. It is evident that with these weights, consumption allocations coincide with those of competitive equilibrium: that is, they satisfy $u'(c_t^{(1)})/u'(c_t^{(2)}) = (y_1\xi_t^{(1)})/(y_2\xi_t^{(2)})$, the ratio of individuals' optimality conditions, and by construction, the goods market clears.

As discussed in David (2008), in equilibrium, $\eta_t = \xi_t^{(1)}/\xi_t^{(2)}$, which is the ratio of the analysts' state price densities (SPDs). Since the SPD is the state price per unit probability of the state at date t, and the analysts agree on the state prices, η_t is the ratio of the probability density of the state arising from the model of analyst 2, divided by the probability density of the state arising from the model of analyst 1. Clearly, η_t belongs to the interval $[0,\infty]$. To obtain a bounded state variable set, we define

$$\varrho_t = \frac{1}{1+\eta_t}, \tag{21}$$

which is in $[0, 1]$. and in the competitive equilibrium, is the posterior probability that the macro fundamental Y_t at date t arises from the model of analyst 1.[11] Similarly $1 - \varrho_t$ is the posterior probability that the model of analyst 2 generates the data. By Ito's Lemma, $\{\varrho_t\}$ satisfies the process:

$$(22) \qquad d\varrho_t = \varrho_t (1 - \varrho_t)^2 \sigma_{\eta t} \, dt - \varrho_t (1 - \varrho_t) \sigma_{\eta t} dW_t^{(1)}.$$

Given the beliefs and the analysts' model probabilities, we can formulate the equilibrium in the production economy as in Cox et al. (1985). David (1997) extends the CIR model by imposing non-negative portfolio weights (which we also use in this paper) in a two-asset economy and analyzes the case with unobserved drifts in the production processes and learning by a representative agent. Here, the representative consumer can invest in the N assets specified. Let $w_i \geq 0$ be the proportion of the representative consumer's wealth invested in asset i for $i = 1, \cdots, N$, and w_0 be the proportion invested in the instantaneous riskless bond in the economy, which offers a rate of return r_t, and is in zero net supply. We will first solve the social planner's problem for the representative agent economy, which does not include investment in the riskless asset, and hence the portfolio weights satisfy $\sum_{i=1}^{N} w_i = 1$. Later, we will find the rate at which a choice of $w_0 = 0$ is optimal. Then, the representative consumer's wealth dynamics can be written under the model of Analyst 1 as

$$(23) \qquad dW_t = -C_t dt + W_t \left[\sum_{i=1}^{N} w_i \alpha_i^{(1)} dt + w_i b_i \sigma_Y d\tilde{W}_{Yt}^{(1)} + w_i \sigma_i d\tilde{W}_i \right],$$

[11] Using Bayes' law over model likelihoods with uninformative priors, the posterior probability that the data at t arises from the model of agent 1 equals $\frac{0.5\,\text{Prob}(Y_t|\text{Analyst 1's model})}{0.5\,\text{Prob}(Y_t|\text{Analyst 1's model}) + 0.5\,\text{Prob}(Y_t|\text{Analyst 2's model})} = \varrho_t$.

where C_t is the rate of consumption.

Proposition 1 *The value function of the representative consumer in the economy that maximizes utility takes the form:*

$$J(W_t, \pi_t^{(1)}, \pi_t^{(2)}, \varrho_t, t) = \exp(-\rho t)\frac{W_t^\gamma}{\gamma} I(\pi_t^{(1)}, \pi_t^{(2)}, \varrho_t), \tag{24}$$

in which the function $I(\pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t)$ satisfies the PDE:

$$\begin{aligned}
0 = \max_{w_i, s.t. w_i \geq 0, \sum_{i=1}^N w_i = 1} \Bigg[& \left(\frac{1}{\gamma} - 1\right) I^{\frac{\gamma}{\gamma-1}} - \frac{\rho}{\gamma} I \\
& + I\left(\sum_{i=1}^N w_i \alpha_i^{(1)} + \frac{1}{2}(\gamma-1)\sum_{i=1}^N w_i^2 (b_i^2 \sigma_Y^2 + \sigma_i^2)\right) \\
& + \sum_{s=1}^{S-1} I_{\pi_s^{(1)}} \left(\frac{\mu_s^{(1)}}{\gamma} + (\sum_{i=1}^N w_i b_i)\sigma_s^{(1)}\sigma_Y\right) + \sum_{s=1}^{S-1} I_{\pi_s^{(2)}} \left(\frac{\mu_s^{(2)} + \sigma_s^{(2)}\sigma_\eta}{\gamma} + (\sum_{i=1}^N w_i b_i)\sigma_s^{(2)}\sigma_Y\right) \\
& + I_\varrho \left(\frac{\varrho(1-\varrho)^2\sigma_\eta}{\gamma} - (\sum_{i=1}^N w_i b_i)\varrho(1-\varrho)\sigma_\eta\right) \\
& + \frac{1}{2}\sum_{s=1}^{S-1} I_{\pi_s^{(1)}\pi_s^{(1)}} (\sigma_s^{(1)})^2 + \frac{1}{2}\sum_{s=1}^{S-1} I_{\pi_s^{(2)}\pi_s^{(2)}} (\sigma_s^{(2)})^2 + \sum_{s=1}^{S-1}\sum_{s'=1}^{S-1} I_{\pi_s^{(1)}\pi_{s'}^{(2)}} \sigma_s^{(1)}\sigma_{s'}^{(2)} \\
& + \frac{1}{2} I_{\varrho\varrho}\varrho^2(1-\varrho)^2\sigma_\eta^2 - \sum_{s=1}^{S-1} I_{\pi_s^{(1)}\varrho}\sigma_s^{(1)}\varrho(1-\varrho)\sigma_\eta - \sum_{s=1}^{S-1} I_{\pi_s^{(2)}\varrho}\sigma_s^{(2)}\varrho(1-\varrho)\sigma_\eta \Bigg].
\end{aligned} \tag{25}$$

The Kuhn-Tucker first-order conditions for the portfolio choices of the consumer are:

$$\begin{aligned}
\alpha_i^{(1)} + (\gamma-1) w_i (b_i^2 \sigma_Y^2 + \sigma_i^2) + \sum_{s=1}^{S-1} \frac{I_{\pi_s^{(1)}}}{I} b_i \sigma_s^{(1)} \sigma_Y + \sum_{s=1}^{S-1} \frac{I_{\pi_s^{(2)}}}{I} b_i \sigma_s^{(2)} \sigma_Y \\
- \frac{I_\varrho}{I} b_i \varrho (1-\varrho) \sigma_\eta \sigma_Y + \kappa_i - \frac{\lambda^{(1)}}{I} \leq 0 \qquad \textit{for} \quad i = 1, \cdots, N
\end{aligned} \tag{26}$$

$$w_i\left[\alpha_i^{(1)} + (\gamma-1)w_i(b_i^2\sigma_Y^2+\sigma_i^2) + \sum_{s=1}^{S-1}\frac{I_{\pi_s^{(1)}}}{I}b_i\sigma_s^{(1)}\sigma_Y + \sum_{s=1}^{S-1}\frac{I_{\pi_s^{(2)}}}{I}b_i\sigma_s^{(2)}\sigma_Y - \frac{I_\varrho}{I}b_i\varrho(1-\varrho)\sigma_\eta\sigma_Y + \kappa_i - \frac{\lambda^{(1)}}{I}\right] = 0 \qquad \textit{for} \quad i=1,\cdots,N \tag{27}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{28}$$

$$w_i \geq 0; i=1,\cdots,N \tag{29}$$

$$w_i\kappa_i = 0; i=1,\cdots,N, \tag{30}$$

where $\lambda^{(1)}$ *is the multiplier associated with the constraint* $\sum_{i=1}^{N} w_i = 1$*, and* κ_i *are the multipliers associated with the constraints* $w_i \geq 0$ *for* $i = 1,\cdots,N$.

The proof is in the Appendix.

We solve the PDE in (25) with Pade polynomials using projection methods. The Pade polynomials (rather than simple polynomials), which are a ratio of finite length simple polynomials, are needed to provide an accurate series approximation of the non-linear leading term: $I^{\frac{\gamma}{\gamma-1}}$. A similar PDE has been solved in David (2008), and we follow the steps outlined there to implement this method. One difference, though, is that the PDE in David (2008) lacked the portfolio choices needed here. We follow a recursive procedure to determine the portfolio choices and the solution to the PDE. Given the nth iteration of the solution, I^n, we solve the portfolio choices w^{n+1} using (26) – (30) using a standard equation solver at each point on the Chebyshev grid. We then use these portfolio choices to obtain the projections onto the polynomials, and hence find I^{n+1}. Using standard contraction mapping arguments, the recursive procedure converges.

As in Buss et al. (2016) and Baker et al. (2016), the Production choices of the two agents are identical to those of the representative agent. Intuitively, the production choices are identical because the two types of consumers agree on the paths of the production processes, once we formulate their drifts under the same measure (we chose the measure of analyst 1). A formal statement and proof are in Proposition 3 in Appendix 1.

D. The Cross Section of Equilibrium Risk Premia in the Economy

We now consider the cross-section of equilibrium risk premia for the different stocks in the economy.

Proposition 2 *In equilibrium, the risk premium for stock i for any stock with* $w_i > 0$ *satisfies:*

$$(31)\quad \alpha_i^{(1)} - r \;=\; \frac{b_i}{b_m}\,(\alpha_m^{(1)} - r) + (1-\gamma)\,w_i\,(b_i^2\sigma_Y^2 + \sigma_i^2) - \frac{b_i}{b_m}\,(1-\gamma)\sum_{i=1}^{N} w_i^2(b_i^2\sigma_Y^2 + \sigma_i^2),$$

in which $b_m = \sum_{i=1}^{N} w_i\, b_i$ *is the cash flow beta of the market portfolio, and* $\alpha_m^{(m)} = \sum_{i=1}^{N} w_i\,\alpha_i^{(1)}$ *is the expected return on the market portfolio; both* b_m *and* $\alpha_m^{(1)}$ *are endogenous. The riskless rate in the economy satisfies:*

$$(32)\quad r = \left(\sum_{i=1}^{N} w_i b_i\right)\left[\alpha_i^{(1)} + \sum_{s=1}^{S-1}\frac{I_{\pi_s^{(1)}}}{I}\,\sigma_s^{(1)}\sigma_Y + \sum_{s=1}^{S-1}\frac{I_{\pi_s^{(2)}}}{I}\,\sigma_s^{(2)}\sigma_Y \;-\; \frac{I_\varrho}{I}\varrho(1-\varrho)\sigma_\eta\sigma_Y\right] + \;(\gamma-1)\sum_{i=1}^{N} w_i^2\,(b_i^2\sigma_Y + \sigma_i^2)$$

The proof is in the Appendix. It is to be noted that we solve the model under the measure of an analyst of type 1, and hence have provided the risk premiums under this measure. From (15) we see that $\alpha_i^{(m)} = b_i \, \bar{\nu}^{(m)}$, and hence the risk premiums can be analogously stated in terms of the measure of analysts of type 2. The relationship between the innovations process in (11) of the two agents implies that the expected return of analysts of type 2 is $\alpha_i^{(2)} = \alpha_i^{(1)} + b_i \sigma_\eta \sigma_Y$.

We provide some comments on the equilibrium cross-sectional pricing equation in (31). It is interesting to note the three terms in each asset's risk premium. The first term compensates the investor for bearing market risk, similar to the CAPM. The only difference from the standard CAPM is the exposure to market risk is measured by the asset's cash flow beta relative to the market cash flow beta, rather than the stock return beta in the CAPM. The second term measures the risk premium for bearing the risk associated with the aggregate macroeconomic factor, as well as any undiversified idiosyncratic risk. The last term is the premium for hedging the risk of shifts in the opportunity set in the ICAPM framework of Merton (1973). In our setup, this term depends on the product of the consumer's risk aversion and the weighted average of the squared cash-flow beta (squared-beta term for brevity). The negative sign implies that in periods when the term is larger, the risk premium for each asset is smaller. The negative sign arises as the consumers' wealth grows faster when they invest in higher cash flow beta assets, that is, their opportunity set is more attractive from a growth perspective.

It is worth noting that in (31) the state variables do not appear explicitly, but only through the portfolio weights. The state variables in the economy are the beliefs of each analyst $\pi_t^{(m)}$ about the state of the macro fundamental, as well as the variable ϱ_t, which is the conditional probability at the time that the fundamentals are generated by the model of agent 1. The risk premia for shifts in these state variables directly affect the portfolio choices in Proposition 1 and, hence, the magnitude of the hedging term in the last paragraph. We will see in the empirical section that the model exhibits an endogenous "flight-to-safety": in periods of higher disagreement, investors tend to choose lower-cash-flow-beta stocks. The intuition for this is in David (2008), which shows that in models with heterogeneous beliefs, investors' beliefs are mutual risk factors for each other (fluctuations in analyst 1's beliefs are a risk factor for analyst 2, and vice versa). During periods of high disagreement, The risk of fluctuations in others' beliefs is greater, and hence investors respond by choosing safer assets.

To build intuition on the way the disagreement affects the opportunity set and the squared-beta term, consider a simple case where there are only two assets in the economy, with $b_1 = 0.1$ and $b_2 = 2$. Suppose the disagreement amongst the consumers is high, and the consumer responds by choosing the safer asset, so that $w_1 = 0.9$, and $w_2 = 0.1$. Now the term $\sum_i w_i^2 b_i^2$ is just 0.0481. Alternatively, when disagreement is low, the weights reverse: $w_1 = 0.1$ and $w_2 = 0.9$, and this term equals 3.24. In the latter case, the variance of the opportunity set is larger, and hence the hedging role of higher b_i assets is larger, so that their overall risk premium is smaller. Conversely, when disagreement is higher (as in the first case), the first term in the risk premium is more important than the hedging term, so assets with a higher b_i will have a higher risk premium. Overall, the example illustrates that the flight-to-safety mechanism is the primary driver underlying the switch in the price of disagreement risk between periods of high and low disagreement. We look at the flight-to-safety effect in more detail in Section C.

V. Empirical Estimation of the Model and its Implications

To address the empirical results in our paper, we first provide a structural estimation of the parameters of our theoretical model, and then proceed to two-stage regressions of individual stock returns on the market return, and aggregate earnings disagreement in the economy.

A. Estimation Methodology and Beliefs of the Analysts and the Representative Agent

We estimate the parameters of our model using the Generalized Method of Moments (GMM), using data on fundamentals as well as disagreement in analysts' forecasts, similar to that in David (2008). A brief description of the estimation method is as follows. The aggregate fundamental is the value-weighted average of firm's actual earnings growth rate in the IBES database. Using a 3-state unobserved regime shifting structure in the mean of the aggregate fundamental, we estimate two sets of parameters (one for each type of analyst), which maximizes the sum of the likelihoods of each analyst type observing the fundamentals, and the difference between aggregate disagreement in the data as shown in Figure 1 and the model-implied disagreement of 5-year ahead expected aggregate earnings growth.[12]

This is implemented in GMM by discretizing the fundamental process and using the likelihood function's scores with respect to each analyst's model parameters as moments. The GMM estimator has a chi-square distribution with two degrees of freedom; one arises from the disagreement moment, and the second from the fact that the analysts agree on the volatility of the fundamental, reducing one parameter in the calculation of the scores. The expected growth rate for each analyst in the model disagreement calculation is computed from each agent type's filtered beliefs.

