How Does Benchmarking Affect Market Efficiency? The Role of Learning Technology

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We study how asset managers' benchmarking affects market efficiency under two learning technologies: separative and integrative. With integrative learning, investors process portfolio-wide signals, optimizing information allocation across assets instead of focusing on individual ones. Therefore, increased benchmarking on an asset with greater uncertainty can *enhance* its price informativeness as investors direct more attention to it. This contrasts with the existing results assuming separative learning. Moreover, our study indicates that benchmarking could increase overall market efficiency, with each learning technology presenting distinct implications for asset pricing. These findings emphasize the critical role of learning technology in understanding the effects of benchmarking.

JEL classification: G11, G14, G23
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I. Introduction

The asset management industry has experienced substantial growth, positioning asset managers as pivotal players in the market.[1] Their performance incentives, tied explicitly or implicitly to benchmarks, generate specific hedging demands.[2] As institutional investors, they may actively seek private information from diverse sources to potentially speculate and outperform their benchmarks. This dynamic raises a crucial question: How does the need to hedge, driven by benchmarking, affect investors' information acquisition and attention allocation across different assets in their portfolios? Addressing this issue is the key to understanding the wider effects of benchmarking on market efficiency and asset pricing. Our paper indicates that the answer to this question is fundamentally influenced by the learning approach investors adopt, whether it is separative or integrative learning.

Under separative learning, where investors receive private signals about individual assets, benchmarking-induced hedging demand reduces the effective supply of the benchmarked asset. This discourages information acquisition and, consequently, diminishes price informativeness, consistent with the findings of Breugem and Buss (2019). In contrast, under integrative learning—where investors process information jointly across multiple assets—benchmarking can enhance price informativeness. Integrative learning introduces a cross-asset attention allocation effect: when investors observe a noisy linear combination of the two assets' payoffs, they optimally allocate attention across assets to minimize posterior portfolio uncertainty. As a result, attention tends to be directed toward the asset with greater uncertainty, since learning more about

[1] Refer to Appel, Gormley, and Keim (2016), Gerakos, Linnainmaa, and Morse (2021), among others.

[2] See, for example, Chevalier and Ellison (1997, 1999), Ma, Tang, and Gómez (2019), Cremers and Petajisto (2009), and Jiang, Verbeek, and Wang (2014).

the more uncertain asset yields a larger reduction in overall uncertainty. This cross-asset attention mechanism can offset the negative effective-supply effect. Consequently, an increase in benchmarking on a more uncertain asset may raise its price informativeness. Moreover, the effects of benchmarking on asset prices may differ substantially depending on investors' learning approach. For example, under separative learning, an increase in an asset's benchmarking level generally raises the expected prices of other assets, whereas under integrative learning, the effects may be reversed.

The economic theory of rational inattention identifies two types of learning technologies. The first, which we refer to as separative learning, imposes a structural restriction on the signal structure such that investors can acquire information about only one asset at a time (e.g., Peng (2005), Peng and Xiong (2006), and Van Nieuwerburgh and Veldkamp (2010)). The second, which we term integrative learning, removes this restriction and allows investors to observe signals about multiple assets simultaneously (e.g., Mondria (2010) and Miao, Wu, and Young (2022)). Both types of learning have theoretical and empirical relevance.[3] Which type most resembles how economic decision makers learn is an open question.[4] In the context of benchmarking, the critical role of learning technology has not been studied. Yet, this is crucial for understanding the effects of benchmarking on asset pricing and market efficiency. Our paper presents an extensive study of this subject and highlights the fundamental differences between these two learning technologies.

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[3]For example, Hameed, Morck, Shen, and Yeung (2015) find that analysts tend to follow assets that contain more valuable market- and industry-wide information (integrative signals).

[4]Veldkamp (2011) points out that "when using information flow measures to quantify information acquisition or research effort, uncorrelated signals may be a realistic assumption. Sims (2006) disagrees; he thinks that the agent cannot observe all the information and must choose the optimal information to observe. If optimized information flows involve constructing correlated signals about independent events, so be it."

We analyze a competitive rational expectations equilibrium under each learning technology for a continuum of investors, of whom a fraction are benchmarked and the rest are non-benchmarked. This approach enables a thorough comparative analysis of the equilibrium implications of separative versus integrative learning. The model features two risky assets with independent noisy supplies and potentially correlated payoffs. A benchmarked investor's performance is evaluated against a uniquely assigned benchmark, which may include one or both assets. The weight assigned to an asset within a benchmark portfolio reflects the investor's concern about their performance relative to that asset. The aggregate benchmarking level of an asset is defined as the sum of these weights across all benchmarked investors. Prior to trading, each investor optimally chooses the precision of his private signal, subject to the constraints of his information processing capacity, as conceptualized in Sims (2003, 2006). In addition to the private information, each investor can also use publicly observed prices to partially infer other investors' private information.

Under separative learning, investors are limited to observing signals about the payoffs of individual assets. Consequently, they will focus their attention (learning capacity) on the asset that yields the highest marginal value of private information. When two assets offer the same marginal value of information, each investor is indifferent about learning either asset. As a result, in equilibrium, an endogenous fraction of investors specialize in learning about one asset, while others concentrate on the other asset. We find that the level of attention an asset receives from investors, and consequently its price informativeness, decreases in its benchmarking level and increases in the benchmarking level of the other asset. This is because investors' hedging needs, driven by benchmarking concerns, reduce the effective supply of the benchmarked asset. As the benchmarking level of an asset increases, the marginal value of information about that asset diminishes. Investors then shift their focus away from it, leading to a decline in its price informativeness. This result is consistent with Breugem and Buss (2019), who examine investors' costly acquisition of information about individual assets rather than their allocation of attention across multiple assets.

Under integrative learning, investors' optimal strategy is not to specialize in acquiring information about a single asset. Rather, as shown by Mondria (2010), they are better off observing a private signal that represents a linear combination of the payoffs from multiple assets. Building upon this, we provide new analytical insights within a linear symmetric equilibrium, where benchmarked investors opt for the same optimal information choices. We show that the impact of benchmarking on price informativeness differs substantially from that under separative learning. In particular, when an asset is substantially more uncertain in its payoff or in its demand/supply noise, an increase in its benchmarking level may enhance its price informativeness. This occurs because the marginal value of private information now reflects two distinct sources, corresponding to two channels through which aggregate benchmarking affects attention allocation and, consequently, price informativeness. The first channel is the reduction in effective supply, as in the separative learning case. When this supply-reduction effect dominates, benchmarking reduces price informativeness. The second channel reflects a cross-asset attention allocation effect, which becomes stronger when the two assets differ more in their overall uncertainty. When this cross-asset mechanism dominates, benchmarking can enhance price informativeness. In this case, investors devote more attention to the benchmarked asset—particularly when it is much more uncertain in its payoff or demand/supply—because shifting attention toward the more uncertain asset most effectively reduces overall portfolio uncertainty and improves inference about both assets.

We then examine how a rise in the fraction of benchmarked investors affects price informativeness and attention allocation. Holding individual benchmarking levels fixed, a larger fraction raises the aggregate benchmarking levels of both assets. Under separative learning, higher benchmarking of an asset lowers its own price informativeness but increases that of the other asset. The overall effect on each asset therefore depends on the relative strength of the negative self-effect and the positive spillover from the other asset's benchmarking.

Under integrative learning, cross-asset attention allocation also matters. When one asset's total uncertainty is sufficiently large, this cross-asset effect dominates, and attention to the more uncertain asset always increases with the fraction of benchmarked investors. When the two assets' uncertainty levels are closer, reduced effective supply plays the primary role. In this case, the asset with the lower benchmarking level benefits more from the positive spillover and tends to receive increased attention as the fraction of benchmarked investors rises.

We next endogenize learning capacity by introducing a quadratic cost function. We show that the effects of benchmarking on learning and information acquisition are fundamentally shaped by the learning technology investors adopt—whether separative or integrative. Importantly, endogenizing learning capacity does not alter our main conclusion: under integrative learning, an increase in the benchmarking level can enhance price informativeness for the more uncertain asset, in contrast to the separative learning case.

Our results underscore the critical role of investors' learning technology in understanding the impact of benchmarking or passive investing on market efficiency. Our model generates several empirical implications. For instance, under integrative learning, our analysis suggests that: (1) An increase in the benchmarking level of a highly risky asset (e.g., growth stocks or meme stocks) can attract more investor attention to this asset and enhance its price informativeness.

Conversely, if the asset has low uncertainties (e.g., value stocks), more benchmarking may result in less investor attention and worse price informativeness. (2) Improvement in price informativeness following enhanced benchmarking can be indicative of investors widely adopting the integrative learning approach.

Separative learning is the equilibrium outcome of a constrained information structure. In practice, such a constraint may arise when learning about different assets is delegated to different portfolio managers, analyst teams, or trading desks, each operating under a narrow mandate and an asset-specific benchmark. It may also reflect fragmented datasets across vendors, markets, or jurisdictions, which make integrative signal design costly, or organizational separation across teams or strategies that impedes joint information production.

A close real-world analogue of our integrative learning framework is the signal blending approach widely adopted in quantitative asset management. In practice, an integrated signal is often constructed by taking a weighted linear combination of multiple factor signals—such as value, momentum, and quality—at the security level. This single composite signal is then used to construct an integrated portfolio. This approach improves investment efficiency by exploiting cross-factor complementarities and eliminating redundant exposures. Empirical studies show that signal blending yields superior performance (e.g., Bender and Wang (2016), Fitzgibbons, Friedman, Pomorski, and Serban (2017) and Ghayur, Heaney, and Platt (2018)), and it has been widely implemented in practice by asset managers. These practices closely parallel the information-processing structure captured by integrative learning in our model.

Our paper is closely related to Mondria (2010), which shows that investors opt to observe a single linear combination of asset payoffs as a private signal when their anticipated investment strategy for the subsequent period involves holding a diversified portfolio. It examines how asset price comovement arises as investors use this composite signal to update information about two assets. In contrast, our paper primarily examines the impact of benchmarking on price informativeness and market efficiency, with a particular focus on the pivotal role of learning technologies in shaping these outcomes.

Our paper adds to the theoretical literature on the implications of benchmarked investors. Brennan (1993) develops a two-factor asset pricing model with a benchmarked fund manager. Cuoco and Kaniel (2011) show that managers' relative performance concerns increase the prices of the benchmarked assets. Basak and Pavlova (2013) show that as the institutional investor optimally holds more of assets included in his benchmark, these assets become more expensive, volatile, and correlated. Buffa, Vayanos, and Woolley (2022) show that benchmarking is part of an optimal contract in the presence of agency frictions. Buffa and Hodor (2023) show that heterogeneous benchmarking can result in negative spillovers across asset returns. Pavlova and Sikorskaya (2023) introduce a novel measure of benchmarking intensity to capture investors' inelastic demand for stocks. They observe that both active and passive funds tend to buy stocks added to benchmarks and sell those removed. Kacperczyk, Nosal, and Sundaresan (2025) study the impact of asset ownership on price informativeness when investors have market power. From a different angle, our paper focuses on how benchmarking affects investors' attention allocation under different learning technologies. This also distinguishes our work from that of Breugem and Buss (2019). We show that quite different implications of investors' benchmarking can arise under different learning technologies.

The implications of investors' benchmarks are also related to the literature on passive investing and indexing. Bond and Garcia (2022) and Baruch and Zhang (2022) find that a rise in index investors diminishes the price informativeness of the index. Conversely, Liu and Wang

(2023) suggest that this effect depends on the causes of indexing, noting that increased indexing could enhance index price informativeness under certain conditions. Lee (2020) expands on the model of Gârleanu and Pedersen (2018) by incorporating strategic trading by asset managers. Coles, Heath, and Ringgenberg (2022) present evidence that indexing does not affect price informativeness. Buss and Sundaresan (2023) find that passive ownership can enhance firms' informational efficiency by reducing the cost of capital and encouraging risk-taking activities, which prompts active investors to gather more precise private information.

The rest of this paper is organized as follows. Section II presents the model. Section III studies the separative learning equilibrium. Section IV studies the integrative learning equilibrium. Section V endogenizes learning capacity. Section VI concludes. Proofs are presented in the Appendix.

II. The Model

Consider a three-period economy with dates $t = 1$, 2, and 3. At $t = 1$, investors choose the precisions of signals about asset payoffs, subject to an information processing capacity. They decide how to allocate their limited learning capacity across assets. At $t = 2$, investors observe the private signals about asset payoffs and then choose their optimal stock holdings. At $t = 3$, investors receive compensations based on their performances relative to preassigned benchmarks.

Asset Market. There is a risk-free asset in perfectly elastic supply. It serves as numeraire and its price is normalized to one. There are two risky assets labeled by $j \in \{1,2\}$. Each risky asset yields a final payoff V_j at $t = 3$ and has a random supply Z_j, where V_1, V_2, Z_1, and Z_2 are independent normal random variables, denoted $V_j \sim \mathcal{N}(\bar{v}_j, \tau_{v,j}^{-1})$ and $Z_j \sim \mathcal{N}(\bar{z}_j, \tau_{z,j}^{-1})$.[5] We can write them as vectors, $V := (V_1, V_2)'$ and $Z := (Z_1, Z_2)'$ with $V \sim \mathcal{N}(\bar{v}, \Sigma_v)$ and $Z \sim \mathcal{N}(\bar{z}, \Sigma_z)$, where

$$\bar{v} := \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \end{pmatrix}, \qquad \Sigma_v := \begin{pmatrix} \tau_{v,1}^{-1} & 0 \\ 0 & \tau_{v,2}^{-1} \end{pmatrix}, \qquad \bar{z} := \begin{pmatrix} \bar{z}_1 \\ \bar{z}_2 \end{pmatrix}, \qquad \Sigma_z := \begin{pmatrix} \tau_{z,1}^{-1} & 0 \\ 0 & \tau_{z,2}^{-1} \end{pmatrix}.$$

Investors. We consider a unit mass of investors (asset managers) indexed by $i \in [0,1]$, each endowed with the same initial wealth W_0^i. A salient feature of the asset management industry is that some asset managers care about their performance relative to predetermined benchmarks such as index portfolios.[6] Accordingly, we divide investors into two groups: (1) a fraction $m \in (0,1]$ of benchmarked investors ($\mathcal{BI}$), and (2) a fraction $1-m$ of non-benchmarked investors ($\mathcal{NI}$). For benchmarked investors ($i \in \mathcal{BI}$), different investment styles are reflected in variations in benchmark composition, which we treat as exogenous in this paper. Let a vector $\gamma^i := (\gamma_1^i, \gamma_2^i)'$ represent the benchmark composition for investor i, where $\gamma_j^i \geq 0$ reflects the strength of investor i's benchmarking concerns regarding asset j. In other words, investor i's benchmark consists of γ_1^i shares of asset 1 and γ_2^i shares of asset 2. If $\gamma_1^i > 0$ and $\gamma_2^i = 0$, there is only asset 1 in his benchmark; if $\gamma_2^i > 0$ and $\gamma_1^i > 0$, there are both assets in his benchmark. For non-benchmarked investors, $i \in \mathcal{NI}$, $\gamma_1^i = \gamma_2^i = 0$.

Portfolio Choice. Let $\mathrm{E}_t^i[\cdot]$ and $\mathrm{Var}_t^i(\cdot)$ denote the posterior expectation and variance, conditional on investor i's information set at time t. This set includes public prices $P := (P_1, P_2)'$ and private

[5] We have studied the case that asset payoffs V_1 and V_2 are correlated. See Internet Appendix B.6 for more details. This extension does not alter our main conclusions. For clarity of exposition, we focus on the model with independent payoffs.

[6] To compare our results with the existing literature (e.g., Breugem and Buss (2019)), we focus on analyzing the impact of benchmarks tied to the performance of index portfolios. Our framework is not suited for studying benchmarks tied to the performance of peer managers, which presents an interesting question for future research.

signals Y^i (to be defined shortly). For tractability and comparability with the existing literature, we follow Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), and Breugem and Buss (2019) to assume that the expected utility at $t = 1$ for investor i is

$$\mathrm{E}_1^i\big[-\ln \mathrm{E}_2^i\big[\exp(-\lambda C^i)\big]\big], \tag{1}$$

where $\lambda > 0$ is the risk-aversion coefficient and C^i is investor i's compensation at $t = 3$,

$$\begin{aligned} C^i &= W_0^i + (\theta^i)'(V - P) - (\gamma^i)'(V - P) \\ &= W_0^i + (\theta^i - \gamma^i)'(V - P), \end{aligned} \tag{2}$$

where $\theta^i := (\theta_1^i, \theta_2^i)'$ represents investor i's asset holdings at $t = 2$.

As discussed in Breugem and Buss (2019), the utility function in (1) is equivalent to $U_1 = \mathrm{E}_1[u_1(\mathrm{E}_2[u_2(.)])]$, where $u_1(x) = -\ln(-x)$ and $u_2(x) = -\exp(-\lambda x)$. The inner utility u_1 is convex, corresponding to a preference for early resolution of uncertainty. This specification is equivalent to that investors maximize a mean-variance objective function. Equation (2) implies that investor i's compensation at $t = 3$ is given by the terminal value of his portfolio minus the performance of benchmark portfolio.[7] At $t = 2$, each trader chooses the optimal asset holdings, θ^i to maximize his expected utility,

$$U_2^i(\gamma^i, Y^i, P) := \max_{\theta^i} \lambda \mathrm{E}_2^i[C^i] - \tfrac{1}{2}\lambda^2 \mathrm{Var}_2^i(C^i). \tag{3}$$

In equilibrium, asset prices P_1 and P_2 satisfy the market-clearing conditions,

$$\int_0^m \theta_j^{\mathcal{BI},i}(\gamma_j^i, Y^i, P_j)\,di + \int_m^1 \theta_j^{\mathcal{NI},i}(0, Y^i, P_j)\,di = Z_j, \quad \text{for } j = 1, 2, \tag{4}$$

[7] This benchmark concerns capture a performance-based fee that adjusts based on outperformance or underperformance relative to a benchmark, as in Breugem and Buss (2019).

where $\theta_j^{\mathcal{BI},i}$ denotes the asset holdings of benchmarked investors and $\theta_j^{\mathcal{NI},i}$ denotes the holdings of non-benchmarked investors. Each investor's demand depends on his benchmark and information set. For asset $j \in 1,2$, the aggregate benchmarking level is defined as

$$\bar{\gamma}_j := \int_0^m \gamma_j^{\mathcal{BI},i}\, di = m\gamma_j, \tag{5}$$

where γ_j denotes the average benchmarking concern for asset j among benchmarked investors ($i \in \mathcal{BI}$). Variation in $\bar{\gamma}_j$ may therefore result from changes in the fraction of benchmarked investors (m), the average benchmarking level (γ_j), or both.

Information Choice. Consider that investors observe private information of the form

$$Y^i = \Lambda^i V + \varepsilon^i, \quad \text{where} \quad \varepsilon^i \sim \mathcal{N}(0, \Sigma^i) \quad \text{and} \quad \Sigma^i := \begin{pmatrix} (\tau_1^i)^{-1} & 0 \\ 0 & (\tau_2^i)^{-1} \end{pmatrix}. \tag{6}$$

Here, Λ^i is a 2×2 matrix that specifies how investor i allocates learning across the two risky assets. This private information contains noise, represented by $\varepsilon^i := (\varepsilon_1^i, \varepsilon_2^i)'$, which is orthogonal to V and independent among different investors. A signal becomes more precise as it receives more attention. Without loss of generality, we assume that the matrix Σ^i is diagonal. Each investor chooses the entries of Σ^i subject to his limited learning capacity. Therefore, Λ^i and Σ^i represent the information choice made by each investor i at $t = 1$ to maximize his expected utility:

$$U_1^i = \max_{\Lambda^i, \Sigma^i} \mathrm{E}_1^i \left[U_2^i(\gamma^i, Y^i, P) \right]. \tag{7}$$

Given the information choices of other investors, taking the first-order conditions of expected utility at $t = 1$ with respect to Λ^i and Σ^i yields investor i's optimal information choice. The resulting (Λ^i, Σ^i) characterize the signal Y^i, which in turn affects portfolio decisions at $t = 2$. Both benchmarked and non-benchmarked investors face the same information acquisition problem, since only the aggregate level of benchmarking—rather than individual levels—affects information choices through equilibrium prices. Accordingly, we focus on the problem of a generic investor $i \in \{\mathcal{BI}, \mathcal{NI}\}$, noting that the optimal information choices (Λ^i, Σ^i) are identical across investor types.

Information Processing Constraint. Following Sims (2003, 2006), we use a standard measure from information theory to quantify the amount of information that an observable signal contains about the asset payoffs. The entropy of a random variable X measures its uncertainty. With a continuous probability density $p(x)$, the entropy is defined as $\mathrm{H}(X) = -\int p(x) \ln p(x) dx$. We can also define the conditional entropy $\mathrm{H}(X|Y) = -\int\int p(x,y) \ln p(x|y) dx dy$, where $p(x,y)$ and $p(x|y)$ are joint and conditional density functions, respectively. The standard information-theoretic measure for uncertainty reduction is the *mutual information* defined as

$$I(X;Y) = H(X) - H(X \mid Y) = \int\int p(x,y) \ln\left(\frac{p(x,y)}{p(x)p(y)}\right) dx\, dy. \tag{8}$$

It is nonnegative $\mathrm{I}(X;Y) \geq 0$, symmetric $\mathrm{I}(X;Y) = \mathrm{I}(Y;X)$, and invariant under any linear transformations of random variables, i.e., $\mathrm{I}(X;Y) = \mathrm{I}(aX + b; cY + d)$ if $ac \neq 0$.

By acquiring private information, investors can reduce their uncertainty about asset payoffs. We assume investors face the following information processing constraint

$$\mathrm{I}(V;Y^i) = \mathrm{H}(V) - \mathrm{H}(V|Y^i) \leq \frac{1}{2} \ln(K), \tag{9}$$

where the parameter $K > 1$ sets the upper limit on how much information each trader can learn. Given the normal distributions of V and Y^i, the above constraint is equivalent to

$$\left|\mathrm{Var}(V \mid Y^i)\right| \geq \frac{1}{K} \left|\mathrm{Var}(V)\right|. \tag{10}$$

For expositional simplicity, we take learning capacity K to be exogenously given in Sections III and IV. In Section V, we endogenize learning capacity by introducing a quadratic cost function and show that doing so does not alter our main conclusion: under integrative learning, an increase in the benchmarking level can enhance price informativeness for the more uncertain asset, in contrast to the separative learning case.

Learning Technology. The general form presented in equation (6) characterizes the private signal $Y^i := (Y_1^i, Y_2^i)'$. If Λ^i is restricted to be a diagonal matrix, it implies that investor i collects information solely about each asset independently, as Y_1^i is uncorrelated with Y_2^i. As a result, investor i's belief about asset 1 is independent of his belief about asset 2. In contrast, if Λ^i is not confined to being a diagonal matrix, each investor i would optimally choose to observe a signal that is a linear combination of the payoffs of the two assets. This allows investors to optimally allocate their information resources for their entire portfolios. We subsequently refer to these two approaches as two distinct learning technologies:

(1) *separative learning*, where Λ^i is constrained to be a diagonal matrix for each investor i.

(2) *integrative learning*, where Λ^i is not constrained and can be non-diagonal for each i.

Definition of Equilibrium. Under either learning technology, an equilibrium is defined by investors' portfolio choices $\{\theta^i\}$ and information choice $\{\Lambda^i, \Sigma^i\}$ as well as asset prices P such that

1. taking Λ^i, Σ^i, and P as given, θ^i solves investor i's optimal portfolio choice problem (3);
2. given $\{\theta^i\}$, the prices P satisfy the market-clearing condition (4);
3. taking the aggregate benchmarking level $\bar{\gamma}_j$ for $j = \{1,2\}$ as given, Λ^i and Σ^i solve investor

i's optimal information choice problem (7) subject to the constraint (10);

4. investors follow rational expectations; their beliefs are consistent with the joint conditional probability distribution in equilibrium.

III. Separative Learning Equilibrium

For comparison, we first briefly present the equilibrium results under the separative learning technology, where Λ^i in (6) is a diagonal matrix, implying that each investor can only observe asset-specific signals. Since Σ^i is also diagonal, the information choice problem is equivalent to the one with $\Lambda^i = I_2$, a 2×2 identity matrix. Then, private information takes the form

$$(11) \qquad Y^i := (Y_1^i, Y_2^i)', \qquad \text{where} \qquad Y_j^i = V_j + \varepsilon_j^i, \quad \varepsilon_j^i \sim \mathcal{N}\big(0, (\tau_j^i)^{-1}\big).$$

Trader i's optimal holding in asset j, obtained by solving (3), is:

$$(12) \qquad \theta_j^i = \gamma_j^i + \frac{\mathrm{E}^i[V_j] - P_j}{\lambda \mathrm{Var}^i(V_j)},$$

where $\mathrm{E}^i[V_j]$ and $\mathrm{Var}^i(V_j)$ denote trader i's posterior expectation and variance of V_j. Given investors' optimal holdings (12), it follows from Appendix A.1 that investors' optimal information choice problem (7) prior to trading can be written as

$$(13) \quad \max_{\tau_1^i, \tau_2^i} \quad \lambda W_0 - 1 + \frac{1}{2} \sum_{j=1,2} \frac{\mathrm{E}\big[(V_j - P_j)^2\big]}{\mathrm{Var}^i(V_j)} \quad \text{s.t.} \quad \prod_{j=1,2} (\tau_{v,j} + \tau_j^i) \leq K \prod_{j=1,2} \tau_{v,j} \quad \text{and} \quad \tau_j^i \geq 0.$$

For each asset j, we define the following quantity as its *Adjusted Squared Sharpe Ratio* (ASSR):

$$(14) \qquad \mathrm{ASSR}_j := \frac{\mathrm{E}[(V_j - P_j)^2]}{\mathrm{Var}(V_j)} = \frac{(\mathrm{E}[V_j - P_j])^2}{\mathrm{Var}(V_j)} + \frac{\mathrm{Var}(V_j - P_j)}{\mathrm{Var}(V_j)},$$

ASSR_j is a dimensionless ratio of reward to risk for each asset j. Under separative learning, ASSR_j measures the marginal value of private information about asset j.

Let $\Gamma_1 \in [0,1]$ denote the endogenous fraction of investors who choose to learn solely about asset 1. Then $\Gamma_2 := 1 - \Gamma_1$ denotes the fraction of investors focusing on asset 2. We thus obtain the following theorem.

Theorem 1. *Under separative learning, there exists a unique linear equilibrium. It includes three possible cases for the overall allocation of investors' attention, denoted by* (Γ_1, Γ_2)*:*

(1) If $\text{ASSR}_1(\Gamma_1) > \text{ASSR}_2(\Gamma_1)$ *holds for any* $\Gamma_1 \in [0,1]$*, then* $\Gamma_1 = 1$ *and* $\Gamma_2 = 0$*;*

(2) If $\text{ASSR}_1(\Gamma_1) = \text{ASSR}_2(\Gamma_1)$ *holds at some* $\Gamma_1^* \in (0,1)$*, then* $\Gamma_1 = \Gamma_1^*$ *and* $\Gamma_2 = 1 - \Gamma_1^*$*;*

(3) If $\text{ASSR}_1(\Gamma_1) < \text{ASSR}_2(\Gamma_1)$ *holds for any* $\Gamma_1 \in [0,1]$*, then* $\Gamma_1 = 0$ *and* $\Gamma_2 = 1$.

The equilibrium prices are determined by:

$$P_j = (\tau_{v,j} + \tau_{p,j} + \bar{\tau}_j)^{-1} [\tau_{v,j}\bar{v}_j + (\tau_{p,j} + \bar{\tau}_j)s_{p,j} - \lambda(\bar{z}_j - \bar{\gamma}_j)], \tag{15}$$

where

$$\tau_{p,j} = (K-1)^2 \Gamma_j^2 \tau_{v,j}^2 \tau_{z,j} / \lambda^2, \quad s_{p,j} := V_j - \lambda(Z_j - \bar{z}_j)/\bar{\tau}_j, \tag{16}$$

and $\bar{\tau}_j := \Gamma_j (K-1)\tau_{v,j}$ *denotes the total precision of signals about asset* j.

