# Common Idiosyncratic Quantile Factors and Asset Prices

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**Abstract**

We investigate whether the tails of firm-level idiosyncratic return distributions are driven by common shocks. We use quantile factor analysis to extract such common idiosyncratic quantile factors with asymmetric pricing effects, and we find a significant premium for innovations to the lower-tail factor: high-beta stocks outperform low-beta stocks by approximately 7-8% per year. This premium remains significant even when controlling for standard factors, idiosyncratic volatility, and tail-risk measures. The downside factor strengthens when intermediary capital is weak and market liquidity is low, and it predicts aggregate market excess returns.

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# I. Introduction

Conventional wisdom holds that firm-level idiosyncratic risk, particularly rare, firm-specific tail shocks, can be diversified away. However, periods of market stress tend to be strikingly consistent: many firms experience unusually negative returns at the same time, and these fluctuations cannot be explained by volatility alone. This means that tail events, which appear diversifiable in isolation, become systemic in aggregate. In this paper, we examine this phenomenon and ask whether a common factor exists in the tails of firm-level idiosyncratic returns that is relevant for investors. We also consider what economic state this tail comovement reflects.

To address these issues, we introduce a nonparametric measure of common idiosyncratic tail risk. After removing standard linear factors from individual stock returns, we follow Chen, Dolado, and Gonzalo (2021) and estimate a quantile factor model on rolling windows to extract the dominant factor of the cross-sectional conditional quantiles of the idiosyncratic return distributions. We refer to this latent factor as the common idiosyncratic quantile (CIQ) factor. Our focus is on innovations, and we orient the factors so that they positively correlate with the corresponding realized cross-sectional quantile of idiosyncratic returns. This means that a decline in the lower-tail factor corresponds to a general deterioration in downside idiosyncratic conditions. The key empirical object is each stock's exposure to innovations in the downside CIQ, which is estimated using time series regressions and is then used to form out-of-sample portfolios.

Our first set of results shows that the downside idiosyncratic tail comovement is priced effectively. Sorting stocks according to their exposure to lower-tail CIQ innovations reveals a consistent pattern of average returns and a high-minus-low spread of approximately 7-8% per year. In contrast, no premium is earned with exposure to median and upper-tail CIQ innovations and the risk price is small and statistically insignificant. This asymmetry is difficult to reconcile with a purely volatility-driven narrative. The premium remains substantial even when controlling for standard factor models, the common idiosyncratic volatility reported in Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016), and significant downside and tail-risk measures, such as the tail-risk factor reported by Kelly and Jiang (2014). The premium is also robust across quantile thresholds: it is concentrated in the lower tail and diminishes as the quantile threshold approaches the center of the distribution.

We find that the lower-tail CIQ deteriorates when intermediary capital is weak and market liquidity is low and that firms with fragile trading conditions and limited financial slack have the strongest downside-CIQ exposures. Consistent with these findings, we provide a potential economic mechanism based on intermediary asset pricing and systemic fragility. In intermediary models such as those developed by He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and He, Kelly, and Manela (2017), shocks to the intermediary net worth tighten risk-bearing constraints and cause the price of risk to increase. We demonstrate that the cross-sectional distribution of idiosyncratic stock returns encompasses a related state variable: a shared element in firms' left-tail idiosyncratic quantiles. In a simple intermediary fragility setting, a single constraint multiplier shifts the lower-tail CIQ factor, affecting corresponding quantiles across assets; the implied quantile representation links innovations in this fragile state to innovations in the lower-tail CIQ factor.

Finally, the same state variable is important for aggregate risk premia. Innovations in the lower-tail CIQ factor forecast future market excess returns; when downside idiosyncratic tail risk worsens, subsequent aggregate returns increase. This predictive relationship is economically significant and remains consistent in both the in-sample and out-of-sample tests. In contrast, the predictability of the upper-tail CIQ factor is weaker and less stable.

Our results contribute to several areas of literature. First, we present a novel, distribution-sensitive measure of common idiosyncratic risk that does not depend on parametric tail assumptions or options data and distinguishes between downside and upside tail comovement. Second, we demonstrate a significant premium exists with exposure to innovations in the downside CIQ that differs from that of standard factors, volatility-based idiosyncratic risk, and existing tail-risk measures. Third, we provide economic content for the factor by linking it to intermediary risk-bearing capacity and liquidity and by demonstrating that firm-level exposures correspond to balance sheet and trading fragility characteristics. Fourth, we demonstrate that innovations in downside CIQ forecast the aggregate equity premium, thereby connecting cross-sectional pricing and time series predictability via a single stress-related state variable.