[12]Using the transition generators for each analyst, the 5-year transition matrix over states for agent m is $P(t, t+5)^{(m)} = \exp(\Lambda^{(m)} \cdot 5)$. Then, the 5-year growth rates esimated by agent m are $g_t^{(m)} = \pi_t^{(m)} P(t, t+5) \cdot \nu^{(m)} \cdot 5$, and the mean 5-year growth rate is $\bar{g}_t = \sum_m 0.5 \cdot g_t^{(m)}$. The model disagreement is calculated as $\sqrt{\sum_m 0.5 \cdot (g_t^{(m)} - \bar{g}_t)^2}$. The data disagreement is the value-weighted average of the standard deviation of analysts' forecasts of the 5-year growth in earnings per share, which is described in detail in Section II.

The estimated parameters are shown in Table 6, and we make some summary comments here.

[Insert Table 1 approximately here]

Each analyst estimates three states for the drift rates of aggregate fundamentals. States 1 and 2 are the 'regular' low and high growth rate states, and the estimates of the two types of analysts are 'fairly' similar. Analysts of type 1 estimate that growth in the two states is about 1.5 percent and 6.4 percent, respectively, while analysts of type 2 estimate that average growth is 4 percent and 7 percent in these two states, i.e., in the regular states, analyst 1 (2) is the relative pessimist (optimist). In the third state, which is described best as a 'disaster' state, agents have vastly different estimates on the severity of growth: type 1 analysts estimate that growth is -42 percent, while type 2 analysts estimate it at -85 percent, i.e., the relative optimism of the analysts flips from the regular states. Overall, analysts of type 2 have more volatile estimates that overshoot those of type 1 in both directions.

The filtered beliefs of the two analyst types are shown in Figure 6.

[Insert Figure 6 approximately here]

The two top panels show that the beliefs about state 1(2) of each type of analyst increase in recessions (booms) of the US economy. The middle-left panel shows that beliefs about the third (disaster) state spiked in both the 2000 and 2007-9 recessions. During the former recession, realized quarterly growth was about -40 percent at an annual rate, while in the second half of 2008, it averaged about -56 percent. In contrast, neither type of analyst assessed a large probability of being in a disaster state in the 1990-91 recession.

The middle-right panel shows the realized quarterly and one-quarter-ahead expected growth rate of each analyst type. The expectations of the two types are highly correlated, and as shown in Table 6, each type's estimates explain close to 50 percent of the variation in realized earnings growth. However, as suggested by the parameter estimates, the expectations of type 2 are more volatile, and the disagreement in their expectations increased sharply in the 2000 and 2007-9 recessions. The bottom-left panel shows the time series of aggregate disagreement in the data and in the model, and, quite significantly, the model explains 75 percent of the variation in disagreement in the data. With a similar 2-state estimation, we are unable to achieve a good fit for the aggregate disagreement during the 2000s recessions, when earnings sharply contracted.

An important input into our model is the process $\{\varrho_t\}$, which is the conditional probability that the data at t arises from the model of analysts of type 1. We show this series in the bottom-right panel of Figure 6. As seen, prior to 2000, ϱ_t fluctuated near 0.5, but in 2001 it jumped to nearly 0.9 when the less pessimistic estimate of analyst 1 was relatively more accurate. Thereafter, it persisted at an elevated level until the end of 2008, when realized growth was closer to Analyst 2's estimate. At this point, ϱ plunged to about 12 percent in the second quarter of 2009, and then slowly rose again until 2011. Thereafter, it remained below 0.5 until the end of the sample period in 2016. The persistence of the ϱ process, as shown in (22), plays a role in the persistent deviations from 0.5 between the two recessions and in the post-2009 period.

B. Risk Premiums and the Price of Disagreement Risk From Two-Stage Regressions in the Model

Our theoretical model assumes that each stock in the economy represents a stochastically linear production process as in Cox et al. (1985). As in Hong and Sraer (2016), to simplify our analysis, we assume that there are 10 linear technologies in the economy. The growth rate of technology i is represented by the cash flow beta b_i multiplied by the aggregate cash flow growth rate of the economy, as in (refeq:dbi). We first discuss our methodology for estimating the cash flow betas for each technology.

We start our estimation at the firm level. The conditional cash flow beta, b_{it}, for each firm in the IBES database is estimated using 12-quarter rolling regressions of firm-level earnings growth on aggregate earnings growth for the period from 1984:3 to 2016:3. To mitigate the effects of rounding due to stock splits, We follow Diether et al. (2002) and use quarterly actual unadjusted earnings from IBES and use the CRSP adjustment factor to adjust the data for split events. In each quarter, we winsorize the firm-level earnings at the 5% and 95% levels. We construct firm i's earnings growth as

$$g_i(t) = \begin{cases} \frac{E_i(t) - E_i(t-4)}{|E_i(t-4)|} & \text{if} \quad E_i(t) < 0 \quad \text{or} \quad E_i(t-4) < 0 \\ \ln\left[\frac{E_i(t)}{E_i(t-4)}\right] & \text{otherwise,} \end{cases} \tag{33}$$

where $E_i(t)$ is firm is actual earning per share at quarter t. Similarly, aggregate earnings growth is calculated as $g(t) = \ln\left[\frac{E(t)}{E(t-4)}\right]$, where $E(t) = \sum_i E_i(t)$. Following Welch (2020), we winsorize each firm earnings' growth $g_i(t)$ to $[(1 \pm \Delta_s) g(t)]$, where $\Delta_s = 3$. Next, in each quarter, we sort the stocks by cash flow beta and calculate the mean cash flow beta for each decile. Finally, we take the time-series average of the cash flow betas of each decile. The resulting ten cash flow betas are -0.411, 0.187, 0.635, 0.896, 1.066, 1.232, 1.391,1.540, 1.736, and 2.044. We did not find statistically significant differences in idiosyncratic volatility across deciles and hence use the same idiosyncratic volatility of 0.35 for each asset.

Using the above cash flow betas for the technologies, we calculate the equity premium and the price of disagreement risk in our model. Our model has the aggregate macro shock, as well as idiosyncratic shocks for the technologies. As for our estimated model, we use the realized process of earnings growth, the analysts' beliefs, and the ϱ_t process in Section A, as shown in Figure 6. These enable us to calculate the risk premium and the risk-free rate for each consumer, as in Proposition 2, which we discuss next.

For calculating the model's prices of risk, we need to assume consumers' preferences. We use a time discount, ρ, of 4 percent, and use a range of values for γ from 0.5 to -15. Our results are presented in Table 7.

[Insert Table 1 approximately here]

While the primary target of our modeling in this paper is not the equity premium, it is worth noting that it can yield sizable equity premiums for each agent and a low riskless rate for a risk aversion coefficient of about 10. Using the specification of the asset processes in (15), the risk premium on the market index for agent m is $\alpha_m^{(m)} = (\sum_i b_i w_i)\nu^{(m)}$. We calculate these at each date, and the time series for the risk premiums for $\gamma = -10$ are shown in Figure 8.

[Insert Figure 8 approximately here]

As seen, the risk premiums for both consumers are procyclical and decline sharply during the 2001 and 2008 recessions. Generally, the more optimistic consumer 1 has a higher risk premium. The average riskless rate and the averages of the risk premiums over the full sample are shown in columns two to four of Table 7. The average riskless rate falls from over 6 percent when $\gamma = 0.5$ to -4.5 percent when $\gamma = -10$, while the risk premiums fall from 3.6 and 2.8 percent respectively, for the two consumers when $\gamma = -10$, to 2.6 and 1.0 percent, when $\gamma = -10$. We provide intuition for these moments next.

Since analysts' (consumers') beliefs are mutual risk factors, the risk of these fluctuations leads them to demand riskless securities, which lowers their equilibrium rate. The rate falls more with higher risk aversion (more negative γ). These beliefs are also priced into equity returns, as they lead to fluctuations in asset returns. As in the heterogeneous beliefs setting of David (2008), the risk premiums decline with risk aversion, as risk-taking in the model is endogenous. Less risk-averse consumers allocate a larger share of their wealth to riskier assets and achieve higher expected returns. Due to the volatilities of their portfolios, though, their consumption growth rates are also higher with lower risk aversion. The aggregate consumption volatility in the model is shown in the 5th column of Table 7. A seen it is as high as 18.5 percent at an annual rate when $\gamma = -.5$ and falls to 3.3 percent when $\gamma = -10$. Mehra and Prescott (1985) uses an estimate of around 3 percent, so the latter number is in a reasonable range. It is helpful to note that David (2008) examined the equity premium in an exchange economy in which the volatility of aggregate consumption is exogenously given and does not vary with risk aversion. The current model has a production economy; hence, the aggregate consumption process is endogenous and varies with γ.

Speculative risk in the economy is evident in the volatility of individual consumption. Columns 6 and 7 show the volatility of consumption for the two consumers, and these numbers are higher than the aggregate consumption volatility, since some of the gains/losses of consumers are speculative (at the expense of the other consumer), and these cancel out in aggregate. It is helpful to note that the ratio of average individual consumption volatilities to aggregate consumption volatility is not as high as in the exchange economy setting of David (2008), where consumers competed for the given quantity of endowment. In the production economy setting here, consumers can increase their consumption by investing in the risky technologies, and hence, there is less pure speculative risk. Instead, we get a boost in the risk premium and a decline in the riskless rate due to the modeling of disaster risk as in Reitz (1988) and Barro (2006). This happens because the marginal utility of consumers rises rapidly in disaster states.

We next estimate the prices of risk using cross-sectional two-stage regressions for our model. The cross-sectional regressions are conditional on the estimated process of aggregate fundamentals and the beliefs of each type of analyst in Section A. As a first step, we simulate individual stock return processes for the 10 assets with the specification in equation (15). We use realized aggregate earnings growth at each date and simulate asset-level shocks, $d\tilde{W}_{it}$, at each date. Using the estimated cash flow betas for the 10 assets provides us with a panel of returns for those assets. Using the filtered beliefs of each analyst type, and the $\{\varrho_t\}$ process, we calculate the market portfolio weights as shown in Proposition 1. Then, similar to the data, we use rolling 12-quarter lagged stock returns for each asset, which we regress on the market return and earnings disagreement from the structural estimation in Section A in the first stage. This gives us β_{it} and δ_{it} for each asset at each date. In the second-stage regression, at each date, we regress the stock returns for each of the 10 assets on beta and delta at that date. These give us estimates of the prices of market risk, π_t, and disagreement risk, ω_t, at each date. We report average prices of risk across 1000 simulated sets of idiosyncratic shocks.

The time series of the model's price of disagreement risk from the second stage and the empirically estimated price of disagreement risk from the 100 delta-beta portfolios are shown in Figure 7.

[Insert Figure 7 approximately here]

As seen in both the model and the data, the signs of the disagreement risk prices change over time. We make three comments on the price of disagreement risk. First, the data and model prices of risk are correlated at 45 percent, despite not targeting the price of risk in our estimation.

Second, the price of disagreement risk and aggregate disagreement are positively correlated at 22 percent in the data and 21 percent in our model; that is, the price of disagreement risk is higher during periods of high aggregate disagreement. Finally, the average price of disagreement risk during high disagreement months (defined as being above one standard deviation from the mean) is relatively invariant to the level of risk aversion (for negative values of γ) and is around 0.25 in high disagreement months and -0.28 in low disagreement months, which as in the data, are relatively high in absolute value, and change signs in these alternative subsamples as we reported for the data in the introduction, and in Section III. We provide further intuition for the sign switch in the price of disagreement risk in the following subsection.

C. The Flight-to-Safety Effect

As described in Proposition 2, a key determinant of an individual asset's risk premia is the squared-beta term, $\sum_i w_{it}^2 b_{it}^2$, which is the hedging term in Merton's ICAPM framework. In this subsection, we examine this term in the data and for our model. For the data, we use the cash flow beta, b_{it}, for each firm in the first stage (as described in the previous subsection), and then calculate the cash flow betas of the 10 assets after sorting firms by their cash flow betas. We use the earnings of the firms in each decile to calculate the weights. The resulting squared-beta term is shown in the top panel of Figure 9.

[Insert Figure 9 approximately here]

The panel also has the aggregate disagreement series from Figure 1. As seen, the two series are negatively correlated, with a correlation coefficient of -0.17.

For our model, we calculate the weights the representative investor allocates to the different assets as shown in Proposition 1. The model weights are calculated at each date using the estimated process for aggregate fundamentals and the beliefs of each analyst type in Section A for $\gamma = -10$. In the bottom panel of Figure 9, we plot the time series of the disagreement in the model and the model's squared-beta term. Similar to the data series, the two series are negatively correlated (correlation of -0.13), i.e, in periods of higher disagreement, the investor chooses lower cash flow beta assets, thus lowering the squared-beta term. Overall, the data support a key implication of our model: a flight-to-safety toward lower cash-flow beta assets in periods of higher disagreement. As discussed in Section D, this flight-to-safety leads to changes in the sign of the price of disagreement risk – it is positive during periods of high disagreement, and negative during periods of low disagreement.

While the flight-to-safety plot is for the case $\gamma = -10$, in the last column of Table 7, we show the correlation between the squared-beta term and aggregate disagreement for the model for alternative levels of γ. As seen, for all negative values of γ, the correlation is quite similar. As pointed out earlier, the magnitudes of the price of disagreement risk in the high and low disagreement samples are also insensitive to the price of disagreement risk. The relative insensitivity of this correlation arises due to two opposing forces in the production economy: with higher risk aversion, consumers take on less risky portfolios, but for a given level of risk, their required risk compensation is higher.

VI. Conclusion

Understanding how disagreement in analysts' beliefs affects the cross-section of stock market returns is one of the most fundamental issues in finance and has led to many important papers. section of stock market returns is one of the most fundamental issues in finance and has led to many important papers. We contribute to this literature, both empirically and theoretically. On the empirical front, we provide robust evidence that the price of disagreement risk changes sign over time: it is significantly positive in periods of high disagreement, significantly negative in periods of low disagreement, and insignificant in the remaining periods (a large part of our sample). Also, the magnitude of the price of disagreement risk is more pronounced in periods of high disagreement compared to periods of low disagreement.

On the theoretical front, we construct a production economy general equilibrium model in the spirit of Cox et al. (1985), in which analysts of different types have heterogeneous beliefs and provide different forecasts of a macroeconomic factor (aggregate earnings growth). The key mechanism of our model that leads to changes in the sign of the price of disagreement risk over time is that disagreement affects the composition of the market portfolio, which affects the price of market risk, as well as the risk of changes in the consumers' investment opportunity set in the spirit of Merton's ICAPM, with opposite signs. During periods of low disagreement, the former effect is more important, leading to a negative price of risk, while during periods of high disagreement, the latter effect dominates, leading to a positive price of risk.

We structurally estimate our model, and in particular, show that analysts' differences in beliefs about regular high- and low-growth-rate states, as well as a disaster state, capture well the time variation in aggregate earnings disagreement. The main implication of our model is that the price of disagreement risk is higher (lower) during periods when investors choose lower (higher) cash flow beta assets are supported in the data for both our estimation period, which ends in Sept-2016, and the out-of-sample period from then to the end of 2021. In particular, aggregate disagreement was mainly in the high range in the out-of-sample period, and the price of disagreement risk was positive and statistically significant. The three Fama-French factors and momentum have statistically insignificant prices of risk in this period.