Proof. See Appendix A.1. □

In equilibrium, each investor optimally allocates all of his learning capacity to the asset that has a higher ASSR. Case (1) of Theorem 1 corresponds to a corner equilibrium and arises when $\text{ASSR}_1(\Gamma_1) > \text{ASSR}_2(\Gamma_1)$ for all $\Gamma_1 \in [0,1]$. In this case, all investors optimally choose to learn about asset 1 and ignore asset 2. For example, this case can arise when the benchmarking level of asset 1 ($\bar{\gamma}_1$) is sufficiently low relative to that of asset 2, or when asset 1 is substantially more uncertain (e.g., $\tau_{v,1}$ or $\tau_{z,1}$ are small), so that $\text{ASSR}_1 > \text{ASSR}_2$ even when all traders are learning about asset 1 (i.e., $\mathit{\Gamma}_1 = 1$). The equilibrium is then characterized by $\mathit{\Gamma}_1 = 1$ and $\mathit{\Gamma}_2 = 0$.

Case (2) corresponds to an interior equilibrium. Here, the equilibrium allocation $\Gamma_1 \in (0,1)$ is characterized by the fixed-point condition $\text{ASSR}_1(\mathit{\Gamma}_1) = \text{ASSR}_2(\mathit{\Gamma}_1)$, so that a marginal investor is indifferent between learning about asset 1 and asset 2. Case (3) is the symmetric corner equilibrium in which all investors choose to learn about asset 2. This occurs when $\text{ASSR}_2(\mathit{\Gamma}_1) > \text{ASSR}_1(\mathit{\Gamma}_1)$ for all $\mathit{\Gamma}_1 \in [0,1]$, which can arise, for example, when the benchmarking level of asset 2 (γ_2) is sufficiently low relative to that of asset 1, or when asset 2 is substantially more uncertain. In this case, the equilibrium is characterized by $\mathit{\Gamma}_1 = 0$ and $\mathit{\Gamma}_2 = 1$.

We measure price informativeness using the mutual information defined by equation (8). Specifically, $\mathrm{I}(V_j; P_j)$ quantifies the information about the asset payoff V_j that is revealed by the asset price P_j. In a linear Gaussian model, $\mathrm{I}(V_j; P_j)$ is informationally equivalent to the correlation coefficient, $\text{Corr}(V_j, P_j)$, and the R-squared measure, $R^2 = 1 - \frac{\text{Var}(V_j|P_j)}{\text{Var}(V_j)}$. In the separative learning equilibrium, the mutual information of asset-j's payoff and price is:

$$\mathrm{I}(V_j; P_j)_{\text{sep}} = \frac{1}{2}\ln\left(\frac{\text{Var}(V_j)}{\text{Var}(V_j \mid P_j)}\right) = \frac{1}{2}\ln\left(1 + \frac{\tau_{p,j}}{\tau_{v,j}}\right). \tag{17}$$

We obtain the following proposition.

Proposition 1. *In the separative learning equilibrium, the price informativeness of asset-j increases with attention $\mathit{\Gamma}_j$, i.e.* $\frac{d\mathrm{I}(V_j;P_j)_{\text{sep}}}{d\mathit{\Gamma}_j} > 0$. *Assume the expected effective supply is positive,* $\bar{z} - \bar{\gamma} > 0$.

1. *With the fraction of benchmarked investors m fixed, we have $\frac{d\Gamma_j}{d\gamma_j} \le 0$ and $\frac{d\Gamma_j}{d\gamma_{-j}} \ge 0$.*[8] *Hence,*

$$\frac{d\mathrm{I}(V_j;P_j)_{\text{sep}}}{d\gamma_j} \le 0, \qquad \frac{d\mathrm{I}(V_j;P_j)_{\text{sep}}}{d\gamma_{-j}} \ge 0. \tag{18}$$

2. *With the benchmarking levels γ_1 and γ_2 held fixed, we have*

   (1) If $\Gamma_1 = 1$, then $\frac{d\mathrm{I}(V_1;P_1)_{\text{sep}}}{dm} \le 0$;

   (2) If $0 < \Gamma_1 < 1$, then $\frac{d\Gamma_1}{dm} \ge 0$ and $\frac{d\mathrm{I}(V_1;P_1)_{\text{sep}}}{dm} \ge 0$ iff $\frac{\partial \ln \mathrm{ASSR}_1}{\partial \ln \bar{\gamma}_1} \ge \frac{\partial \ln \mathrm{ASSR}_2}{\partial \ln \bar{\gamma}_2}$;

   (3) If $\Gamma_1 = 0$, then $\frac{d\mathrm{I}(V_1;P_1)_{\text{sep}}}{dm} \ge 0$.

Proof. See Appendix A.2. □

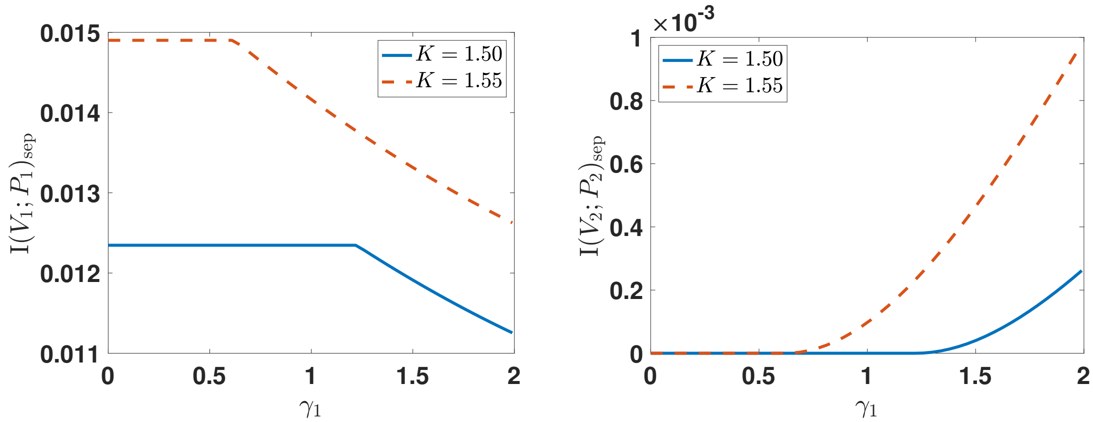
Throughout the paper, we focus on the case in which the expected effective supply $\bar{z}_j - \bar{\gamma}_j$ is positive for each asset j. This case is economically more interesting because it ensures a positive equilibrium risk premium.

Part 1 of Proposition 1 shows that, as illustrated in Figure 1, as the benchmarking level for asset 1, γ_1, increases, the mass of investors who focus solely on learning about asset 1 weakly decreases, thereby weakly lowering the price informativeness of asset 1. Conversely, a higher γ_1 shifts attention toward asset 2 and weakly raises its price informativeness.

Intuitively, an increase in the benchmarking level of asset 1 reduces its expected effective supply. This lowers the expected squared return, $\mathrm{E}[(V_1 - P_1)^2]$, and hence the marginal value of information about asset 1, as measured by ASSR_1 defined in (14). As a result, some investors reallocate their limited attention away from asset 1 toward asset 2, leading to a lower endogenous attention mass on asset 1, Γ_1, and a corresponding decline in its price informativeness. Moreover, with limited attention, a higher benchmarking level of asset 1 induces more investors to focus on

[8] We use subscript "$-j$" to refer to the "other" asset. For example, if $j = 1$, then "$-j$" refers to asset 2.

FIGURE 1. Price Informativeness $\mathrm{I}(V_1; P_1)_{\mathrm{sep}}$ and $\mathrm{I}(V_2; P_2)_{\mathrm{sep}}$ under separative learning as a function of γ_1 for different learning capacities. Other parameters are: $\lambda = 1$, $\tau_{v,1} = 1$, $\tau_{z,1} = 0.1$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 2$, $m = 0.5$, and $\gamma_2 = 0$.

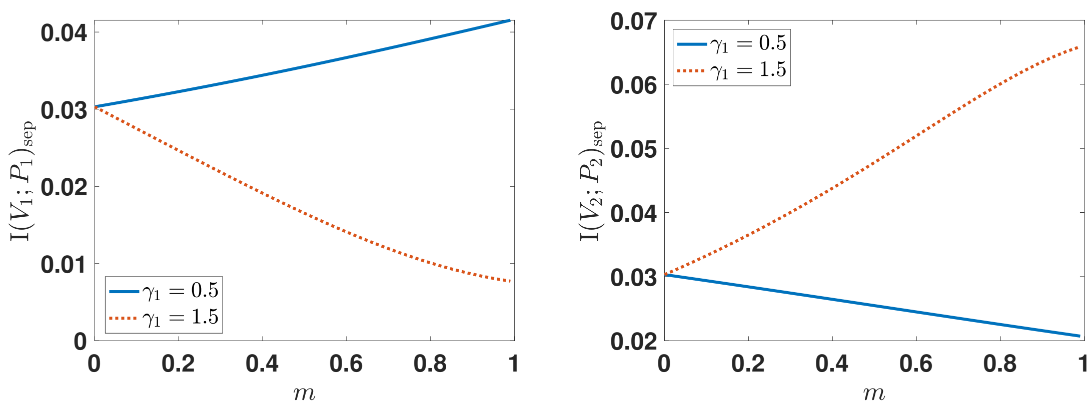


asset 2, increasing its price informativeness. Figure 1 also shows that a higher learning capacity K increases price informativeness for both assets, while leaving the qualitative effect of benchmarking unchanged.

Holding γ_1 and γ_2 fixed, a higher fraction of benchmarked investors m raises the aggregate benchmarking levels $\bar{\gamma}_j$ for $j \in \{1,2\}$. Since increasing the benchmarking level of an asset reduces its own price informativeness while raising that of the other asset, the net effect of increasing m on price informativeness depends on whether the negative impact of an asset's own benchmarking is outweighed by the positive spillover effect of the other asset's benchmarking.

Part 2 of Proposition 1 implies that if $\Gamma_1 = 1$ (i.e., $\mathrm{ASSR}_1 > \mathrm{ASSR}_2$ for all $\Gamma_1 \in [0,1]$), then both the attention allocation Γ_1 and the price informativeness of asset 1 are weakly decreasing in the fraction of benchmarked investors m (Case (1)). This result is immediate in a corner solution: because $\Gamma_1 \le 1$, an increase in m cannot raise Γ_1 but may reduce it. Symmetrically, if $\Gamma_1 = 0$, then attention allocation Γ_1 and the price informativeness of asset 1 are weakly increasing in m (Case (3)). In Case (2), where $0 < \Gamma_1 < 1$ and $\mathrm{ASSR}_1 = \mathrm{ASSR}_2$, the signs of $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}m}$ and $\frac{d\mathrm{I}(V_1;P_1)_{\mathrm{sep}}}{dm}$ are determined by the difference between the elasticities of the marginal value of information with respect to benchmarking: $\frac{\partial \ln \mathrm{ASSR}_1}{\partial \ln \bar{\gamma}_1} - \frac{\partial \ln \mathrm{ASSR}_2}{\partial \ln \bar{\gamma}_2}$. Because increasing $\bar{\gamma}_j$ lowers the marginal value of information about asset j, we have $\frac{\partial \ln \mathrm{ASSR}_j}{\partial \ln \bar{\gamma}_j} < 0$ for $j \in \{1,2\}$. Therefore, a higher m raises the price informativeness of asset j when the negative impact of asset j's own benchmarking ($\frac{\partial \ln \mathrm{ASSR}_j}{\partial \ln \bar{\gamma}_j} < 0$) is outweighed by the positive spillover effect of the other asset ($-\frac{\partial \ln \mathrm{ASSR}_{-j}}{\partial \ln \bar{\gamma}_{-j}} > 0$).

FIGURE 2. Price Informativeness $\mathrm{I}(V_1;P_1)_{\mathrm{sep}}$ and $\mathrm{I}(V_2;P_2)_{\mathrm{sep}}$ under separative learning as a function of m, for different values of γ_1. Other parameters are: $\lambda = 1$, $\tau_{v,1} = \tau_{z,1} = 1$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 2$, $K = 1.5$, and $\gamma_2 = 0.75$.



As illustrated in Figure 2, when asset 1 has a relatively low γ_1 (e.g., solid lines), the slower increase of $\bar{\gamma}_1$ with respect to m implies that the negative effect of its own benchmarking can be dominated by the positive spillover effect from asset 2. In this case, the attention allocated to asset 1—and therefore its price informativeness—tends to weakly increase with m. By contrast, when asset 1 has a relatively high γ_1 (e.g., dotted lines), the negative effect of its own benchmarking is more likely to dominate, so the attention allocated to asset 1 and its price informativeness tend to decline with m, while those for asset 2 increase.

It is worth mentioning some other results under separative learning. Section A.3 in the Appendix shows that the endogenous fraction of traders who learn about asset j increases with the volatility of asset j in terms of its payoff or supply. This is because private information about more uncertain outcomes is more valuable and highly sought after, consistent with Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016). Section A.4 in the Appendix examines the effects of benchmarking on asset prices and return volatility under separative learning. As the benchmarking level γ_j increases, investors shift their attention to the other asset, increasing its price informativeness and reducing its return volatility and risk premium, leading to a higher expected price for the other asset. For asset j, a higher benchmarking level raises hedging demand and, consequently, its expected price, but reduces price informativeness while increasing return volatility and the risk premium. Despite this, for a broad range of parameters, the demand effect dominates, resulting in an overall rise in the expected price.

IV. Integrative Learning Equilibrium

Now consider a general specification that the matrix Λ^i in equation (6) is not constrained to be diagonal. This implies that investors are not restricted to observing signals specific to individual assets. In Internet Appendix B.1, we confirm that it is optimal for trader i to observe a signal that is a linear combination of the payoffs of two assets, plus a noise term. We can express Λ^i as $(1, \omega^i)$ such that the private signal becomes a scalar variable:

$$Y^i = V_1 + \omega^i V_2 + \varepsilon^i, \qquad \varepsilon^i \sim \mathcal{N}(0, 1/\tau^i), \tag{19}$$

where both ω^i and τ^i are chosen by trader i.

A. The Equilibrium

The equilibrium can be characterized in three steps. First, conjecture the existence of a linear equilibrium where each investor i optimally chooses the integrative learning technology with $\Lambda^i = (1, \omega^i)$ and thus observes an integrative signal as in (19). Given investors' information choices, derive their optimal asset holdings and the market-clearing prices. Second, given these results, find the optimal attention weight ω^i and the signal precision τ^i. Last, confirm that it is an equilibrium under the choice of integrative learning.

Proposition 2. *Given traders' information choices $\{\omega^i, \tau^i\}$, the equilibrium price $P := (P_1, P_2)'$ is*

$$P = C + B\left(\Omega V - \lambda\left(Z - \bar{z}\right)\right), \qquad \text{where} \qquad \Omega := \int \tau^i \begin{pmatrix} 1 & w^i \\ w^i & (w^i)^2 \end{pmatrix} di, \tag{20}$$

C is a constant vector and B is a 2×2 matrix given by

$$C := \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\left(\Sigma_v^{-1}\bar{v} - \lambda(\bar{z} - \bar{\gamma})\right), \quad B := \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\left(I_2 + \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}\right). \tag{21}$$

Investor i's optimal asset holding $\theta^i := (\theta_1^i, \theta_2^i)'$ is

$$\theta^i = \gamma^i + (\lambda\hat{\Sigma}_v^i)^{-1}(\hat{V}^i - P), \tag{22}$$

where trader i's posterior variance-covariance matrix ($\hat{\Sigma}_v^i$) and posterior mean ($\hat{V}^i$) are given by (A-25) in the Appendix.

Proof. See Appendix A.5. □

Proposition 2 shows that equilibrium prices depend on the aggregate signal precision, through Ω, and on the aggregate benchmarking levels, $\bar{\gamma}$, rather than on the signal precision or benchmarking levels of individual investors. Given this, each trader faces the same decision problem when choosing their information structure. We thus focus on a symmetric equilibrium where $\omega^i = \omega$ and $\tau^i = \tau$ under integrative learning, given its uniqueness and analytical tractability.

Theorem 2. *A linear symmetric equilibrium exists if and only if the expected product of returns for two assets is positive:*

$$A_{12} := \mathrm{E}\big[(V_1 - P_1)(V_2 - P_2)\big] > 0. \tag{23}$$

In such an equilibrium, each investor optimally chooses to observe a private signal that represents a linear combination of the payoffs from both assets, denoted as $Y^i = V_1 + \omega V_2 + \varepsilon^i$, where $\varepsilon^i \sim \mathcal{N}(0, 1/\tau)$ is a noise term. The weight ω and precision τ are given as

$$\omega = \alpha + \sqrt{\alpha^2 + \frac{\tau_{v,2}}{\tau_{v,1}}} \qquad \textit{and} \qquad \tau = \frac{K-1}{\tau_{v,1}^{-1} + \omega^2 \tau_{v,2}^{-1}}, \quad \textit{where} \tag{24}$$

$$\alpha := \frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{2A_{12}\tau_{v,1}} = \frac{\tau_{v,2}^{-1}\tau_{z,2}^{-1} + \tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2 - \tau_{v,1}^{-1}\tau_{z,1}^{-1} - \tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2}{2(\bar{z}_1 - \bar{\gamma}_1)(\bar{z}_2 - \bar{\gamma}_2)\tau_{v,2}^{-1}}, \tag{25}$$

and $\mathrm{ASSR}_j = \tau_{v,j}\mathrm{E}[(V_j - P_j)^2]$ *is the adjusted squared Sharpe ratio for asset j, as defined in* (14)*. Given the optimal weight ω and precision τ, the equilibrium prices are determined by equation* (20) *and the optimal portfolio choice for each investor is given by equation* (22)*.*

Proof. See Appendix A.6. □

Condition (23) is required to satisfy the second-order condition for the optimal information choice. It holds when investors' risk aversion (λ) lies within a certain range—neither too small nor too large. As shown in Appendix A.6, a sufficient (though not necessary) condition for $A_{12} > 0$ is $\lambda_l < \lambda < \lambda_h$, where λ_l and λ_h are defined in Equation (A-41) of the Appendix. This condition depends solely on exogenous parameters and implies that the optimal attention weight ω in the integrative signal is positive. It is equivalent to requiring that the equilibrium covariance between the returns of the two assets not be excessively negative.

Theorem 2 implies that the aggregate benchmarking level of asset j ($\bar{\gamma}_j$) influences investors' optimal information choices, ω and τ, through the asset's expected effective supply, $\bar{z}_j - \bar{\gamma}_j$. This reflects aggregate non-benchmark demand, as each trader i incorporates a hedging demand of γ^i into his optimal demand due to benchmarking concerns, as shown in Equation (22).

To help understand the results in Theorem 2, note that the total payoff uncertainty associated with asset j's effective supply is given by $\mathrm{Var}\big((V_j - \bar{v}_j)\cdot(Z_j - \bar{\gamma}_j)\big) = \tau_{v,j}^{-1}\tau_{z,j}^{-1} + \tau_{v,j}^{-1}(\bar{z}_j - \bar{\gamma}_j)^2$. This uncertainty plays an important role in determining how investors allocate their learning capacities across two assets.

In the limiting case where $\alpha \to +\infty$, we have $\omega \to \infty$, implying that all investors focus on learning about asset 2. Conversely, if $\alpha \to -\infty$, then $\omega \to 0$, indicating that all investors shift their focus to asset 1. The limit $|\alpha| \to \infty$ arises when asset j's expected effective supply becomes negligible ($\bar{\gamma}_j \uparrow \bar{z}_j$). For example, assume $\bar{z}_j - \bar{\gamma}_j > 0$. In the limit $\bar{\gamma}_1 \uparrow \bar{z}_1$, if asset 1 has much greater uncertainty than asset 2, such that $\tau_{v,1}^{-1}\tau_{z,1}^{-1} > \tau_{v,2}^{-1}\tau_{z,2}^{-1} + \tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2$, then almost all investors choose to learn about asset 1 (i.e., $\alpha \to -\infty$ and $\omega \to 0$). This result is intuitive: learning about the asset with greater uncertainty—either because of higher payoff volatility or because of greater demand/supply noise—is more valuable.

Next, we examine the effects of benchmarking on investors' attention allocation and the informativeness of prices.

B. The Effects of Benchmarking on Investors' Attention Allocation

Define

$$\Delta := \mathrm{Var}\left(\left(V_2-\bar{v}_2\right)\left(Z_2-\bar{z}_2\right)\right)-\mathrm{Var}\left(\left(V_1-\bar{v}_1\right)\left(Z_1-\bar{z}_1\right)\right)=\tau_{v,2}^{-1}\tau_{z,2}^{-1}-\tau_{v,1}^{-1}\tau_{z,1}^{-1}. \tag{26}$$

The quantity Δ measures the difference in total uncertainty between the two assets, capturing variation arising from both payoff volatility and demand/supply noise. In particular, the uncertainty in the total payoff from all outstanding shares of asset j is given by $\mathrm{Var}\left(\left(V_j-\bar{v}_j\right)\left(Z_j-\bar{z}_j\right)\right)$. Since $V_j-\bar{v}_j$ and $Z_j-\bar{z}_j$ are independent and have zero means,

$$\mathrm{Var}\left(\left(V_j-\bar{v}_j\right)\left(Z_j-\bar{z}_j\right)\right)=\mathrm{Var}\left(V_j-\bar{v}_j\right)\mathrm{Var}\left(Z_j-\bar{z}_j\right)=\tau_{v,j}^{-1}\tau_{z,j}^{-1}. \tag{27}$$

Define the total payoff uncertainty of the portfolio, consisting of $\left(\bar{z}_1-\bar{\gamma}_1\right)$ shares of asset 1 and $\left(\bar{z}_2-\bar{\gamma}_2\right)$ shares of asset 2, as

$$\xi := \mathrm{Var}\left(\left(\bar{z}_1-\bar{\gamma}_1\right)V_1+\left(\bar{z}_2-\bar{\gamma}_2\right)V_2\right)=\left(\bar{z}_1-\bar{\gamma}_1\right)^2\tau_{v,1}^{-1}+\left(\bar{z}_2-\bar{\gamma}_2\right)^2\tau_{v,2}^{-1}. \tag{28}$$

We then obtain the following proposition.

Proposition 3. *1. Investors' optimal attention allocation on asset-2 satisfies*

$$\omega \geq \frac{\bar{z}_2-\bar{\gamma}_2}{\bar{z}_1-\bar{\gamma}_1}, \quad \textit{if and only if} \quad \Delta \geq 0, \quad \textit{where equality holds when} \quad \Delta = 0. \tag{29}$$

2. With the fraction of benchmarked investors m fixed, we have

(1) If $\frac{\Delta}{\xi} \geq 1$, then $\frac{d\omega}{d\gamma_1}>0$, $\frac{d\omega}{d\gamma_2}\geq 0$;

(2) If $\frac{\Delta}{\xi} \leq -1$, then $\frac{d\omega}{d\gamma_1}\leq 0$, $\frac{d\omega}{d\gamma_2}< 0$;

(3) If $-1<\frac{\Delta}{\xi} < 1$, then $\frac{d\omega}{d\gamma_1}>0$, $\frac{d\omega}{d\gamma_2}< 0$.

3. *With the benchmarking levels γ_1 and γ_2 held fixed, we have*

$$\frac{\mathrm{d}\omega}{\mathrm{d}m} \geq 0, \quad \textit{iff} \quad \frac{\Delta}{\xi} \geq q, \quad \textit{where} \quad q := \frac{\frac{\bar{z}_1}{\bar{\gamma}_1} - \frac{\bar{z}_2}{\bar{\gamma}_2}}{\frac{\bar{z}_1}{\bar{\gamma}_1} + \frac{\bar{z}_2}{\bar{\gamma}_2} - 2} \in (-1, 1). \tag{30}$$

Proof. See Appendix A.7. □

Result 1 of Proposition 3 is intuitive: when the two assets have equal total uncertainty ($\Delta = 0$), investors' optimal attention weight exactly matches the ratio of effective supplies, with $\omega = \frac{\bar{z}_2 - \bar{\gamma}_2}{\bar{z}_1 - \bar{\gamma}_1}$. When asset 2 is more uncertain than asset 1 ($\Delta > 0$) in terms of payoff or noise, investors allocate more attention to asset 2, leading to $\omega > \frac{\bar{z}_2 - \bar{\gamma}_2}{\bar{z}_1 - \bar{\gamma}_1}$. Conversely, when asset 1 is more uncertain than asset 2 ($\Delta < 0$), more attention is directed towards asset 1, resulting in $\omega < \frac{\bar{z}_2 - \bar{\gamma}_2}{\bar{z}_1 - \bar{\gamma}_1}$.

Result 2 of Proposition 3 shows that when $|\Delta| > \xi$, the effect of benchmarking on attention allocation differs qualitatively from the separative learning case. For example, in Case (1)—when asset 2 is significantly more uncertain than asset 1 (in payoff or demand/supply noise)— an increase in its benchmarking level γ_2 can induce investors to allocate more attention to asset 2, in contrast to the separative learning equilibrium.

The reason is that, in addition to the reduced effective-supply effect present under separative learning—where ASSR_j decreases with γ_j and increases with γ_{-j}, so that raising γ_j tends to shift attention toward the other asset—integrative learning introduces a cross-asset attention allocation effect. Observing a noisy linear combination of the two assets' payoffs, investors optimally allocate attention across assets to minimize posterior portfolio uncertainty.

When $\text{ASSR}_2 > \text{ASSR}_1$, a rise in γ_2 can, through this mechanism, increase attention to asset 2. In Appendix A.8, we show that the cross-asset attention effect rises with the difference in the assets' adjusted squared Sharpe ratios, $\text{ASSR}_2 - \text{ASSR}_1$. Hence, if $\text{ASSR}_2 > \text{ASSR}_1$, the positive cross-asset allocation effect may lead investors to allocate more attention to asset 2 even as its benchmarking rises. When ASSR_2 is sufficiently larger than ASSR_1 (equivalently, $\Delta > \xi$, i.e., asset 2 is substantially more uncertain than asset 1), this positive cross-asset attention allocation effect dominates the negative supply-reduction effect, so attention to asset 2 increases with γ_2. Intuitively, under integrative learning, investors optimally observe a signal combining information from both assets and allocate limited attention to reduce overall portfolio uncertainty. When asset 2 is substantially more uncertain than asset 1, shifting attention toward asset 2 reduces total uncertainty and improves inference about both assets.

As in the separative learning case, holding γ_1 and γ_2 fixed, a higher fraction m raises the aggregate benchmarking levels $\bar{\gamma}_j$ for $j \in \{1,2\}$. Since increasing the benchmarking level of an asset tends to shift attention toward the other asset, the net effect of increasing m on attention allocation depends on whether the negative impact of an asset's own benchmarking is outweighed by the positive spillover effect of the other asset's benchmarking. Under integrative learning, the cross-asset attention allocation effect also influences this net effect.

Result 3 of Proposition 3 implies that when asset 2's total uncertainty is sufficiently large (e.g., Case (1) of Result 2 where $\frac{\Delta}{\xi} \geq 1 > q$), the cross-asset attention allocation effect dominates. In this case, the attention allocated to asset 2 always increases in m (i.e., $\frac{\mathrm{d}\omega}{\mathrm{d}m} > 0$). Conversely, when asset 1's total uncertainty is sufficiently large (e.g., Case (2) of Result 2 where $\frac{\Delta}{\xi} \leq -1 < q$), the attention allocated to asset 1 always increases in m (i.e., $\frac{\mathrm{d}\omega}{\mathrm{d}m} < 0$).