# II. Common Idiosyncratic Quantile Factors: Asymmetric Idiosyncratic Risk

Standard linear factor models summarize risk through variation, implicitly treating downside and upside outcomes as symmetric mirror images. Our approach, however, focuses on common shifts in the tails of the cross-sectional distribution of idiosyncratic returns. This approach retains directional information in adverse and favorable states, providing a more detailed characterization of systematic risk than volatility-based measures do.

## A. Factor Specification

For each month $m$ and every stock $i$, we use a rolling window of 60 months and retain stocks for which complete observations within the window are available. Within each window, we estimate a baseline factor model and save the residuals.

$$(1)\quad r_{i,t} = \alpha_{i,m} + \beta_{i,m}^{MKT} MKT_t + \beta_{i,m}^{SMB} SMB_t + \beta_{i,m}^{HML} HML_t + e_{i,t,m}, \quad t = m - 59, \ldots, m.$$

We use the three-factor model reported in Fama and French (1993) (FF3) as our baseline model, as it aligns our approach with the literature on commonality in idiosyncratic risks and tail exposures (e.g., Ang, Hodrick, Xing, and Zhang (2006b), Kelly and Jiang (2014)). Moreover, increasing the baseline factor set has only a modest effect on the time variation in realized idiosyncratic quantiles, and FF3 provides a transparent benchmark for evaluating the additional information provided by tail-based factors. In Section V.B, we demonstrate that our pricing results remain robust when richer linear specifications, including the five- and six-factor models reported in Fama and French (2015) and Fama and French (2018), are used.[1]

We study common tail variation using the residuals $e_{i,t}$ from equation (1). As in principal component analysis, we standardize these residuals by their within-window standard deviation.[2] For a probability level $\tau \in (0, 1)$ in the current month $m$, we estimate a common idiosyncratic

[1] Internet Appendix E reports variance ratios for realized idiosyncratic quantiles across alternative factor models and shows that residual quantiles exhibit significant time variation.

[2] This standardization makes the cross-sectional quantiles comparable across stocks and stabilizes estimation in the rolling windows.

quantile factor $CIQ_{t,m}(\tau)$ via the following:

(2) $$e_{i,t,m} = \gamma_{i,m}(\tau) CIQ_{t,m}(\tau) + u_{i,t,m}(\tau), \quad t = m-59, \ldots, m$$

where $u_{i,t,m}(\tau)$ satisfies the quantile restriction $P\{u_{i,t,m}(\tau) < 0 \mid CIQ_{t,m}(\tau)\} = \tau$. We estimate $CIQ_{t,m}(\tau)$ and $\gamma_{i,m}(\tau)$ using the quantile factor analysis procedure detailed in Chen et al. (2021).[3] At each $\tau$, we use only the first (most informative) factor. In our applications, using the factor-number selection procedures detailed in Chen et al. (2021), a single factor for the residual-quantile panels is almost always selected.

In the baseline analysis, we focus on three CIQ factors. We capture common movements in adverse idiosyncratic outcomes using the *lower-tail* factor at $\tau = 0.2$, $CIQ^{LT} \equiv CIQ(0.2)$. This choice is motivated by (i) evidence that disappointment-type preferences are linked to approximately the worst 20% of outcomes in several asset-pricing settings[4], (ii) its close relationship to other downside quantiles below the median (e.g., $\tau = 0.1$ and $\tau = 0.3$), and (iii) the practical trade-off between focusing on tail events and retaining sufficient observations for precise estimation. As a counterpart capturing common upside movements, we define the *upper-tail* factor at $\tau = 0.8$, $CIQ^{UT} \equiv CIQ(0.8)$, and we define the *central* factor at $\tau = 0.5$, $CIQ^{C} \equiv CIQ(0.5)$, to summarize movements in the middle of the idiosyncratic return distribution.

Quantile factors can be identified only up to their sign. We impose a normalization that

[3] Details of the algorithm and implementation are provided in Internet Appendix A.