Appendix

Proof of Proposition 1

The following moments are used to formulate the value function of consumers below:

$$
\begin{aligned}
(34)\quad E[\frac{dX_{it}}{X_{it}}\, d\pi_{st}^{(m)}] &= b_i\, \sigma_{st}^{(m)}\, \sigma_Y\, dt, \quad \text{for } i=1,\cdots,N; s=1,\cdots,S-1; m=1,2, \\
(35)\quad E[\left(\frac{dX_{it}}{X_{it}}\right)^2] &= b_i^2\, \sigma_Y^2 + \sigma_i^2, \quad \text{for } i=1,\cdots,N, \\
(36)\quad E[d\varrho_t\, d\pi_{st}^{(m)}] &= -\varrho_t(1-\varrho_t)\sigma_{\eta t}\sigma_{st}^{(m)}, \quad \text{for } s=1,\cdots,S-1; m=1,2.
\end{aligned}
$$

The value function $J(W_t, \pi_t^{(1)}, \pi_t^{(2)}, \varrho_t, t)$ under the measure of analyst 1 satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$
\begin{aligned}
(37)\quad 0 = \max_{C, w_i, s.t. w_i \geq 0, \sum_{i=1}^N w_i = 1} \Bigg[& U(C) + J_t - J_W C \\
& + J_W W \left(\sum_{i=1}^N w_i \alpha_i^{(1)}\right) + \sum_{s=1}^{S-1} J_{\pi_s^{(1)}} \mu_s^{(1)} + \sum_{s=1}^{S-1} J_{\pi_s^{(2)}} (\mu_s^{(2)} + \sigma_s^{(2)}\sigma_\eta) + J_\varrho \varrho(1-\varrho)^2 \sigma_\eta \\
& + \sum_{s=1}^{S-1} J_{W\pi_s^{(1)}} W \left(\sum_{i=1}^N w_i b_i\right) \sigma_s^{(1)} \sigma_Y + \sum_{s=1}^{S-1} J_{W\pi_s^{(2)}} W \left(\sum_{i=1}^N w_i b_i\right) \sigma_s^{(2)} \sigma_Y - J_{W\varrho} W \left(\sum_{i=1}^N w_i b_i\right) \varrho(1-\varrho)\sigma_\eta \\
& + \frac{1}{2}\sum_{s=1}^{S-1} J_{\pi_s^{(1)}\pi_s^{(1)}} (\sigma_s^{(1)})^2 + \frac{1}{2}\sum_{s=1}^{S-1} J_{\pi_s^{(2)}\pi_s^{(2)}} (\sigma_s^{(2)})^2 + \sum_{s=1}^{S-1}\sum_{s'=1}^{S-1} J_{\pi_s^{(1)}\pi_{s'}^{(2)}} \sigma_s^{(1)} \sigma_{s'}^{(2)} \\
& + \frac{1}{2} J_{\varrho\varrho} \varrho^2 (1-\varrho)^2 \sigma_\eta^2 - \sum_{s=1}^{S-1} J_{\pi_s^{(1)}\varrho} \sigma_s^{(1)} \varrho(1-\varrho)\sigma_\eta - \sum_{s=1}^{S-1} J_{\pi_s^{(2)}\varrho} \sigma_s^{(2)} \varrho(1-\varrho)\sigma_\eta \\
& + \frac{1}{2} J_{WW} W^2 \left(\sum_{i=1}^N w_i^2 (b_i^2 \sigma_Y^2 + \sigma_i^2)\right) \Bigg].
\end{aligned}
$$

The envelope optimality condition for consumption is:

$$
(38)\qquad U_C = J_W
$$

Given the CRRA preference of the representative consumer, her value function takes the form:

$$J(W_t, \pi_t^{(1)}, \pi_t^{(2)}, \varrho_t, t) = \exp(-\rho t)\frac{W_t^{\gamma}}{\gamma} I(\pi_t^{(1)}, \pi_t^{(2)}, \varrho_t). \tag{39}$$

The partial derivatives of J therefore satisfy: $J_t = -\rho J$; $J_W = (\gamma J)/W$;

$J_{WW} = (\gamma(\gamma-1)J)/W^2$; $J_{\pi_s^{(m)}} = (I_{\pi_s^{(m)}}/I)J$; $J_{\pi_s^{(m)}\pi_s^{(m)}} = (I_{\pi_s^{(m)}\pi_s^{(m)}}/I)J$; $J_{\varrho} = (I_{\varrho}/I)J$,

$J_{\varrho\varrho} = (I_{\varrho\varrho}/I)J$; $J_{W\pi_s^{(m)}} = (\gamma J)/W(I_{\pi_s^{(m)}}/I)$; $J_{W\varrho} = (\gamma J)/W(I_{\varrho}/I)$, for $s = 1, \cdots, S-1$,

$m = 1, 2$. Substituting these and the optimality condition for consumption (38) into the HJB equation (37) implies the PDE in (25). The Kuhn-Tucker first-order conditions for the portfolio choices of the consumer follow. ■

Proof of Proposition 2

First consider the derived utility of wealth function in the decentralized economy. By following the same steps as for the central planner's problem, the function $I(\pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t)$

satisfies the PDE:

$$
\begin{aligned}
(40)\quad 0 = \max_{w_i, s.t. w_i \geq 0,\ \sum_{i=0}^{N} w_i = 1} \Bigg[& \left(\frac{1}{\gamma} - 1\right) I^{\frac{\gamma}{\gamma-1}} - \frac{\rho}{\gamma} I \\
& + I \left(w_0\, r + \sum_{i=1}^{N} w_i \alpha_i^{(1)} + \frac{1}{2}(\gamma - 1) \sum_{i=1}^{N} w_i^2 (b_i^2 \sigma_Y^2 + \sigma_i^2) \right) \\
& + \sum_{s=1}^{S-1} I_{\pi_s^{(1)}} \left(\frac{\mu_s^{(1)}}{\gamma} + (\sum_{i=1}^{N} w_i b_i) \sigma_s^{(1)} \sigma_Y \right) + \sum_{s=1}^{S-1} I_{\pi_s^{(2)}} \left(\frac{\mu_s^{(2)} + \sigma_s^{(2)} \sigma_\eta}{\gamma} + (\sum_{i=1}^{N} w_i b_i) \sigma_s^{(2)} \sigma_Y \right) \\
& + I_\varrho \left(\frac{\varrho(1-\varrho)^2 \sigma_\eta}{\gamma} - (\sum_{i=1}^{N} w_i b_i) \varrho (1-\varrho) \sigma_\eta \right) \\
& + \frac{1}{2} \sum_{s=1}^{S-1} I_{\pi_s^{(1)} \pi_s^{(1)}} (\sigma_s^{(1)})^2 + \frac{1}{2} \sum_{s=1}^{S-1} I_{\pi_s^{(2)} \pi_s^{(2)}} (\sigma_s^{(2)})^2 + \sum_{s=1}^{S-1} \sum_{s'=1}^{S-1} I_{\pi_s^{(1)} \pi_{s'}^{(2)}} \sigma_s^{(1)} \sigma_{s'}^{(2)} \\
& + \frac{1}{2} I_{\varrho\varrho} \varrho^2 (1-\varrho)^2 \sigma_\eta^2 - \sum_{s=1}^{S-1} I_{\pi_s^{(1)} \varrho} \sigma_s^{(1)} \varrho (1-\varrho) \sigma_\eta - \sum_{s=1}^{S-1} I_{\pi_s^{(2)} \varrho} \sigma_s^{(2)} \varrho (1-\varrho) \sigma_\eta \Bigg].
\end{aligned}
$$

where now w_0 is the portfolio choice in the riskless asset. We allow for w_0 to be positive (riskless lending) or negative (riskless borrowing). The first order condition for w_0 is

$$
(41)\qquad r = \frac{\lambda^{(1)}}{I}.
$$

Now set the optimal choices for w_i for $i = 1, \cdots, N$, as for the central planner, and in addition, let r satisfy (41) Then, $w_0 = 0$ is optimal in the decentralized choice. Now, summing (27) over $i = 1, \cdots, N$, and using (28) and (30) implies that

$$
\begin{aligned}
(42)\quad \left(\sum_{i=1}^{N} w_i b_i \right) & \left[\alpha_i^{(1)} + \sum_{s=1}^{S-1} \frac{I_{\pi_s^{(1)}}}{I} \sigma_s^{(1)} \sigma_Y + \sum_{s=1}^{S-1} \frac{I_{\pi_s^{(2)}}}{I} \sigma_s^{(2)} \sigma_Y - \frac{I_\varrho}{I} \varrho (1-\varrho) \sigma_\eta \sigma_Y \right] \\
& + (\gamma - 1) \sum_{i=1}^{N} w_i^2 \left(b_i^2 \sigma_Y + \sigma_i^2 \right) = \frac{\lambda^{(1)}}{I} = r
\end{aligned}
$$

Now using (26), which holds with equality for any asset with $w_i > 0$, and (42), implies (31). ■

Proposition 3 *Consumers agree on the investment choices in the N assets.*

Proof of Proposition 3

Consumer m maximizes the utility in (16) subject to the wealth dynamics of agent m

$$dW_t^{(m)} = -C_t^{(m)}dt + W_t^{(m)}\left[\sum_{i=1}^{N} w_i\alpha_i^{(m)}dt + w_i b_i \sigma_Y d\tilde{W}_{Yt}^{(m)} + w_i\sigma_i d\tilde{W}_i\right], \tag{43}$$

which is similar to (23) that was stated under the measure of consumer 1. By the definition of the Radon-Nikodym derivative, the sum of expected utilities of the two consumers

$$E^{(1)}[\int_0^\infty \exp(-\rho s)\cdot u_(c_{1s})ds] + E^{(2)}[\int_0^\infty \exp(-\rho s)\cdot \frac{\eta_s}{\eta_t}u_2(c_{2s})ds] \tag{44}$$

$$= E^{(1)}[\int_0^\infty \exp(-\rho s)\cdot u_(c_{1s}) + \frac{\eta_s}{\eta_t}u_2(c_{2s})ds] = E^{(1)}[\int_0^\infty \exp(-\rho s)\cdot U[c_s]ds], \tag{45}$$

which is the objective function of the planner. Similarly for any given consumption and wealth processes $\{C_t\}$, and $\{W_t\}$, respectively. the budget equation

$$\begin{aligned} dW_t &= -C_t dt + W_t\left[\sum_{i=1}^{N} w_i\alpha_i^{(2)}dt + w_i b_i\sigma_Y d\tilde{W}_{Yt}^{(2)} + w_i\sigma_i d\tilde{W}_i\right], \\ &= -C_t dt + W_t\left[\sum_{i=1}^{N} w_i(\alpha_i^{(2)} + b_i\sigma_\eta\sigma_Y)dt + w_i b_i\sigma_Y d\tilde{W}_{Yt}^{(1)} + w_i\sigma_i d\tilde{W}_i\right] \\ &= -C_t dt + W_t\left[\sum_{i=1}^{N} w_i\alpha_i^{(1)}dt + w_i b_i\sigma_Y d\tilde{W}_{Yt}^{(1)} + w_i\sigma_i d\tilde{W}_i\right], \end{aligned} \tag{46}$$

where the second equality arises from the definition of the innovation processes of the two consumers in (11), and the third equality follows from due to the agreement by consumers on the path of the ith in (15). Therefore, under the measure of consumer 1, the dynamics of wealth that are assessed by consumer 2 agree with the wealth dynamics of consumer 1. Since the consumers agree on the value of the objective function as well as the wealth constraint under the measure of agent 1, they agree on the portfolio weights. ■

Padé Approximation of Nonlinear Term

As mentioned in the comments following Proposition 1, we approximate the term with Padé polynomials; specifically, $\left(\frac{1}{\gamma}-1\right) I^{\frac{\gamma}{\gamma-1}}(x) = \frac{I_p(x)}{I_q(x)}$, where $I_p(x)$ and $I_q(x)$ are polynomials of order m and n, respectively. In particular, we find very accurate approximations using $m = 5$ and $n = 5$ for alternative levels of γ. Multiplying (25) by $I_q(\pi, \varrho)$, we then have a linear partial differential equation:

$$
\begin{aligned}
(47)\quad 0 = \max_{w_i, s.t. w_i \geq 0, \sum_{i=1}^N w_i = 1} & I_p - I_q \Bigg[\frac{\rho}{\gamma} I \\
& + I \left(\sum_{i=1}^N w_i b_i \alpha_i^{(1)} + \frac{1}{2}(\gamma - 1) \sum_{i=1}^N w_i^2 (b_i^2 \sigma_Y^2 + \sigma_i^2) \right) \\
& + \sum_{s=1}^{S-1} I_{\pi_s^{(1)}} \left(\frac{\mu_s^{(1)}}{\gamma} + (\sum_{i=1}^N w_i b_i) \sigma_s^{(1)} \sigma_Y \right) + \sum_{s=1}^{S-1} I_{\pi_s^{(2)}} \left(\frac{\mu_s^{(2)} + \sigma_s^{(2)} \sigma_\eta}{\gamma} + (\sum_{i=1}^N w_i b_i) \sigma_s^{(2)} \sigma_Y \right) \\
& + I_\varrho \left(\frac{\varrho(1-\varrho)^2 \sigma_\eta}{\gamma} - (\sum_{i=1}^N w_i b_i) \varrho (1-\varrho) \sigma_\eta \right) \\
& + \frac{1}{2} \sum_{s=1}^{S-1} I_{\pi_s^{(1)} \pi_s^{(1)}} (\sigma_s^{(1)})^2 + \frac{1}{2} \sum_{s=1}^{S-1} I_{\pi_s^{(2)} \pi_s^{(2)}} (\sigma_s^{(2)})^2 + \sum_{s=1}^{S-1} \sum_{s'=1}^{S-1} I_{\pi_s^{(1)} \pi_{s'}^{(2)}} \sigma_s^{(1)} \sigma_{s'}^{(2)} \\
& + \frac{1}{2} I_{\varrho\varrho} \varrho^2 (1-\varrho)^2 \sigma_\eta^2 - \sum_{s=1}^{S-1} I_{\pi_s^{(1)} \varrho} \sigma_s^{(1)} \varrho (1-\varrho) \sigma_\eta - \sum_{s=1}^{S-1} I_{\pi_s^{(2)} \varrho} \sigma_s^{(2)} \varrho (1-\varrho) \sigma_\eta \Bigg].
\end{aligned}
$$

We then proceed so solve an approximate solution to (47) using projection methods (Judd, 1999, Chapter 11).

STEP 1. Choice of individual basis functions. We choose the Chebyshev polynomials in each of the 5 dimensions (2 dimensions for the beliefs of each analyst, and one for the weighting process, ϱ. The Chebyshev polynomials on $[-1,1]$ for the basis for each dimension are given by

$$q_m(x) \;=\; \cos(m \cos^{-1} x),$$

for $m = 1, 2, \cdots$, which satisfy the recursive scheme

$$q_{m+1}(x) = 2\,x q_m(x) - q_{m-1}(x). \tag{48}$$

These polynomials are restricted for the interval $[a, b]$ using the transformation

$$p_m, (x) = \frac{q_m(\frac{2x-a-b}{b-a})}{||q_m(\frac{2x-a-b}{b-a})||}.$$

For the belief variables, $a = 0$ and $b = 1$. For η, we use the interval $[0, 25]$. The family $\{p_m(x)\}_{m=1,2,\cdots}\}$ are orthonormal polynomials over the chosen intervals.