When the difference between the two assets' total uncertainties is not significant (e.g., Case (3) of Result 2 where $-1 < \frac{\Delta}{\xi} < 1$), the reduced effective-supply plays the dominant role. In this case, the asset with the lower benchmarking level γ_j is more likely to experience increased attention allocation as m increases. For example, if asset 2 has a relatively low benchmarking level γ_2, the attention allocated to asset 2 is more likely to increase in m because q is lower when

γ_2 is low, making it more likely that $\frac{\Delta}{\xi} > q$ as m increases. Intuitively, if γ_2 is relatively low, the negative impact of asset 2's own benchmarking (since the aggregate benchmarking level $\bar{\gamma}_2$ rises more slowly with m) is outweighed by the positive spillover effect of asset 1's benchmarking. As a result, the attention allocated to asset 2 tends to increase as m increases.

C. The Effects of Benchmarking on Asset Price Informativeness

We now examine how benchmarking affects the informativeness of asset prices. To compare with the results under separative learning and with the existing literature, we first analyze how benchmarking affects the informativeness of individual asset prices.

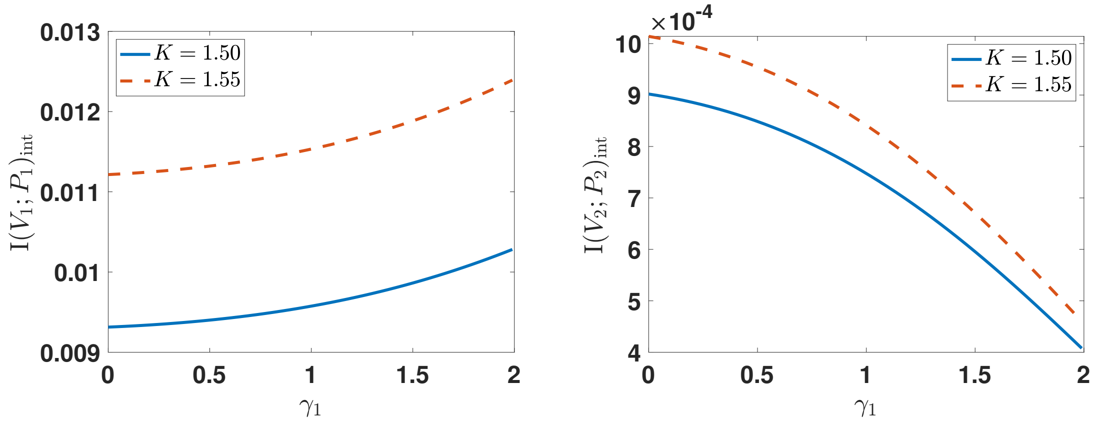
We first examine the impact of benchmarking level γ_j on price informativeness, holding fixed the fraction of benchmarked investors m. Under integrative learning, each investor i chooses to observe a signal $Y^i = V_1 + \omega V_2 + \varepsilon^i$. The conditional variance of the asset payoff V_j given the signal Y^i is

$$\text{(31)} \qquad \mathrm{Var}(V_1 \mid Y^i) = \tau_{v,1}^{-1}\left(1 - \frac{K-1}{K}\frac{1}{1+\omega^2\tau_{v,1}\tau_{v,2}^{-1}}\right), \qquad \mathrm{Var}(V_2 \mid Y^i) = \tau_{v,2}^{-1}\left(1 - \frac{K-1}{K}\frac{\omega^2}{\omega^2+\tau_{v,1}^{-1}\tau_{v,2}}\right).$$

An increase in ω implies that investors allocate more attention to asset 2. Equation (31) shows that increasing ω raises each investor's posterior uncertainty about asset 1 while reducing it for asset 2. Consequently, the price informativeness of asset 1, measured by the mutual information $\mathrm{I}(V_j;P_j)_{\text{int}}$, decreases as ω increases, while the price informativeness of asset 2 increases.

Therefore, Proposition 3 implies that the price informativeness of asset j, $\mathrm{I}(V_j;P_j)_{\text{int}}$, may increase with γ_j, particularly when $|\Delta| > \xi$. As illustrated in Figure 3, when asset 1 has a substantially higher ASSR than asset 2, the price informativeness of asset 1, $\mathrm{I}(V_1;P_1)_{\text{int}}$, increases

FIGURE 3. Price Informativeness $\mathrm{I}(V_1;P_1)_{\text{int}}$ and $\mathrm{I}(V_2;P_2)_{\text{int}}$ under integrative learning as a function of γ_1 for different learning capacities. Other parameters are: $\lambda = 1$, $\tau_{v,1} = 1$, $\tau_{z,1} = 0.1$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 2$, $m = 0.5$, and $\gamma_2 = 0$.



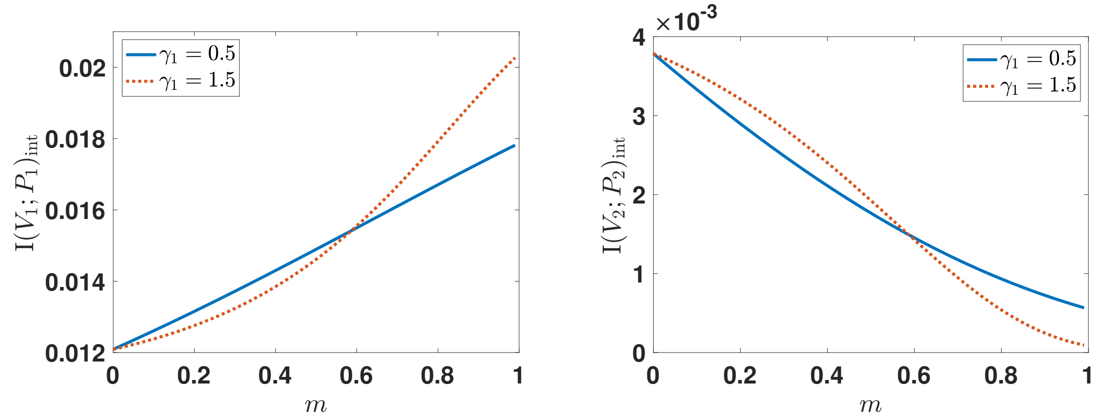
with γ_1.[9] In contrast, the price informativeness of asset 2, $\mathrm{I}(V_2;P_2)_{\text{int}}$, decreases with γ_1. Notably, the effect of benchmarking on price informativeness under integrative learning is opposite to that under separative learning.

Similar to the separative learning case, Figure 3 also shows that a higher learning capacity K increases price informativeness for both assets, while leaving the qualitative effect of benchmarking unchanged.

We next examine how varying the fraction of benchmarked investors m affects price informativeness, holding fixed γ_1 and γ_2. Figure 4 illustrates a case in which asset 1 exhibits greater total uncertainty, that is, $\frac{\Delta}{\xi} < -1 < q$. From Result 3 of Proposition 3, the cross-asset attention allocation effect dominates in this case. Consequently, the attention allocated to asset 1 always increases with m. As a result, an increase in m raises the price informativeness of asset 1 while reducing that of asset 2.

[9] The analytical expressions for the mutual information measure are presented in equations (A-51) and (A-52) in Appendix A.9. The general patterns shown in this figure remain qualitatively consistent when $|\Delta| > \xi$ holds.

FIGURE 4. Price Informativeness $\mathrm{I}(V_1;P_1)_{\mathrm{int}}$ and $\mathrm{I}(V_2;P_2)_{\mathrm{int}}$ under integrative learning as a function of m, for different values of γ_1. Other parameters are: $\lambda = 1$, $\tau_{v,1} = 0.9$, $\tau_{z,1} = 0.2$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 2$, $K = 1.5$, and $\gamma_2 = 0.75$.



When the difference between the two assets' total uncertainties is not significant (e.g., Case (3) of Result 2 where $-1 < \frac{\Delta}{\xi} < 1$), the reduced effective supply plays the dominant role. In this case, the price informativeness of the asset with the lower benchmarking level γ_j is more likely to increase as m increases.

D. Market Informational Efficiency

For robustness, we next examine how the benchmarking level γ_j affects the price informativeness of asset j, measured by the mutual information $\mathrm{I}(V_j;P)_{\mathrm{int}}$, where $P = (P_1, P_2)'$ denotes the vector of asset prices. As illustrated in Figures 5 and 6, the price informativeness of asset j, measured by $\mathrm{I}(V_j;P)_{\mathrm{int}}$, can still increase with the benchmarking level γ_1 (holding m fixed) and with the fraction of benchmarked investors m (holding γ_1 and γ_2 fixed).

The analytical expression of $\mathrm{I}(V_j;P)_{\mathrm{int}}$ is algebraically complex and is provided in (A-55) in Appendix A.9. We provide more detailed analytical results and intuition using overall price efficiency measure $\mathrm{I}(V;P)_{\mathrm{int}}$ in Appendix A.10. The main result is consistent with previous

FIGURE 5. Price informativeness $\mathrm{I}(V_j; P)_{\text{int}}$ under integrative learning as a function of γ_1. Parameters are: $\lambda = 1$, $K = 1.5$, $\tau_{v,1} = 0.9$, $\tau_{z,1} = 0.2$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 2$, $m = 0.5$, and $\gamma_2 = 0$.

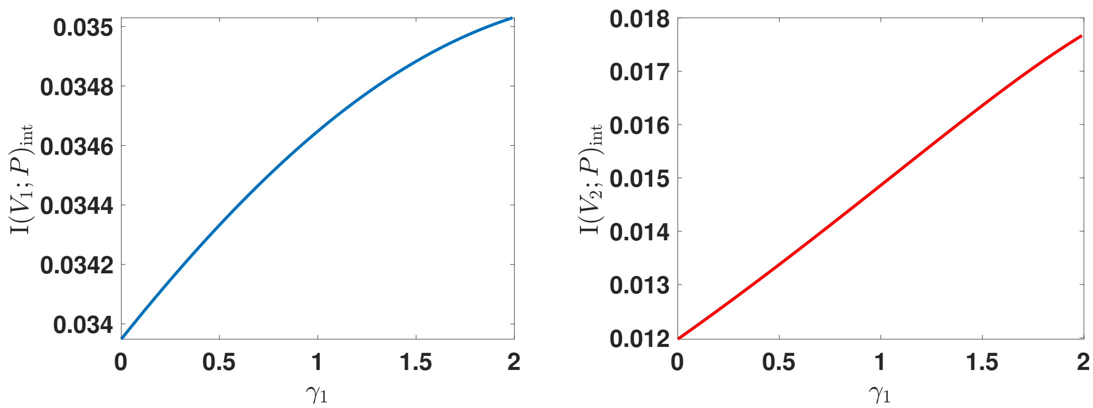
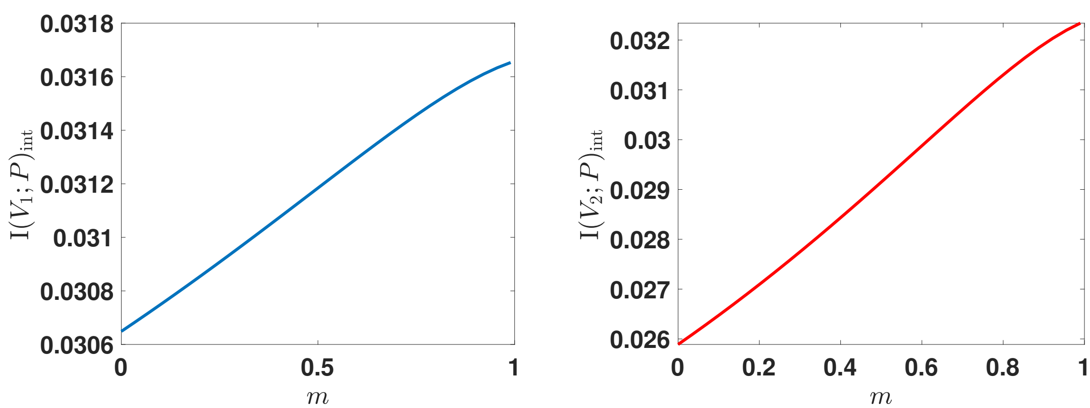


FIGURE 6. Price Informativeness $\mathrm{I}(V_j; P)_{\text{int}}$ under integrative learning as a function of m. Parameters are: $\lambda = 1$, $K = 1.5$, $\tau_{v,1} = 0.2$, $\tau_{z,1} = 0.9$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 3$, $\gamma_1 = 1.5$, and $\gamma_2 = 1$.

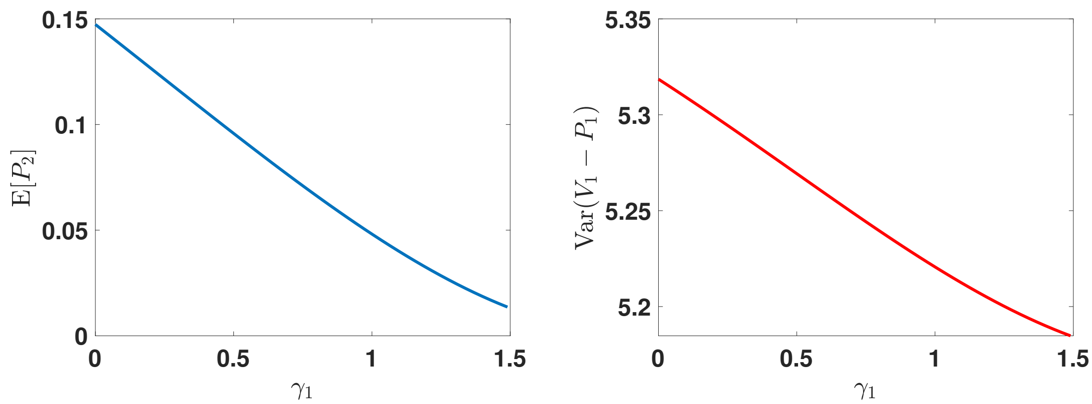


findings: under integrative learning, a higher benchmarking on the asset with greater uncertainty may enhance its price informativeness, in contrast to the case under separative learning.

E. The Effects on Asset Prices

We next examine how benchmarking affects asset prices. As under separative learning, an asset's average price generally rises with its benchmarking level because stronger benchmarking increases hedging demand and reduces effective supply, driving up expected prices. This aligns with the empirical evidence in Pavlova and Sikorskaya (2023).

FIGURE 7. Expected price of asset 2 and return volatility of asset 1 versus γ_1 under integrative learning. Parameters are: $\lambda = 1$, $K = 1.5$, $\bar{V}_1 = \bar{V}_2 = 1$, $\tau_{v,1} = \tau_{v,2} = 1$, $\tau_{z,1} = 0.1$, $\tau_{z,2} = 2$, $\bar{z}_1 = 1$, $\bar{z}_2 = 2$, $\gamma_2 = 0$, and $m = 0.5$.



The benchmarking level of one asset also affects the other asset's price. Under integrative learning, when asset j is much more uncertain ($|\Delta| > \xi$), a rise in its benchmarking level can lower the expected price of the other asset. The reason is that greater benchmarking shifts attention toward asset j, reducing price informativeness and raising the other asset's risk premium. This contrasts with separative learning, where higher benchmarking of one asset typically raises the other's expected price. In addition, under integrative learning, when the difference in asset uncertainty is sufficiently large ($|\Delta| > \xi$), stronger benchmarking of the riskier asset can reinforce attention to it, thereby reducing its return volatility. As shown in Figure 7, when asset 1 is much more uncertain than asset 2, increasing γ_1 lowers the price of asset 2 and reduces the volatility of asset 1.

V. Endogenous Learning Capacity

In this section, we endogenize learning capacity and compare traders' ex-ante utility across equilibria of different games: a separative-learning game with information-structure constraints and an integrative-learning game without such constraints.

For any given learning capacity K, whether exogenously fixed or endogenously chosen at date 0, if the information structure is unrestricted at date 1, each trader strictly prefers to acquire private information through an integrative signal rather than through separative signals. This result is established in Internet Appendix B.1, which characterizes traders' optimal date-1 learning choice conditional on K. We endogenize the learning capacity K by introducing a quadratic cost function, $\frac{1}{2}c(K-1)^2$, where $c>0$. From the proofs of Theorems 1 and 2, signal precision is proportional to $K-1$ in both the separative and integrative learning cases. Accordingly, we assume that the cost function is quadratic in $K-1$. After incorporating the quadratic cost function, investor i's utility function becomes

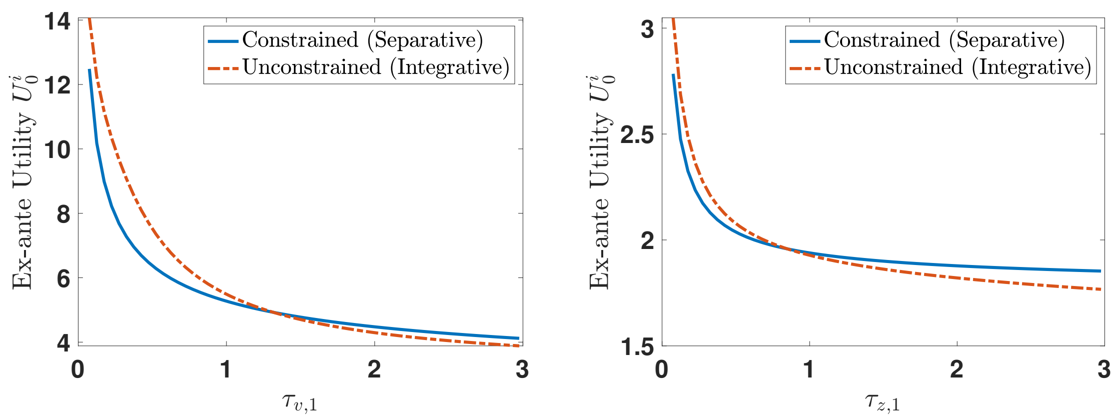
$$\lambda[W_0+(\theta^i-\gamma^i)'(\hat{V}^i-P)]-\frac{\lambda^2}{2}(\theta^i-\gamma^i)'\hat{\Sigma}_v^i(\theta^i-\gamma^i)-\frac{1}{2}c(K-1)^2, \tag{32}$$

where $\hat{V}^i$ and $\hat{\Sigma}_v^i$ represent investor i's posterior mean and variance-covariance matrix of V, respectively. A detailed analysis of trader i's ex-ante expected utility under separative and integrative learning is provided in Internet Appendix B.2.

We next solve numerically for the learning capacity K and compare traders' ex-ante utility under separative versus integrative learning. When learning capacity K is endogenously determined at date 0, we show that, when one asset is sufficiently more uncertain than the other, the ex-ante expected utility under integrative learning exceeds that under separative learning.

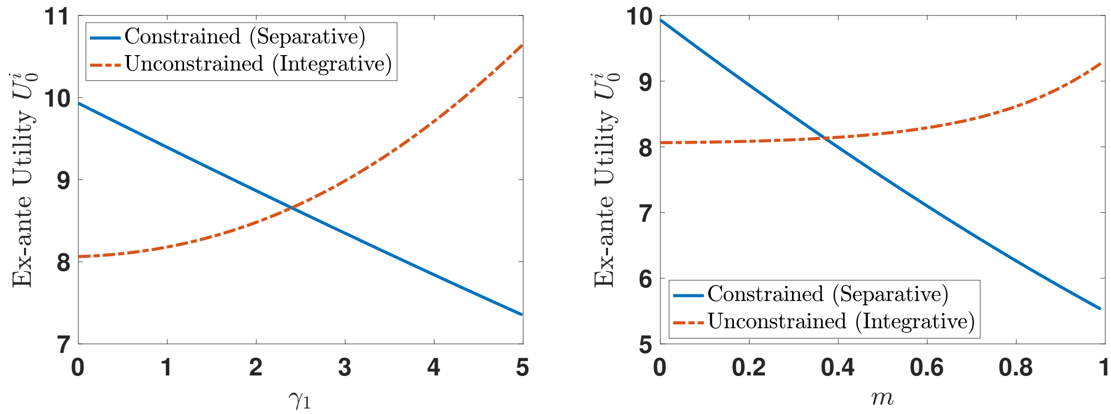
By contrast, when the two assets have similar levels of uncertainty, the equilibrium with a restriction on the information structure (i.e., separative learning) can deliver higher ex-ante utility than the unrestricted (i.e., integrative learning) equilibrium. Importantly, this result does not imply that separative learning would be chosen when the information structure is unrestricted. Starting from the endogenously chosen K at date 0, if traders are allowed at date 1 to choose any information structure, each trader has an incentive to deviate to integrative learning. Consequently, separative learning is not a Nash equilibrium of the date-1 learning subgame unless traders can coordinate on and commit to a restriction requiring Λ to be diagonal.

FIGURE 8. Ex-ante utility under constrained (separative) and unconstrained (integrative) learning technologies. Parameters are $\lambda = 1$, $c = 1$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 5$ and $\bar{\gamma}_1 = \bar{\gamma}_2 = 3$. In the left panel, $\tau_{z,1} = 0.2$, and in the right panel, $\tau_{v,1} = 3$.



As numerically illustrated in Figures 8 and 9, with endogenous K, separative learning (i.e., a constrained information structure) yields higher ex-ante utility when the uncertainties of the two assets are comparable. By contrast, integrative learning yields higher ex-ante utility when one asset is significantly more uncertain—for example, when $\tau_{v,1}$ or $\tau_{z,1}$ is relatively low in Figure 8, or when the benchmarking level γ_1 or m is relatively high in Figure 9, making the condition $|\Delta| > \xi$ more likely to hold. In the numerical exercises, this condition appears to be sufficient,

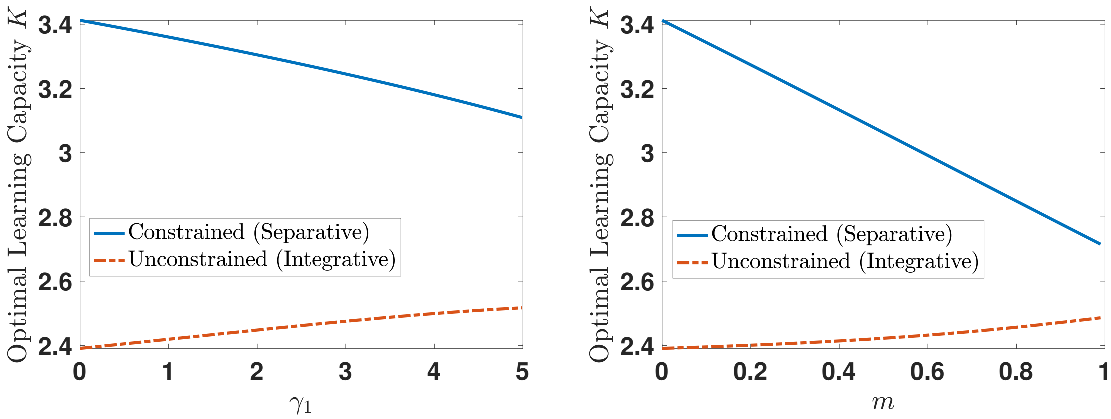
FIGURE 9. Ex-ante utility under constrained (separative) and unconstrained (integrative) learning technologies. Parameters are $\lambda = 1$, $c = 1$, $\tau_{v,1} = 1$, $\tau_{z,1} = 0.1$, $\tau_{v,2} = \tau_{z,2} = 1$, and $\bar{z}_1 = \bar{z}_2 = 5$. In the left panel, $\gamma_2 = 0$ and $m = 0.5$, and in the right panel, $\gamma_1 = \gamma_2 = 2.5$.



though not necessary, for integrative learning to yield higher ex-ante utility. Proposition 3 identifies this as precisely the case in which the effect of benchmarking on price informativeness differs substantially from that under separative learning. In particular, under integrative learning, an increase in the benchmarking level can enhance price informativeness for the more uncertain asset, in contrast to the separative learning case.

The ex-ante utility comparison across equilibria reflects a general-equilibrium effect operating through prices, risk reduction, and endogenous learning capacity choices. For example, under separative learning (i.e., a constrained information structure), traders may optimally choose a higher learning capacity K, as numerically illustrated in Figure 10. Although this choice entails higher learning costs ex ante, it can lead to greater reductions in asset-specific uncertainty and, consequently, in the posterior variance of traders' terminal wealth. Under certain conditions (e.g., when the two assets have similar levels of uncertainty), the resulting reduction in posterior variance can outweigh the higher learning costs, thereby raising traders' ex-ante utility relative to the unrestricted integrative learning equilibrium. Importantly, separative learning is not individually optimal absent commitment, since traders would deviate to integrative learning when the information structure is unrestricted.

FIGURE 10. Optimal learning capacity under constrained (separative) and unconstrained (integrative) learning technologies. Parameters are $\lambda = 1$, $c = 1$, $\tau_{v,1} = 1$, $\tau_{z,1} = 0.1$, $\tau_{v,2} = \tau_{z,2} = 1$, and $\bar{z}_1 = \bar{z}_2 = 5$. In the left panel, $\gamma_2 = 0$ and $m = 0.5$, and in the right panel, $\gamma_1 = \gamma_2 = 2.5$.



Interestingly, Figure 10 shows that while an increase in the benchmarking level tends to reduce the endogenous learning capacity K under separative learning, it may increase K under integrative learning. Under separative learning, a higher benchmarking level for an asset lowers the marginal value of acquiring information about that asset, leading traders to optimally reduce their demand for signal precision. By contrast, under integrative learning, investors observe a signal that combines information about both assets. When asset 1 is significantly more uncertain than asset 2—due to higher payoff volatility or noisier demand/supply—an increase in the benchmarking level of asset 1 can induce traders to reallocate attention toward learning more about asset 2. In this case, acquiring a more precise integrative signal can reduce overall uncertainty and improve inference about both assets, resulting in a higher optimal learning capacity K.

These results reinforce our main finding: the effects of benchmarking on learning and information acquisition are fundamentally shaped by the type of learning technology investors adopt—whether separative or integrative.

FIGURE 11. Price informativeness under constrained (separative) and unconstrained (integrative) learning technologies. Parameters are $\lambda = 1$, $c = 1$, $\tau_{v,1} = 1$, $\tau_{z,1} = 0.1$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 5$, $\gamma_2 = 0$ and $m = 0.5$.

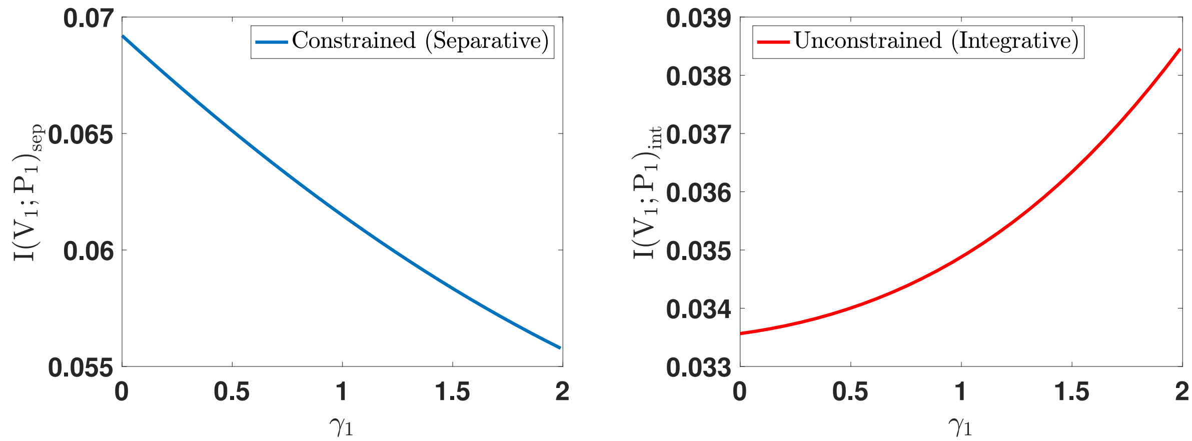
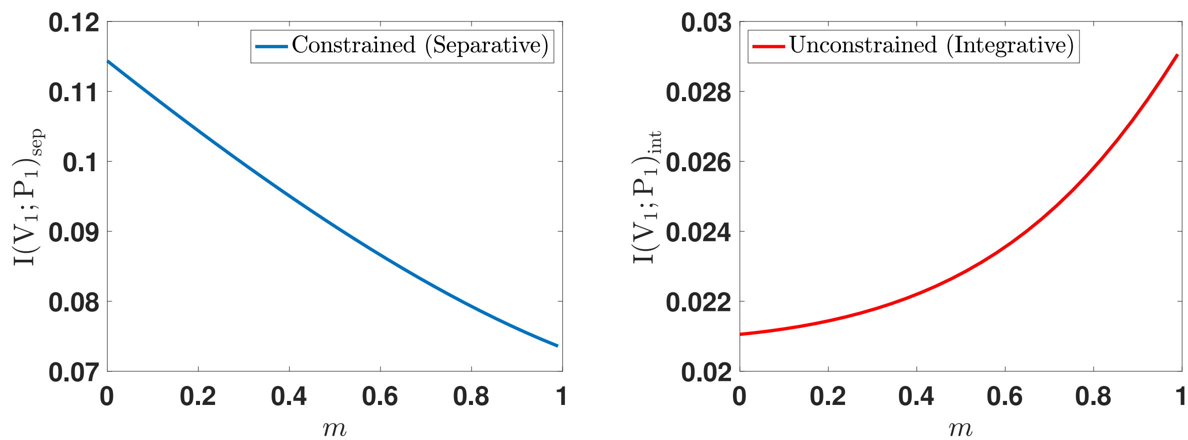


FIGURE 12. Price informativeness under constrained and unconstrained learning technologies. Parameters are $\lambda = 1$, $c = 1$, $\tau_{v,1} = 1$, $\tau_{z,1} = 0.1$, $\tau_{v,2} = \tau_{z,2} = 1$, $\bar{z}_1 = \bar{z}_2 = 5$, $\gamma_1 = \gamma_2 = 2.5$.