[4] See, for example, Giglio, Kelly, and Pruitt (2016), Delikouras and Kostakis (2019), Massacci, Sarno, and Trapani (2025), and Farago and Tédongap (2018).

gives their movements an economically meaningful direction. Specifically, each CIQ factor is oriented so that it is positively correlated with the corresponding cross-sectional realized quantile of idiosyncratic returns. For the lower-tail factor, this means that a decline in $CIQ_t^{LT}$ indicates a deterioration in downside idiosyncratic outcomes across firms. For the upper-tail factor, an increase in $CIQ_t^{UT}$ corresponds to an improved residual upside potential. This directional information is precisely what cannot be preserved by symmetric dispersion measures.

In line with the volatility factor literature, we focus on innovations in conditional idiosyncratic quantile factors,

$$\Delta CIQ_{t,m}(\tau) \equiv CIQ_{t,m}(\tau) - CIQ_{t-1,m}(\tau), \quad t = m - 58, \ldots, m, \tag{3}$$

and conduct our baseline analysis using $\Delta CIQ_{t,m}(\tau)$ rather than levels.[5] This choice reflects the standard idea that investors are compensated for exposure to innovations in systematic risk, as set out in the ICAPM of Merton (1973) and commonly used in volatility-based settings (e.g., Ang et al. (2006b), Herskovic et al. (2016)). In our setting, factor levels can combine persistent distributional features with higher-frequency stress fluctuations. First differences emphasize the latter and yield a sharper proxy for shifts in tail conditions.[6] We repeat the steps in equations (1) to (3) until the whole dataset is exhausted.

[5] We report pricing results using levels or AR(1) innovations in Section V.B and obtain qualitatively similar conclusions in both cases.

[6] Differences are computed within each 60-month rolling window using information up to time $m$. The resulting 59 differences are used to estimate stock-level exposures, which are then related to out-of-sample returns in month $m + 1$. For time series predictability tests, the final difference from each window, $\Delta CIQ_{m,m}(\tau)$, is used to predict the market return in month $m + 1$.

We estimate quantile-specific factors using idiosyncratic returns rather than raw returns for several reasons. First, this approach is consistent with the literature on common movements in idiosyncratic volatility and tail risk.[7] Second, quantiles depend on both location and scale; thus, by removing well-documented linear risk exposures, distributional variation is isolated beyond the mean. Third, linear factor models provide a familiar benchmark against which the additional content of tail-based factors can be evaluated. Finally, since exposures to common linear factors are often easy to hedge, working with residuals enables us to study tail variations that are not mechanically spanned by standard factor structures.

Our framework is nonparametric: it relies on conditional quantiles of observed returns and does not impose a parametric structure for nonlinear dependence.[8] Quantiles provide information about a range of distributional features, including but not limited to volatility. Under a location-scale model, common movements in quantiles are tightly linked to common movements in volatility, which is consistent with the factor structure in idiosyncratic volatility (e.g., Ang et al. (2006b)). In that special case, a volatility factor can be recovered by principal components applied to squared residuals (PCA-SQ).

When higher-order features matter, however, volatility-based methods may miss economically relevant tail risks. Quantile factor models remain informative by directly targeting distributional shifts beyond the first two moments. Internet Appendix B illustrates this distinction in a simple theoretical example.

[7] For example, Ang et al. (2006b) construct idiosyncratic volatility relative to FF3, and Kelly and Jiang (2014) study tail-risk exposures in a similar residual-return framework.

[8] For instance, Gorodnichenko and Ng (2017) study joint level and volatility factors; other approaches use copulas to model tail dependence (see Amengual and Sentana (2020), Oh and Patton (2017)).

## B. Data and Descriptive Statistics

We estimate CIQ factors using monthly CRSP stock returns from January 1963 to December 2024. We include common stocks (share codes 10 and 11), adjust them for delistings following Bali, Engle, and Murray (2016), and keep only nonpenny stocks with prices above $1 to mitigate microstructure-related biases.[9] With a 60-month rolling window, the first set of returns that we are able to predict occurred in January 1968.