STEP 2. Choose a basis of 'complete' polynomials over the space $[0, 1]^5$. The basis of degree M over the 5 dimensions is given by

$$\mathcal{P}_M = \{p_{1,i_1}(\pi_1^{(1)}) \cdot p_{2,i_2}(\pi_2^{(1)}) \cdot p_{3,i_3}(\pi_1^{(2)}) \cdot p_{4,i_4}(\pi_2^{(2)}) \cdot p_{5,i_5}(\varrho) | \sum_{n=1}^{5} i_n \;\leq\; M, 0 \leq i_1, \cdots, i_5\}$$

We will write the generic element of $\mathcal{P}_M^N$ as ϕ_m, $m = 1, 2, \cdots M^c$, where M^c is the length of the complete polynomial basis. The set of complete polynomials for an N dimensional problem grows polynomially in N, as opposed to the tensor product basis which would use every possible product of the degree-M individual basis functions, and hence would grow at the rate of M^N see, e.g., pp. 239 in (Judd, 1999). The complete polynomials asymptotically, as M becomes large, provide as good an approximation as the tensor product, but with far fewer elements. For example, we solve each PDE using $M = 5$. Using the tensor product basis, we would have a total of 3125 elements, but using the complete basis, we have far fewer, 252 elements. Extending the L^2 norm over the 5-dimensional space as the 5-fold integral, it can be verified that the basis of complete polynomials is orthonormal on $[0, 1]^5$.

STEP 3 Let $\mathcal{D}(y)$ be the differential operator associated with the PDE (47). Write the candidate solution as $\hat{y} = \sum_{m=1}^{M^c} a_m \phi_m$. Then any solution to the PDE, $\hat{y}$, will be written as $\mathcal{D}(\hat{y}) = 0$.

STEP 4 We appeal to the Chebyshev Interpolation Theorem (see, e.g, Judd (1992),Den Haan (1997))to find an approximate solution to the PDE. The approximation is made by evaluating the operator $\mathcal{D}(\hat{y})$ at a chosen set of points, and setting it equal to zero at each of these points. Each interpolation point therefore provides us a linear equation in the coefficients $(a_m)_{m=1}^{M^c}$. The chosen points for the 5-dimensional space are the Cartesian product of the zeros of the Chebyshev polynomial of the highest degree chosen in Step 2 in each dimension. In general the $m + 1$ zeros of the mth polynomial are given by

$$x_k = (-\cos \frac{2k-1}{2m}\pi) + 1)\frac{b-a}{2} + a, \qquad \text{for} \quad k = 0, 1, \cdots m.$$

For example by choosing polynomials of order 5 in each dimension, we obtain 3125 interpolation points. With M^I interpolation points, we have an overidentified system of equations in M^c unknown coefficients. We note that, since the HJB equation in (47) is non-homogeneous, the system of equations is non-homogeneous. Denote the $M^I \times 1$ vector of constants from each equation as c, and the $M^I \times M^c$ coefficient matrix as A. Analogous to regression coefficients, the best-fitting set of coefficients satisfies:

$$\hat{a} = (A^T A)^{-1} A^T c.$$

Tables

TABLE 1

Summary Statistics of Aggregate Disagreement (December 1981 to September 2016).

Aggregate disagreement (in percentage points) is the value-weighted average of stock-level disagreement. Stock-level disagreement is the standard deviation of analyst forecasts of the earning-per-share long-term growth rate (EPS LTG). We define low (high) disagreement months as the months where aggregate disagreement is lower (higher) than the average aggregate disagreement at $t-1$ minus (plus) one standard deviation.

	Observations	Mean	Standard Deviation
Full Sample	418	3.51	0.63
Low Disagreement Months	55	2.80	0.09
Medium Disagreement Months	276	3.35	0.38
High Disagreement Months	87	4.48	0.32

TABLE 2

Ken French's Sets of Portfolios

Number	Name
1	25 Portfolios Based on Size and Accruals
2	25 Portfolios Based on Book-to-Market and Investment
3	25 Portfolios based on Book-to-Market and Operating Profitability
4	25 Portfolios Based on Operating Profitability and Investment
5	100 Portfolios Based on Size and Book-to-Market
6	25 Portfolios Based on Size and Book-to-Market
7	100 Portfolios Based on Size and Investment
8	25 Portfolios Based on Size and Investment
9	100 Portfolios Based on Size and Operating Profitability
10	25 Portfolios Based on Size and Operating Profitability
11	10 Industry Portfolios
12	12 Industry Portfolios
	17 Industry Portfolios
14	30 Industry Portfolios
15	38 Industry Portfolios
16	48 Industry Portfolios
17	49 Industry Portfolios
18	10 Portfolios Based on Long-Term Reversal
19	10 Portfolios Based on Momentum
20	25 Portfolios based on Size and Long-Term Reversal
21	25 Portfolios Based on Size and Momentum
22	25 Portfolios Based on Size and Short-Term Reversal
23	10 Portfolios Based on Short Term Reversal
24	25 Portfolios Based on Size and Market Beta
25	25 Portfolios Based on Size and Net Share Issuance
26	25 Portfolios Based on Size and Residual Variance
27	25 Portfolios Based on Size and Variance
28	32 Portfolios Based on Book-to-Market, Investment, and Size
29	32 Portfolios Based on Book-to-Market, Operating Profitability and Size
30	32 Portfolios Based on Operating Profitability, Investment, and Size
31	20 Portfolios Based on Book-to-Market
32	20 Portfolios Based on Cash Flow Divided by Price
33	20 Portfolios Based on Dividend Yield
34	20 Portfolios Based on Earnings Divided by Price
35	20 Portfolios Based on Investment
36	20 Portfolios Based on Operating Profitability

TABLE 3

Portfolio Returns, Controlling for Firm Characteristics/Risk-Loadings

This table reports average monthly value-weighted return differentials (in percent) between the highest and lowest disagreement-loading portfolios (Q5–Q1) across double-sorted portfolios formed on aggregate disagreement loading and one of the seven firm characteristics/risk-loadings: market beta, size, book-to-market (B/M), intangibles-augmented book-to-market (B/M^{INT}), momentum, the mean of analysts' forecasts and sentiment. For each characteristic, stocks are first sorted into quintiles based on the characteristic value at the end of month $t-1$. Within each characteristic quintile, stocks are then sorted into quintiles based on their disagreement loadings. Market beta $\beta_{i,t}$ and disagreement loading $\delta_{i,t}$ are estimated monthly using rolling three-year regressions of individual stock returns on the market excess return and the aggregate disagreement measure:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} R_{m,t} + \delta_{i,t} \text{ Aggregate Disagreement}_{t-1} + \epsilon_{i,t},$$

with the first two years of the sample filled using rolling one-year regressions. Size is market capitalization (price $\times$ shares outstanding). As in Asness and Frazzini (2013), B/M is defined as the ratio of their book equity and the current month market value of equity. Every June-end, BE_t is updated using the previous fiscal year's book equity value. The intangibles-augmented ratio B/M^{INT} uses intangible-adjusted book equity. Each June-end, BE_t^{INT} is updated using the previous fiscal year's book equity value obtained from the website of Andrea Eisfeldt and described in and described in Eisfeldt et al. (2022). Momentum is the cumulative six-month return from $t-11$ to $t-6$. The mean forecast is the cross-sectional average of analysts' earnings forecasts at the end of month $t-1$. The sentiment loading, $\theta_{i,t}$, is estimated each month using a rolling three-year window of lagged individual stock returns through the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \theta_{i,t} \times \text{ Aggregate Sentiment}_{t-1} + \epsilon_{i,t}.$$

The monthly aggregate sentiment data is obtained from Huang et al. (2015). "All months" uses the full sample of months. "Low disagreement months" ("High disagreement months") are defined as the months when aggregate disagreement is higher (lower) than the average aggregate disagreement plus (minus) one standard deviation. $t(5–1)$ reports the Newey–West t-statistic. * and ** denote statistical significance at the 5% and 1% levels, respectively.

Characteristic/Risk-Loading	Market β		Size		B/M		B/M^{INT}		Momentum		Mean Forecasts		Sentiment-Loading	
Quintile	5–1	$t(5–1)$	5–1	$t(5–1)$	5–1	$t(5–1)$	5–1	$t(5–1)$	5–1	$t(5–1)$	5–1	$t(5–1)$	5–1	$t(5–1)$
Panel A: All Months														
1	0.836	1.391	1.304	1.515	0.715	1.018	0.739	1.049	1.353	1.663	0.957	1.800	0.692	0.961
2	0.785	1.255	1.355	1.735	1.497*	2.274	1.147	1.709	1.466*	2.167	1.120	1.878	0.339	0.590
3	1.293*	2.017	1.128	1.537	1.127	1.764	1.401*	2.170	1.185	1.928	1.075	1.760	0.483	0.895
4	1.241	1.884	1.114	1.571	1.293	1.837	1.214	1.900	1.034	1.706	1.299	1.813	0.379	0.642
5	1.342	1.551	0.911	1.596	1.642*	2.034	1.524*	2.038	1.194	1.696	1.084	1.207	1.124	1.379
All	1.088	1.665	1.088	1.665	1.088	1.665	1.088	1.665	1.088	1.665	1.088	1.665	1.088	1.665
Panel B: Low Disagreement Months														
1	-2.518**	-4.190	-3.891**	-4.522	-2.907**	-4.141	-3.293**	-4.673	-2.368**	-2.911	-0.858	-1.614	-3.419**	-4.748
2	-1.647**	-2.632	-2.695**	-3.452	-1.836**	-2.789	-1.604*	-2.390	-1.984**	-2.933	-1.774**	-2.974	-1.644**	-2.862
3	-1.423*	-2.220	-3.116**	-4.246	-1.142	-1.788	-1.186	-1.837	-1.777**	-2.891	-1.986**	-3.252	-1.738**	-3.216
4	-2.438**	-3.701	-2.506**	-3.534	-1.913**	-2.718	-2.139**	-3.348	-2.115**	-3.489	-2.303**	-3.213	-2.654**	-4.492
5	-3.123**	-3.609	-1.400*	-2.453	-1.359	-1.683	-1.565*	-2.093	-3.093**	-4.393	-3.540**	-3.944	-2.605**	-3.194
All	-1.986**	-3.037	-1.986**	-3.037	-1.986**	-3.037	-1.986**	-3.037	-1.986**	-3.037	-1.986**	-3.037	-1.986**	-3.037
Panel C: High Disagreement Months														
1	4.305**	7.164	6.873**	7.988	4.398**	6.265	4.408**	6.255	5.252**	6.457	3.769**	7.087	4.236**	5.884
2	3.701**	5.915	5.795**	7.423	4.921**	7.475	4.557**	6.790	5.051**	7.467	4.181**	7.009	1.790**	3.115
3	5.356**	8.354	4.729**	6.443	4.459**	6.981	5.172**	8.011	4.084**	6.645	4.725**	7.736	2.437**	4.511
4	5.079**	7.710	5.098**	7.188	5.639**	8.013	4.098**	6.414	4.347**	7.170	4.490**	6.263	2.799**	4.738
5	5.470**	6.321	3.697**	6.477	5.327**	6.597	5.550**	7.421	4.988**	7.085	5.778**	6.438	3.866**	4.740
All	4.608**	7.049	4.608**	7.049	4.608**	7.049	4.608**	7.049	4.608**	7.049	4.608**	7.049	4.608**	7.049

Figures

FIGURE 1

Time-Series of Aggregate Disagreement

The figure plots the monthly aggregate disagreement measured as the value-weighted average of stock-level disagreement. Stock-level disagreement is the standard deviation of analysts' forecasts of the long-term growth rate of earnings-per-share (EPS).High (low) disagreement months are defined as the months when aggregate disagreement is higher (lower) than the average aggregate disagreement plus (minus) one standard deviation; medium disagreement months are those when aggregate disagreement is in the intermediate range. The one-standard-deviation bands are shown by the horizontal lines.

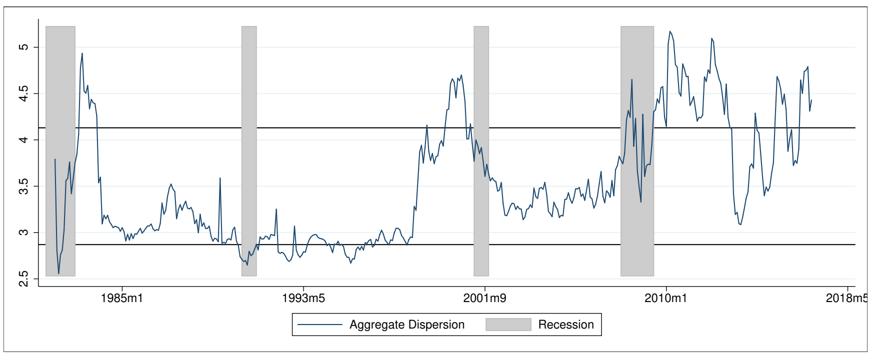


TABLE 4

Average Prices of Market and Disagreement Risk for the 100 δ-β Portfolios: Full Sample (December May 1982 – September 2016).

The table reports the time-series averages of the prices of market and disagreement risk from Fama-Macbeth regressions of the 100 $\beta - \delta$ sorted portfolios constructed as described in the footnote of Table A2. Each month, post-ranking risk loadings are estimated using the current and previous 11 monthly returns. For example, for the fourth specification, we estimate the first-stage equation:

$$R^P_{t-s} = \alpha_{P,t} + \beta^{MKT}_{P,t} \times \boldsymbol{R}^m_{t-s} + \delta_{P,t} \times \textit{Agg\_disp}_{t-s-1} \\ + \beta^{SMB}_{P,t} \times \textit{SMB}_{t-s} + \beta^{HML}_{P,t} \times \textit{HML}_{t-s} + \beta^{UMD}_{P,t} \times \textit{UMD}_{t-s} + \epsilon_{P,t-s},$$

for $P = 1, ..., 100$, and $s = 0, \cdots, 11$. $\boldsymbol{R}^P_t$ is the value-weighted monthly return of the P^{th} $\beta - \delta$-sorted portfolio at t. Next, for the second-stage, each month we estimate the prices of risk at date t using the equation

$$R^P_t = \kappa_t + \pi_t \times \beta^{MKT}_{P,t} + \omega_t \times \delta_{P,t} \\ + \phi^{SMB}_t \times \beta^{SMB}_{P,t} + \phi^{HML}_t \times \beta^{HML}_{P,t} + \phi^{UMD}_t \times \beta^{UMD}_{P,t} + \epsilon_{t,P},$$

for $P = 1, ..., 100$. Finally, we calculate the time-series mean of the estimated prices of risk and the estimated variance (our equations illustrate the case of the price of disagreement risk), as

$$\hat{\omega} = \frac{1}{T}\sum_{t=1}^{T}\hat{\omega}_t, \quad \text{and,} \quad \sigma^2(\hat{\omega}) = \frac{1}{T^2}\sum_{t=1}^{T}(\hat{\omega}_t - \hat{\omega})^2,$$

respectively. The prices of risk of the other factors are estimated analogously. * and ** denote significance at 5% and 1%, respectively.