Importantly, Figures 11 and 12 show that endogenizing K does not affect our main conclusion: under integrative learning, increasing the benchmarking level can enhance price informativeness for the more uncertain asset, in contrast to the results under separative learning.

VI. Conclusion

This paper studies how asset managers' benchmarking concerns affect market efficiency and asset pricing in a multi-asset economy. We analyze and compare rational-expectations equilibria under two types of learning technologies, which determine how asset managers acquire information about assets in their managed portfolios given their limited attention.

We find that under separative learning, where the information structure is constrained, an asset's price informativeness decreases with its benchmarking level. In contrast, under integrative learning, where the information structure is unconstrained, price informativeness may increase with the benchmarking level of the more uncertain asset. This occurs because integrative learning enables investors to optimize information allocation across assets in their portfolios. The distinction suggests that an improvement in price informativeness, alongside enhanced benchmarking, can be indicative of investors adopting an integrative learning approach. In addition, our analysis suggests that benchmarking can potentially enhance the overall informational efficiency of the financial market. The impacts of benchmarking on asset prices also differ significantly between cases with different learning modes. Overall, our results highlight the pivotal role of learning technology in fully understanding the effects of benchmarking in financial markets.

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Appendix

A.1. Proof of Theorem 1

Substituting equation (2) into (3) yields

$$\max_{\theta^i} \lambda[W_0 + (\theta^i - \gamma^i)'(\hat{V}^i - P)] - \frac{\lambda^2}{2}(\theta^i - \gamma^i)'\hat{\Sigma}_v^i(\theta^i - \gamma^i), \tag{A-1}$$

where $\hat{V}^i$ and $\hat{\Sigma}_v^i$ represent investor i's posterior mean and variance-covariance matrix of V, respectively. From (A-1), investor i's optimal asset holding is

$$\theta^i = \gamma^i + \frac{1}{\lambda}\left(\hat{\Sigma}_v^i\right)^{-1}\left(\hat{V}^i - P\right). \tag{A-2}$$

Substituting (A-2) into the market clearing condition, $\int_0^1 \theta^i = Z$, we get

$$P = \left[\int_0^1 (\hat{\Sigma}_v^i)^{-1} di\right]^{-1}\left[\int_0^1 (\hat{\Sigma}_v^i)^{-1}\hat{V}^i di - \lambda(Z - \bar{\gamma})\right], \tag{A-3}$$

where $\bar{\gamma} := (\bar{\gamma}_1, \bar{\gamma}_2)'$ and $\bar{\gamma}_j := \int_0^1 \gamma_j^i di$ is the aggregate benchmarking level of asset j. The price vector P is informationally equivalent to the signal $s_p = V + \varepsilon_p$, where $\varepsilon_p \sim \mathcal{N}(0, \Sigma_p)$ has a diagonal variance matrix $\Sigma_p = \begin{pmatrix} \tau_{p,1}^{-1} & 0 \\ 0 & \tau_{p,2}^{-1} \end{pmatrix}$. Investor i's posterior belief about V_j is

$$\hat{V}_j^i = \mathrm{E}^i[V_j] = \mathrm{E}[V_j \mid Y^i, s_p] = \frac{\tau_{v,j}\bar{v}_j + \tau_j^i Y_j^i + \tau_{p,j}s_{p,j}}{\tau_{v,j} + \tau_j^i + \tau_{p,j}}, \quad \mathrm{Var}^i(V_j) = (\tau_{v,j} + \tau_j^i + \tau_{p,j})^{-1}, \quad \mathrm{Cov}^i(V_1, V_2) = 0.$$

We have

$$\left(\hat{\Sigma}_v^i\right)^{-1}\hat{V}^i = \begin{pmatrix} \tau_{v,1}\bar{v}_1 + \tau_1^i Y_1^i + \tau_{p,1}s_{p,1} & 0 \\ 0 & \tau_{v,2}\bar{v}_2 + \tau_2^i Y_2^i + \tau_{p,2}s_{p,2} \end{pmatrix}.$$

We integrate over all investors (with $\int_0^1 \varepsilon^i di = 0$) and use the notation $\bar{\tau}_j = \int_0^1 \tau_j^i di$ to obtain

$$\int_0^1 \left(\hat{\Sigma}_v^i\right)^{-1} di = \begin{pmatrix} \tau_{v,1} + \bar{\tau}_1 + \tau_{p,1} & 0 \\ 0 & \tau_{v,2} + \bar{\tau}_2 + \tau_{p,2} \end{pmatrix},$$

$$\int_0^1 \left(\hat{\Sigma}_v^i\right)^{-1} \hat{V}^i di = \begin{pmatrix} \tau_{v,1}\bar{v}_1 + \bar{\tau}_1 V_1 + \tau_{p,1} s_{p,1} & 0 \\ 0 & \tau_{v,2}\bar{v}_2 + \bar{\tau}_2 V_2 + \tau_{p,2} s_{p,2} \end{pmatrix}.$$

It follows that the equilibrium price is:

(A-4) $$P_j = \left(\tau_{v,j} + \tau_{p,j} + \bar{\tau}_j\right)^{-1} \left[\tau_{v,j}\bar{v}_j + (\tau_{p,j} + \bar{\tau}_j) s_{p,j} - \lambda(\bar{z}_j - \bar{\gamma}_j)\right],$$

where $\bar{\tau}_j := \int_0^1 \tau_j^i di$ measures the total signal precision about asset j and $\bar{\gamma}_j := \int_0^1 \gamma_j^i di$ measures the aggregate benchmarking level of asset j. The price P_j is informationally equivalent to the signal $s_{p,j} = V_j + \varepsilon_{p,j}$, where

(A-5) $\varepsilon_{p,j} = -\frac{\lambda}{\bar{\tau}_j}(Z_j - \bar{z}_j) \sim \mathcal{N}(0, \tau_{p,j}^{-1})$, and the precision $\tau_{p,j} := \mathrm{Var}(\varepsilon_{p,j})^{-1} = \bar{\tau}_j^2 \tau_{z,j}/\lambda^2$.

Given the private information and benchmarking needs, trader i's optimal holding on asset j is

(A-6) $$\theta_j^i = \gamma_j^i + \frac{\mathrm{E}^i[V_j] - P_j}{\lambda \mathrm{Var}^i(V_j)}.$$

Given investors' optimal holdings (A-6), the information choice problem (7) prior to trading is equivalent to:

(A-7) $$\max_{\tau_1^i, \tau_2^i} \mathrm{E}\left[\frac{1}{2} \sum_{j=1,2} \frac{(\mathrm{E}^i[V_j] - P_j)^2}{\mathrm{Var}^i(V_j)}\right] \quad \text{s.t.} \quad \prod_{j=1,2} (\tau_{v,j} + \tau_j^i) \leq K \prod_{j=1,2} \tau_{v,j} \quad \text{and} \quad \tau_j^i \geq 0.$$

Since

(A-8) $$\mathrm{E}\left[\left(\mathrm{E}^i[V_j] - P_j\right)^2\right] = \mathrm{Var}\left(\mathrm{E}^i[V_j] - P_j\right) + \left(\mathrm{E}\left[\mathrm{E}^i[V_j] - P_j\right]\right)^2,$$

and by the law of total variance,

(A-9) $$\mathrm{Var}\left(\mathrm{E}^i[V_j]-P_j\right)=\mathrm{Var}(V_j-P_j)-\mathrm{E}\left[\mathrm{Var}^{\mathrm{i}}(\mathrm{V_j}-\mathrm{P_j})\right]=\mathrm{Var}(V_j-P_j)-\mathrm{Var}^i(V_j),$$

it follows that

(A-10) $$\mathrm{E}\left[\left(\mathrm{E}^i[V_j]-P_j\right)^2\right]=\mathrm{Var}(V_j-P_j)+(\mathrm{E}[V_j-P_j])^2-\mathrm{Var}^{\mathrm{i}}(\mathrm{V_j})=\mathrm{E}\left[(V_j-P_j)^2\right]-\mathrm{Var}^i(V_j).$$

Therefore, investors' objective function (7) becomes

(A-11) $$\max_{\tau_1^i,\tau_2^i} \quad \lambda W_0-1+\frac{1}{2}\sum_{j=1,2}\frac{\mathrm{E}\left[(V_j-P_j)^2\right]}{\mathrm{Var}^i(V_j)} \quad \text{s.t.} \quad \prod_{j=1,2}(\tau_{v,j}+\tau_j^i)\le K\prod_{j=1,2}\tau_{v,j} \quad \text{and} \quad \tau_j^i\ge 0,$$

where

(A-12) $$\begin{aligned}\mathrm{Var}^i(V_j)&=\left(\tau_{v,j}+\tau_{p,j}+\tau_j^i\right)^{-1},\\ \mathrm{E}\left[(V_j-P_j)^2\right]&=(\tau_{v,j}+\tau_{p,j}+\bar{\tau}_j)^{-2}\left[\lambda^2(\bar{z}_j-\bar{\gamma}_j)^2+\lambda^2\tau_{z,j}^{-1}+\bar{\tau}_j\right]+(\tau_{v,j}+\tau_{p,j}+\bar{\tau}_j)^{-1}.\end{aligned}$$

The information choice problem, as described in equation (A-11), is not a concave objective function, leading to a corner solution. The optimal precision of his signal on asset j is

(A-13) $$\tau_j^i=\begin{cases}(K-1)\tau_{v,j} & \text{if} \quad \mathrm{ASSR}_j=\max\{\mathrm{ASSR}_1,\mathrm{ASSR}_2\},\\ 0 & \text{if} \quad \mathrm{ASSR}_j\ne\max\{\mathrm{ASSR}_1,\mathrm{ASSR}_2\}.\end{cases}$$

When $\mathrm{ASSR}_1=\mathrm{ASSR}_2$, each investor becomes indifferent to learning about asset 1 or asset 2. Note that ASSR_j depends on the total precision of signals regarding asset j, and this precision depends on the endogenous fraction of traders opting to learn about asset j.

Let $\Gamma_1\in[0,1]$ denote the endogenous fraction of investors who choose to learn solely about asset 1. Then $\Gamma_2:=1-\Gamma_1$ denotes the fraction of investors focusing on asset 2. Investors' optimal choice of signal precision can be written as

$$\tau_1^i = (K-1)\tau_{v,1}, \quad \tau_2^i = 0, \quad \text{for} \quad i \in [0, \mathit{\Gamma}_1],$$

$$\tau_1^i = 0, \quad \tau_2^i = (K-1)\tau_{v,2}, \quad \text{for} \quad i \in (\mathit{\Gamma}_1, 1]. \tag{A-14}$$

Clearly, ASSR_j depends endogenously on the value of $\mathit{\Gamma}_j$. We thus obtain Theorem 1.

A.2. Proof of Proposition 1

Equation (17) follows from the linear-Gaussian relationship of V_j and P_j, where P_j is as given in equation (A-4) and $\tau_{p,j} = \left(\mathit{\Gamma}_j \tau_{v,j} \frac{K-1}{\lambda}\right)^2 \tau_{z,j}$, is as given in equation (A-5). From the proof of Theorem 1 and given the notation $\bar{\tau}_j = \mathit{\Gamma}_j (K-1)\tau_{v,j}$, we can compute

$$\mathrm{ASSR}_j = \tau_{v,j}\left(\tau_{v,j} + \bar{\tau}_j^2 \tau_{z,j}/\lambda^2 + \bar{\tau}_j\right)^{-1}\left[\frac{\lambda^2(\bar{z}_j - \bar{\gamma}_j)^2 + \lambda^2 \tau_{z,j}^{-1} + \bar{\tau}_j}{\tau_{v,j} + \bar{\tau}_j^2 \tau_{z,j}/\lambda^2 + \bar{\tau}_j} + 1\right]. \tag{A-15}$$

Clearly, ASSR_j depends on the fraction of investors who choose to learn about asset 1 ($\mathit{\Gamma}_1$) and on the benchmarking level of asset j ($\bar{\gamma}_j$), i.e., $\mathrm{ASSR}_j = \mathrm{ASSR}_j(\mathit{\Gamma}_1, \bar{\gamma}_j)$. Suppose $\bar{z}_j - \bar{\gamma}_j > 0$. We have

$$\frac{\partial \mathrm{ASSR}_j}{\partial \bar{\gamma}_j} < 0, \qquad \frac{\partial \mathrm{ASSR}_1}{\partial \mathit{\Gamma}_1} < 0, \qquad \frac{\partial \mathrm{ASSR}_2}{\partial \mathit{\Gamma}_1} > 0. \tag{A-16}$$

Equation (A-16) shows that the value of information about asset 1 decreases with the fraction of investors learning about asset 1. Since $\mathit{\Gamma}_j$ is a function of $\bar{\gamma}_1$ and $\bar{\gamma}_2$, we examine how the aggregate benchmarking levels affect $\mathit{\Gamma}_j$. In the interior equilibrium with $0 < \mathit{\Gamma}_j < 1$, we have $\mathrm{ASSR}_1(\mathit{\Gamma}_1, \bar{\gamma}_1) = \mathrm{ASSR}_2(\mathit{\Gamma}_2, \bar{\gamma}_2)$. Taking the total derivative of both sides with respect to γ_1 gives

$$\frac{\mathrm{dASSR}_1}{\mathrm{d}\gamma_1} = \frac{\partial \mathrm{ASSR}_1}{\partial \mathit{\Gamma}_1}\frac{\mathrm{d}\mathit{\Gamma}_1}{\mathrm{d}\gamma_1} + \frac{\partial \mathrm{ASSR}_1}{\partial \bar{\gamma}_1} m = \frac{\partial \mathrm{ASSR}_2}{\partial \mathit{\Gamma}_2}\frac{\mathrm{d}\mathit{\Gamma}_2}{\mathrm{d}\gamma_1} = \frac{\partial \mathrm{ASSR}_2}{\partial \mathit{\Gamma}_1}\frac{\mathrm{d}\mathit{\Gamma}_1}{\mathrm{d}\gamma_1}, \tag{A-17}$$

where in the last equality we have used the definition $\Gamma_2 = 1 - \Gamma_1$. Rearranging (A-17), we have

$$\text{(A-18)} \qquad \frac{\mathrm{d}\Gamma_1}{\mathrm{d}\gamma_1} = -m\Big(\underbrace{\frac{\partial \mathrm{ASSR}_1}{\partial \Gamma_1}}_{-} - \underbrace{\frac{\partial \mathrm{ASSR}_2}{\partial \Gamma_1}}_{+}\Big)^{-1} \underbrace{\frac{\partial \mathrm{ASSR}_1}{\partial \bar{\gamma}_1}}_{-} < 0.$$

By a similar argument, we have $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\gamma_2} = m\left(\frac{\partial \mathrm{ASSR}_1}{\partial \Gamma_1} - \frac{\partial \mathrm{ASSR}_2}{\partial \Gamma_1}\right)^{-1} \frac{\partial \mathrm{ASSR}_2}{\partial \bar{\gamma}_2} > 0$ and symmetrically $\frac{\mathrm{d}\Gamma_2}{\mathrm{d}\bar{\gamma}_1} > 0$. Including results in the corner equilibrium where $\Gamma_1 = 0$ or $\Gamma_1 = 1$, we have $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\bar{\gamma}_1} \leq 0$ and $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\bar{\gamma}_2} \geq 0$. Since $\tau_{p,j} = \left(\Gamma_j \tau_{v,j} \frac{K-1}{\lambda}\right)^2 \tau_{z,j}$ is an increasing function of Γ_j, we have $\frac{\mathrm{d}\tau_{p,j}}{\mathrm{d}\bar{\gamma}_j} \leq 0$ and $\frac{\mathrm{d}\tau_{p,j}}{\mathrm{d}\bar{\gamma}_{-j}} \geq 0$. Since $\mathrm{I}(V_j; P_j)_{\mathrm{sep}}$ is an increasing function of $\tau_{p,j}$, we also have $\frac{\mathrm{d}\,\mathrm{I}(V_j;P_j)_{\mathrm{sep}}}{\mathrm{d}\bar{\gamma}_j} \leq 0$ and $\frac{\mathrm{d}\,\mathrm{I}(V_j;P_j)_{\mathrm{sep}}}{\mathrm{d}\bar{\gamma}_{-j}} \geq 0$.

By taking the total derivative of both sides of $\mathrm{ASSR}_1(\Gamma_1, \bar{\gamma}_1) = \mathrm{ASSR}_2(\Gamma_2, \bar{\gamma}_2)$ with respect to m and rearranging, we obtain

$$\text{(A-19)} \qquad \frac{\mathrm{d}\Gamma_1}{\mathrm{d}m} = -\frac{1}{m}\Big(\underbrace{\frac{\partial\, \mathrm{ASSR}_1}{\partial \Gamma_1}}_{-} - \underbrace{\frac{\partial\, \mathrm{ASSR}_2}{\partial \Gamma_1}}_{+}\Big)^{-1}\Big(\underbrace{\frac{\partial\, \ln \mathrm{ASSR}_1}{\partial \ln \bar{\gamma}_1}}_{-}\mathrm{ASSR}_1 - \underbrace{\frac{\partial\, \ln \mathrm{ASSR}_2}{\partial \ln \bar{\gamma}_2}}_{-}\mathrm{ASSR}_2\Big),$$

where $\frac{\partial\, \mathrm{ASSR}_1}{\partial \Gamma_1} < 0$, $\frac{\partial\, \mathrm{ASSR}_2}{\partial \Gamma_1} > 0$, the negative signs of $\frac{\partial\, \mathrm{ASSR}_j}{\partial \bar{\gamma}_j}$ and $\frac{\partial\, \ln \mathrm{ASSR}_j}{\partial \ln \bar{\gamma}_j}$ follow from (A-16).

If $\Gamma_1 = 1$ (i.e., $\mathrm{ASSR}_1 > \mathrm{ASSR}_2$ for all $\Gamma_1 \in [0,1]$), then both Γ_1 and the price informativeness of asset 1 are weakly decreasing in the fraction of benchmarked investors m (Case (1)). Since $\Gamma_1 \leq 1$, increases in m cannot raise Γ_1 and may lower it. Symmetrically, if $\Gamma_1 = 0$, then Γ_1 and the price informativeness of asset 1 are weakly increasing in m (Case (3)). In Case (2), where $0 < \Gamma_1 < 1$ and $\mathrm{ASSR}_1 = \mathrm{ASSR}_2$, from (A-19), the sign of $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}m}$ is determined by the sign of $\frac{\partial \ln \mathrm{ASSR}_1}{\partial \ln \bar{\gamma}_1} - \frac{\partial \ln \mathrm{ASSR}_2}{\partial \ln \bar{\gamma}_2}$.

A.3. The Effects of Payoff Volatility on the Endogenous Fraction Γ_j

Corollary A-1. *Assume $\bar{z}-\bar{\gamma}>0$. The greater the volatility of an asset in terms of its payoff or supply, the more investors learn about it:*

$$\text{(A-20)} \qquad \frac{\mathrm{d}\Gamma_j}{\mathrm{d}\tau_{v,j}} \leq 0, \qquad \frac{\mathrm{d}\Gamma_j}{\mathrm{d}\tau_{z,j}} \leq 0, \qquad \frac{\mathrm{d}\Gamma_j}{\mathrm{d}\tau_{v,-j}} \geq 0, \qquad \frac{\mathrm{d}\Gamma_j}{\mathrm{d}\tau_{z,-j}} \geq 0.$$

Proof. In equilibrium, Γ_i endogenously depends on $\tau_{v,j}$ and $\tau_{z,j}$. From equation (A-15), we have

$$\mathrm{ASSR}_j = \frac{1}{1+\Gamma_j^2(K-1)^2\tau_{v,j}\tau_{z,j}/\lambda^2+\Gamma_j(K-1)}\left(1+\frac{\lambda^2(\bar{z}_j-\bar{\gamma}_j)^2/\tau_{v,j}+\lambda^2/(\tau_{v,j}\tau_{z,j})+\Gamma_j(K-1)}{1+\Gamma_j^2(K-1)^2\tau_{v,j}\tau_{z,j}/\lambda^2+\Gamma_j(K-1)}\right),$$

which explicitly depends on Γ_j, $\tau_{v,j}$, and $\tau_{z,j}$. One can easily see that $\frac{\partial \mathrm{ASSR}_j}{\partial \tau_{v,j}}<0$ and $\frac{\partial \mathrm{ASSR}_j}{\partial \tau_{z,j}}<0$. In the interior equilibrium, $\mathrm{ASSR}_1(\tau_{v,1},\tau_{z,1},\Gamma_1)=\mathrm{ASSR}_2(\tau_{v,2},\tau_{z,2},\Gamma_2)$. Similar to (A-17), we can take the total derivative of both sides with respect to $\tau_{v,1}$ or $\tau_{z,1}$. This yields

$$\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\tau_{v,1}} = -\left(\frac{\partial \mathrm{ASSR}_1}{\partial \Gamma_1}-\frac{\partial \mathrm{ASSR}_2}{\partial \Gamma_1}\right)^{-1}\frac{\partial \mathrm{ASSR}_1}{\partial \tau_{v,1}}<0, \qquad \frac{\mathrm{d}\Gamma_1}{\mathrm{d}\tau_{z,1}} = -\left(\frac{\partial \mathrm{ASSR}_1}{\partial \Gamma_1}-\frac{\partial \mathrm{ASSR}_2}{\partial \Gamma_1}\right)^{-1}\frac{\partial \mathrm{ASSR}_1}{\partial \tau_{z,1}}<0.$$

Since the equality holds in the corner solution, we can write $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\tau_{v,1}}\leq 0$ and $\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\tau_{z,1}}\leq 0$. □

A.4. The Effects of Benchmarking on Asset Prices and Return Volatility

Corollary A-2. *(1) An increase in the benchmarking level of asset j increases the expected prices of the other assets:* $\frac{\mathrm{dE}[P_{-j}]}{\mathrm{d}\bar{\gamma}_j}\geq 0$.

(2) An increase in the benchmarking level of an asset increases its return volatility and decreases the return volatility of the other asset: $\frac{\mathrm{dVar}(V_j-P_j)}{\mathrm{d}\bar{\gamma}_j}\geq 0$ *and* $\frac{\mathrm{dVar}(V_{-j}-P_{-j})}{\mathrm{d}\bar{\gamma}_j}\leq 0$.

We compute the impact of benchmarking on the (unconditional) expected asset prices:

$$\begin{aligned}
\text{(A-21)} \qquad \frac{\mathrm{dE}[P_j]}{\mathrm{d}\bar{\gamma}_j} &= \frac{\lambda}{\tau_{v,j}+\tau_{p,j}+\bar{\tau}_j}\left(1+\frac{(1+2\bar{\tau}_j\tau_{z,j}/\lambda^2)(\bar{z}_j-\bar{\gamma}_j)(K-1)\tau_{v,j}}{\tau_{v,j}+\tau_{p,j}+\bar{\tau}_j}\frac{\mathrm{d}\Gamma_j}{\mathrm{d}\bar{\gamma}_j}\right),\\
\text{(A-22)} \qquad \frac{\mathrm{dE}[P_j]}{\mathrm{d}\bar{\gamma}_{-j}} &= \frac{\lambda(1+2\bar{\tau}_j\tau_{z,j}/\lambda^2)(\bar{z}_j-\bar{\gamma}_j)(K-1)\tau_{v,j}}{(\tau_{v,j}+\tau_{p,j}+\bar{\tau}_j)^2}\frac{\mathrm{d}\Gamma_j}{\mathrm{d}\bar{\gamma}_{-j}}.
\end{aligned}$$

An increase in benchmarking concerns about the other asset "$-j$" increases the fraction of investors who specialize in learning about asset j. As private information about asset j increases, the risk premium decreases and thus the expected price of asset j increases. Thus, an increase in the benchmarking level of one asset tends to increase the other asset price. By symmetry, we can write $\frac{\mathrm{dE}[P_{-j}]}{\mathrm{d}\bar{\gamma}_j} \geq 0$. The variance of asset return is

$$\text{(A-23)} \qquad \mathrm{Var}(V_j - P_j) = \frac{1}{\tau_{v,j} + \bar{\tau}_j + \tau_{p,j}} + \frac{\bar{\tau}_j + \lambda^2 \tau_{z,j}^{-1}}{(\tau_{v,j} + \bar{\tau}_j + \tau_{p,j})^2}.$$

From Proposition 1, as $\bar{\gamma}_j$ increases, $\tau_{p,j}$ and $\bar{\tau}_j$ decrease, while $\tau_{p,-j}$ and $\bar{\tau}_{-j}$ increase. Thus, $\mathrm{Var}(V_j - P_j)$ increases in $\bar{\gamma}_j$ and decreases in $\bar{\gamma}_{-j}$.