Figure 1 plots the lower-tail, central, and upper-tail CIQ factors alongside a volatility-based benchmark (PCA-SQ).[10] Two features stand out. First, CIQ factors exhibit pronounced variation and spikes around major intermediary-based stress episodes, such as those between 2007 and 2009. This is consistent with state-like shifts in cross-sectional tail conditions. Second, the lower- and upper-tail factors are not mirror images, indicating that the downside and upside tail states are distinct.[11]

Panel A of Table 1 summarizes the distributional properties of the $\Delta CIQ$ factors.[12] The standard deviations are large relative to the means, kurtosis exceeds three, and first-order autocorrelations are negative, indicating that tail innovations are short-lived and mean reverting. Panel B shows that $\Delta CIQ^{LT}$ and $\Delta CIQ^{UT}$ are only weakly correlated (0.13), rejecting the view

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[9] See, e.g., Amihud (2002).

[10] The method used to estimate the PCA-SQ factor is similar to that used to estimate the CIQ factors, i.e., principal component analysis on a balanced panel of squared FF3 residuals over a rolling window of 60 months. Only the last observation from each window is plotted.

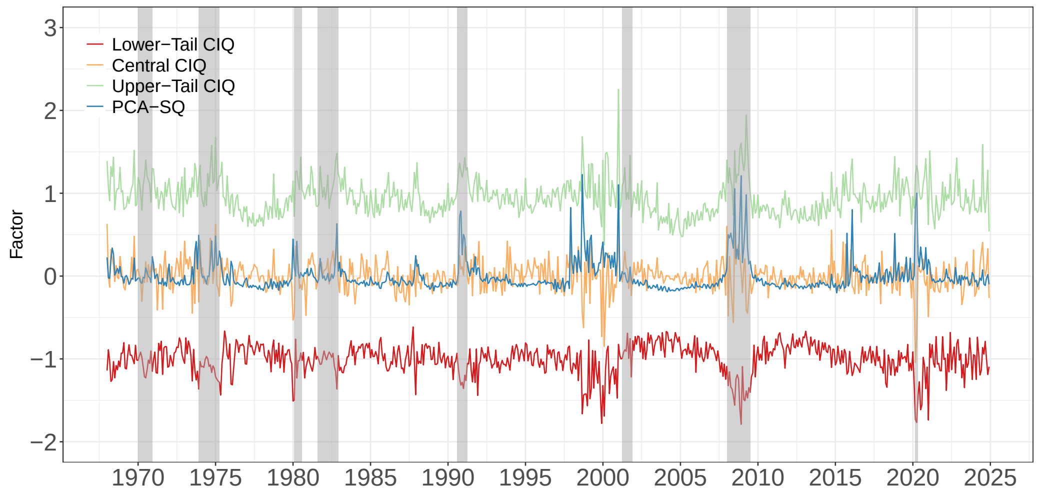
[11] The CIQ factors are available to download and use at https://github.com/matejnevrla/Common-Idiosyncratic-Quantile-Risk.

[12] Table A3 in Internet Appendix G reports summary statistics and the correlation structure across $\Delta CIQ(\tau)$ across a fine grid of $\tau$ values.

FIGURE 1

## CIQ Factors

The figure captures the lower-tail ($\tau = 0.2$), central ($\tau = 0.5$), and upper-tail ($\tau = 0.8$) $CIQ(\tau)$ factors, alongside the PCA-SQ factor. The factors are estimated on monthly idiosyncratic returns with respect to the three-factor model developed by Fama and French (1993) on the basis of either the quantile factor analysis suggested by Chen et al. (2021) (CIQ) or a principal component analysis using squared idiosyncratic returns (PCA-SQ). Factors are estimated using a rolling window of 60 months. In each window, the last estimated value is plotted. Central CIQ a PCA-SQ are standardized to have zero mean and same volatility as the lower-tail CIQ factor. The data come from the CRSP and cover the period from January 1968 to December 2024. We exclude penny stocks with prices of $1 or less. The shaded areas represent NBER recessions.




that the two series simply rescale a common dispersion factor. The correlations with ΔPCA-SQ are economically meaningful and vary by tail, as expected, since quantiles partly reflect variance but also contain tail-specific information. The correlations with prominent idiosyncratic volatility and tail-risk factors are moderate to small, highlighting that CIQ innovations are related to, but not subsumed by, existing measures. Panel C links $\Delta CIQ$ to forward-looking uncertainty measures. A higher $\mathrm{VIX}^2$ at the end of month $t$ predicts a decline in the lower-tail factor in $t + 1$, i.e., a worsening of downside idiosyncratic tail conditions, with similar patterns for its components. In contrast, the upper-tail factor has a weak or no predictive relationship with these measures. Similar asymmetry appears for narrative-based volatility (Manela and Moreira, 2017): topics related to *Securities Markets* and *Intermediation* predict deterioration in lower-tail idiosyncratic conditions, while the upper tail remains largely unrelated.