	(1)	(2)	(3)	(4)
$\bar{\pi}$	0.001	-0.052	-0.244	-0.130
	(0.258)	(-0.200)	(-0.935)	(-0.524)
$\bar{\omega}$		0.032		0.099**
		(0.975)		(3.471)
$\bar{\phi}^{SMB}$			0.403	0.403*
			(1.851)	(2.179)
$\bar{\phi}^{HML}$			-0.287	-0.353**
			(-1.311)	(-2.014)
$\bar{\phi}^{UMD}$			-0.283	-0.041
			(-0.834)	(-0.146)
Constant	0.014**	0.016**	0.016**	0.015**
	(10.359)	(10.92	(9.545)	(12.281)
Adjusted-R^2	0.117	0.290	0.283	0.371

TABLE 5

Average Prices of Market and Disagreement Risk for 100 β-δ Portfolios: Alternative Subsamples Between May 1982 and September 2016.

The table reports the time-series averages of the prices of market and disagreement risk for alternative subsamples from Fama-Macbeth regressions of the 100 $\beta - \delta$ sorted portfolios constructed as described in the footnote of Table 4. The estimators for the prices of risk are as in the footnote to Table 4. * and ** denote significance at 5% and 1%, respectively.

Panel A: Low Disagreement Months	(1)	(2)	(3)	(4)
$\bar{\pi}$	0.668	0.837	0.837	0.764
	(1.906)	(1.321)	(1.491)	(1.333)
$\bar{\omega}$		-0.153**		-0.113**
		(-3.680)		(-4.830)
$\bar{\phi}^{SMB}$			-1.225**	-0.583
			(-2.458)	(-1.153)
$\bar{\phi}^{HML}$			-1.109	-0.545
			(-2.255)	(-1.342)
$\bar{\phi}^{UMD}$			0.173	0.249
			(0.288)	(0.435)
Constant	0.011**	0.010*	0.010**	0.011**
	(3.188)	(2.197)	(3.228)	(3.191)
Adjusted-R^2	0.115	0.244	0.311	0.347
Panel B: Medium Disagreement Months				
$\bar{\pi}$	-0.309	-0.236	-0.406	-0.397
	(-0.981)	(-0.748)	(-1.236)	(-1.261)
$\bar{\omega}$		-0.063		-0.013
		(-1.855)		(-0.449)
$\bar{\phi}^{SMB}$			0.611**	0.499**
			(2.463)	(2.561)
$\bar{\phi}^{HML}$			0.152	-0.179
			(0.68)	(-0.857)
$\bar{\phi}^{UMD}$			-0.950**	-0.258
			(-2.372)	(-0.804)
Constant	0.015**	0.016**	0.014**	0.015**
	(9.433)	(9.240)	(10.446)	(18.047)
Adjusted-R^2	0.127	0.299	0.289	0.378
Panel C: High Disagreement Months				
$\bar{\pi}$	0.580	-0.039	-0.371	0.153
	(0.944)	(-0.062)	(-0.615)	(0.286)
$\bar{\omega}$		0.466**		0.607**
		(4.564)		(7.504)
$\bar{\phi}^{SMB}$			0.815	0.739
			(1.357)	(1.333)
$\bar{\phi}^{HML}$			-0.939	-0.796
			(-1.532)	(-1.735)
$\bar{\phi}^{UMD}$			1.201	0.475
			(1.303)	(0.565)
Constant	0.0138**	0.020**	0.026**	0.016**
	(3.612)	(5.238)	(5.074)	(5.851)
Adjusted-R^2	0.089	0.294	0.248	0.369

TABLE 6

GMM Estimates of the Model's Parameters

Top Panel: GMM estimates of the following (discretized) model for real earnings growth, $y_t = \log(Y_t)$,

$$y_{t+1} = y_t \cdot e^{(\theta_t^{(m)} - \frac{1}{2}\sigma_Y^2)\Delta t + \sigma_Y \varepsilon_{t+1}}$$

where $(\theta_t^{(m)})$ follows a three-state regime-switching model. The estimates of the quarterly transition probability matrix are shown. The implied generator (not shown) can be calculated from: $\Lambda^{(m)} = \sum_{i=1}^{\infty} (-1)^{i+1} \cdot \left((P^{(m)}(0.25))^4 - I \right)^i / i$, whose value is estimated using a series approximation of length 10 (see Israel, Rosenthal and Wei (2001)). The GMM errors include the scores of the likelihood function of each type of agent and the difference in model-implied and historical disagreement in forecasts of analysts' disagreement. Standard errors are in parenthesis. **Bottom Panel:** Linear regression for model fits. Standard errors of parameter estimates are in parentheses. Units of measurement are quarterly and in percentage points. *T*-statistics are in parentheses. All *t*-statistics are adjusted for heteroskedasticity and autocorrelation using the methodology of Newey and West (1987). The top panel of Figure 6 shows the belief processes of the two agents. The bottom-left panel of Figure 6 shows the actual and model-implied 20-quarter-ahead disagreements of earnings growth, which are in the third regression. h-quarter-ahead filtered probabilities of the analysts are calculated as $\pi^{(m)}(t, t+h) = \pi_t^{(m)}(P^{(m)}(t, t+1))^h$.

Series Used: Value-Weighted Aggregate Earnings Growth (IBES)
and Value-Weighted Disagreement of Earnings Growth Forecasts
Time Span (Quarterly): 1988 – 2016:Q1

	Analyst 1			**Analyst 2**		
Drifts:	$\theta_1^{(1)}$	$\theta_2^{(1)}$	$\theta_3^{(1)}$	$\theta_1^{(2)}$	$\theta_2^{(2)}$	$\theta_3^{(2)}$
	0.014	0.064	-0.422	0.043	0.073	-0.855
	(0.011)	(0.024)	(0.005)	(0.009)	(0.020)	(0.011)
Transition Matrix Analyst 1:	$P_{12}^{(1)}$	$P_{13}^{(1)}$	$P_{21}^{(1)}$	$P_{23}^{(1)}$	$P_{31}^{(1)}$	$P_{32}^{(1)}$
	0.000	0.121	0.023	0.000	0.132	0.062
	(0.000)	(.073)	(0.033)	(0.000)	(0.069)	(0.071)
Transition Matrix Analyst 2:	$P_{12}^{(2)}$	$P_{13}^{(2)}$	$P_{21}^{(2)}$	$P_{23}^{(2)}$	$P_{31}^{(2)}$	$P_{32}^{(2)}$
	0.000	0.069	0.005	0.010	0.128	0.029
	(0.000)	(0.033)	(0.003)	(0.010)	(0.035)	(0.036)
Earnings Volatility:	σ_Y					
	0.066					
	(0.001)					

Chi-Square (2 d.f) = 0.931 **P-Value = 0.627**

Model Fits: $\Delta \log(q)\,(t) = \alpha + \beta \cdot (\theta_1^{(m)} \pi_1^{(m)}(t|t) + \theta_2^{(m)} \pi_1^{(m)}(t|t)) + \epsilon(t)$, $m = 1,2$

Analyst 1			**Analyst 2**		
$\hat{\alpha}$	$\hat{\beta}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	R^2
0.078	0.643	0.439	0.007	1.105	0.518
(0.149)	(17.573)		(0.013)	(15.654)	

Disagreement: $d(t, 20) = \alpha + \beta \cdot d(\pi^{(1)}(t, 20), \pi^{(2)}(t, 4)) + \epsilon(t)$

$\hat{\alpha}$	$\hat{\beta}$	R^2
0.007	1.022	0.765
(0.013)	(15.654)	

TABLE 7

Equity Premium of Different Consumers, the Price of Disagreement Risk, and the Flight-to-Safety in the Model for Alternative Levels of Risk Aversion

The riskless rate (r) is in Equation (32). The risk-premiums for each consumer are calculated using Proposition 2. In particular, we calculate the risk premium of the ith asset (whose returns follow the process in Equation (15)) at time t as $\alpha_{it}^{(m)} = b_i \nu_t^{(m)}$, for $m = 1, 2$. The risk premium on the market index assessed by consumer m is $\alpha_{Mt}^{(m)} = \left(\sum_{i=1}^{N} w_i b_i\right) \nu_t^{(m)} - r$. Consumption flows for each agent are calculated using (18) and (19), and portfolio weights used to construct the market portfolio are calculated using Proposition 1. These are all calculated using the estimated process of aggregate fundamentals, the beliefs and the process ϱ_t (probability of the data at t being generated from the model of analyst 1) as shown in Section A and displayed in Figure 6. The flight-to-safety (FLS) is the correlation between the squared-beta ($\sum_{i=1}^{N} w_i^2 b_i^2$ and aggregate disagreement among the analysts in the model, which we show for the case $\gamma = -10$ in Figure 9.

γ	r	E.P.$^{(1)}$	E. P.$^{(2)}$	σ_C	$\sigma_C^{(1)}$	$\sigma_C^{(2)}$	DPR$_H$	DPR$_L$	FLS
0.5	0.061	0.036	0.028	1.442	1.545	1.994	0.184	-0.015	-0.033
-0.5	0.043	0.026	0.017	0.185	0.393	0.676	0.264	-0.259	-0.158
-2	0.023	0.024	0.013	0.089	0.134	0.216	0.262	-0.259	-0.161
-5	-0.005	0.022	0.008	0.086	0.126	0.108	0.246	-0.285	-0.189
-10	-0.045	0.026	0.01	0.033	0.057	0.049	0.245	-0.286	-0.127
-15	-0.081	0.032	0.017	0.025	0.042	0.034	0.246	-0.286	-0.152

FIGURE 2

Disagreement Loadings and Portfolio Returns

This figure plots the average monthly value-weighted portfolio returns formed by sorting stocks on their pre-ranking δs each month. Pre-ranking β and δ are obtained each month using lagged three-year rolling data on individual stock returns using the regression: $R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \delta_{i,t} \times$ Aggregate Disagreement$_{t-1} + \epsilon_{i,t}$. High (low) disagreement months are defined as the months when aggregate disagreement is higher (lower) than the average aggregate disagreement plus (minus) one standard deviation; medium disagreement months are those when aggregate disagreement is in the intermediate range.

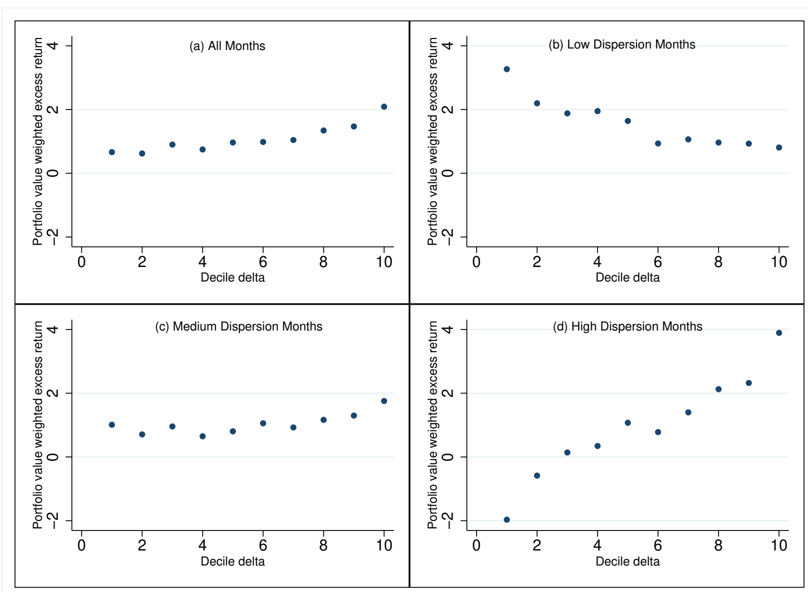


FIGURE 3

Aggregate Disagreement and the Price of Disagreement Risk

The figure plots the time series of the price of disagreement risk, ω_t. The price of risk is estimated from Fama-Macbeth regressions of the 100 $\beta - \delta$ sorted portfolios constructed as described in the footnote of Table A2. In each month, t, we run the two-stage Fama-Macbeth regression to estimate the price of market and disagreement risk as described in the footnote to Table 4. We plot here ω_t, which is the price of disagreement risk at time t.

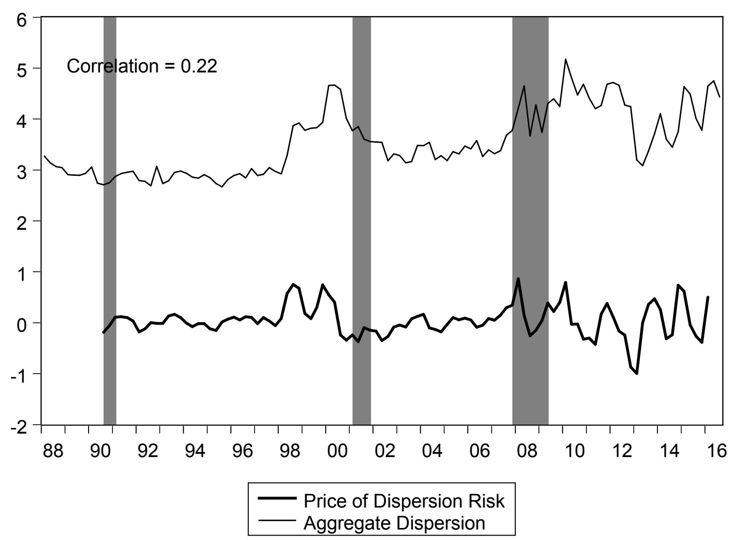


FIGURE 4

Aggregate Disagreement and Investor Sentiment

The figure plots the time series of aggregate disagreement as well the investor sentiment index of Huang et al. (2015). Aggregate disagreement is measured as the value-weighted average of stock-level disagreement. Stock-level disagreement is the standard deviation of analysts' forecasts of the long-term growth rate of earnings-per-share (EPS).

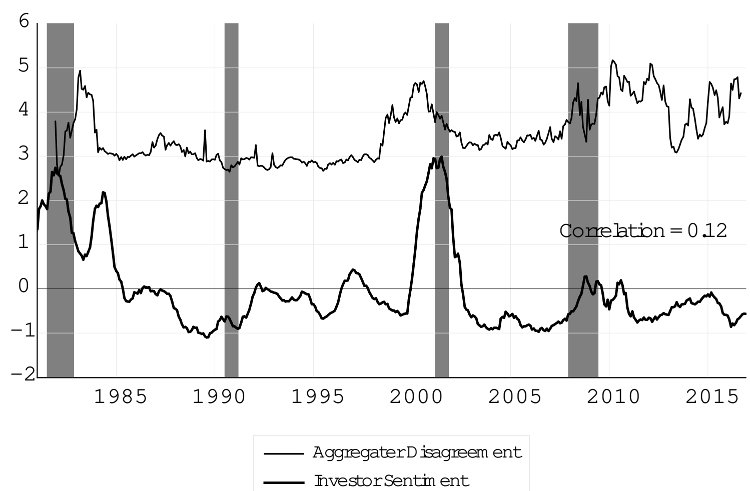


FIGURE 5

Price of Disagreement Risk in Different Sets of Portfolios Created by Kenneth French

The price of risk is estimated from Fama-Macbeth two-stage regressions for each set of portfolios listed in Table 2. The numbers on the x-axis correspond to the number of the portfolio set in this table. For each portfolio P in a set, we estimate the first-stage equation:

$$R^P_{t-s} = \alpha_{P,t} + \beta^{MKT}_{P,t} \times R^m_{t-s} + \delta_{P,t} \times Agg\_disp_{t-s-1} + \beta^{SMB}_{P,t} \times SMB_{t-s} + \beta^{HML}_{P,t} \times HML_{t-s} + \beta^{UMD}_{P,t} \times UMD_{t-s} + \epsilon_{P,t-s},$$

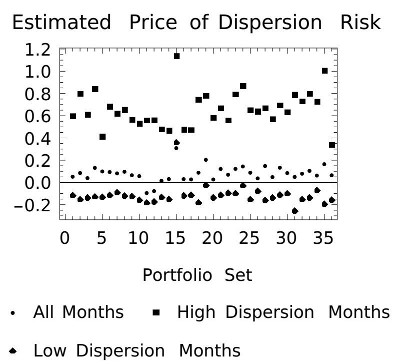
for $P = 1, ..., N_P$, and $s = 0, \cdots, 35$. R^P_t. Next, for the second-stage, each month we estimate the prices of risk at date t using the equation

$$R^P_t = \kappa_t + \pi_t \times \beta^{MKT}_{P,t} + \omega_t \times \delta_{P,t} + \phi^{SMB}_t \times \beta^{SMB}_{P,t} + \phi^{HML}_t \times \beta^{HML}_{P,t} + \phi^{UMD}_t \times \beta^{UMD}_{P,t} + \epsilon_{t,P},$$

for $P = 1, ..., N_P$. Finally, we calculate the time-series mean of the estimated prices of risk and the estimated variance (our equations illustrate the case of the price of disagreement risk), as

$$\hat{\omega} = \frac{1}{T} \sum_{t=1}^{T} \hat{\omega}_t, \qquad \text{and,} \qquad \sigma^2(\hat{\omega}) = \frac{1}{T^2} \sum_{t=1}^{T} (\hat{\omega}_t - \hat{\omega})^2,$$

respectively. The top panel has estimated coefficients, and the bottom panel has t-statistics.