A.5. Proof of Proposition 2

Following Admati (1985), we can express each asset price as a linear function of the asset payoff and the noisy supply. The price vector can be written as

$$P = C + B(\Omega V - \lambda(Z - \bar{z})), \qquad \Omega := \int \tau^i \begin{pmatrix} 1 & w^i \\ w^i & (w^i)^2 \end{pmatrix} di,$$

where the constant vector C and the 2×2 matrix B are given by Equation (21). Similar to the proof of Theorem 1, we can derive investor i's optimal holding as:

$$\text{(A-24)} \qquad \theta^i = \gamma^i + \frac{1}{\lambda}(\hat{\Sigma}_v^i)^{-1}(\hat{V}^i - P),$$

where trader i's posterior variance-covariance matrix and mean of V are:

(A-25)

$$\hat{\Sigma}_v^i = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + (\Lambda^i)'(\Sigma^i)^{-1}\Lambda^i\right)^{-1}, \quad \text{where} \quad \Lambda^i = (1, \omega^i)$$

$$\hat{V}^i = \hat{\Sigma}_v^i\left(\left(I_2 - \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}\left(I_2 + \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}\right)^{-1}\right)\left(\Sigma_v^{-1}\bar{v} + \tfrac{1}{\lambda}\Omega\Sigma_z^{-1}\bar{z}\right) + (\Lambda^i)'(\Sigma^i)^{-1}Y^i + \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}B^{-1}P\right).$$

A.6. Proof of Theorem 2

Using Proposition 2, we compute the unconditional mean and variance of $\hat{V}^i - P$ as follows

$$\mathrm{E}\left(\hat{V}^i - P\right) := R = (R_1, R_2)' = \left(\Sigma_v^{-1} + \lambda^{-2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\lambda(\bar{z} - \bar{\gamma}),$$

(A-26) $$\mathrm{Var}\left(\hat{V}^i - P\right) = \mathrm{Var}(V - P) - \mathrm{E}\left(\mathrm{Var}^i(V - P)\right) = \mathrm{Var}(V - P) - \hat{\Sigma}_v^i, \text{ where}$$

$$\mathrm{Var}(V - P) := Q = \left(\begin{smallmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22}\end{smallmatrix}\right) = (I_2 - B\Omega)\Sigma_v(I_2 - B\Omega)' + \lambda^2 B\Sigma_z B'.$$

For notation simplicity, we further define

(A-27)

$$A_{12} := R_1R_2 + Q_{12} = \mathrm{E}[(V_1 - P_1)(V_2 - P_2)], \;\; A_{11} := R_1^2 + Q_{11} = \mathrm{E}[(V_1 - P_1)^2], \;\; A_{22} := R_2^2 + Q_{22} = \mathrm{E}[(V_2 - P_2)^2].$$

Therefore, $\mathrm{ASSR}_1 = A_{11}\tau_{v,1}$ and $\mathrm{ASSR}_2 = A_{22}\tau_{v,2}$. Investor i's utility at time-1 can be written as

$$U_1^i = \lambda W_0 + \frac{1}{2}\mathrm{Tr}\left[(\hat{\Sigma}_v^i)^{-1}Q - I_2\right] + \frac{1}{2}R'(\hat{\Sigma}_v^i)^{-1}R = \lambda W_0 - 1 + \frac{1}{2}\mathrm{Tr}\left[(\hat{\Sigma}_v^i)^{-1}Q\right] + \frac{1}{2}R'(\hat{\Sigma}_v^i)^{-1}R.$$

Since $|\mathrm{Var}(V)| = (\tau_{v,1}\tau_{v,2})^{-1}$ and $|\mathrm{Var}(V \mid Y^i)| = (\tau_{v,1}\tau_{v,2}\tau^i)^{-1}(\tau_{v,1}^{-1} + (\omega^i)^2\tau_{v,2}^{-1} + (\tau^i)^{-1})^{-1}$, the learning capacity constraint (10) implies that $\tau^i \le (K-1)/(\tau_{v,1}^{-1} + (\omega^i)^2\tau_{v,2}^{-1})$.

Obviously, for optimality, the information capacity constraint is always binding, i.e.,

(A-28) $$\tau^i = \frac{K-1}{\tau_{v,1}^{-1} + (\omega^i)^2\tau_{v,2}^{-1}}.$$

Therefore, we can rewrite the investor i's information choice problem as

(A-29) $$\max_{\omega^i} \;\; \frac{1}{2}\left(\tau_{v,1}^{-1} + (\omega^i)^2\tau_{v,2}^{-1}\right)^{-1}\left[A_{22}(\omega^i)^2 + 2A_{12}\omega^i + A_{11}\right](K-1) + A_0,$$

where

$$\text{(A-30)} \qquad \mathrm{A}_0 = \lambda W_0 - 1 + \frac{1}{2}\mathrm{Tr}\left((\Sigma_v^{-1} + \lambda^{-2}\Omega\Sigma_z^{-1}\Omega)Q\right) + \frac{1}{2}R'(\Sigma_v^{-1} + \lambda^{-2}\Omega\Sigma_z^{-1}\Omega)R.$$

Trader i's objective function at $t = 1$ is equivalent to maximizing the reward-to-risk ratio:

$$\max_{\omega^i} \frac{\mathrm{E}\left[\left((V_1 - P_1) + \omega^i(V_2 - P_2)\right)^2\right]}{\mathrm{Var}(\varepsilon^i)} = \max_{\omega^i} \frac{\mathrm{E}[(V_1 - P_1)^2 + (\omega^i)^2(V_2 - P_2)^2] + 2\omega^i \mathrm{E}[(V_1 - P_1)(V_2 - P_2)]}{\left(\tau_{v,1}^{-1} + (\omega^i)^2\tau_{v,2}^{-1}\right)/(K-1)}.$$

The first order condition of the above problem is given by

$$\text{(A-31)} \qquad \left(-A_{12}(\omega^i)^2 - \left(A_{11} - A_{22}\tau_{v,1}^{-1}\tau_{v,2}\right)\omega^i + A_{12}\tau_{v,1}^{-1}\tau_{v,2}\right)\left(\tau_{v,1}^{-1} + \tau_{v,2}^{-1}(\omega^i)^2\right)^{-2}\tau_{v,2}^{-1} = 0.$$

We can solve this equation and verify the second-order condition.

If $A_{12} = R_1R_2 + Q_{12} = \mathrm{E}[(V_1 - P_1)(V_2 - P_2)] \neq 0$, then there are two solutions to (A-31). By checking the second order condition (SOC) at the optimal ω^i,

$$\text{(A-32)} \quad SOC = (-(R_1R_2 + Q_{12})\omega^i - ((R_1^2 + Q_{11}) - (R_2^2 + Q_{22})\tau_{v,1}^{-1}\tau_{v,2}))\frac{\tau_{v,2}^{-1}}{(\tau_{v,1}^{-1}+\tau_{v,2}^{-1}(\omega^i)^2)^2},$$

we can determine the optimal ω^i as

$$\text{(A-33)} \qquad \omega^i = \frac{(R_2^2+Q_{22})\tau_{v,1}^{-1}\tau_{v,2} - (R_1^2+Q_{11}) + \sqrt{\left((R_1^2+Q_{11}) - (R_2^2+Q_{22})\tau_{v,1}^{-1}\tau_{v,2}\right)^2 + 4(R_1R_2+Q_{12})^2\tau_{v,1}^{-1}\tau_{v,2}}}{2(R_1R_2+Q_{12})}.$$

Equation (A-33) can be written as

$$\text{(A-34)} \qquad \omega^i = \frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{2A_{12}\tau_{v,1}} + \sqrt{\left(\frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{2A_{12}\tau_{v,1}}\right)^2 + \frac{\tau_{v,2}}{\tau_{v,1}}},$$

where $\mathrm{ASSR}_j = \tau_{v,j}\mathrm{E}[(V_j - P_j)^2]$ is the adjusted squared Sharpe ratio for asset j, as defined in (14). The SOC at this solution (A-33) is indeed negative,

$$SOC(\omega^i) = -\sqrt{\left((R_1^2 + Q_{11}) - (R_2^2 + Q_{22})\tau_{v,1}^{-1}\tau_{v,2}\right)^2 + 4(R_1R_2 + Q_{12})^2\tau_{v,1}^{-1}\tau_{v,2}}\frac{\tau_{v,2}^{-1}}{\left(\tau_{v,1}^{-1}+\tau_{v,2}^{-1}(\omega^i)^2\right)^2} < 0.$$

Therefore, when $A_{12} := \mathrm{E}[(V_1 - P_1)(V_2 - P_2)] \neq 0$, the best response for investor i is to observe a signal about the linear combination of both assets, with attention weight ω^i given by (A-34).

If Λ^i is not restricted to be diagonal, there does not exist an equilibrium where a finite fraction of investors specialize in learning only about one asset (Proposition 2 of Mondria (2010)). Instead, the best response for each investor is to follow integrative learning by observing a signal in the form of (19) which is informative about a linear combination of two asset payoffs.

This can be proved by contradiction. Assume investors have specialized in learning about either asset 1 or 2. The asset prices and returns would be uncorrelated, ex-ante and ex-post, so that $A_{12} = \mathrm{E}[(V_1 - P_1)(V_2 - P_2)] = \mathrm{E}[V_1 - P_1]\mathrm{E}[V_2 - P_2]$. Given $\bar{z}_j - \bar{\gamma}_j > 0$, we have $\mathrm{E}[V_j - P_j] > 0$ and $A_{12} > 0$. We show that the best response of each investor i is to learn about $V_1 + \omega^i V_2$, where $\omega^i \in (0, \infty)$ is given by (A-34). It is not optimal for each investor to follow separative learning by observing an asset-specific signal. Thus, each investor would deviate from the conjectured specialization in learning about one asset.

Next, we conjecture a symmetric linear equilibrium where all investors choose the same attention weight in the private signal, $\omega^i = \omega$. Then they must choose the same signal precision,

(A-35)
$$\tau^i = \tau = \frac{K-1}{\tau_{v,1}^{-1} + \omega^2 \tau_{v,2}^{-1}}.$$

If such an ω exists, then Ω is a singular matrix and we need to rewrite equation (A-26). One can substitute the symmetric ω into R and Q in the first order condition (A-31), and obtain

(A-36)
$$\frac{\lambda^4(\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1})\left[\omega(\tau_{v,1}^{-1}\tau_{z,1}^{-1} - \tau_{v,2}^{-1}\tau_{z,2}^{-1}) + (\omega(\bar{z}_1 - \bar{\gamma}_1) - (\bar{z}_2 - \bar{\gamma}_2))\left(\omega(\bar{z}_2 - \bar{\gamma}_2)\tau_{v,2}^{-1} + (\bar{z}_1 - \bar{\gamma}_1)\tau_{v,1}^{-1}\right)\right]}{\tau_{v,1}\left(\lambda^2 K\left(\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}\right) + (K-1)^2(\tau_{z,1} + \omega^2\tau_{z,2})\right)} = 0.$$

There is only one real solution that satisfies the second order condition (A-32):

(A-37)
$$\omega = \alpha + \sqrt{\alpha^2 + \frac{\tau_{v,2}}{\tau_{v,1}}},$$

where

$$\text{(A-38)} \qquad \alpha = \frac{\tau_{v,2}^{-1}\tau_{z,2}^{-1} + \tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2 - \tau_{v,1}^{-1}\tau_{z,1}^{-1} - \tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2}{2(\bar{z}_1 - \bar{\gamma}_1)(\bar{z}_2 - \bar{\gamma}_2)\tau_{v,2}^{-1}}.$$

Since ω has to be positive by equation (A-37), it implies that $\mathrm{E}[(V_1 - P_1)(V_2 - P_2)] \neq 0$ is necessary but not sufficient for the existence of the conjectured symmetric equilibrium. We also need

$$\text{(A-39)} \qquad A_{12}(\omega) := R_1(\omega)R_2(\omega) + Q_{12}(\omega) = \mathrm{E}[(V_1 - P_1)(V_2 - P_2)] > 0,$$

to ensure that the solution (A-33) is positive. Therefore, the sufficient and necessary condition for the existence and uniqueness of ω is that $A_{12} = R_1(\omega)R_2(\omega) + Q_{12}(\omega) > 0$. Note that both $R(\omega)$ and $Q(\omega)$ depend on the optimal ω through Ω,

$$\text{(A-40)} \qquad \Omega(\omega) = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} = \int \tau \begin{pmatrix} 1 & \omega \\ \omega & \omega^2 \end{pmatrix} di = \frac{K-1}{\tau_{v,1}^{-1} + \omega^2 \tau_{v,2}^{-1}} \begin{pmatrix} 1 & \omega \\ \omega & \omega^2 \end{pmatrix}.$$

Define

$$\mathrm{Det} := (\tau + \tau_{v,1})\tau_{v,2} + \tfrac{\tau^2}{\lambda^2}\left(\tau_{v,2}\tau_{z,1} + \omega^4 \tau_{v,1}\tau_{z,2}\right) + \omega^2 \tau\left(\tau_{v,1} + \tfrac{\tau}{\lambda^2}\left(\tau_{v,1}\tau_{z,1} + \tau_{v,2}\tau_{z,2}\right)\right),$$

where τ is given by equation (A-35). Let $\beta = \tau(\tau_{z,1} + \omega^2 \tau_{z,2})$, we have

$$\begin{aligned} R_1 R_2 = & \frac{\lambda^2}{\mathrm{Det}}(\bar{z}_1 - \bar{\gamma}_1)(\bar{z}_2 - \bar{\gamma}_2) - \frac{\lambda^2 \Omega_{12}}{\mathrm{Det}^2}\left(1 + \frac{\beta}{\lambda^2}\right)\left[\tau_{v,2}(\bar{z}_1 - \bar{\gamma}_1)^2 + \tau_{v,1}(\bar{z}_2 - \bar{\gamma}_2)^2\right] \\ & - \frac{\lambda^2 \Omega_{12}}{\mathrm{Det}^2}\left(1 + \frac{\beta}{\lambda^2}\right)^2 \left[\sqrt{\Omega_{22}}(\bar{z}_1 - \bar{\gamma}_1) - \sqrt{\Omega_{11}}(\bar{z}_2 - \bar{\gamma}_2)\right]^2, \end{aligned}$$

where Ω_{11}, Ω_{12}, and Ω_{22} are defined in equation (A-40). Since $Q_{12} = \mathrm{Cov}(P_1, P_2) - \mathrm{Cov}(V_1, P_2) - \mathrm{Cov}(V_2, P_1)$, a sufficient condition for $A_{12} = R_1 R_2 + Q_{12} > 0$ is that $\mathrm{Cov}(P_1, P_2) > 0$ and $R_1 R_2 > \mathrm{Cov}(V_1, P_2) + \mathrm{Cov}(V_2, P_1)$. First, a sufficient condition for

$R_1R_2 > \text{Cov}(V_1,P_2)+\text{Cov}(V_2,P_1)$ is that

$$\frac{1}{\lambda^2} < \frac{(\bar{z}_1-\bar{\gamma}_1)(\bar{z}_2-\bar{\gamma}_2)}{\sqrt{\tau_{v,1}\tau_{v,2}}}\sqrt{\frac{2}{(1+\beta/\lambda^2)(1+(K-1)(1+\beta/\lambda^2))}} - \frac{1}{2}\left[\frac{(\bar{z}_1-\bar{\gamma}_1)^2}{\tau_{v,1}}+\frac{(\bar{z}_2-\bar{\gamma}_2)^2}{\tau_{v,2}}\right].$$

Second, a sufficient condition for positive price covariance is that $\lambda^{-2} > \tau_{v,1}^{-1}\tau_{z,1}^{-1}+\tau_{v,2}^{-1}\tau_{z,2}^{-1}$. Thus, a sufficient condition for $A_{12}(\omega)>0$ is $\lambda_l<\lambda<\lambda_h$, where

(A-41)

$$\lambda_h := \left(\frac{1}{\tau_{v,1}\tau_{z,1}}+\frac{1}{\tau_{v,2}\tau_{z,2}}\right)^{1/2},$$

$$\lambda_l := \frac{1}{\sqrt{2\beta}}\left(-1-\frac{\beta}{2}\left(\frac{(\bar{z}_1-\bar{\gamma}_1)^2}{\tau_{v,1}}+\frac{(\bar{z}_2-\bar{\gamma}_2)^2}{\tau_{v,2}}\right)+\sqrt{\left(1-\frac{\beta}{2}\left(\frac{(\bar{z}_1-\bar{\gamma}_1)^2}{\tau_{v,1}}+\frac{(\bar{z}_2-\bar{\gamma}_2)^2}{\tau_{v,2}}\right)\right)^2+4\beta\sqrt{\frac{2}{K}}\frac{(\bar{z}_1-\bar{\gamma}_1)(\bar{z}_2-\bar{\gamma}_2)}{\sqrt{\tau_{v,1}\tau_{v,2}}}}\right)^{1/2}.$$

Therefore, the equilibrium outlined in Theorem 2 exists within a well-defined range of risk aversion parameters.

A.7. Proof of Proposition 3

When $\Delta=0$, it is straightforward to calculate that $\omega=\frac{\bar{z}_2-\bar{\gamma}_2}{\bar{z}_1-\bar{\gamma}_1}$ based on equations (24) and (25). Moreover, from equation (25), one can see that α is an increasing function of $\tau_{z,1}$ and a decreasing function of $\tau_{z,2}$. By equation (24), ω is an increasing function of α and depends on $\tau_{z,j}$ only through α. It follows that $\frac{d\omega}{d\tau_{z,1}}>0$ and $\frac{d\omega}{d\tau_{z,2}}<0$. Given an arbitrary set of parameters $\{\tau_{v,1},\tau_{v,2},\tau_{z,1},\tau_{z,2}\}$, we define a "break-even" $\tau_{z,1}^* := \tau_{v,2}\tau_{z,2}/\tau_{v,1}$ and compare it with the actual $\tau_{z,1}$. When $\Delta>0$, we have $\tau_{v,2}^{-1}\tau_{z,2}^{-1}>\tau_{v,1}^{-1}\tau_{z,1}^{-1}$ which implies $\tau_{z,1}>\tau_{z,1}^*$. Since ω increases in $\tau_{z,1}$, we must have $\omega(\tau_{v,1},\tau_{v,2},\tau_{z,1},\tau_{z,2})>\omega(\tau_{v,1},\tau_{v,2},\tau_{z,1}^*,\tau_{z,2})$, that is, $\omega>\frac{\bar{z}_2-\bar{\gamma}_2}{\bar{z}_1-\bar{\gamma}_1}$. When $\Delta<0$, we must have $\tau_{z,1}<\tau_{z,1}^*$ and $\omega<\frac{\bar{z}_2-\bar{\gamma}_2}{\bar{z}_1-\bar{\gamma}_1}$. The above results are summarized by (29). Taking derivative of equation (24) with respect to $\bar{\gamma}_j$ yields

(A-42) $$\frac{\mathrm{d}\omega}{\mathrm{d}\bar{\gamma}_j} = \left[1 + \alpha\left(\alpha^2 + \frac{\tau_{v,2}}{\tau_{v,1}}\right)^{-\frac{1}{2}}\right]\frac{\mathrm{d}\alpha}{\mathrm{d}\bar{\gamma}_j} = \frac{\omega}{\omega-\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\bar{\gamma}_j}.$$

Since $\omega \geq \alpha$ and $\omega > 0$, the sign of $\frac{\mathrm{d}\omega}{\mathrm{d}\bar{\gamma}_j}$ should be the same as that of $\frac{\mathrm{d}\alpha}{\mathrm{d}\bar{\gamma}_j}$. Direct calculations yield

(A-43)
$$\frac{\mathrm{d}\alpha}{\mathrm{d}\bar{\gamma}_1} = \frac{\tau_{v,1}\tau_{z,1}\left(1+(\bar{z}_2-\bar{\gamma}_2)^2\tau_{z,2}\right) - \tau_{v,2}\tau_{z,2}\left(1-(\bar{z}_1-\bar{\gamma}_1)^2\tau_{z,1}\right)}{2(\bar{z}_1-\bar{\gamma}_1)^2(\bar{z}_2-\bar{\gamma}_2)\tau_{v,1}\tau_{z,1}\tau_{z,2}} = \frac{\Delta+\xi}{2(\bar{z}_1-\bar{\gamma}_1)^2(\bar{z}_2-\bar{\gamma}_2)\tau_{v,2}^{-1}},$$

(A-44)
$$\frac{\mathrm{d}\alpha}{\mathrm{d}\bar{\gamma}_2} = \frac{\tau_{v,1}\tau_{z,1}\left(1-(\bar{z}_2-\bar{\gamma}_2)^2\tau_{z,2}\right) - \tau_{v,2}\tau_{z,2}\left(1+(\bar{z}_1-\bar{\gamma}_1)^2\tau_{z,1}\right)}{2(\bar{z}_1-\bar{\gamma}_1)(\bar{z}_2-\bar{\gamma}_2)^2\tau_{v,1}\tau_{z,1}\tau_{z,2}} = \frac{\Delta-\xi}{2(\bar{z}_1-\bar{\gamma}_1)(\bar{z}_2-\bar{\gamma}_2)^2\tau_{v,2}^{-1}}.$$

The sign pattern of $\frac{\mathrm{d}\omega}{\mathrm{d}\bar{\gamma}_j}$ in Proposition 3 follows from equations (A-42), (A-43), and (A-44).

Similarly, direct calculations yield that $\frac{\mathrm{d}\omega}{\mathrm{d}m} \geq 0$ is equivalent to

(A-45) $$\Delta(\bar{z}_2\bar{\gamma}_1 + \bar{z}_1\bar{\gamma}_2 - 2\bar{\gamma}_1\bar{\gamma}_2) + \xi(\bar{z}_2\bar{\gamma}_1 - \bar{z}_1\bar{\gamma}_2) \geq 0, \quad i.e., \quad \frac{\Delta}{\xi} \geq q.$$

A.8. The Impact of Benchmarking on Attention Allocation

Optimal attention allocation equalizes this marginal value across assets, that is,

(A-46) $$\mathrm{ASSR}_1 + A_{12}\tau_{v,1}\omega = \mathrm{ASSR}_2 + A_{12}\tau_{v,2}/\omega,$$

where $\mathrm{ASSR}_j = \mathrm{E}[(V_j - P_j)^2]\tau_{v,j}$ and $A_{12} = \mathrm{E}[(V_1-P_1)(V_2-P_2)]$. Equation (A-46) can be written as

(A-47) $$\tau_{v,1}\omega - \frac{\tau_{v,2}}{\omega} = \frac{\mathrm{ASSR}_2 - \mathrm{ASSR}_1}{A_{12}}.$$

To analyze the effects of benchmarking on attention allocation, we calculate the first derivative of equation (A-47) with respect to the aggregate benchmarking level of asset 2:

$$\text{(A-48)} \qquad \left(\tau_{v,1} + \frac{\tau_{v,2}}{\omega^2}\right)\frac{\mathrm{d}\omega}{\mathrm{d}\gamma_2} = \frac{m}{A_{12}}\Bigg(\underbrace{\frac{\mathrm{d}(\mathrm{ASSR}_2 - \mathrm{ASSR}_1)}{\mathrm{d}\bar{\gamma}_2}}_{\text{effect of reduced effective supply}} + \underbrace{(\mathrm{ASSR}_2 - \mathrm{ASSR}_1)\frac{-\mathrm{d}\ln A_{12}}{\mathrm{d}\bar{\gamma}_2}}_{\text{effect of cross-asset allocation}}\Bigg).$$

The first term on the right-hand side of equation (A-48) represents the effect of reduced effective supply. Similar to the implication under separative learning, an asset's ASSR decreases in its benchmarking level and increases in the other asset's benchmarking level, $\frac{\mathrm{dASSR}_2}{\mathrm{d}\bar{\gamma}_2} < 0$ and $\frac{\mathrm{dASSR}_1}{\mathrm{d}\bar{\gamma}_2} > 0$. Therefore, this first term is negative given the equilibrium condition $A_{12} > 0$. Intuitively, as in the case under separative learning, benchmarking tends to decrease the marginal value of information about this asset, thereby shifting investors' attention toward the other asset.

The second term of (A-48) represents the effect of cross-asset allocation. In Internet Appendix B.3, we show $\frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_j} < 0$ in various cases and confirmed by extensive numerical analysis. Intuitively, increased benchmarking on asset j raises its excess demand and price, thereby lowering its expected return. This benchmarking may indirectly impact the expected return of the other asset by influencing its price informativeness and, consequently, its risk premium. However, the direct effect of increased excess demand on asset j's own expected return dominates. As a result, the expected product of returns for the two assets tends to decrease as the benchmarking level of either asset rises. Therefore, the coefficient $\frac{-\mathrm{d}\ln A_{12}}{\mathrm{d}\bar{\gamma}_2}$ is positive, and the cross-asset allocation effect rises with the difference in the assets' adjusted squared Sharpe ratios, $\mathrm{ASSR}_2 - \mathrm{ASSR}_1$.

A.9. The Price Informativeness of Assets

Each asset price incorporates private information about both V_1 and V_2 and the noise terms are also correlated. Equation (20) can be written as

$$P_1 = C_1 + \tau(B_{11} + B_{12}\omega)(V_1 + \omega V_2) - \lambda\left(B_{11}(Z_1 - \bar{z}_1) + B_{12}(Z_2 - \bar{z}_2)\right), \tag{A-49}$$

$$P_2 = C_2 + \tau(B_{21} + B_{22}\omega)(V_1 + \omega V_2) - \lambda\left(B_{21}(Z_1 - \bar{z}_1) + B_{22}(Z_2 - \bar{z}_2)\right). \tag{A-50}$$

The elements of the matrix B can be expressed as:

$$\begin{aligned}
B_{11} &= \frac{\omega^2}{\tau_{v,2}+\omega^2\tau_{v,1}} + \frac{\tau_{v,2}(\lambda^2+(K-1)\tau_{v,1}\tau_{z,1})}{\tau_{v,1}(K\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)^2\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2})}, \\
B_{12} &= -\frac{(K-1)\omega((\lambda^2-\tau_{v,2}\tau_{z,2})(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2}))}{(\tau_{v,2}+\omega^2\tau_{v,1})(K\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)^2\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2}))}, \\
B_{21} &= -\frac{\omega}{\tau_{v,2}+\omega^2\tau_{v,1}} + \frac{\omega(\lambda^2+(K-1)\tau_{v,1}\tau_{z,1})}{K\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)^2\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2})}, \\
B_{22} &= \frac{\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})(\omega^2\tau_{v,1}+K\tau_{v,2})+(K-1)\tau_{v,1}\tau_{v,2}((K-1)\tau_{z,1}\tau_{v,2}+\omega^2\tau_{z,2}(\omega^2\tau_{v,1}+K\tau_{v,2}))}{\tau_{v,2}(\tau_{v,2}+\omega^2\tau_{v,1})(K\lambda^2(\tau_{v,2}+\omega^2\tau_{v,1})+(K-1)^2\tau_{v,1}\tau_{v,2}(\tau_{z,1}+\omega^2\tau_{z,2})}.
\end{aligned}$$

We can see from (A-49) and (A-50) that both asset prices contain information about $V_1 + \omega V_2$. The price informativeness of asset j can be derived from equations (A-49) and (A-50):

$$\mathrm{I}(V_1; P_1) = \frac{1}{2}\ln\left(1 + \frac{\tau^2\tau_{v,1}^{-1}(B_{11}+\omega B_{12})^2}{\omega^2\tau^2(B_{11}+\omega B_{12})^2\tau_{v,2}^{-1}+\lambda^2\left(B_{11}^2\tau_{z,1}^{-1}+B_{12}^2\tau_{z,2}^{-1}\right)}\right), \tag{A-51}$$

$$\mathrm{I}(V_2; P_2) = \frac{1}{2}\ln\left(1 + \frac{\omega^2\tau^2\tau_{v,2}^{-1}(B_{21}+\omega B_{22})^2}{\tau^2(B_{21}+\omega B_{22})^2\tau_{v,1}^{-1}+\lambda^2\left(B_{21}^2\tau_{z,1}^{-1}+B_{22}^2\tau_{z,2}^{-1}\right)}\right), \tag{A-52}$$

where ω and τ are given by (24). Under integrative learning, we measure the *total* price informativeness using $\mathrm{I}(V_j; P)_{\text{int}} = \mathrm{I}(V_j; P_1, P_2) = \mathrm{H}(V_j) - \mathrm{H}(V_j | P_1, P_2)$. To calculate $\mathrm{I}(V_j; P)_{\text{int}}$, we use the result of variance reduction by standard Kalman filtering:

$$\Theta := \mathrm{Var}(V) - \mathrm{Var}(V|P) = B\Omega\Sigma_v(B\Omega\Sigma_v\Omega B' + \lambda^2 B\Sigma_z B')^{-1}\Sigma_v\Omega B'. \tag{A-53}$$

This matrix Θ is symmetric and full-rank. So the conditional variance for asset $j = \{1,2\}$ is

(A-54)
$$\operatorname{Var}(V_1 \mid P_1, P_2) = \tau_{v,1}^{-1} - \Theta_{11}, \qquad \operatorname{Var}(V_1 \mid P_1, P_2) = \tau_{v,2}^{-1} - \Theta_{22}.$$

The *total* price informativeness is shown to be:

(A-55)
$$\mathrm{I}(V_j; P)_{\text{int}} = -\frac{1}{2}\ln\left(1 - \frac{\omega^{2(j-1)}\tau_{v,j}^{-1}(\tau_{z,1}+\omega^2\tau_{z,2})}{\frac{\lambda^2}{(K-1)^2}\left(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}\right)^2+(\tau_{z,1}+\omega^2\tau_{z,2})\left(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}\right)}\right).$$

A.10. Market Informational Efficiency

We define the *market informational efficiency* by the mutual information:

(A-56)
$$\mathrm{I}(V;P) := \mathrm{H}(V) - \mathrm{H}(V|P) = \frac{1}{2}\ln\left(\frac{|\operatorname{Var}(V)|}{|\operatorname{Var}(V|P)|}\right) = \frac{1}{2}\ln\left(\frac{|\Sigma_v|}{|\Sigma_v - \Theta|}\right).$$

Here, $\Theta := \operatorname{Var}(V) - \operatorname{Var}(V|P)$ is the reduction in variance of V conditional on P, with the expression given by equation (A-53). Thus, $\mathrm{I}(V;P)$ measures the amount of information about the payoff vector obtained by observing the price vector.