## TABLE 1

## Summary of the ΔCIQ Factors

The table provides summary statistics of the estimated lower-tail ($\tau = 0.2$), central ($\tau = 0.5$), and upper-tail ($\tau = 0.8$) $\Delta CIQ(\tau)$ factors. Factors are estimated using idiosyncratic returns with respect to the three-factor model of Fama and French (1993) and using the quantile factor analysis of Chen et al. (2021). Factors are estimated using a 60-month moving window, with differences being computed during that period and the last value being used. In Panel A, we report descriptive statistics of the ΔCIQ($\tau$) factors, including their means, standard deviations, skewness, kurtosis and autocorrelation coefficients of order between one and three. In Panel B, we report contemporaneous correlations between the ΔCIQ($\tau$) factors themselves, as well as with other related factors: differences of the PCA-SQ factor (ΔPCA-SQ), differences of the common idiosyncratic variance factor (ΔCIV) of Herskovic et al. (2016), differences of the cross-sectional bivariate idiosyncratic volatility (ΔCBIV) factor of Han and Li (2025), differences of the tail risk factor (ΔTR) of Kelly and Jiang (2014), and differences of the sentiment factor of Baker and Wurgler (2006) and Baker and Wurgler (2007). In Panel C, we report correlations between the ΔCIQ($\tau$) factors at time $t+1$ and forward-looking variance measures at time $t$. First, we report correlations with the squared CBOE volatility index ($VIX^2$) and its two components: conditional variance (CV) and variance premium (VP) of Bekaert and Hoerova (2014). Second, we employ the squared news-implied volatility index ($NVIX^2$) of Manela and Moreira (2017) and its topic decomposition. The data generally cover the period from January 1968 to December 2024 with the following exceptions: January 1968 to December 2022 for the CBIV, January 1968 to December 2023 for the sentiment, February 1990 to December 2024 for the VIX, February 1990 to February 2022 for the CV and VP, and January 1968 to April 2016 for the NVIX. * indicates $p < 0.1$, ** indicates $p < 0.05$ and *** indicates $p < 0.01$.

| | Lower-Tail | Central | Upper-Tail |
|---|---|---|---|
| *Panel A. Descriptive Statistics* | | | |
| Mean $\times 10^3$ | -3.21 | -41.48 | -8.34 |
| St. Dev. | 0.19 | 1.46 | 0.23 |
| Skewness | 0.06 | -0.35 | -0.05 |
| Kurtosis | 5.09 | 6.41 | 6.86 |
| AR(1) | -0.44 | -0.27 | -0.42 |
| AR(2) | -0.02 | 0.07 | -0.07 |
| AR(3) | 0.03 | -0.02 | 0.07 |
| *Panel B. Contemporaneous Correlations* | | | |
| Lower-Tail | 1.00 | 0.22*** | 0.13*** |
| Central | | 1.00 | 0.41*** |
| Upper-Tail | | | 1.00 |
| ΔPCA-SQ | -0.42*** | 0.14*** | 0.50*** |
| ΔCIV | -0.26*** | 0.10*** | 0.29*** |
| ΔCBIV | -0.00 | 0.07* | -0.02 |
| ΔTR | 0.09** | -0.03 | -0.24*** |
| ΔSentiment | 0.10*** | 0.02 | 0.02 |
| *Panel C. Forward Correlations* | | | |
| $VIX^2$ | -0.17*** | -0.19*** | 0.07 |
| - CV | -0.19*** | -0.21*** | 0.07 |
| - VP | -0.12** | -0.18*** | 0.07 |
| $NVIX^2$ | -0.13*** | -0.07* | 0.04 |
| - Securities Markets | -0.14*** | -0.06 | 0.06 |
| - Intermediation | -0.10** | -0.03 | 0.03 |
| - Unclassified | -0.14*** | -0.08* | 0.05 |
| - Government | 0.02 | 0.01 | 0.03 |
| - Natural Disaster | -0.02 | 0.02 | 0.04 |
| - War | 0.04 | 0.07* | -0.01 |