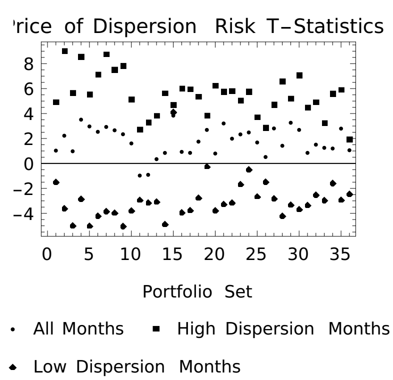


FIGURE 6

Investor Beliefs, Expected Growth Rates of Earnings From Estimated Model, Data and Model Analysts' Disagreement, and the Conditional Probability that the Data are Generated from the Model of Analyst 1 (1988 - 2016:Q1)

The **top-left**, **top-right**, and **middle-left** panels display the time-series of filtered beliefs about real earnings growth 1-quarter ahead of the two types of analysts, for states 1, 2, and 3, respectively. Filtered beliefs of the two agents are obtained from the discretized version of the belief processes as shown in equation (7). The estimated parameters for each type of agent shown in Table 6. The **middle-right** panel displays the actual and one-quarter ahead expected earnings growth of the two types of agents using these filtered beliefs. The **bottom-left** panel shows the disagreement in analysts' beliefs in the data, which is the value-weighted disagreement of analysts' forecasts of the long-term (5-year ahead) growth rate of EPS of firms in the IBES database, and the model-implied 5-year ahead disagreement, from the filtered beliefs of each analyst type. Finally, the **bottom-right** panel shows the process $\{\varrho_t\}$, which is the conditional probability that the data at t is generated from the model of Analyst 1.

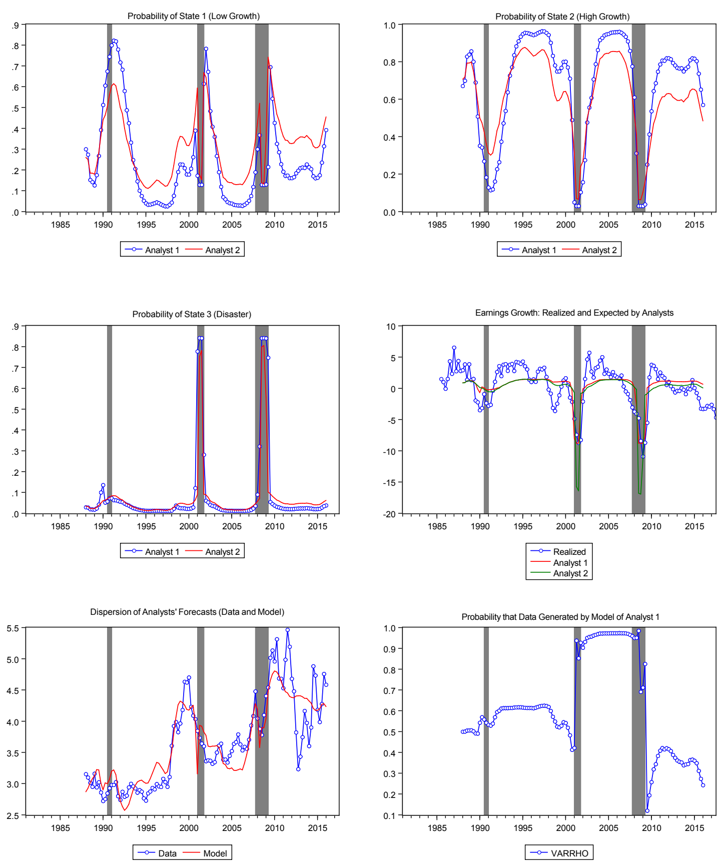


FIGURE 7

Time-Series of the Price of Disagreement Risk (Data and Model)

We show the time-series of the price of disagreement estimated using Fama-Macbeth regressions as described in Section III. For the model the estimated price of disagreement risk is similarly generated using simulated stock returns as described in Section A using the calibrated model with parameters in Table 6.

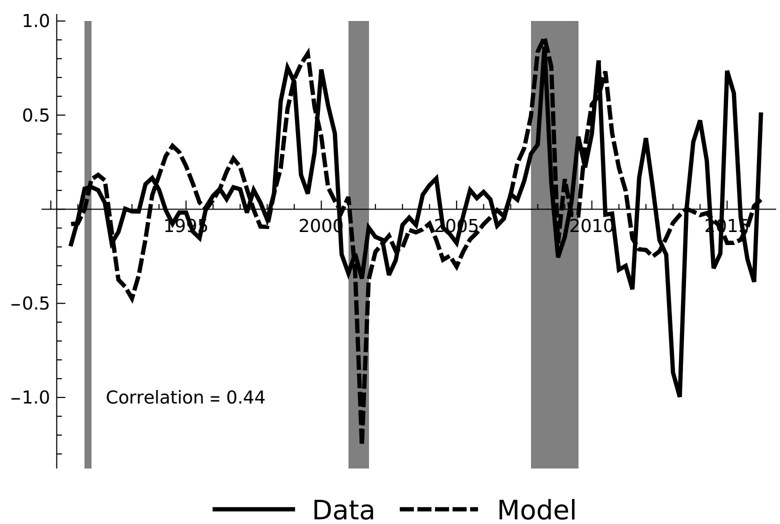


FIGURE 8

Risk Premiums of Consumers

We show the time-series of the conditional risk premiums of the two consumers. Using the calibrated model with

parameters in Table 6 and the estimated belief and model probability series in Figure 6, we calculate the risk premium of the i^{th} asset (whose returns follow the process in Equation (15)) at time t as $\alpha_{it} = b_i \nu_t$. The risk premium on the market index assessed by consumer m is $\alpha_{Mt} = \left(\sum_{i=1}^{N} w_i b_i \right) \nu_t - r$, where the riskless rate is in Equation (32).

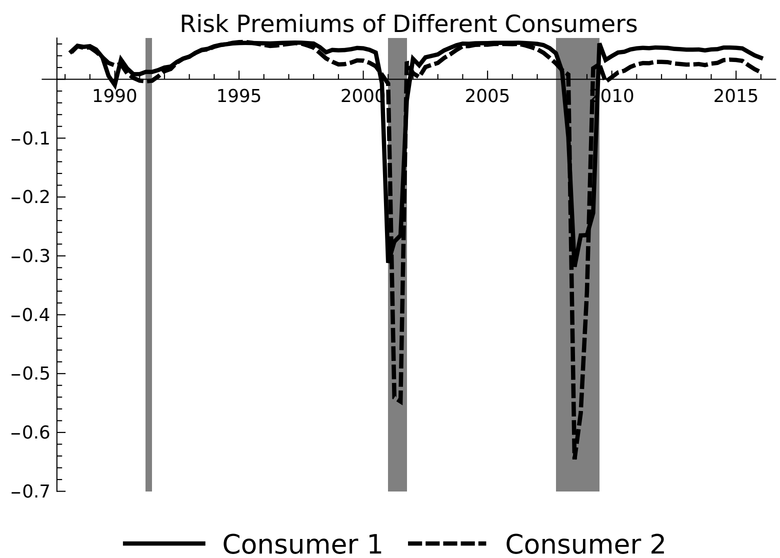
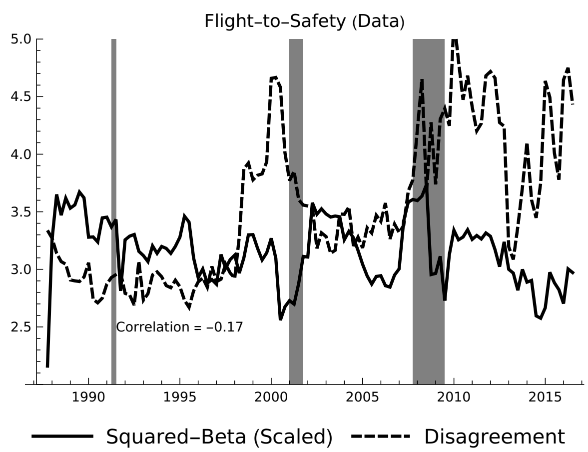
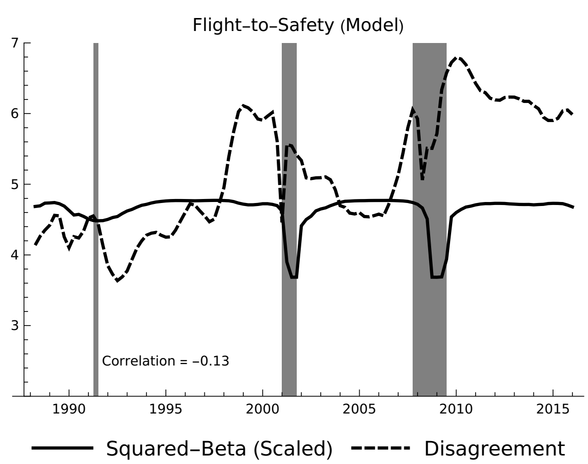


FIGURE 9

The Flight-to-Safety Phenomenon (Data and Model)

The **left** and **right** panels show the time-series of aggregate disagreement, and the "Squared Beta', which equals $\sum_i w_{it}^2 b_i^2$ in the model and data, respectively. The cash flow beta, b_i for each firm in the IBES database is estimated using 5-year rolling windows of firm level earnings growth on aggregate earnings growth. The weight of each stock, w_i, is the earnings of each company divided by the aggregate earnings for each stock in the CRSP database. We analogously calculate the market cash flow beta in the model using ten assets, as described in Section B. The weights of each asset in the market portfolio in the model are calculated as shown in Proposition 1. The model's estimated parameters are in Table 6. We use a risk-aversion parameter $\gamma = -10$, and time-discount parameter $\rho = 0.04$





Internet Appendix

In this online appendix, we present three sets of results. The first set has additional summary statistics on disagreement and betas. The second set has the full tables on double-sorting by aggregatge disagreement-loading and alternative characteristics/risk-loadings. Finally, the third set extends our model's implications to an out-of-sample period from 2017 – 2021.

A. Additional Summary Statistics on Disagreement

In Table A1 we report the correlation betweenone month lagged aggregate disagreement , stock market excess returns, and the standard pricing factors SMB, HML and UMD. We note that aggregate disagreement is weakly correlated with other variables using the full sample. In the full sample, as well as in the alternative subsamples, disagreement is positively correlated with the market and SMB factors, but negatively correlated with HML. The correlation with momentum is mostly negative, however, it switches sign in high disagreement months.

In Table A2, we present summary statistics on pre- and post-ranking betas and deltas.

B. Double-Sorting on Disagreement and Alternative Characteristics

In this subsection, we present the complete tables for the double-sorting exercises that are presented in Section B.

C. Out-of-Sample Predictions of Model

In this section, we provide the results for the out-of-sample predictions for the model, which are mentioned in the introduction as well as the conclusion.

TABLE A1

Correlation Between Aggregate Dispersion and Standard Factors in the Asset Pricing Literature.

High (low) dispersion months are defined as the months when aggregate dispersion is higher (lower) than the average aggregate dispersion plus (minus) one standard deviation; medium dispersion months are those when aggregate dispersion is in the intermediate range.

Panel A: Full sample

	L1.Agg. dispersion	$R_m - R_f$	SMB	HML	UMD
L1.Agg. dispersion	1				
$R_m - R_f$	-0.0363	1			
SMB	0.0606	0.205	1		
HML	0.0251	-0.261	-0.304	1	
UMD	-0.0588	-0.187	0.0643	-0.176	1

Panel B: Low dispersion months

	L1.Agg. dispersion	$R_m - R_f$	SMB	HML	UMD
L1.Agg. dispersion	1				
$R_m - R_f$	0.181	1			
SMB	0.117	0.409	1		
HML	-0.0541	-0.455	-0.446	1	
UMD	-0.115	-0.467	-0.357	0.286	1

Panel C: Medium dispersion months

	L1.Agg. dispersion	$R_m - R_f$	SMB	HML	UMD
L1.Agg. dispersion	1				
$R_m - R_f$	-0.0214	1			
SMB	0.111	0.158	1		
HML	-0.0796	-0.266	-0.129	1	
UMD	-0.0706	-0.246	-0.153	-0.0567	1

Panel D: High dispersion months

	L1.Agg. dispersion	$R_m - R_f$	SMB	HML	UMD
L1.Agg. dispersion	1				
$R_m - R_f$	0.130	1			
SMB	-0.0507	0.265	1		
HML	0.0276	-0.220	-0.493	1	
UMD	-0.0695	0.0371	0.542	-0.477	1

TABLE A2

Average Pre-Ranking and Post-Ranking Betas and Deltas (December 1981 - September 2016).