FIGURE A-1. Market informational efficiency $\mathrm{I}(V;P)_{\text{int}}$ and $\mathrm{I}(V;P)_{\text{sep}}$ as functions of $\bar{\gamma}_1$ and $\bar{\gamma}_2$. Parameters are: $\lambda = 1$, $K = 1.5$, $\tau_{v,1} = \tau_{v,2} = \tau_{z,2} = 1$, $\tau_{z,1} = 0.5$, $\bar{z}_1 = \bar{z}_2 = 5$.

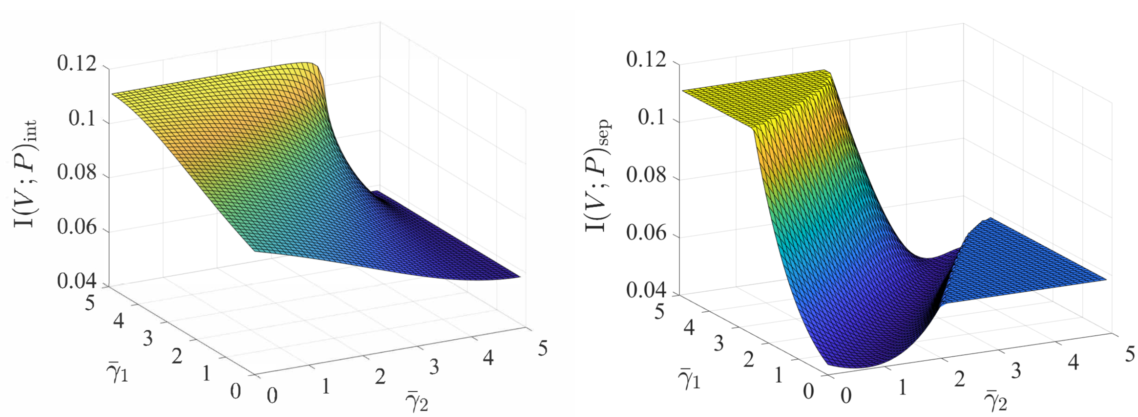


Figure A-1 illustrates how market informational efficiency under integrative learning, $\mathrm{I}(V;P)_{\text{int}}$ (as given in Equation (A-57)), and under separative learning,

$I(V;P)_{sep} = I(V_1;P_1)_{sep} + I(V_2;P_2)_{sep}$ (as given in Equation (B-53) of the Internet Appendix), vary with $\bar{\gamma}_1$ and $\bar{\gamma}_2$ when asset 1 is more uncertain than asset 2 (i.e., $\Delta < 0$).

Under integrative learning (left panel), $I(V;P)_{int}$ increases with $\bar{\gamma}_1$ and decreases with $\bar{\gamma}_2$. Under separative learning (right panel), the surface of $I(V;P)_{sep}$ exhibits two plateaus with a pronounced dip in the middle. This provides a comprehensive view of how benchmarking affects market-wide informational efficiency and highlights its strong dependence on the underlying learning technology.

Proposition A-1. *1. Under integrative learning, the market informational efficiency is*

$$I(V;P)_{int} = \frac{1}{2}\ln\left(1+\left(\frac{K-1}{\lambda}\right)^2 \frac{\tau_{z,1}+\omega^2\tau_{z,2}}{\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}}\right) = I(V_1+\omega V_2;P)_{int}. \tag{A-57}$$

2. With the fraction of benchmarked investors m fixed, we have

(1) If $-\xi < \Delta < 0$, then $\frac{dI(V;P)_{int}}{d\gamma_1} > 0$, $\frac{dI(V;P)_{int}}{d\gamma_2} < 0$;

(2) If $0 < \Delta < \xi$, then $\frac{dI(V;P)_{int}}{d\gamma_1} < 0$, $\frac{dI(V;P)_{int}}{d\gamma_2} > 0$;

(3) If $|\Delta| \geq \xi$, then $\frac{dI(V;P)_{int}}{d\gamma_1} \leq 0$, $\frac{dI(V;P)_{int}}{d\gamma_2} \leq 0$.

3. With the benchmarking levels γ_1 and γ_2 held fixed, we have

(1) If $0 < \frac{\Delta}{\xi} < q$ or $q < \frac{\Delta}{\xi} < 0$, then $\frac{dI(V;P)_{int}}{dm} > 0$;

(2) If $0 \leq q \leq \frac{\Delta}{\xi}$ or $\frac{\Delta}{\xi} \leq q \leq 0$, then $\frac{dI(V;P)_{int}}{dm} \leq 0$, where q is as defined in (30).

Proof. See Internet Appendix B.4. □

Result 1 of Proposition A-1 implies that integrative learning essentially enables investors to obtain information about a portfolio of assets, aligning with the objective of portfolio management. The equality $I(V;P)_{int} = I(V_1+\omega V_2;P)_{int}$ shows the informational equivalence between the payoff vector and a hypothetical portfolio paying $V_1 + \omega V_2$. While this hypothetical portfolio reflects investors' attention allocation $(1, \omega)$, it may not align with the average portfolio consisting of $(\bar{z}_j - \bar{\gamma}_j)$ shares of each asset $j = 1, 2$. These two portfolios align only when assets are equally uncertain ($\Delta = 0$), leading to an endogenous choice $\omega = \frac{\bar{z}_2 - \bar{\gamma}_2}{\bar{z}_1 - \bar{\gamma}_1}$. We define investors' *real* attention, denoted as ω^*, by taking $\omega(\Delta = 0)$ as the baseline:

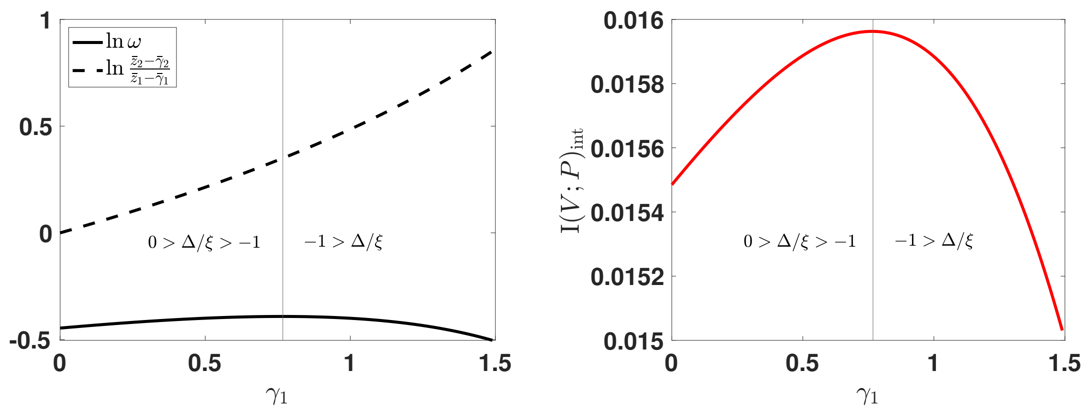
$$\omega^* := \frac{\omega}{\omega(\Delta = 0)} = \omega \cdot \frac{\bar{z}_1 - \bar{\gamma}_1}{\bar{z}_2 - \bar{\gamma}_2}.$$

It follows from Proposition 3 that $\omega^* > 1$ if and only if $\Delta > 0$ and $\omega^* = 1$ when $\Delta = 0$. In Internet Appendix B.4, we show that the real attention ω^* shifts toward the asset with higher uncertainty.

Result 2 of Proposition A-1 shows that $\frac{\mathrm{dI}(V;P)_{\mathrm{int}}}{\mathrm{d}\gamma_j} > 0$ holds for asset 1 when $-\xi < \Delta < 0$ and for asset 2 when $0 < \Delta < \xi$. This implies that increasing the benchmarking level of the riskier asset (characterized by greater uncertainty in payoff or noisy demand/supply) can enhance market informational efficiency if $|\Delta| < \xi$. In this case, as shown in Figure A-2 (left) where, as γ_1 increases, investors' nominal attention ω (solid line) shifts toward asset 2. This changes in the same direction as the baseline $\frac{\bar{z}_2 - \bar{\gamma}_2}{\bar{z}_1 - \bar{\gamma}_1}$ (dotted line) but to a lesser extent. As a result, $\omega^* = \omega \cdot \frac{\bar{z}_1 - \bar{\gamma}_1}{\bar{z}_2 - \bar{\gamma}_2}$ decreases in γ_1, that is, investors' real attention actually shifts towards asset 1, the more uncertain asset given $\Delta < 0$. This shift in real attention effectively reduces the posterior uncertainty about the attention-targeted portfolio. Consequently, the market informational efficiency, $\mathrm{I}(V;P)_{\mathrm{int}} = \mathrm{I}(V_1 + \omega V_2; P)_{\mathrm{int}}$, increases with the benchmarking level of the riskier asset (γ_1) until the point $\Delta = -\xi$. This can be seen in the right panel of Figure A-2, which corresponds to Case (1) of Result 2 in Proposition A-1.

When $\Delta < -\xi$, investors shift substantially more attention towards asset 1, even though its expected number of information-sensitive shares is diminishing when $\bar{\gamma}_1$ approaches $\bar{z}_1$. As

FIGURE A-2. The attention $\ln\omega$ and the average portfolio $\ln\frac{\bar{z}_2-\bar{\gamma}_2}{\bar{z}_1-\bar{\gamma}_1}$ as functions of γ_1 (left), and the market informational efficiency $\mathrm{I}(V;P)_{\mathrm{int}}$ as a function of γ_1 (right). Parameters are: $\lambda=1$, $K=1.5$, $\tau_{v,1}=0.1$, $\tau_{z,1}=0.9$, $\tau_{v,2}=\tau_{z,2}=1$, $\bar{z}_1=\bar{z}_2=1.5$, $\gamma_2=0$, and $m=0.5$.



shown in Figure A-2 (left), this leads to a directional divergence between the attention-targeted portfolio $(1,\omega)'$ and the average speculative portfolio $\left(1,\frac{\bar{z}_2-\bar{\gamma}_2}{\bar{z}_1-\bar{\gamma}_1}\right)'$. Consequently, as shown in the right panel of Figure A-2, market efficiency decreases with γ_1 in this regime, consistent with Case (3) of Result 2 in Proposition A-1.

Result 3 of Proposition A-1 follows directly from Result 2. Holding γ_1 and γ_2 fixed, a higher fraction of benchmarked investors m raises the aggregate benchmarking levels $\bar{\gamma}_j$ for $j\in\{1,2\}$. For example, in the case when $-\xi<\Delta<0$—i.e., when asset 1 exhibits greater uncertainty in payoff or noisy demand/supply— an increase in asset 1's benchmarking can enhance market informational efficiency, whereas an increase in the benchmarking of the less uncertain asset 2 tends to reduce it. If γ_1 is relatively high compared to γ_2 (making it more likely $q<\frac{\Delta}{\xi}$ as m rises, as discussed in Result 3 of Proposition 3), the higher growth of $\bar{\gamma}_1$ with respect to m implies that the effect of increasing $\bar{\gamma}_1$ on overall market efficiency, $\mathrm{I}(V;P)_{\mathrm{int}}$, dominates. As a result, market informational efficiency tends to increase with m, as shown in Case (1) of Result 3 in Proposition A-1 and illustrated in Figure 6.

Internet Appendix for "How Does Benchmarking Affect Market Efficiency? The Role of Learning Technology"

Section B.1 shows that each investor allocates all of the learning capacity to learn about one linear combination of asset payoffs. Section B.2 solves for the endogenous learning capacity K numerically under both integrative and separative learning technologies. Section B.3 provides a detailed analysis of the effects of benchmarking on ASSR_j and A_{12} under integrative learning. Section B.4 provides the detailed proof of Proposition A-1. Section B.5 examines how benchmarking affects return comovement as a model implication. Section B.6 provides details about the case with correlated asset payoffs under separative and integrative learning.

B.1. Proof of the Optimality of Integrative Learning

Investor i's private information is defined as:

(B-1) $$Y^i = \Lambda^i V + \varepsilon^i, \qquad \varepsilon^i \sim \mathcal{N}(0, \Sigma^i),$$

where Λ^i is a 2×2 matrix and Σ^i is the covariance matrix of the error term. In this section, we solve the equilibrium without restricting the matrix Λ^i to be diagonal. We show that when there is no restriction on the information structure Λ^i, investors optimally choose to acquire private information about the two asset payoffs through a single linear combination. This implies that, in the absence of constraints on investors' learning technologies, integrative learning emerges as the first-best strategy.

The expression for the equilibrium price is the same as in Proposition 2 of the paper:

(B-2) $$P = C + B\left(\Omega V - \lambda(Z - \bar{z})\right),$$

where C and B are:

(B-3)
$$C = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1} \left(\Sigma_v^{-1}\bar{v} - \lambda(\bar{z} - \bar{\gamma})\right),$$
$$B = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1} \left(I_2 + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\right),$$

and

(B-4)
$$\Omega = \int_0^1 \left(\Lambda^i\right)^\top \left(\Sigma^i\right)^{-1} \Lambda^i di.$$

Similar to the proof of Theorem 2 in Appendix A.6, investor i's objective function at $t = 1$ is given as

(B-5)
$$U_1^i = \lambda W_0^i + \frac{1}{2}\mathrm{Tr}\left[(\hat{\Sigma}_v^i)^{-1}Q - I_2\right] + \frac{1}{2}R'(\hat{\Sigma}_v^i)^{-1}R,$$

where

(B-6)
$$R = (R_1, R_2)' := \mathrm{E}\left(\hat{V}^i - P\right) = \left(\Sigma_v^{-1} + \lambda^{-2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\lambda(\bar{z} - \bar{\gamma}),$$
$$Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix} := \mathrm{Var}\left(V - P\right) = (I_2 - B\Omega)\Sigma_v(I_2 - B\Omega)' + \lambda^2 B\Sigma_z B',$$

and

(B-7)
$$\hat{\Sigma}_v^i = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + (\Lambda^i)'(\Sigma^i)^{-1}\Lambda^i\right)^{-1},$$
$$\hat{V}^i = \hat{\Sigma}_v^i\left(\left(I_2 - \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}\left(I_2 + \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}\right)^{-1}\right)\left(\Sigma_v^{-1}\bar{v} + \tfrac{1}{\lambda}\Omega\Sigma_z^{-1}\bar{z}\right) + (\Lambda^i)'(\Sigma^i)^{-1}Y^i + \tfrac{1}{\lambda^2}\Omega\Sigma_z^{-1}B^{-1}P\right).$$

Investors choose the private information structure and precision, i.e., matrix Λ^i and the variance-covariance matrix of the error term Σ^i, by maximizing the expected utility given by (B-5) subject to the learning capacity (10). The following lemma shows that investors choose to observe one linear combination of asset payoffs as their private information.

Lemma B-1. *In the first period, each investor allocates all of his learning capacity to learn about one linear combination of asset payoffs.*

Proof. Following Mondria (2010), we use three steps to show that each investor allocates all of his learning capacity to learn about one linear combination of asset payoffs.

Step 1: Normalization of Σ^i.

Given that the variance-covariance matrix of the error term, Σ^i, is symmetric and positive semi-definite, it can be decomposed as

$$\Sigma^i = A^i D^i (A^i)^T, \tag{B-8}$$

where D^i is a diagonal matrix and $(A^i)^\top = (A^i)^{-1}$. We will show that for every non-diagonal Σ^i, there exists an equivalent private information $\tilde{Y}^i$ defined as

$$\tilde{Y}^i = (A^i)^{-1} Y^i = (A^i)^{-1} \Lambda^i V + (A^i)^{-1} \varepsilon^i, \tag{B-9}$$

where $\mathrm{Var}((A^i)^{-1}\varepsilon^i) = D^i$ is a diagonal matrix. The transformed private signal $\tilde{Y}^i$ satisfies the learning capacity constraint. This follows from the fact that mutual information is invariant under invertible linear transformations, which implies that

$$I(X; Y^i) = I(X; (A^i)^{-1} Y^i) = H(X) - H(X \mid Y^i) \leq K. \tag{B-10}$$

Since $(\Lambda^i)^\top (\Sigma^i)^{-1} \Lambda^i = ((A^i)^{-1}\Lambda^i)^\top (D^i)^{-1} ((A^i)^{-1}\Lambda^i)$ and $(\Lambda^i)^\top (\Sigma^i)^{-1} Y^i = ((A^i)^{-1}\Lambda^i)^\top (D^i)^{-1} \tilde{Y}^i$, it follows from Equations (B-3), (B-4), and (A-25) that $\tilde{Y}^i$ is an equivalent private signal, leading to the same equilibrium prices and portfolio holdings. Therefore, for any information structure Λ^i, we can let investors choose a diagonal variance-covariance matrix for the signal error term, given by:

(B-11) $$\Sigma^i = \begin{pmatrix} (\tau_1^i)^{-1} & 0 \\ 0 & (\tau_2^i)^{-1} \end{pmatrix}.$$

Step 2: Normalization of Λ^i.

Next, we show that for any matrix of signal weights, Λ^i, and any diagonal variance-covariance matrix of the error term Σ^i, there exists an equivalent private signal $\tilde{Y}^i$ defined as

(B-12) $$\tilde{Y}^i = \Gamma Y^i = \Gamma \Lambda^i V + \Gamma \varepsilon^i,$$

where Γ is a diagonal, non-singular matrix. The variance-covariance matrix of the new error term, $\mathrm{Var}(\Gamma \varepsilon^i)$, is still diagonal and given by $\Gamma \Sigma^i \Gamma$.

As in Step 1 of the proof, this new private signal also satisfies the learning capacity constraint. Since $(\Lambda^i)^\top (\Sigma^i)^{-1} \Lambda^i = (\Gamma \Lambda^i)^\top (\Gamma \Sigma^i \Gamma)^{-1} (\Gamma \Lambda^i)$ and $(\Lambda^i)^\top (\Sigma^i)^{-1} Y^i = (\Gamma \Lambda^i)^\top (\Gamma \Sigma^i \Gamma)^{-1} \tilde{Y}^i$, it follows from Equations (B-3), (B-4), and (A-25) that $\tilde{Y}^i$ is an equivalent private signal. Therefore, we can normalize the first column of Λ^i to be a column of ones such that the matrix of the weights is given by

(B-13) $$\Lambda^i = \begin{pmatrix} 1 & w_1^i \\ 1 & w_2^i \end{pmatrix}.$$

Step 3: Optimization problem of an infinitesimal investor.

From Steps 1 and 2, for a given normalized matrix of Λ^i and Σ^i, Ω can be expressed as

(B-14) $$\Omega = \int_0^1 (\Lambda^i)^\top (\Sigma^i)^{-1} \Lambda^i di = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{12} & \pi_{22} \end{pmatrix},$$

where

(B-15)

$$\pi_{11} = \int_0^1 \left(\tau_1^i + \tau_2^i\right) di, \quad \pi_{12} = \int_0^1 \left(w_1^i \tau_1^i + w_2^i \tau_2^i\right) di, \quad \pi_{22} = \int_0^1 \left((w_1^i)^2 \tau_1^i + (w_2^i)^2 \tau_2^i\right) di.$$

From Equation (A-26), the elements of $R = \mathrm{E}\left(\hat{V}^i - P\right)$ and the elements of $Q = \mathrm{Var}\left(V - P\right)$ can be expressed in terms of π_{11}, π_{12}, and π_{22}. Therefore, investors' objective function (B-5) is equivalent to maximize the following expression

$$\text{(B-16)} \qquad \max_{\tau_1^i \geq 0, \tau_2^i \geq 0} \left(A_{11} + 2A_{12} w_1^i + A_{22} (w_1^i)^2\right) \tau_1^i + \left(A_{11} + 2A_{12} w_2^i + A_{22} (w_2^i)^2\right) \tau_2^i,$$

subject to the learning capacity (10),

$$\text{(B-17)} \qquad \left(\tau_{v,1}^{-1} + (w_1^i)^2 \tau_{v,2}^{-1}\right) \tau_1^i + \left(\tau_{v,1}^{-1} + (w_2^i)^2 \tau_{v,2}^{-1}\right) \tau_2^i + \tau_{v,1}^{-1} \tau_{v,2}^{-1} (w_2^i - w_1^i)^2 \tau_1^i \tau_2^i = K - 1,$$

where A_{11}, A_{12}, and A_{22} are as defined in Equation (A-27) and can also be expressed in terms of π_{11}, π_{12}, and π_{22}.

Denote a_1^i and a_2^i as the coefficients of τ_1^i and τ_2^i, respectively, in Equation (B-16). Similarly, denote b_1^i, b_2^i, and b_3^i as the coefficients of τ_1^i, τ_2^i, and $\tau_1^i \tau_2^i$, respectively, in Equation (B-17). This optimization problem can be written as

$$\text{(B-18)} \qquad \max_{\tau_1^i \geq 0, \tau_2^i \geq 0} a_1^i \tau_1^i + a_2^i \tau_2^i, \quad \text{subject to} \quad b_1^i \tau_1^i + b_2^i \tau_2^i + b_3^i \tau_1^i \tau_2^i = K - 1.$$

Solving for τ_2^i from the constraint and substituting it into the objective function, the maximization problem can be rewritten as

$$\text{(B-19)} \qquad \max_{0 \leq \tau_1^i \leq (K-1)/b_1^i} a_1^i \tau_1^i + a_2^i \left(\frac{K - 1 - b_1^i \tau_1^i}{b_2^i + b_3^i \tau_1^i}\right).$$

It is straightforward to show that if $b_3^i > 0$, then the objective function in Equation (B-19) is a strictly convex. This implies that the solution is a corner solution, meaning either $\tau_1^i = 0$ or $\tau_2^i = 0$.

In this case, investors allocate all their attention to a single linear combination of asset payoffs. If $b_3^i = 0$, then $w_1^i = w_2^i$ and investors are indifferent between any two private signals with identical weights on asset payoffs. This also implies that investors choose to learn about one linear combination of asset payoffs. Therefore, the weight matrix Λ^i in the private signal reduces to a 1×2 matrix,

(B-20) $$\Lambda^i = \begin{pmatrix} 1 & \omega^i \end{pmatrix},$$

and the error variance matrix reduces to the scalar $(\tau^i)^{-1}$.

$\square$

B.2. Endogenous Learning Capacity K

In this subsection, we endogenize the learning capacity $K > 1$ by introducing a quadratic cost function: $\frac{1}{2}c(K-1)^2$, where c is a positive constant. From the proof of Theorem 1, under separative learning, the optimal precision of investor i's signal about asset j is given by:

(B-21) $$\tau_j^i = \begin{cases} (K-1)\tau_{v,j} & \text{if} \quad \text{ASSR}_j = \max\{\text{ASSR}_1, \text{ASSR}_2\}, \\ 0 & \text{if} \quad \text{ASSR}_j \neq \max\{\text{ASSR}_1, \text{ASSR}_2\}. \end{cases}$$

For the integrative learning case, from the proof of Theorem 2, the information capacity constraint binds, implying:

(B-22) $$\tau^i = \frac{K-1}{\tau_{v,1}^{-1} + (\omega^i)^2 \tau_{v,2}^{-1}}.$$

Thus, we assume that the cost function is quadratic in $K-1$, since the precision is proportional to $K-1$ in both cases.

After incorporating the quadratic cost function, investor i's utility function becomes

$$\text{(B-23)} \qquad \lambda[W_0 + (\theta^i - \gamma^i)'(\hat{V}^i - P)] - \frac{\lambda^2}{2}(\theta^i - \gamma^i)'\hat{\Sigma}_v^i(\theta^i - \gamma^i) - \frac{1}{2}c(K-1)^2,$$

where $\hat{V}^i$ and $\hat{\Sigma}_v^i$ represent investor i's posterior mean and variance-covariance matrix of V, respectively. Since $c > 0$, the second order condition is satisfied, ensuring the existence of an optimal learning capacity K.

1. Separative Learning

From Equations (A-11) and (A-13), trader i's ex-ante expected utility can be written as:

$$\text{(B-24)} \qquad U_1^i = W_0 - 1 + \frac{1}{2}\sum_{j=1,2}\text{ASSR}_j\left(1 + \frac{\tau_{p,j}}{\tau_{v,j}}\right) + \frac{1}{2}\max\{\text{ASSR}_1, \text{ASSR}_2\}(K-1) - \frac{1}{2}c(K-1)^2,$$

where $\text{ASSR}_j = \text{E}[(V_j - P_j)^2]\tau_{v,j}$.

Trader i chooses the optimal K to maximize the expected utility U_1^i. Since $ASSR_j$ and $\tau_{p,j}$ depend on K only through the equilibrium price, and traders are price takers, the first-order condition does not apply to $ASSR_j$ and $\tau_{p,j}$. The first-order condition with respect to K therefore yields:

$$\text{(B-25)} \qquad K = 1 + \frac{\max\{\text{ASSR}_1, \text{ASSR}_2\}}{2c},$$

where the right-hand-side is itself a function of K. We will solve the optimal K numerically. Replacing $\max\{\text{ASSR}_1, \text{ASSR}_2\}$ with $2(K-1)c$ in Equation (B-24), we obtain trader i's ex-ante utility as:

$$\text{(B-26)} \qquad U_0^i = \arg\max_K \; U_1^i = \lambda W_0 - 1 + \frac{1}{2}\sum_{j=1,2}\text{ASSR}_j\left(1 + \frac{\tau_{p,j}}{\tau_{v,j}}\right) + \frac{1}{2}c(K-1)^2,$$

where $\tau_{p,j} = (K-1)^2 \Gamma_j^2 \tau_{v,j}^2 \tau_{z,j}/\lambda^2$. The ex-ante utility can be rewritten as:

(B-27) $$U_0^i = \lambda W_0 - 1 + \frac{1}{2} \sum_{j=1,2} \text{ASSR}_j + \frac{1}{2}(K-1)^2 \left[c + \frac{1}{\lambda^2} \sum_{j=1,2} \text{ASSR}_j \Gamma_j^2 \tau_{v,j} \tau_{z,j} \right].$$

From Theorem 1, if $\text{ASSR}_1 > \text{ASSR}_2$ holds for any $\Gamma_1 \in [0,1]$, then $\Gamma_1 = 1$ and $\Gamma_2 = 0$. If $\text{ASSR}_1 = \text{ASSR}_2$ holds at some $\Gamma_1^* \in (0,1)$, then $\Gamma_1 = \Gamma_1^*$ and $\Gamma_2 = 1 - \Gamma_1^*$. If $\text{ASSR}_1 < \text{ASSR}_2$ holds for any $\Gamma_1 \in [0,1]$, then $\Gamma_1 = 0$ and $\Gamma_2 = 1$. It follows that, Equation (B-27) can be written as:

(B-28) $$U_0^i = \lambda W_0 - 1 + \frac{1}{2} \sum_{j=1,2} \text{ASSR}_j + \frac{1}{2} c(K-1)^2 + \frac{c(K-1)^3}{\lambda^2} \left(\Gamma_1^2 \tau_{v,1} \tau_{z,1} + \Gamma_2^2 \tau_{v,2} \tau_{z,2} \right),$$

where K is the optimally solved capacity constraint.