Finally, CIQ factors capture realized cross-sectional tail quantiles more precisely than volatility does: the lower-tail CIQ factor explains a greater proportion of the variation in realized lower-tail quantiles than cross-sectional volatility does. Furthermore, monthly Anderson–Darling tests consistently reject the normality of the cross-sectional idiosyncratic return distribution, emphasizing the need to move beyond variance-based summaries. The incremental value of CIQ is particularly evident in the lower tail, whereas upper-tail quantiles are closer to those that can be captured by volatility alone.[13]

# III. Pricing the CIQ Risks in the Cross-Section

We now investigate the pricing implications of exposure to CIQ risks. We hypothesize that there will be significant price heterogeneity associated with the lower, central, and upper portions of the cross-sectional distribution of idiosyncratic returns. Specifically, we show a strong asymmetric effect of only exposure to common lower-tail CIQ innovations being systematically priced and distinct from existing volatility, intermediary, and liquidity factors. Importantly, tail-based proxies cannot account for this pricing effect either.

## A. ΔCIQ Betas

To measure stock sensitivities to the lower-tail ($\Delta CIQ^{LT}$), central ($\Delta CIQ^{C}$), and upper-tail ($\Delta CIQ^{UT}$) factors, each month $m$, we separately estimate factor loadings using time series regressions of excess returns on the corresponding CIQ innovations as follows:

$$r_{i,t} = a_{i,m} + \beta^{CIQ}_{i,j,m}, \Delta CIQ^{j}_{t,m} + v_{i,t,m}, \quad t = m-59, \ldots, m, \tag{4}$$

[13]Panel B of Table A2 in Internet Appendix E reports the corresponding regression results.

for each $j \in LT, C, UT$, where $\beta_{i,j,m}^{CIQ}$ captures stock $i$'s exposure to quantile-specific innovations in common idiosyncratic risk.[14] Betas are estimated over the same 60-month rolling window used to construct the CIQ factors, including stocks with at least 48 monthly observations. Factor realizations and betas estimated using information up to time $m$ are used to predict returns in month $m + 1$, ensuring no overlap between estimation and prediction periods. Unless stated otherwise, the dependent variable is the one-month-ahead out-of-sample return. We also examine longer-horizon returns using portfolios to assess the persistence of the $\Delta CIQ$ exposures and, indirectly, the role of trading frictions.

The data cover the usual cross-sectional asset pricing period between January 1963 and December 2024. The first returns that we predict pertain to January 1968. In total, our baseline dataset without penny stocks consists of 2,082,857 stock-month observations.

[14] Although the quantile factor model developed by Chen et al. (2021) delivers quantile-specific loadings $\gamma_{i,m}(\tau)$ as part of the cross-sectional estimation, we do not interpret these objects as pricing betas. The $\gamma_{i,m}(\tau)$ coefficients characterize how individual returns contribute to the conditional cross-sectional quantile at a given $\tau$ and are therefore cross-sectional objects tied to the level of the quantile factor. In contrast, asset pricing requires a measure of exposure to innovations in systematic risk. Since our analysis focuses on $\Delta CIQ(\tau)$—which captures changes in tail conditions rather than persistent distributional states—the economically relevant exposure is a stock's time-series covariance with these innovations. Estimating betas via time series regressions therefore provides a direct measure of sensitivity to quantile-specific risk shocks that enter the stochastic discount factor, whereas the $\gamma_{i,m}(\tau)$ loadings need not coincide with exposure to time series innovations in tail risk. In this sense, the time series betas and the quantile factor loadings capture distinct economic concepts, and only the former is appropriate for pricing tests based on risk innovations.

## B. Univariate Portfolio Sorts

We start our analysis by investigating the performance of portfolios formed on the basis of the $\Delta CIQ$ betas. Each month, stocks are sorted into five or ten portfolios according to their ΔCIQ betas estimated using information available up to time $m$.[15] The portfolios are formed at the end of month $m$ and held during month $m+1$, with returns computed using either equal- or value-weighted schemes based on market capitalization at formation. Portfolios are rebalanced monthly as the betas are re-estimated using a rolling window. These univariate portfolio results are summarized in Table 2. In Panel A, we report decile sorts. For the lower-tail $\Delta CIQ$ factor, we observe a monotonic increase in returns from low- to high-exposure portfolios, whereas no significant return spread is observed for portfolios sorted on the central or upper-tail $\Delta CIQ$ factors.