The table reports the summary statistics of pre-ranking and post-ranking βs and δs for alternative subsamples). Pre-ranking β_i and δ_i are obtained each month using lagged one-year rolling data on individual stock returns using the regression:

$$R^i_{t-s} = \alpha_{i,t} + \beta^{MKT}_{i,t} \times R^m_{t-s} + \delta_{i,t} \times Agg\_disp_{t-s-1} + \epsilon_{i,t-s},$$

for $s = 0, \cdots, 11$. We create 100 $\beta - \delta$ portfolios by double-sorting stocks on pre-ranking βs and δs. Post-ranking βs and δs are estimated analogously each month using rolling data for the 100 $\beta - \delta$ portfolios and running the regressions:

$$R_{p,t} = \alpha_{p,t} + \beta_{p,t} \times R_{m,t} + \delta_{p,t} \times \text{Aggregate Disagreement}_{t-1} + \epsilon_{p,t}, \quad \text{for } p = 1, \cdots, 100.$$

We report the the time-series averages of the cross-sectional mean, standard deviation (sd), as well as the 10^{th} to 90^{th} percentiles (p10 to p90) of each variables in the different subsamples. High (low) disagreement months are defined as the months when aggregate disagreement is higher (lower) than the average aggregate disagreement plus (minus) one standard deviation;Medium disagreement months are those when aggregate disagreement is in the intermediate range.

	mean	sd	p10	p25	Median	p75	p90
Panel A: Full Sample							
pre-ranking β	1.09	0.94	0.04	0.49	1.01	1.60	2.27
pre-ranking δ	-0.01	0.24	-0.29	-0.14	-0.01	0.12	0.20
post-ranking β	1.01	0.76	0.06	0.49	0.95	1.47	2.05
post-ranking δ	-0.00	0.19	-0.24	-0.12	-0.01	0.11	0.24
Panel B: Low disagreement months							
pre-ranking β	1.12	1.04	-0.03	0.45	1.03	1.69	2.43
pre-ranking δ	0.00	0.33	-0.37	-0.18	0.00	0.19	0.38
post-ranking β	1.03	0.78	0.06	0.47	0.94	1.52	2.13
post-ranking δ	0.01	0.28	-0.33	-0.17	-0.00	0.17	0.37
Panel C: Medium disagreement months							
pre-ranking β	1.10	0.95	0.04	0.49	1.01	1.61	2.29
pre-ranking δ	-0.02	0.26	-0.32	-0.16	-0.01	0.13	0.28
post-ranking β	1.01	0.77	0.06	0.50	0.96	1.46	2.05
post-ranking δ	-0.01	0.20	-0.26	-0.13	-0.01	0.11	0.25
Panel D: High disagreement months							
pre-ranking β	1.06	0.84	0.09	0.50	0.98	1.52	2.12
pre-ranking δ	-0.00	0.11	-0.13	-0.06	0.00	0.06	0.12
post-ranking β	0.99	0.75	0.07	0.47	0.93	1.44	2.03
post-ranking δ	0.00	0.09	-0.11	-0.05	0.01	0.06	0.12

TABLE A3

Portfolio Returns Controlling for Market Risk

The table reports average monthly value-weighted portfolio returns formed by sorting stocks on their market beta and their disagreement loadings. Disagreement loading, δ and market β are obtained each month using lagged three-year rolling data on individual stock returns using the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times \boldsymbol{R}_{m,t} + \delta_{i,t} \times \text{Aggregate Disagreement}_{t-1} + \epsilon_{i,t}.$$

The columns (5-1) and rows (5-1) in each table report the return differential across market β quintiles and across the disagreement loading quintiles, respectively. t(5-1)is the robust Newey-West t-statistic. *and **denote significance at 5% and 1%, respectively.

All Months

	Disagreement Loadings							
Market β	1	2	3	4	5	5-1	t(5-1)	All
1	0.888	0.962	1.322	1.304	1.711	0.823	1.358	1.312
2	1.051	1.071	1.033	1.414	1.768	0.717	1.141	1.337
3	0.738	0.995	1.295	1.583	1.996	1.259*	1.984	1.485
4	0.839	1.170	1.346	1.660	2.118	1.279	1.934	1.584
5	1.304	1.628	1.765	1.814	2.722	1.418	1.601	2.070
5–1	0.415	0.666	0.443	0.509	1.011			0.758
t(5–1)	1.031	1.787	1.099	1.101	1.782			1.891
All	0.959	1.144	1.258	1.571	2.086	1.126	1.722	

Low Disagreement Months

	Disagreement Loadings							
Market β	1	2	3	4	5	5-1	t(5-1)	All
1	2.627	1.957	1.731	1.307	0.037	-2.590**	-4.272	1.676
2	2.880	1.806	1.160	0.822	1.312	-1.568*	-2.495	1.753
3	2.850	2.096	1.450	1.194	1.333	-1.516*	-2.389	1.964
4	3.486	2.701	2.241	1.527	1.076	-2.409**	-3.643	2.383
5	4.789	3.942	3.034	2.210	1.952	-2.837**	-3.204	3.307
5–1	2.162**	1.986**	1.303**	0.903	1.914**			1.631**
t(5–1)	5.369	5.327	3.231	1.953	3.374			4.069
All	3.255	2.302	1.669	1.330	1.289	-1.965**	-3.004	

High Disagreement Months

	Disagreement Loadings							
Market β	1	2	3	4	5	5-1	t(5-1)	All
1	-0.322	0.598	1.593	1.842	3.818	4.140**	6.829	1.479
2	-0.439	0.606	1.111	2.318	3.174	3.613**	5.749	1.248
3	-1.661	0.108	1.282	2.366	3.655	5.316**	8.377	1.305
4	-1.686	-0.139	0.621	1.623	3.453	5.140**	7.773	0.918
5	-1.821	-0.159	1.499	1.510	3.987	5.808**	6.559	1.151
5–1	-1.499**	-0.757*	-0.094	-0.332	0.169			-0.328
t(5–1)	-3.722	-2.031	-0.233	-0.718	0.298			-0.818
All	-1.305	0.357	1.118	2.086	3.345	4.651**	7.109	

TABLE A4

Portfolio Returns Controlling for Size

The table reports average monthly value-weighted portfolio returns formed by sorting stocks on their size and their disagreement loadings. Each month, we sort stocks into five quintiles based on their market capitalization at the end of the previous month. Then within each quintile, we sort stocks into five quintiles based on their aggregate disagreement loading. Size is defined as the market capitalization (price $\times$ share outstanding). Disagreement loading, δ , is obtained each month using lagged three-year rolling data on individual stock returns using the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times \boldsymbol{R}_{m,t} + \delta_{i,t} \times \text{Aggregate Disagreement}_{t-1} + \epsilon_{i,t}.$$

The first two years of data is filled used one-year rolling regressions. The columns (5-1) and rows (5-1) in each table report the return differential across size quintiles and across the disagreement loading quintiles, respectively. t(5-1)is the robust Newey-West t-statistic. *and **denote significance at 5% and 1%, respectively.

			Disagreement Loadings					
All Months								
Size	1	2	3	4	5	5-1	t(5-1)	All
1	1.985	2.019	2.134	2.288	3.404	1.418	1.619	2.485
2	1.556	1.706	1.751	2.126	3.010	1.455	1.788	2.116
3	1.467	1.581	1.721	1.984	2.540	1.073	1.418	1.940
4	1.344	1.476	1.628	1.858	2.411	1.068	1.509	1.818
5	0.960	1.142	1.243	1.442	1.865	0.905	1.609	1.412
5–1	-1.025**	-0.876**	-0.891**	-0.846**	-1.539 **			-1.073**
t(5–1)	-3.678	-3.936	-4.094	-3.387	-4.533			-5.077
All	0.992	1.150	1.265	1.587	2.112	1.121	1.715	

			Disagreement Loadings					
Low Disagreement Months								
Size	1	2	3	4	5	5-1	t(5-1)	All
1	4.869	3.372	2.402	2.144	0.894	-3.976**	-4.538	2.825
2	4.490	2.627	2.380	1.902	1.776	-2.713**	-3.336	2.702
3	3.902	2.904	2.188	2.085	0.831	-3.071**	-4.059	2.431
4	3.706	2.705	2.299	1.637	0.975	-2.731**	-3.860	2.317
5	2.807	2.267	1.896	1.361	1.377	-1.430*	-2.543	2.011
5–1	-2.062**	-1.105**	-0.506*	-0.783**	0.484			-0.814**
t(5–1)	-7.401	-4.961	-2.325	-3.133	1.425			-3.851
All	3.263	2.288	1.681	1.360	1.301	-1.962**	-3 .003	

			Disagreement Loadings					
High Disagreement Months								
Size	1	2	3	4	5	5-1	t(5-1)	All
1	-1.063	1.123	2.149	3.168	5.904	6.967**	7.953	2.443
2	-0.615	0.848	1.483	2.701	5.213	5.829**	7.167	2.050
3	-0.395	0.949	1.853	2.419	4.648	5.044**	6.667	1.987
4	-0.974	0.651	1.654	2.294	4.019	4.993**	7.058	1.651
5	-0.929	0.173	1.039	1.755	2.670	3.599**	6.403	1.015
5–1	0.134	-0.950**	-1.110**	-1.413**	-3.234**			-1.428**
t(5–1)	0.480	-4.268	-5.098	-5.654	-9.529			-6.755
All	-1.292	0.357	1.123	2.095	3.390	4.682**	7.165	

TABLE A5

Portfolio Returns Controlling for Value

The table reports average monthly value-weighted portfolio returns formed by sorting stocks on their B/M ratios and their disagreement loadings. Each month, we sort stocks into five quintiles based on their B/M ratio. Then within each quintile, we sort stocks into five quintiles based on their aggregate disagreement loading. As in Asness and Frazzini (2013), B/M is defined as the ratio of their book equity and the current month market value of equity. Every June-end, BE_t is updated using the previous fiscal year's book equity value. The disagreement loading, δ , is obtained each month using lagged three-year rolling data on individual stock returns using the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \delta_{i,t} \times \text{Aggregate Disagreement}_{t-1} + \epsilon_{i,t}.$$

The columns (5-1) and rows (5-1) in each table report the return differential across B/M quintiles and across the disagreement loading quintiles, respectively. t(5-1)is the robust Newey-West t-statistic. *and **denote significance at 10%, 5% and 1%, respectively.

All Months

	Disagreement Loadings							
B/M	1	2	3	4	5	5-1	t(5-1)	All
1	1.043	1.205	1.265	1.542	1.795	0.753	1.075	1.461
2	0.868	1.233	1.218	1.505	2.410	1.542*	2.343	1.485
3	1.014	1.137	1.591	1.597	2.170	1.156	1.795	1.523
4	1.058	1.230	1.440	1.584	2.314	1.256	1.792	1.510
5	0.993	1.810	1.743	2.158	2.751	1.758*	2.159	2.037
5–1	-0.049	0.605	0.478	0.616	0.955**			0.576
t(5–1)	-0.154	1.903	1.503	1.937	3.004			1.812
All	0.946	1.145	1.251	1.532	2.053	1.106	1.697	

Low disagreement months

	Disagreement Loadings							
B/M	1	2	3	4	5	5-1	t(5-1)	All
1	3.891	2.581	1.649	1.409	0.910	-2.982**	-4.257	2.158
2	2.827	2.357	1.357	1.334	1.054	-1.773**	-2.694	1.870
3	2.886	2.147	1.513	1.281	1.714	-1.172	-1.820	2.125
4	3.012	2.406	2.648	1.924	1.105	-1.908**	-2.722	2.280
5	3.013	2.558	1.826	2.108	1.619	-1.394	-1.712	2.165
5–1	-0.878**	-0.024	0.177	0.699*	0.709*			0.008
t(5–1)	-2.761	-0.075	0.557	2.198	2.230			0.025
All	3.222	2.294	1.662	1.295	1.297	-1.926**	-2.955	

High disagreement months

	Disagreement Loadings							
B/M	1	2	3	4	5	5-1	t(5-1)	All
1	-1.229	0.290	1.270	1.934	3.120	4.349**	6.208	0.947
2	-0.869	0.535	1.068	1.682	4.087	4.956**	7.531	1.131
3	-1.116	0.389	1.772	2.104	3.389	4.505**	6.994	1.157
4	-1.373	0.359	1.356	1.907	4.338	5.711**	8.147	1.353
5	-0.444	1.409	1.548	2.815	5.122	5.566**	6.837	2.287
5–1	0.785*	1.118**	0.278	0.881**	2.002**			1.339**
t(5–1)	2.469	3.516	0.874	2.771	6.297			4.211
All	-1.342	0.357	1.122	2.028	3.285	4.627**	7.100	

TABLE A6

Portfolio Returns Controlling for Intangibles-Augmented Value

The table reports average monthly value-weighted portfolio returns formed by sorting stocks on their B^{INT}/M ratios and their disagreement loadings. B^{INT}/M is defined as the ratio of their book equity adjusted for intangibles, and the current month market value of equity. Each month, we sort stocks into five quintiles based on their B/M^{INT} ratio; then within each quintile, we sort stocks into five quintiles based on their aggregate disagreement loading. More specifically, each June-end, BE_t^{INT} is updated using the previous fiscal year's book equity value obtained from the website of Andrea Eisfeldt and described in Eisfeldt et al. (2022). Disagreement loading, δ , is obtained each month using lagged three-year rolling data on individual stock returns using the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \delta_{i,t} \times \text{Aggregate Disagreement}_{t-1} + \epsilon_{i,t}.$$

The columns (5-1) and rows (5-1) in each table report the return differential across B/M^{INT} quintiles and across the disagreement loading quintiles, respectively. t(5-1)is the robust Newey-West t-statistic. *and **denote significance at 5% and 1%, respectively.

All Months								
	Disagreement Loadings							
B/M^{INT}	1	2	3	4	5	5-1	t(5-1)	All
1	0.905	1.120	1.235	1.363	1.807	0.902	1.529	1.389
2	1.122	1.279	1.457	1.720	2.116	0.993	1.447	1.651
3	1.213	1.513	1.612	1.845	2.351	1.138	1.547	1.815
4	1.269	1.686	1.667	2.003	2.677	1.408	1.909	2.003
5	1.908	2.042	2.035	2.100	3.180	1.271	1.562	2.390
5–1	0.696**	0.529**	0.423 *	0.255	0.829**			0.575**
t(5–1)	3.442	2.616	2.092	1.261	4.100			2.844
All	0.950	1.147	1.253	1.535	2.044	1.095	1.681	

Low Disagreement Months								
			Disagreement Loadings					
B/M^{INT}	1	2	3	4	5	5-1	t(5-1)	All
1	3.048	2.288	1.865	1.175	1.339	-1.709**	-2.896	1.980
2	3.471	2.564	2.256	1.576	1.278	-2.193**	-3.196	2.246
3	3.862	2.715	2.316	2.030	0.964	-2.898**	-3.940	2.376
4	3.716	2.948	2.134	1.999	1.463	-2.253**	-3.054	2.481
5	4.459	3.362	2.303	1.888	1.061	-3.398**	-4.176	2.690
5–1	0.597**	0.647**	-0.013	-0.142	0.097			0.315
t(5–1)	2.952	3.200	-0.064	-0.702	0.480			1.558
All	3.228	2.292	1.662	1.291	1.298	-1.930**	-2.963	

High disagreement months								
			Disagreement Loadings					
B/M^{INT}	1	2	3	4	5	5-1	t(5-1)	All
1	-1.094	0.204	1.041	1.652	2.676	3.770**	6.389	0.992
2	-1.183	0.729	1.390	2.185	3.468	4.651**	6.779	1.456
3	-1.032	0.850	1.634	2.467	3.992	5.024**	6.830	1.730
4	-0.671	0.965	1.447	2.484	4.544	5.216**	7.071	1.974
5	-0.658	0.938	2.069	2.942	5.340	5.998**	7.371	2.364
5–1	0.374	0.088	0.435*	0.475*	1.348**			0.634**
t(5–1)	1.850	0.435	2.151	2.349	6.667			3.135
All	-1.340	0.363	1.127	2.050	3.250	4.590**	7.046	

TABLE A7

Portfolio Returns, Controlling for Momentum

The table reports average monthly value-weighted portfolio returns formed by sorting stocks on their momentum values and their disagreement loadings. Each month, we sort stocks into five quintiles based on their momentum values. Then within each quintile, we sort stocks into five quintiles based on their aggregate disagreement loadings. The momentum value is defined as the six-month lagged moving average of returns:
$\text{ret}(t-6) + \text{ret}(t-7) + \cdots + \text{ret}(t-11)$. The disagreement loading, δ , is obtained each month using lagged three-year rolling data on individual stock returns using the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times \boldsymbol{R}_{m,t} + \delta_{i,t} \times \text{Aggregate Disagreement}_{t-1} + \epsilon_{i,t}.$$

The columns (5-1) and rows (5-1) in each table report the return differential across momentum quintiles and across the disagreement loading quintiles, respectively. t(5-1)is the robust Newey-West t-statistic. *and **denote significance at 10%, 5% and 1%, respectively.