2. Integrative Learning

From the proof of Theorem 2, Equation (A-34), we know that the optimal attention allocation ω^i is given as:

(B-29) $$\omega^i = \frac{\text{ASSR}_2 - \text{ASSR}_1}{2A_{12}\tau_{v,1}} + \sqrt{\left(\frac{\text{ASSR}_2 - \text{ASSR}_1}{2A_{12}\tau_{v,1}} \right)^2 + \frac{\tau_{v,2}}{\tau_{v,1}}},$$

where $\text{ASSR}_j = \text{E}[(V_j - P_j)^2]/\text{Var}(V_j)$ and $A_{ij} = \text{E}[(V_i - P_i)(V_j - P_j)]$. Therefore, ω^i depends on learning capacity K through equilibrium price P_j.

From Equation (A-29), after incorporating the quadratic cost function, trader i's utility function becomes:

(B-30) $$\max_K \; A_0 + \frac{1}{2} \left(\tau_{v,1}^{-1} + (\omega^i)^2 \tau_{v,2}^{-1} \right)^{-1} \left[A_{22}(\omega^i)^2 + 2A_{12}\omega^i + A_{11} \right] (K-1) - \frac{1}{2} c(K-1)^2,$$

where

(B-31) $$\text{A}_0 = \lambda W_0 - 1 + \frac{1}{2} \text{Tr}\left((\Sigma_v^{-1} + \lambda^{-2} \Omega \Sigma_z^{-1} \Omega) Q \right) + \frac{1}{2} R' (\Sigma_v^{-1} + \lambda^{-2} \Omega \Sigma_z^{-1} \Omega) R,$$

depends on K only through the equilibrium price, and traders are price takers, the first-order condition does not apply to A_0, A_{11}, A_{12}, A_{22}, or ASSR_j. The first-order condition with respect to K therefore yields:

$$K = 1 + \frac{1}{2c}\frac{A_{22}(\omega^i)^2 + 2A_{12}\omega^i + A_{11}}{\tau_{v,1}^{-1} + (\omega^i)^2\tau_{v,2}^{-1}}, \tag{B-32}$$

where the right-hand-side is itself a function of K. We focus on the symmetric equilibrium with

$$\tau^i = \tau = \frac{K-1}{\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}}, \quad \omega^i = \omega = \alpha + \sqrt{\alpha^2 + \frac{\tau_{v,2}}{\tau_{v,1}}}, \tag{B-33}$$

where

$$\alpha = \frac{\tau_{v,2}^{-1}\tau_{z,2}^{-1} + \tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2 - \tau_{v,1}^{-1}\tau_{z,1}^{-1} - \tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2}{2(\bar{z}_1 - \bar{\gamma}_1)(\bar{z}_2 - \bar{\gamma}_2)\tau_{v,2}^{-1}}. \tag{B-34}$$

We solve for the optimal K numerically in the following subsection. Substituting Equation (B-32) into Equation (B-30) yields:

$$U_0^i = A_0 + \frac{1}{2}c(K-1)^2. \tag{B-35}$$

It can be shown that

$$\begin{aligned} A_0 &= \lambda W_0 - 1 + \frac{1}{2}\sum_{j=1,2}\text{ASSR}_j + \frac{1}{2}\lambda^{-2}\tau^2(\tau_{z,1} + \omega^2\tau_{z,2})\left(A_{22}\omega^2 + 2A_{12}\omega + A_{11}\right) \\ &= \lambda W_0 - 1 + \frac{1}{2}\sum_{j=1,2}\text{ASSR}_j + \frac{c(K-1)^3}{\lambda^2}\frac{\tau_{z,1} + \omega^2\tau_{z,2}}{\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}}. \end{aligned} \tag{B-36}$$

Therefore, the ex-ante utility function (B-35) can be rewritten as

$$U_0^i = \lambda W_0 - 1 + \frac{1}{2}\sum_{j=1,2}\text{ASSR}_j + \frac{1}{2}c(K-1)^2 + \frac{c(K-1)^3}{\lambda^2}\frac{\tau_{z,1} + \omega^2\tau_{z,2}}{\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}}. \tag{B-37}$$

B.3. The Effects of Benchmarking on ASSR_j and A_{12}

We next show $\frac{\partial \text{ASSR}_j}{\partial \bar{\gamma}_j} < 0$, $\frac{\partial \text{ASSR}_j}{\partial \bar{\gamma}_{-j}} > 0$. We also show that $\frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_j} < 0$ for some special cases.

Define b_1, b_2, b_3, and Det as,

$$b_1 := \tau + \tau_{v,1} + \tfrac{\tau^2}{\lambda^2}(\tau_{z,1} + \omega^2\tau_{z,2}),\ \ b_2 := \omega\tau + \omega\tfrac{\tau^2}{\lambda^2}(\tau_{z,1} + \omega^2\tau_{z,2}),\ \ b_3 := \tau_{v,2} + \omega^2\tau + \omega^2\tfrac{\tau^2}{\lambda^2}(\tau_{z,1} + \omega^2\tau_{z,2}),$$

$$\text{Det} := (\tau + \tau_{v,1})\tau_{v,2} + \tfrac{\tau^2}{\lambda^2}\left(\tau_{v,2}\tau_{z,1} + \omega^4\tau_{v,1}\tau_{z,2}\right) + \omega^2\tau\left(\tau_{v,1} + \tfrac{\tau}{\lambda^2}\left(\tau_{v,1}\tau_{z,1} + \tau_{v,2}\tau_{z,2}\right)\right).$$

Note that, as each investor is atomic, we have $\frac{\partial\Omega}{\partial\omega^i} = 0$. Even though $\bar{\gamma}_j$ affects the equilibrium ω, it does not affect Ω through each investor's choice of ω^i. Hence, by equation (A-26), we have $\frac{\partial Q(\Omega)}{\partial\bar{\gamma}_j} = Q'(\Omega)\frac{\partial\Omega}{\partial\bar{\gamma}_j} = 0$. It can be shown that

$$\text{(B-38)} \qquad R = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\lambda(\bar{z} - \bar{\gamma}) = \frac{\lambda}{\text{Det}}\begin{pmatrix} b_3(\bar{z}_1 - \bar{\gamma}_1) - b_2(\bar{z}_2 - \bar{\gamma}_2) \\ -b_2(\bar{z}_1 - \bar{\gamma}_1) + b_1(\bar{z}_2 - \bar{\gamma}_2) \end{pmatrix},$$

$$\text{(B-39)} \qquad \frac{\partial R_1}{\partial\bar{\gamma}_1} = -\frac{\lambda b_3}{\text{Det}} < 0, \quad \frac{\partial R_1}{\partial\bar{\gamma}_2} = \frac{\lambda b_2}{\text{Det}} > 0, \quad \frac{\partial R_2}{\partial\bar{\gamma}_1} = \frac{\lambda b_2}{\text{Det}} > 0, \quad \frac{\partial R_2}{\partial\bar{\gamma}_2} = -\frac{\lambda b_1}{\text{Det}} < 0.$$

From equation (A-27), $\text{ASSR}_j = \left(\text{Var}(V_j - P_j) + R_j^2\right)\tau_{v,j}$, which implies

$$\text{(B-40)} \qquad \frac{\partial\text{ASSR}_j}{\partial\bar{\gamma}_j} = 2\tau_{v,j}R_j\frac{\partial R_j}{\partial\bar{\gamma}_j} < 0, \qquad \frac{\partial\text{ASSR}_j}{\partial\bar{\gamma}_{-j}} = 2\tau_{v,j}R_j\frac{\partial R_j}{\partial\bar{\gamma}_{-j}} > 0,$$

$$\text{(B-41)} \qquad \frac{\partial(\text{ASSR}_1 - \text{ASSR}_2)}{\partial\bar{\gamma}_1} < 0, \qquad \frac{\partial(\text{ASSR}_1 - \text{ASSR}_2)}{\partial\bar{\gamma}_2} > 0.$$

By equation (A-27), we have

$$\text{(B-42)} \qquad \frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_1} = R_1\frac{\partial R_2}{\partial\bar{\gamma}_1} + \frac{\partial R_1}{\partial\bar{\gamma}_1}R_2 = \frac{\lambda^2}{\text{Det}^2}\left[2b_2b_3(\bar{z}_1 - \bar{\gamma}_1) - (b_1b_3 + b_2^2)(\bar{z}_2 - \bar{\gamma}_2)\right],$$

$$\text{(B-43)} \qquad \frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_2} = R_1\frac{\partial R_2}{\partial\bar{\gamma}_2} + \frac{\partial R_1}{\partial\bar{\gamma}_2}R_2 = \frac{\lambda^2}{\text{Det}^2}\left[-(b_1b_3 + b_2^2)(\bar{z}_1 - \bar{\gamma}_1) + 2b_1b_2(\bar{z}_2 - \bar{\gamma}_2)\right].$$

Unlike in the separative learning scenario, when it comes to integrative learning, the benchmarking of one asset influences its expected return negatively, while simultaneously impacting the expected return of the other asset positively. Therefore, the overall effect of benchmarking on A_{12} hinges on the relative dominance of these contrasting impacts.

For cases with $\omega \to 0$ or $\omega \to +\infty$, we can show $b_2 \to 0$, $b_1 > 0$ and $b_3 > 0$ and thus

$$\lim_{\omega\to 0 \text{ or } \omega\to+\infty} \frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_1} = -\frac{\lambda^2 b_1 b_3}{\mathrm{Det}^2}(\bar{z}_2 - \bar{\gamma}_2) < 0, \qquad \lim_{\omega\to 0 \text{ or } \omega\to+\infty} \frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_2} = -\frac{\lambda^2 b_1 b_3}{\mathrm{Det}^2}(\bar{z}_1 - \bar{\gamma}_1) < 0.$$

For the case where the risk profiles of two assets are identical: $\tau_{v,1} = \tau_{v,2}$, $\tau_{z,1} = \tau_{z,2}$, and $\bar{z}_1 - \bar{\gamma}_1 = \bar{z}_2 - \bar{\gamma}_2$, we have $\omega = 1$, $b_1 = b_3$, and then

$$\frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_1} = \frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_2} = -\frac{\lambda^2(\bar{z}_1 - \bar{\gamma}_1)}{\mathrm{Det}^2}(b_1 - b_2)^2 < 0.$$

From our extensive numerical analysis, we find that the negative effect of benchmarking on its own expected return dominates, and $\frac{\mathrm{d}A_{12}}{\mathrm{d}\bar{\gamma}_j} < 0$ holds for a wide range of parameter values.

B.4. Proof of Proposition A-1

Since both V and P are vectors of Gaussian random variables, the entropy measure of informational efficiency in the integrative learning case is given by

$$\text{(B-44)} \qquad \mathrm{I}(V;P)_{\text{int}} = \frac{1}{2}\ln\left(\frac{|\mathrm{Var}(V)|}{|\mathrm{Var}(V|P)|}\right) = \frac{1}{2}\ln\left(\frac{|\Sigma_v|}{|\Sigma_v - \Theta|}\right),$$

where $\Theta := \mathrm{Var}(V) - \mathrm{Var}(V|P)$ is the variance reduction for the payoff vector V conditional on the price vector P, as given in equation (A-53). We derive and find a simple expression for the determinant of variance-covariance matrix $\mathrm{Var}(V|P)$, i.e.,

$$\text{(B-45)} \qquad |\Sigma_v - \Theta| = \left(\tau_{v,1}^{-1} - \Theta_{11}\right)\left(\tau_{v,2}^{-1} - \Theta_{22}\right) - \Theta_{12}\Theta_{21} = \frac{1}{\tau_{v,1}\tau_{v,2}}\left(1 + \left(\frac{K-1}{\lambda}\right)^2 \frac{\tau_{z,1} + \omega^2\tau_{z,2}}{\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}}\right)^{-1}.$$

This leads to equation (A-57). Using equations (A-54) and (B-45), we have

$$\mathrm{I}(V;P)_{\text{int}} - \mathrm{I}(V_1;P)_{\text{int}} - \mathrm{I}(V_2;P)_{\text{int}} = \mathrm{H}(V_1|P_1,P_2)_{\text{int}} + \mathrm{H}(V_2|P_1,P_2)_{\text{int}} - \mathrm{H}(V|P)_{\text{int}}$$
$$= \frac{1}{2}\ln\left(\mathrm{Var}(V_1|P_1,P_2)\mathrm{Var}(V_2|P_1,P_2)\right) - \frac{1}{2}\ln\left(|\mathrm{Var}(V|P)|\right)$$

$$= \frac{1}{2}\ln\left((\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22})\right) - \frac{1}{2}\ln\left((\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22}) - \Theta_{12}\Theta_{21}\right)$$

$$= -\frac{1}{2}\ln\left(1 - \frac{\Theta_{12}^2}{(\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22})}\right), \tag{B-46}$$

where the last step follows from the fact that Θ is a symmetric non-diagonal matrix such that $\Theta_{12}\Theta_{21} = \Theta_{12}^2 > 0$. We find the explicit expression

$$\frac{\Theta_{12}\Theta_{21}}{(\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22})} = \left(1 + \frac{\lambda^4(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1})^4 + (K-1)^2\lambda^2(\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1})^3(\tau_{z,1}+\omega^2\tau_{z,2})}{(K-1)^4\omega^2\tau_{v,1}^{-1}\tau_{v,2}^{-1}(\tau_{z,1}+\omega^2\tau_{z,2})^2}\right)^{-1}, \tag{B-47}$$

which is bounded between 0 and 1, indicating that $\ln\left(1 - \frac{\Theta_{12}^2}{(\tau_{v,1}^{-1}-\Theta_{11})(\tau_{v,2}^{-1}-\Theta_{22})}\right) < 0$ in (B-46). Thus the first inequality holds, $\mathrm{I}(V;P)_{\text{int}} > \mathrm{I}(V_1;P)_{\text{int}} + \mathrm{I}(V_2;P)_{\text{int}}$. Since $\mathrm{Var}(V_j|P_1,P_2) < \mathrm{Var}(V_j|P_j)$ for each asset j, it follows that $\mathrm{I}(V_j;P)_{\text{int}} > \mathrm{I}(V_j;P_j)_{\text{int}}$. Combining both inequalities, we have

$$\mathrm{I}(V;P)_{\text{int}} > \mathrm{I}(V_1;P)_{\text{int}} + \mathrm{I}(V_2;P)_{\text{int}} > \mathrm{I}(V_1;P_1)_{\text{int}} + \mathrm{I}(V_2;P_2)_{\text{int}}. \tag{B-48}$$

Next, we prove $\mathrm{I}(V;P)_{\text{int}} = \mathrm{I}(V_1+\omega V_2;P)_{\text{int}}$. Equation (20) can be written as

$$P = C + B\left(\Omega V - \lambda(Z-\bar{z})\right) = C + B\left(\tau\Lambda(V_1+\omega V_2) - \lambda(Z-\bar{z})\right), \qquad \Lambda = (1,\omega)'. \tag{B-49}$$

Thus, the price $P = (P_1,P_2)'$ is informational equivalent to a signal vector $s_p = (s_{p,1}, s_{p,2})'$ where

$$s_{p,1} = V_1 + \omega V_2 - \frac{\lambda}{\tau}(Z_1 - \bar{z}_1) = V_\omega - \frac{\lambda}{\tau}(Z_1-\bar{z}_1), \tag{B-50}$$

$$s_{p,2} = V_1 + \omega V_2 - \frac{\lambda}{\omega\tau}(Z_2 - \bar{z}_2) = V_\omega - \frac{\lambda}{\omega\tau}(Z_2-\bar{z}_2). \tag{B-51}$$

Using the notation $V_\omega := V_1 + \omega V_2 \sim \mathcal{N}(\bar{v}_1 + \omega\bar{v}_2, \tau_{v,\omega}^{-1})$ and $\tau_{v,\omega}^{-1} := \tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}$, we can derive

$$\mathrm{I}(V_1+\omega V_2;P)_{\text{int}} = \mathrm{I}(V_\omega;s_p)_{\text{int}} = \frac{1}{2}\ln\left(\frac{\mathrm{Var}(V_\omega)}{\mathrm{Var}(V_\omega|s_{p,1},s_{p,2})}\right)$$

$$= \frac{1}{2}\ln\left(\tau_{v,\omega}^{-1}(\tau_{v,\omega} + \tau^2\tau_{z,1}/\lambda^2 + \omega^2\tau^2\tau_{z,2}/\lambda^2)\right)$$

$$= \frac{1}{2}\ln\left(1 + \left(\frac{K-1}{\lambda}\right)^2 \frac{\tau_{z,1}+\omega^2\tau_{z,2}}{\tau_{v,1}^{-1}+\omega^2\tau_{v,2}^{-1}}\right) = \mathrm{I}(V;P)_{\text{int}}. \tag{B-52}$$

In a linear Gaussian model, $I(V;P)$ can be empirically estimated in three steps: (1) Regress the payoff vector V on the price vector P using multivariate linear regression: $V = AP + \varepsilon$; (2) Compute the covariance of residuals $\widehat{\mathrm{Var}}(V|P) = \frac{1}{N}\sum_{i=1}^{N} \hat{\varepsilon}^{(i)} \left(\hat{\varepsilon}^{(i)}\right)^{\top}$, where $\hat{\varepsilon}^{(i)} = V^{(i)} - AP^{(i)}$; (3) Compute $\widehat{I}(V;P) = \frac{1}{2}\ln\left(\left|\widehat{\mathrm{Var}}(V)\right| / \left|\widehat{\mathrm{Var}}(V|P)\right|\right)$ using the log of determinant ratio. As a multivariate analog to R^2, this information-theoretic measure captures the joint reduction in uncertainty across all payoffs due to observing prices.

In the separative learning equilibrium of Proposition 1,

(B-53) $$\mathrm{I}(V;P)_{\mathrm{sep}} = \frac{1}{2}\ln\left(1 + \Gamma_1^2\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,1}\tau_{z,1}\right) + \frac{1}{2}\ln\left(1 + \Gamma_2^2\left(\frac{K-1}{\lambda}\right)^2 \tau_{v,2}\tau_{z,2}\right).$$

This is exactly equal to the sum of price informativeness of individual assets:

(B-54) $$\mathrm{I}(V;P)_{\mathrm{sep}} = \mathrm{I}(V_1;P)_{\mathrm{sep}} + \mathrm{I}(V_2;P)_{\mathrm{sep}} = \mathrm{I}(V_1;P_1)_{\mathrm{sep}} + \mathrm{I}(V_2;P_2)_{\mathrm{sep}}.$$

From equation (A-57), we can calculate $\frac{\mathrm{dI}(V;P)_{\mathrm{int}}}{\mathrm{d}\gamma_j}$ and $\frac{\mathrm{dI}(V;P)_{\mathrm{int}}}{\mathrm{d}m}$ as

(B-55) $$\frac{\mathrm{dI}(V;P)_{\mathrm{int}}}{\mathrm{d}\gamma_j} = \frac{\mathrm{dI}(V;P)_{\mathrm{int}}}{\mathrm{d}\omega}\frac{\mathrm{d}\omega}{\mathrm{d}\gamma_j} = -C_I\,\Delta\,\frac{\mathrm{d}\omega}{\mathrm{d}\gamma_j}, \qquad \frac{\mathrm{dI}(V;P)_{\mathrm{int}}}{\mathrm{d}m} = -C_I\,\Delta\,\frac{\mathrm{d}\omega}{\mathrm{d}m}$$

and the constant

(B-56) $$C_I := \frac{\omega\tau_{z,1}\tau_{z,2}}{\left(\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}\right)\left(\tau_{z,1} + \omega^2\tau_{z,2}\right) + \left(\frac{\lambda}{K-1}\right)^2\left(\tau_{v,1}^{-1} + \omega^2\tau_{v,2}^{-1}\right)^2}.$$

Proposition A-1 then follows directly from Proposition 3.

The real attention ω^* always shifts toward the asset with higher uncertainty:

(B-57) $$\frac{\mathrm{d}\omega^*}{\mathrm{d}\bar{\gamma}_j} > 0 \quad \text{if and only if} \quad \Delta > 0,$$

the equality $\frac{\mathrm{d}\omega^*}{\mathrm{d}\bar{\gamma}_j} = 0$ holds when $\Delta = 0$.

Proof. From equation (24) and the definition of real attention $\omega^* = \omega \cdot \frac{\bar{z}_1 - \bar{\gamma}_1}{\bar{z}_2 - \bar{\gamma}_2}$, we can derive

$$\begin{aligned}
\frac{\mathrm{d}\omega^*}{\mathrm{d}\bar{\gamma}_1} &= \frac{\Delta - \xi + \sqrt{4\tau_{v,1}^{-1}\tau_{v,2}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2(\bar{z}_2 - \bar{\gamma}_2)^2 + (\Delta + \xi_-)^2}}{\sqrt{4\tau_{v,1}^{-1}\tau_{v,2}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2(\bar{z}_2 - \bar{\gamma}_2)^2 + (\Delta + \xi_-)^2}} \frac{\tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)}{\tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2} \\
&= \left(1 + \frac{\Delta - \xi}{\sqrt{\xi^2 + 2\Delta\xi_- + \Delta^2}}\right) \frac{\tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)}{\tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2}, \qquad \text{(B-58)}
\end{aligned}$$

where

$$\xi := \tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2 + \tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2 \ \text{ and } \xi_- := \tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2 - \tau_{v,2}^{-1}(\bar{z}_2 - \bar{\gamma}_2)^2.$$

From equation (B-58), when $\Delta = 0$ (and thus $\omega^* = 1$), $\frac{\mathrm{d}\omega^*}{\mathrm{d}\bar{\gamma}_1} = 0$. It can be shown that

$$(\Delta - \xi)^2 - (\xi^2 + 2\Delta\xi_- + \Delta^2) = -2\Delta(\xi + \xi_-) = -4\Delta\tau_{v,1}^{-1}(\bar{z}_1 - \bar{\gamma}_1)^2. \qquad \text{(B-59)}$$

Therefore, $|\Delta - \xi| < \sqrt{\xi^2 + 2\Delta\xi_- + \Delta^2}$ if and only if $\Delta > 0$. Since $\xi > 0$, we have

$$-1 < \frac{\Delta - \xi}{\sqrt{\xi^2 + 2\Delta\xi_- + \Delta^2}} < 1 \quad \text{if} \quad \Delta > 0, \qquad \frac{\Delta - \xi}{\sqrt{\xi^2 + 2\Delta\xi_- + \Delta^2}} < -1 \quad \text{if} \quad \Delta < 0. \qquad \text{(B-60)}$$

With positive effective supplies, we thus obtain the monotonic dependence of ω^* on $\bar{\gamma}_1$,

$$\frac{\mathrm{d}\omega^*}{\mathrm{d}\bar{\gamma}_1} > 0 \quad \text{if} \quad \Delta > 0, \qquad \frac{\mathrm{d}\omega^*}{\mathrm{d}\bar{\gamma}_1} < 0 \quad \text{if} \quad \Delta < 0. \qquad \text{(B-61)}$$

Similar arguments can be applied to asset 2, leading to the same pattern,

$$\frac{\mathrm{d}\omega^*}{\mathrm{d}\bar{\gamma}_2} > 0 \quad \text{if} \quad \Delta > 0, \qquad \frac{\mathrm{d}\omega^*}{\mathrm{d}\bar{\gamma}_2} < 0 \quad \text{if} \quad \Delta < 0. \qquad \text{(B-62)}$$

□

B.5. Model Implications on Return Comovements

To enhance our understanding of the impact of benchmarking on asset comovements under different learning technologies, we modify the original model by treating one asset as a common risk factor and the other as an idiosyncratic risk factor. The payoffs of these factors are independent, represented as V_C for the common risk and V_I for the idiosyncratic risk. We consider two new risky assets, denoted as a and b. Their payoffs are defined as

$$V_a = V_C + V_I, \qquad V_b = V_C.$$

This formulation allows us to analyze how different learning mechanisms influence the return correlation of these assets. Under separative learning, investors can only observe factor-specific private signals. The asset return correlation is

$$\mathrm{Corr}(V_a - P_a, V_b - P_b)_{\mathrm{sep}} = \left(1 + \frac{\mathrm{Var}(V_I - P_I)}{\mathrm{Var}(V_C - P_C)}\right)^{-1/2}.$$

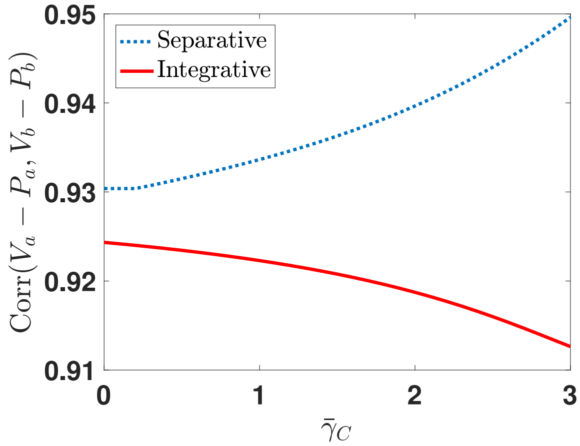
We find that a higher benchmarking level of the common risk factor (V_C) always leads to a lower attention allocated to this factor. The reduction in attention raises the return volatility of the common risk factor, $\mathrm{Var}(V_C - P_C)$. This heightened volatility, in turn, drives up the correlation of asset returns, $\mathrm{Corr}(V_a - P_a, V_b - P_b)_{\mathrm{sep}}$. Consequently, enhanced benchmarking of the common risk factor tends to increase the comovement of asset returns under separative learning.

Under integrative learning, investors optimally choose to observe a private signal about a linear combination of V_C and V_I. The correlation between asset returns is shown to be

$$\begin{aligned}\mathrm{Corr}(V_a - P_a, V_b - P_b)_{\mathrm{int}} = &\left(\sqrt{\frac{\mathrm{Var}(V_C - P_C)}{\mathrm{Var}(V_I - P_I)}} + \mathrm{Corr}(V_I - P_I, V_C - P_C)\right) \times \\ &\left(\left(\sqrt{\frac{\mathrm{Var}(V_C - P_C)}{\mathrm{Var}(V_I - P_I)}} + 2\mathrm{Corr}(V_I - P_I, V_C - P_C)\right)\sqrt{\frac{\mathrm{Var}(V_C - P_C)}{\mathrm{Var}(V_I - P_I)}} + 1\right)^{-1/2}.\end{aligned}$$

When the common risk factor becomes significantly more volatile than the idiosyncratic one, an increase in the benchmarking level of the common risk factor can induce investors to allocate more attention to it. This increased attention reduces the return volatility of the common risk

FIGURE B-1. Asset return correlation versus the benchmarking level of common risk factor $\bar{\gamma}_C$. Parameters values are: $\lambda = 1$, $K = 1.5$, $\tau_{v,C} = 0.5$, $\tau_{v,I} = \tau_{z,C} = \tau_{z,I} = 5$, $\bar{z}_C = \bar{z}_I = 5$, and $\bar{\gamma}_I = 3$.