Moreover, to formally assess the presence of compensation for bearing the CIQ risks, we present the returns of the high-minus-low portfolios obtained as the difference between the returns of portfolios with the highest ΔCIQ betas and those of portfolios with the lowest ΔCIQ betas. These returns correspond to the investment strategy that involves buying stocks with high exposures and selling stocks with low exposures to the ΔCIQ factors. These portfolios are zero-cost portfolios and capture the risk premium associated with the specific joint movements of the idiosyncratic returns. We observe a significant positive premium for the difference portfolio only for the lower-tail ΔCIQ factor. This premium is both economically and statistically

[15] In the baseline analysis, portfolio breakpoints are constructed using the full cross-section of stocks to ensure that portfolio assignments reflect the entire investable universe rather than being driven by disproportionately large and mature firms. The results using the NYSE-based breakpoints are reported as a robustness check in Table 9 to confirm that the findings are not attributable to microcaps.

TABLE 2

## Portfolios Sorted on Exposures to the $\Delta$CIQ Factors

The table reports the annualized out-of-sample excess returns of portfolios sorted on the exposure to the lower-tail ($\tau = 0.2$), central ($\tau = 0.5$) and upper-tail ($\tau = 0.8$) $\Delta$CIQ factors. We also report the returns of zero-cost portfolios obtained by buying the high-exposure portfolio and selling the low-exposure portfolio (High - Low). The corresponding $t$-statistics (in parentheses) are computed using the robust standard errors suggested by Newey and West (1987) with six lags. Stocks are either sorted into ten portfolios in Panel A or into five portfolios in Panel B, with portfolio returns obtained by either equally weighting stock returns (EW) or value weighting by their market capitalization (VW). The portfolios are formed each month based on the sensitivity to the $\Delta$CIQ factors estimated using time-series regression over the previous 60 months. The corresponding next-period out-of-sample return is then recorded for each portfolio. We also report time-series averages of equally-weighted stock-level $\Delta$CIQ betas within portfolios during the formation. The return sample covers period between January 1968 and December 2024. Each month, we use all the CRSP stocks for which at least 48 monthly observations are available over the last 60 months and exclude penny stocks with prices of $1 or less.

| | Lower-Tail | | | Central | | | Upper-Tail | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\beta^{CIQ}_{LT}$ | EW | VW | $\beta^{CIQ}_{C}$ | EW | VW | $\beta^{CIQ}_{UT}$ | EW | VW |
| *Panel A. Decile Sorts* | | | | | | | | | |
| Low $\beta^{CIQ}$ | -0.29 | 4.36 | 3.36 | -0.03 | 6.90 | 5.02 | -0.13 | 8.86 | 7.23 |
| 2 | -0.17 | 7.94 | 7.28 | -0.01 | 9.06 | 5.22 | -0.04 | 10.10 | 9.12 |
| 3 | -0.12 | 9.46 | 6.66 | -0.01 | 10.34 | 6.62 | -0.01 | 10.45 | 8.15 |
| 4 | -0.09 | 9.63 | 6.57 | -0.01 | 9.05 | 6.58 | 0.01 | 9.61 | 7.06 |
| 5 | -0.07 | 9.45 | 7.16 | -0.00 | 10.23 | 7.52 | 0.03 | 9.74 | 8.13 |
| 6 | -0.04 | 10.37 | 8.12 | 0.00 | 9.86 | 8.43 | 0.05 | 9.70 | 7.83 |
| 7 | -0.02 | 10.41 | 7.97 | 0.00 | 10.32 | 8.15 | 0.07 | 9.91 | 6.81 |
| 8 | 0.01 | 10.74 | 8.22 | 0.01 | 10.23 | 7.73 | 0.09 | 8.98 | 6.36 |
| 9 | 0.04 | 11.40 | 9.85 | 0.01 | 10.30 | 6.49 | 0.13 | 9.75 | 5.88 |
| High $\beta^{CIQ}$ | 0.14 | 11.77 | 10.73 | 0.03 | 9.26 | 9.39 | 0.21 | 8.43 | 6.35 |
| High - Low | | 7.41 | 7.36 | | 2.36 | 4.37 | | -0.43 | -0.88 |
| $t$-statistic | | (4.30) | (2.73) | | (1.36) | (1.82) | | (-0.26) | (-0.35) |
| *Panel B. Quintile Sorts* | | | | | | | | | |
| Low $\beta^{CIQ}$ | -0.23 | 6.15 | 5.89 | -0.02 | 7.98 | 5.08 | -0.09 | 9.48 | 8.34 |
| 2 | -0.11 | 9.54 | 6.62 | -0.01 | 9.69 | 6.54 | -0.00 | 10.03 | 7.74 |
| 3 | -0.06 | 9.91 | 7.69 | -0.00 | 10.04 | 7.89 | 0.04 | 9.72 | 7.75 |
| 4 | -0.01 | 10.57 | 7.96 | 0.01 | 10.28 | 8.04 | 0.08 | 9.45 | 6.61 |
| High $\beta^{CIQ}$ | 0.09 | 11.59 | 9.98 | 0.02 | 9.78 | 7.57 | 0.17 | 9.09 | 6.35 |
| High - Low | | 5.44 | 4.09 | | 1.80 | 2.49 | | -0.39 | -1.99 |
| $t$-statistic | | (3.76) | (2.04) | | (1.36) | (1.44) | | (-0.29) | (-1.12) |