All Months

Momentum	Disagreement Loadings 1	2	3	4	5	5-1	t(5-1)	All
1	0.692	1.13	1.292	1.585	2.030	1.338	1.602	1.441
2	0.705	1.214	1.256	1.349	2.093	1.388*	2.057	1.409
3	0.688	0.891	1.422	1.605	1.941	1.253*	1.960	1.401
4	1.119	1.168	1.299	1.556	2.043	0.924	1.504	1.465
5	1.614	1.644	1.864	1.851	2.755	1.141	1.654	2.002
5–1	0.922**	0.514	0.572	0.266	0.724*			0.561*
t(5–1)	3.004	1.807	1.860	0.877	2.215			2.234
All	0.984	1.158	1.257	1.579	2.089	1.105	1.682	

Low Disagreement Months

Momentum	Disagreement Loadings 1	2	3	4	5	5-1	t(5-1)	All
1	3.277	2.099	1.454	1.660	0.949	-2.328**	-2.787	2.011
2	2.812	2.396	1.677	0.945	0.906	-1.906**	-2.824	1.888
3	2.840	1.901	1.864	1.591	0.844	-1.996**	-3.123	1.976
4	3.526	2.164	1.837	1.654	1.320	-2.206**	-3.590	2.164
5	4.448	3.199	2.313	1.805	1.241	-3.207**	-4.648	2.675
5–1	1.171**	1.100**	0.859**	0.145	0.292			0.663**
t(5–1)	3.815	3.867	2.793	0.478	0.894			2.640
All	3.281	2.307	1.705	1.317	1.251	-2.029**	-3.089	

High Disagreement Months

Momentum	Disagreement Loadings 1	2	3	4	5	5-1	t(5-1)	All
1	-1.634	0.086	0.733	2.287	3.846	5.481**	6.561	0.938
2	-1.101	0.878	1.093	1.856	3.752	4.854**	7.192	1.164
3	-0.974	0.442	1.499	2.211	3.455	4.429**	6.929	1.257
4	-1.286	0.449	1.046	2.369	2.798	4.084**	6.646	0.954
5	-0.745	0.977	1.605	1.497	4.148	4.893**	7.091	1.506
5–1	0.889**	0.891**	0.872**	-0.79**	0.302			0.568*
t(5–1)	2.896	3.132	2.835	-2.604	0.924			2.261
All	-1.313	0.412	1.150	2.116	3.375	4.688**	7.137	

TABLE A8

Portfolio Returns, Controlling for Mean Analysts' Forecasts

The table reports average monthly value-weighted portfolio returns formed by sorting stocks on the mean of analysts' forecasts and disagreement loadings. Each month, we sort stocks into five quintiles based on the mean of analysts' forecasts for the end of the previous month. Then within each quintiles, we sort stocks into five quintiles based on their aggregate disagreement loadings. Disagreement loading, δ , is obtained each month using lagged three-year rolling data on individual stock returns using the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times \boldsymbol{R}_{m,t} + \delta_{i,t} \times \text{Aggregate Disagreement}_{t-1} + \epsilon_{i,t}.$$

The first two years of data is filled using one-year rolling regressions. Columns (5-1) and lines (5-1) in each table report the return differential across mean quintiles and across the disagreement loading quintiles, respectively. t(5-1)is the robust Newey-West t-statistic. **and ***denote significance at 5% and 1%, respectively.

All Months

Mean	1	2	Disagreement Loadings 3	4	5	5-1	t(5-1)	All
1	0.866	1.188	1.342	1.326	1.852	0.986	1.796	1.419
2	0.938	0.970	1.250	1.442	2.113	1.174**	1.977	1.365
3	0.798	1.041	1.411	1.549	1.892	1.094	1.784	1.407
4	0.829	1.035	1.289	1.590	2.262	1.434**	2.005	1.585
5	1.362	1.687	1.683	1.952	2.428	1.066	1.171	2.016
5–1	0.497	0.498	0.341	0.626	0.577			0.598
t(5–1)	1.571	1.684	1.056	1.778	1.101			2.008**
All	0.992	1.150	1.265	1.587	2.112	1.121	1.715	

Low Disagreement Months

Mean	1	2	Disagreement Loadings 3	4	5	5-1	t(5-1)	All
1	2.355	2.114	1.725	1.592	1.600	-0.755	-1.375	1.996
2	2.987	2.253	1.440	1.369	1.126	-1.861***	-3.133	2.054
3	2.781	2.346	1.787	1.254	0.894	-1.887***	-3.078	2.082
4	3.393	2.207	1.938	1.486	1.336	-2.058***	-2.878	2.252
5	4.093	3.030	1.614	1.637	0.363	-3.730***	-4.097	2.361
5–1	1.737***	0.916***	-0.111	0.045	-1.237**			0.365
t(5–1)	5.497	3.096	-0.344	0.128	-2.363			1.226
All	3.263	2.288	1.681	1.360	1.301	-1.962***	-3.003	

High Disagreement Months

Mean	1	2	Disagreement Loadings 3	4	5	5-1	t(5-1)	All
1	-0.666	0.460	1.064	1.804	3.174	3.839***	6.993	1.124
2	-0.817	0.372	1.602	1.606	3.382	4.199***	7.071	1.180
3	-1.174	0.082	1.147	2.185	3.568	4.742***	7.733	1.093
4	-1.267	0.249	1.114	2.024	3.732	4.999***	6.993	1.216
5	-1.180	0.318	1.739	2.561	4.322	5.502***	6.043	1.428
5–1	-0.514	-0.141	0.675**	0.757**	1.148**			0.304
t(5–1)	-1.627	-0.478	2.089	2.151	2.192			1.021
All	-1.292	0.357	1.123	2.0949	3.390	4.682***	7.1652	

TABLE A9

Portfolio returns, controlling for Sentiment

The table reports average monthly value-weighted portfolio returns formed by sorting stocks on their Sentiment loadings and their disagreement loadings. Each month, we sort stocks into five quintiles based on their Sentiment loadings. Then, within each quintile, we sort stocks by their aggregate disagreement loading. The sentiment loading, θ, is obtained each month using lagged three-year rolling data on individual stock returns using the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times \boldsymbol{R}_{m,t} + \theta_{i,t} \times \text{Aggregate Sentiment}_{t-1} + \epsilon_{i,t}.$$

The disagreement loading, δ, is obtained each month using lagged three-year rolling data on individual stock returns using the regression:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times \boldsymbol{R}_{m,t} + \delta_{i,t} \times \text{Aggregate Disagreement}_{t-1} + \epsilon_{i,t}.$$

The columns (5-1) and rows (5-1) in each table report the return differential across the sentiment loadings quintiles and across the disagreement loadings quintiles, respectively. t(5-1)is the robust Newey-West t-statistic.

All Months			Disagreement Loadings Quintile					
Sentiment Loading Quintile	1	2	3	4	5	5-1	t(5-1)	All
1	1.461	1.494	1.634	1.681	2.183	0.722	1.03	1.851
2	1.396	1.042	1.293	1.412	1.860	0.464	0.815	1.482
3	1.133	1.312	1.553	1.396	1.606	0.472	0.865	1.506
4	1.081	1.183	1.372	1.556	1.505	0.425	0.725	1.419
5	1.247	1.221	1.459	1.972	2.345	1.098	1.360	1.837
5–1	-0.214	-0.273	-0.174	0.291	0.162			-0.014
t(5–1)	-0.275	-0.446	-0.258	0.417	0.236			-0.020
All	0.984	1.148	1.254	1.583	2.112	1.128	1.727	

Low Disagreement Months			Disagreement Loadings					
Sentiment Loading Quintile	1	2	3	4	5	5-1	t(5-1)	All
1	4.666	3.346	2.953	1.759	1.292	-3.374***	-4.813	2.786
2	3.406	2.271	1.962	1.670	1.717	-1.689***	-2.966	2.287
3	3.042	2.234	2.041	1.610	0.956	-2.086***	-3.819	2.084
4	3.346	2.125	1.657	1.572	0.689	-2.657***	-4.536	2.047
5	2.774	1.647	1.384	0.874	0.208	-2.566***	-3.178	1.512
5–1	-1.892**	-1.699***	-1.569**	-0.885	-1.084			-1.274
t(5–1)	-2.438	-2.775	-2.319	-1.269	-1.573			-1.898
All	3.313	2.428	1.929	1.619	1.431	-1.882***	-2.882	

High Disagreement Months			Disagreement Loadings					
Sentiment Loading Quintile	1	2	3	4	5	5-1	t(5-1)	All
1	-0.810	0.521	1.464	1.778	2.951	3.761***	5.365	1.319
2	1.811	1.267	1.851	1.847	2.490	0.679	1.192	1.822
3	1.688	1.631	2.255	2.155	2.713	1.026	1.878	2.056
4	1.215	2.085	2.654	2.791	2.616	1.401**	2.393	2.232
5	2.034	2.624	2.880	3.980	4.543	2.509***	3.108	3.126
5–1	2.844***	2.104***	1.416**	2.203***	1.592**			1.807
t(5–1)	3.666	3.436	2.093	3.160	2.310			2.692
All	-0.309	1.269	1.985	2.616	3.46	3.769***	5.773	

TABLE A10

Average Prices of Market and Disagreement Risk for 100 β-δ Portfolios: Out-of-Sample From September 2016 to December 2021.

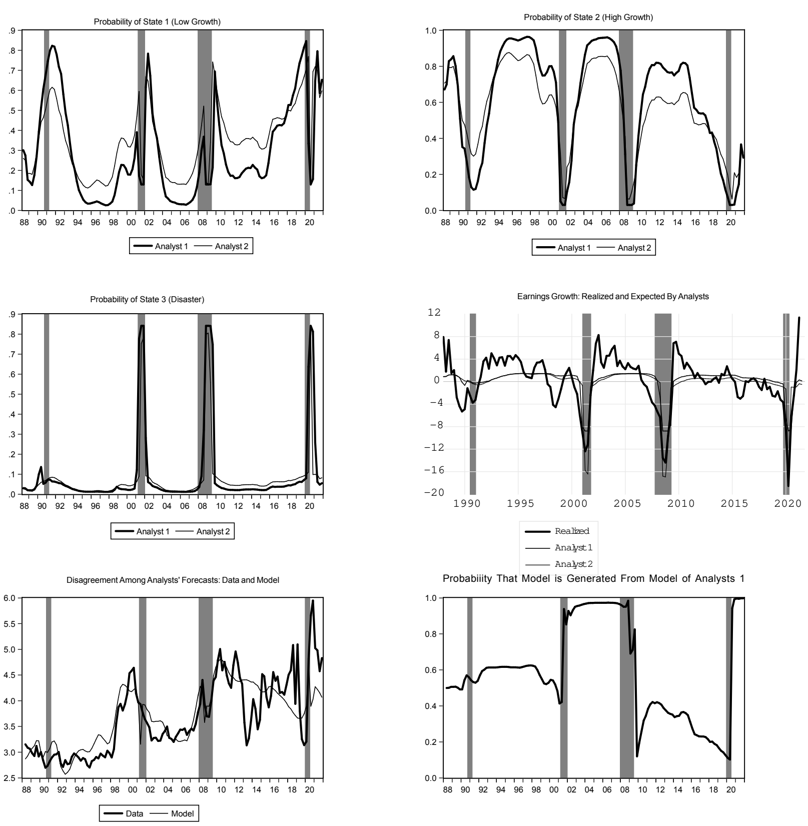
The table reports the time-series averages of the prices of market and disagreement risk for alternative subsamples from Fama-Macbeth regressions of the 100 $\beta - \delta$ sorted portfolios constructed as described in the footnote of Table A2. The estimators for the prices of risk are as in the footnote to Table 4. * and ** denote significance at 5% and 1%, respectively.

	(1)	(2)	(3)	(4)
Panel A: All Months (63 months)				
$\bar{\pi}$	0.597	0.268	0.542	0.152
	(0.461)	(0.221)	(0.412)	(0.124)
$\bar{\omega}$		0.578**		0.602**
		(2.778)		(2.745)
$\bar{\phi}^{SMB}$			0.001	-0.002
			(0.002)	(-0.004)
$\bar{\phi}^{HML}$			0.142	0.759
			(0.130)	(0.757)
$\bar{\phi}^{UMD}$			-0.588	-0.635
			(0.615)	(0.696)
Constant	0.000**	0.000*	0.000**	0.011**
	(15.289)	(17.387)	(16.946)	(3.191)
Adjusted-R^2	0.007	0.018	0.033	0.049
Panel B: High Disagreement Months (43 months)				
$\bar{\pi}$	1.558	0.806	1.324	0.537
	(1.493)	(0.765)	(1.152)	(0.471)
$\bar{\omega}$		0.887**		0.948 **
		(4.184)		(4.248)
$\bar{\phi}^{SMB}$			-0.112	0.070
			(-.142)	(0.087)
$\bar{\phi}^{HML}$			1.304	2.009*
			(1.293)	(2.087)
$\bar{\phi}^{UMD}$			-0.919	-1.106
			(-0.870)	(-0.956)
Constant	0.000**	0.000**	0.014**	0.000**
	(15.321)	(16.276)	(17.213)	(17.865)
Adjusted-R^2	0.001	0.020	0.025	0.037
Panel C: Medium Disagreement Months (20 months)				
$\bar{\pi}$	-6.589	-1.435	-6.092	-1.069
	(-1.796)	(-0.358)	(-1.791)	(-0.276)
$\bar{\omega}$		0.466**		-0.232
		(4.564)		(-0.247)
$\bar{\phi}^{SMB}$			-0.947	-3.205
			(0.745)	(-1.379)
$\bar{\phi}^{HML}$			-4.522	0.572
			(-1.312)	(0.310)
$\bar{\phi}^{UMD}$			2.043	-0.495
			(0.874)	(-1.802)
Constant	0.000**	0.000**	0.000**	0.000**
	(4.773)	(6.589)	(5.394)	(7.197)
Adjusted-R^2	0.023	0.027	0.059	0.068

FIGURE A1

Investor Beliefs, Expected Growth Rates of Earnings From Estimated Model, Data and Model Analysts' Disagreement, and the Conditional Probability that the Data are Generated from the Model of Analyst 1. In-sample (1988 – 2016:Q1) and Out-of-sample (2016:2 – 2021)

The **top-left**, **top-right**, and **middle-left** panels display the time-series of filtered beliefs about real earnings growth 1-quarter ahead of the two types of analysts, for states 1, 2, and 3, respectively. Filtered beliefs of the two agents are obtained from the discretized version of the belief processes as shown in equation (7). The estimated parameters for each type of agent shown in Table 6. The **middle-right** panel displays the actual and one-quarter ahead expected earnings growth of the two types of agents using these filtered beliefs. The **bottom-left** panel shows the disagreement in analysts' beliefs in the data, which is the value-weighted disagreement of analysts' forecasts of the long-term (5-year ahead) growth rate of EPS of firms in the IBES database, and the model-implied 5-year ahead disagreement, from the filtered beliefs of each analyst type. Finally, the **bottom-right** panel shows the process $\{\varrho_t\}$, which is the conditional probability that the data at t is generated from the model of Analyst 1.



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