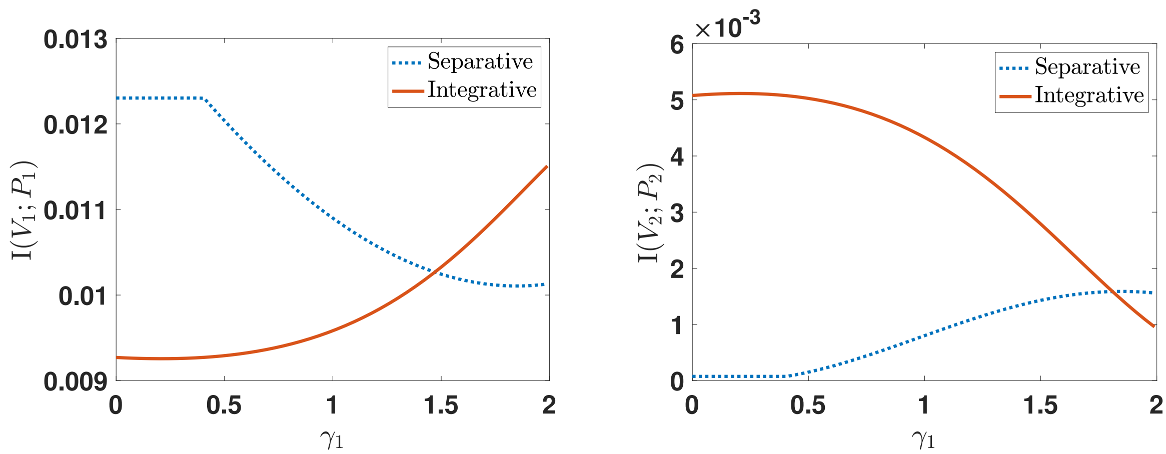


factor, $\mathrm{Var}(V_C - P_C)$ and the variance ratio, $\mathrm{Var}(V_C - P_C)/\mathrm{Var}(V_I - P_I)$. For a wide range of parameter values, a decrease in the variance ratio, $\mathrm{Var}(V_C - P_C)/\mathrm{Var}(V_I - P_I)$, is observed to reduce the correlation $\mathrm{Corr}(V_a - P_a, V_b - P_b)_{\mathrm{int}}$. As illustrated in Figure B-1, the comovement of asset returns can decrease with the benchmarking level of the common risk factor under integrative learning. This contrasts with the pattern observed under separative learning. Our finding suggests that in the context of integrative learning, an increase in the benchmarking level of the common risk factor can potentially mitigate asset comovement, especially during turbulent times when the common risk factor experiences heightened volatility.

B.6. Model Extension with Correlated Asset Payoffs

For clarity of exposition, we have thus far focused on a setting where asset payoffs are independent. It might be conjectured that the differences between the equilibria under separative learning and integrative learning would be minimal if asset payoffs were correlated. However, this is not the case. In this section, we expand our analysis to the general cases where asset payoffs are correlated. We have derived closed-form solutions and used numerical analysis to demonstrate that the implications under different learning technologies remain qualitatively different in the general case with correlated asset payoffs.

FIGURE B-2. Price Informativeness $\mathrm{I}(V_1; P_1)$ and $\mathrm{I}(V_2; P_2)$ under separative learning (dashed line) or integrative learning (solid line) versus asset 1's benchmarking level, $\bar{\gamma}_1$. The correlation of asset payoffs is $\rho = 0.2$. Other parameters are the same as those used in Figure 3.



In this extension, the primary change from our main model presented in Section II is the assumption of a correlation between V_1 and V_2, denoted as $\rho = \mathrm{Corr}(V_1, V_2)$. This modification does not affect the definitions of the two learning technologies or their respective forms of private information Y^i, as characterized by (11) and (19).

In the extreme case where asset payoffs become perfectly correlated ($\rho \to 1$), the two-asset economy converges to the one-asset economy, thereby eliminating any distinction between separative and integrative learning. Otherwise, our extensive numerical analysis shows that the differences between these two learning approaches remain significant if the correlation of asset payoffs is not extremely high. Figure B-2 presents the price informativeness, $\mathrm{I}(V_1; P_1)$ and $\mathrm{I}(V_2; P_2)$, against benchmarking level $\bar{\gamma}_1$ under either separative or integrative learning, with a payoff correlation $\rho = 0.2$. This example shows that, under separative learning, the price informativeness of an asset decreases in its benchmarking level. In contrast, under integrative learning, the price informativeness of an asset can rise with its benchmarking level. This aligns with our main results obtained in the case with independent asset payoffs.

The detailed analysis is given in the following subsections.

1. Separative Learning Equilibrium

Under separate learning technology, each investor i observes a signal about the payoff of one asset, $Y^i = \Lambda^i V + \varepsilon^i$, with $\varepsilon^i \sim \mathcal{N}(0, \Sigma^i)$, where

$$\Lambda^i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad \Sigma^i = \begin{pmatrix} (\tau_1^i)^{-1} & 0 \\ 0 & (\tau_2^i)^{-1} \end{pmatrix}.$$

In equilibrium, the vector of asset prices is $P = C + B\left(\Omega V - \lambda(Z - \bar{Z})\right)$, where

$$\Omega = \int_0^1 (\Lambda^i)^\top (\Sigma^i)^{-1} \Lambda^i di := \begin{pmatrix} \bar{\tau}_1 & 0 \\ 0 & \bar{\tau}_2 \end{pmatrix},$$

$$C = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1} \left(\Sigma_v^{-1}\bar{V} - \lambda(\bar{Z} - \bar{\gamma})\right), \quad B = \left(\Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1} \left(I_2 + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\right).$$

For each investor i, the time-1 utility function is

$$U_1^i = \lambda W_0^i - 1 + \frac{1}{2}\mathrm{Tr}\left[(\hat{\Sigma}_V^i)^{-1} Q\right] + \frac{1}{2} R^\top (\hat{\Sigma}_V^i)^{-1} R, \tag{B-63}$$

where

$$R := \mathrm{E}(V - P) = \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} \mathrm{E}(V_1 - P_1) \\ \mathrm{E}(V_2 - P_2) \end{pmatrix}, \tag{B-64}$$

$$Q := \mathrm{Var}(V - P) = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix} = \begin{pmatrix} \mathrm{Var}(V_1 - P_1) & \mathrm{Cov}(V_1 - P_1, V_2 - P_2) \\ \mathrm{Cov}(V_1 - P_1, V_2 - P_2) & \mathrm{Var}(V_2 - P_2) \end{pmatrix}, \tag{B-65}$$

$$(\hat{\Sigma}_v^i)^{-1} = \Sigma_v^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + (\Lambda^i)^\top(\Sigma^i)^{-1}\Lambda^i. \tag{B-66}$$

Investor i's information choice problem becomes

$$\max_{\tau_1^i,\tau_2^i} U_1^i, \quad \text{s.t.} \quad (\tau_1^i + \tau_{v,1})(\tau_2^i + \tau_{v,2}) - \rho^2\tau_1^i\tau_2^i \le K\tau_{v,1}\tau_{v,2}, \quad \tau_1^i \ge 0 \quad \text{and} \quad \tau_2^i \ge 0. \tag{B-67}$$

Given that each trader acts as a price taker, his information choice does not influence Ω, Q or R. Instead, it solely affects his own posterior variance, $(\hat{\Sigma}_v^i)^{-1}$, via the selection of his information matrix, $(\Sigma^i)^{-1}$. Therefore, the information choice problem can be simplified to

$$\begin{aligned} &\max_{\tau_1^i,\tau_2^i} \mathrm{E}(V_1 - P_1)^2\tau_1^i + \mathrm{E}(V_2 - P_2)^2\tau_2^i, \\ &\text{s.t.} \quad (\tau_1^i + \tau_{v,1})(\tau_2^i + \tau_{v,2}) - \rho^2\tau_1^i\tau_2^i \le K\tau_{v,1}\tau_{v,2}, \quad \tau_1^i \ge 0 \quad \text{and} \quad \tau_2^i \ge 0, \end{aligned} \tag{B-68}$$

which has corner solutions

$$\tau_1^i = (K-1)\tau_{v,1}, \quad \tau_2^i = 0, \quad \text{if} \quad \mathrm{ASSR}_1 \ge \mathrm{ASSR}_2, \tag{B-69}$$

$$\tau_1^i = 0, \quad \tau_2^i = (K-1)\tau_{v,2}, \quad \text{if} \quad \mathrm{ASSR}_1 \le \mathrm{ASSR}_2, \tag{B-70}$$

$$\text{with} \quad \mathrm{ASSR}_1 = \frac{\mathrm{E}(V_1 - P_1)^2}{\mathrm{Var}(V_1)} = A_{11}\tau_{v,1}, \quad \mathrm{ASSR}_2 = \frac{\mathrm{E}(V_2 - P_2)^2}{\mathrm{Var}(V_2)} = A_{22}\tau_{v,2}, \tag{B-71}$$

where $A_{11} = \mathrm{E}(V_1 - P_1)^2$ and $A_{22} = \mathrm{E}(V_2 - P_2)^2$. By solving $A_{11}(\mathit{\Gamma})\tau_{v,1} = A_{22}(1-\mathit{\Gamma})\tau_{v,2}$, for the endogenous fraction of investors who learn about asset-1 only ($\mathit{\Gamma}$), we obtain the equilibrium information choice

$$\begin{aligned} &\tau_1^i = (K-1)\tau_{v,1}, \quad \tau_2^i = 0, \quad \text{for} \quad i \in [0, \mathit{\Gamma}], \\ &\tau_1^i = 0, \quad \tau_2^i = (K-1)\tau_{v,2}, \quad \text{for} \quad i \in (\mathit{\Gamma}, 1]. \end{aligned} \tag{B-72}$$

This result is similar to Theorem 1. The total precisions of the signals about assets 1 and 2 are:

(B-73) $$\bar{\tau}_1 = \mathit{\Gamma}(K-1)\tau_{v,1} \quad \text{and} \quad \bar{\tau}_2 = (1-\mathit{\Gamma})(K-1)\tau_{v,2}.$$

Then we calculate both R and Q as functions of $\mathit{\Gamma}$, given by

(B-74) $$R = \left(\Sigma_V^{-1} + \frac{1}{\lambda^2}\Omega\Sigma_z^{-1}\Omega + \Omega\right)^{-1}\lambda(\bar{Z}-\bar{\gamma}), \quad Q = (I_2 - B\Omega)\Sigma_v(I_2 - B\Omega)^\top + \lambda^2 B\Sigma_z B^\top.$$

Although the results of separative learning with correlated asset payoffs resemble those in scenarios without such correlation, key differences become apparent in the equilibrium price. Due to the presence of non-zero off-diagonal elements in matrix B, both V_2 and Z_2 influence P_1, and similarly, both V_1 and Z_1 influence P_2. This implies that P_1 and P_2 are correlated. Denote

$$C := \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \quad \text{and} \quad B := \begin{pmatrix} B_{11} & B_{12} \\ B_{12} & B_{22} \end{pmatrix}, \text{ then}$$

$$P_1 = C_1 + B_{11}\bar{\tau}_1 V_1 + B_{12}\bar{\tau}_2 V_2 - \lambda B_{11}(Z_1 - \bar{Z}_1) - \lambda B_{12}(Z_2 - \bar{Z}_2),$$

$$P_2 = C_2 + B_{12}\bar{\tau}_1 V_1 + B_{22}\bar{\tau}_2 V_2 - \lambda B_{12}(Z_1 - \bar{Z}_1) - \lambda B_{22}(Z_2 - \bar{Z}_2).$$

Therefore, the price informativeness of each asset can be expressed as

(B-75) $$\mathrm{I}(V_1, P_1) = -\frac{1}{2}\ln\left(1 - \mathrm{Corr}^2(V_1, P_1)\right), \qquad \mathrm{I}(V_2, P_2) = -\frac{1}{2}\ln\left(1 - \mathrm{Corr}^2(V_2, P_2)\right),$$

and the correlations are given as

$$\mathrm{Corr}^2(V_1, P_1) = \frac{(B_{11}\bar{\tau}_1\sigma_1^2 + B_{12}\bar{\tau}_2\rho\sigma_1\sigma_2)^2}{\sigma_1^2\left[B_{11}^2\bar{\tau}_1^2\sigma_1^2 + B_{12}^2\bar{\tau}_2^2\sigma_2^2 + 2B_{11}B_{12}\bar{\tau}_1\bar{\tau}_2\rho\sigma_1\sigma_2 + \lambda^2\left(B_{11}^2/\tau_{Z,1} + B_{12}^2/\tau_{Z,2}\right)\right]},$$
$$\mathrm{Corr}^2(V_2, P_2) = \frac{(B_{22}\bar{\tau}_2\sigma_2^2 + B_{12}\bar{\tau}_1\rho\sigma_1\sigma_2)^2}{\sigma_2^2\left[B_{22}^2\bar{\tau}_2^2\sigma_2^2 + B_{12}^2\bar{\tau}_1^2\sigma_1^2 + 2B_{12}B_{22}\bar{\tau}_1\bar{\tau}_2\rho\sigma_1\sigma_2 + \lambda^2\left(B_{12}^2/\tau_{Z,1} + B_{22}^2/\tau_{Z,2}\right)\right]},$$

where $\sigma_1^2 := 1/\tau_{v,1}$ and $\sigma_2^2 := 1/\tau_{v,2}$.

2. Integrative Learning Equilibrium

In the integrative learning context, the private signal is represented as $Y^i = V_1 + \omega^i V_2 + \varepsilon^i$, where $\varepsilon^i \sim \mathcal{N}(0, \Sigma^i)$, and Σ^i is a scalar. For computational simplicity, we define

$$F := \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = T^{-1} V \sim \mathcal{N}\left(\bar{F}, \Sigma_F\right), \text{ where} \tag{B-76}$$

$$T = \begin{pmatrix} 1 & 0 \\ \frac{\rho\sigma_2}{\sigma_1} & 1 \end{pmatrix}, \quad \bar{F} = \begin{pmatrix} \bar{F}_1 \\ \bar{F}_2 \end{pmatrix} = \begin{pmatrix} \bar{V}_1 \\ -\frac{\rho\sigma_2}{\sigma_1}\bar{V}_1 + \bar{V}_2 \end{pmatrix}, \quad \Sigma_F = \begin{pmatrix} \tau_{F,1}^{-1} & 0 \\ 0 & \tau_{F,2}^{-1} \end{pmatrix}, \tag{B-77}$$

and $\tau_{F,1}^{-1} = \tau_{v,1}^{-1} = \sigma_1^2$, $\tau_{F,2}^{-1} = \tau_{v,2}^{-1}(1-\rho^2) = \sigma_2^2(1-\rho^2)$, $\mathrm{Cov}(F_1, F_2) = 0$. The risk factor F_1 can be interpreted as the common risk factor, while F_2 represents the idiosyncratic risk factor,

$$V_1 = F_1, \quad V_2 = \frac{\rho\sigma_2}{\sigma_1} F_1 + F_2. \tag{B-78}$$

We use the superscript "f" to label quantities for these risk factors. Their effective supplies are

$$Z^f = T^\top (Z - \bar{\gamma}) = \begin{pmatrix} (Z_1 - \bar{\gamma}_1) + \frac{\rho\sigma_2}{\sigma_1}(Z_2 - \bar{\gamma}_2) \\ Z_2 - \bar{\gamma}_2 \end{pmatrix} \sim \mathcal{N}\left(\bar{Z}^f, \Sigma_z^f\right), \tag{B-79}$$

where

$$\bar{Z}^f = \begin{pmatrix} (\bar{Z}_1 - \bar{\gamma}_1) + \frac{\rho\sigma_2}{\sigma_1}(\bar{Z}_2 - \bar{\gamma}_2) \\ \bar{Z}_2 - \bar{\gamma}_2 \end{pmatrix}, \qquad \Sigma_z^f = \begin{pmatrix} \tau_{z,1}^{-1} + \frac{\rho^2\sigma_2^2}{\sigma_1^2}\tau_{z,2}^{-1} & \frac{\rho\sigma_2}{\sigma_1}\tau_{z,2}^{-1} \\ \frac{\rho\sigma_2}{\sigma_1}\tau_{z,2}^{-1} & \tau_{z,2}^{-1} \end{pmatrix}. \tag{B-80}$$

Therefore, investor i's private information Y^i is equivalent to $Y^i = \Lambda^i F + \epsilon^i$, where

$$\Lambda^i = \begin{pmatrix} 1 & \nu^i \end{pmatrix}, \quad \epsilon^i \sim \mathcal{N}(0, 1/\tau^i), \quad \nu^i = \frac{\omega^i}{1 + \frac{\rho\sigma_2}{\sigma_1}\omega^i}, \quad \epsilon^i = \left(1 - \frac{\rho\sigma_2}{\sigma_1}\nu^i\right)^{-1} \epsilon^i. \tag{B-81}$$

This defines a direct, one-to-one correspondence between the information choice regarding risk factors (ν^i) and the information choice related to asset payoffs (ω^i). Define

(B-82) $$\Omega = \int_0^1 (\Lambda^i)^\top \tau^i \Lambda^i di = \int_0^1 \tau^i \begin{pmatrix} 1 & \nu^i \\ \nu^i & (\nu^i)^2 \end{pmatrix} di.$$

Following Admati (1985), the equilibrium prices of the risk factors F are given by

(B-83) $$\begin{aligned} P^f &= C + B\left(\Omega F - \lambda(Z^f - \bar{Z}^f)\right), \text{ where} \\ C &= \left(\Sigma_F^{-1} + \tfrac{1}{\lambda^2}\Omega(\Sigma_z^f)^{-1}\Omega + \Omega\right)^{-1}(\Sigma_F^{-1}\bar{F} - \lambda\bar{Z}^f), \\ B &= \left(\Sigma_F^{-1} + \tfrac{1}{\lambda^2}\Omega(\Sigma_z^f)^{-1}\Omega + \Omega\right)^{-1}\left(I_2 + \tfrac{1}{\lambda^2}\Omega(\Sigma_z^f)^{-1}\right). \end{aligned}$$

Thus, investor i's conditional variance about F is

(B-84) $$\hat{\Sigma}_F^i = \mathrm{Var}(F \mid P^f, Y^i) = \left(\Sigma_F^{-1} + \frac{1}{\lambda^2}\Omega(\Sigma_z^f)^{-1}\Omega + (\Lambda^i)^\top \tau^i \Lambda^i\right)^{-1}.$$

Investor i's conditional expectation about F is denoted by $\hat{F}^i = \mathrm{E}[F \mid P^f, Y^i]$, then the unconditional expectation of factor returns is

$$\mathrm{E}\left(\hat{F}^i - P^f\right) = R = \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \left(\Sigma_F^{-1} + \frac{1}{\lambda^2}\Omega(\Sigma_z^f)^{-1}\Omega + \Omega\right)^{-1}\left(\lambda\bar{Z}^f\right),$$

$$\mathrm{Var}\left(\hat{F}^i - P^f\right) = \mathrm{Var}\left(F - P^f\right) - \hat{\Sigma}_F^i = \mathrm{Var}\left(F - P^f\right) - \left(\Sigma_F^{-1} + \frac{1}{\lambda^2}\Omega(\Sigma_z^f)^{-1}\Omega + (\Lambda^i)^\top \tau^i \Lambda^i\right)^{-1},$$

$$\mathrm{Var}\left(F - P^f\right) = Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix} = (I_2 - B\Omega)\,\Sigma_F\,(I_2 - B\Omega)^\top + \lambda^2 B\Sigma_z^f B^\top.$$

For the risk factors, we define

$$A_{11} := \mathrm{E}(F_1 - P_1^f)^2 = R_1^2 + Q_{11}, \quad A_{22} := \mathrm{E}(F_2 - P_2^f)^2 = R_2^2 + Q_{22},$$
$$A_{12} := \mathrm{E}\left[(F_1 - P_1^f)(F_2 - P_2^f)\right] = R_1 R_2 + Q_{12}. \tag{B-85}$$

Therefore, investor i's expected utility function at $t = 1$ is

$$U_1^i = \lambda W_0 - 1 + \frac{1}{2}\mathrm{Tr}\left[(\hat{\Sigma}_F^i)^{-1} Q\right] + \frac{1}{2} R^\top (\hat{\Sigma}_F^i)^{-1} R. \tag{B-86}$$

Note that $\Sigma_F^{-1} = \left(\begin{smallmatrix} \tau_{F,1} & 0 \\ 0 & \tau_{F,2} \end{smallmatrix}\right)$ is also a diagonal matrix. Thus,

$$|\mathrm{Var}(F)| = \tau_{F,1}^{-1}\tau_{F,2}^{-1}, \quad |\mathrm{Var}(F \mid Y^i)| = \tau_{F,1}^{-1}\tau_{F,2}^{-1}\left(\tau_{F,1}^{-1} + (\nu^i)^2 \tau_{F,2}^{-1} + (\tau^i)^{-1}\right)^{-1}. \tag{B-87}$$

Since F and V are informationally equivalent, the learning capacity remains the same

(B-88)
$$\frac{|\mathrm{Var}(V)|}{|\mathrm{Var}(V \mid Y^i)|} = \frac{|\mathrm{Var}(TF)|}{|\mathrm{Var}(T\,F \mid Y^i)|} = \frac{|T\ \mathrm{Var}(F)\ T^\top|}{|T\ \mathrm{Var}(F \mid Y^i) T^\top|} = \frac{|T|\ |\mathrm{Var}(F)|\ |T^\top|}{|T|\ |\mathrm{Var}(F \mid Y^i)|\ |T^\top|} = \frac{|\mathrm{Var}(F)|}{|\mathrm{Var}(F \mid Y^i)|} \le K,$$

which implies the optimal signal precision chosen by investor i

$$\tau^i = \frac{K-1}{\tau_{F,1}^{-1} + (\nu^i)^2 \tau_{F,2}^{-1}}. \tag{B-89}$$

Therefore, investor i's information choice problem becomes

$$\max_{\nu^i} \frac{K-1}{\tau_{F,1}^{-1} + (\nu^i)^2 \tau_{F,2}^{-1}} \left(A_{22}(\nu^i)^2 + 2A_{12}\nu^i + A_{11}\right), \tag{B-90}$$

and the first order condition yields

$$\left[-A_{12}(\nu^i)^2 - \left(A_{11} - A_{22}\frac{\tau_{F,1}^{-1}}{\tau_{F,2}^{-1}}\right)\nu^i + A_{12}\frac{\tau_{F,1}^{-1}}{\tau_{F,2}^{-1}}\right]\left(\tau_{F,1}^{-1} + \tau_{F,2}^{-1}(\nu^i)^2\right)^{-2}\tau_{F,2}^{-1} = 0. \tag{B-91}$$

In equilibrium, the weight on the second factor is found to be

$$\nu = \frac{\tau_{z,2}}{2\tau_{v,1}\left(\rho\sqrt{\frac{\tau_{v,1}}{\tau_{v,2}}}+\tau_{z,2}(\bar{Z}_2-\bar{\gamma}_2)\left(\rho(\bar{Z}_2-\bar{\gamma}_2)\sqrt{\frac{\tau_{v,1}}{\tau_{v,2}}}+(\bar{Z}_1-\bar{\gamma}_1)\right)\right)}$$

$$\text{(B-92)}\quad \left\{\frac{\rho^2\tau_{v,1}\tau_{z,1}+\tau_{v,2}\tau_{z,2}}{\rho^2\tau_{z,1}\tau_{z,2}-\tau_{z,1}\tau_{z,2}}+\frac{\tau_{v,2}\left(\rho(\bar{Z}_2-\bar{\gamma}_2)\sqrt{\frac{\tau_{v,1}}{\tau_{v,2}}}+(\bar{Z}_1-\bar{\gamma}_1)\right)^2}{\rho^2-1}+\frac{\tau_{v,1}}{\tau_{z,2}}+\tau_{v,1}(\bar{Z}_2-\bar{\gamma}_2)^2\right.$$

$$+\left[\left(\frac{\tau_{v,2}\tau_{z,2}\left(\tau_{z,1}\left(\rho(\bar{Z}_2-\bar{\gamma}_2)\sqrt{\frac{\tau_{v,1}}{\tau_{v,2}}}+(\bar{Z}_1-\bar{\gamma}_1)\right)^2+1\right)+\left(2\rho^2-1\right)\tau_{v,1}\tau_{z,1}}{\left(\rho^2-1\right)\tau_{z,1}\tau_{z,2}}+\tau_{v,1}(\bar{Z}_2-\bar{\gamma}_2)^2\right)^2\right.$$

$$\left.\left.-\frac{4\tau_{v,1}\tau_{v,2}\left(\rho\sqrt{\frac{\tau_{v,1}}{\tau_{v,2}}}+\tau_{z,2}(\bar{Z}_2-\bar{\gamma}_2)\left(\rho(\bar{Z}_2-\bar{\gamma}_2)\sqrt{\frac{\tau_{v,1}}{\tau_{v,2}}}+(\bar{Z}_1-\bar{\gamma}_1)\right)\right)^2}{\left(\rho^2-1\right)\tau_{z,2}^2}\right]^{\frac{1}{2}}\right\}.$$

The vector of asset prices is $P^f = C + B\left(\Omega F - \lambda(Z^f - \bar{Z}^f)\right)$. Specifically,

$$P_1 = P_1^f, \qquad P_2 = P_2^f + \frac{\rho\sigma_2}{\sigma_1}P_1^f,$$

in terms of the factor prices

$$\text{(B-93)}\quad \begin{aligned} P_1^f &= C_1 + \tau(B_{11}+B_{12}\nu)(F_1+\nu F_2) - \lambda\left(B_{11}(Z_1^f-\bar{Z}_1^f)+B_{12}(Z_2^f-\bar{Z}_2^f)\right),\\ P_2^f &= C_2 + \tau(B_{21}+B_{22}\nu)(F_1+\nu F_2) - \lambda\left(B_{21}(Z_1^f-\bar{Z}_1^f)+B_{22}(Z_2^f-\bar{Z}_2^f)\right). \end{aligned}$$

The price informativeness of each asset can be expressed as the mutual information

$$\text{(B-94)}\quad \mathrm{I}(V_1,P_1) = -\frac{1}{2}\ln\left(1-\mathrm{Corr}^2(V_1,P_1)\right), \quad \mathrm{I}(V_2,P_2) = -\frac{1}{2}\ln\left(1-\mathrm{Corr}^2(V_2,P_2)\right),$$

where $\mathrm{Corr}(V_1,P_1)=\mathrm{Corr}(F_1,P_1^f)$ and $\mathrm{Corr}(V_2,P_2)=\mathrm{Corr}\left(F_2+\frac{\rho\sigma_2}{\sigma_1}F_1, P_2^f+\frac{\rho\sigma_2}{\sigma_1}P_1^f\right)$. We find

$$\mathrm{Corr}^2(V_1,P_1) = \frac{\left(\tau(B_{11}+B_{12}\nu)\sigma_1^2\right)^2}{\sigma_1^2\left[\tau^2(B_{11}+B_{12}\nu)^2(\sigma_1^2+\nu^2\sigma_2^2(1-\rho^2))+\lambda^2\left(B_{11}^2(\tau_{z,1}^{-1}+\frac{\rho^2\sigma_2^2}{\sigma_1^2}\tau_{z,2}^{-1})+B_{12}^2\tau_{z,2}^{-1}+2B_{11}B_{12}(\frac{\rho\sigma_2}{\sigma_1})\tau_{z,2}^{-1}\right)\right]},$$

$$\begin{aligned}\mathrm{Corr}^2(V_2,P_2) &= \tau^2\left[(B_{21}+B_{22}\nu)+\frac{\rho\sigma_2}{\sigma_1}(B_{11}+B_{12}\nu)\right]^2\left[\nu\sigma_2^2(1-\rho^2)+\rho\sigma_1\sigma_2\right]^2\sigma_2^{-2}\\ &\times\left[\tau^2\left((B_{21}+B_{22}\nu)+\frac{\rho\sigma_2}{\sigma_1}(B_{11}+B_{12}\nu)\right)^2\left(\sigma_1^2+\nu^2\sigma_2^2(1-\rho^2)\right)+\lambda^2\left(\left(B_{21}+\frac{\rho\sigma_2}{\sigma_1}B_{11}\right)^2\left(\tau_{z,1}^{-1}+\frac{\rho^2\sigma_2^2}{\sigma_1^2}\tau_{z,2}^{-1}\right)\right.\right.\\ &\left.\left.+\left(B_{22}+\frac{\rho\sigma_2}{\sigma_1}B_{12}\right)^2\tau_{z,2}^{-1}+2\left(B_{21}+\frac{\rho\sigma_2}{\sigma_1}B_{11}\right)\left(B_{22}+\frac{\rho\sigma_2}{\sigma_1}B_{12}\right)\tau_{z,2}^{-1}\right)\right]^{-1}.\end{aligned}$$