significant. In the case of the decile equal-weighted portfolio, the premium reaches 7.41% on an annual basis with a robust $t$-statistic on the basis of standard error correction suggested by Newey and West (1987) with six lags of 4.30. The premium of the value-weighted portfolio achieves a similar performance of 7.36% p.a. with a $t$-statistic of 2.73. The stocks that hedge the lower-tail CIQ movements possess particularly low excess returns. Investors clearly value this feature, which increases the prices of these stocks and decreases their expected returns.

We also observe a sizable spread in the exposures across the portfolios captured by the time series average of equally weighted stock-level betas within portfolios during the formation period. The low-exposure portfolio has an average $\beta_{LT}^{CIQ}$ of -0.29, which is the greatest difference with respect to a neighborhood portfolio across all the portfolios (the second-lowest-exposure portfolio has an average $\beta_{LT}^{CIQ}$ of -0.17, a difference of 0.12).

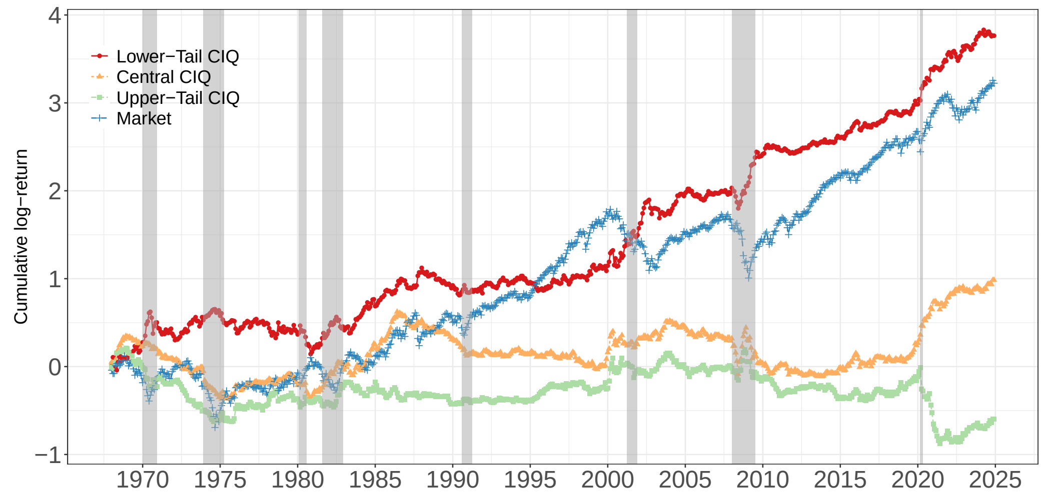
To show that the premium is not driven by a particular choice of the sorting scheme, in Panel B, we report the results from sorting the stocks into quintiles. Although the premium is smaller than in the case of decile sorting, it remains both economically and statistically significant at 5.44% p.a. with a $t$-statistic of 3.76 in the case of an equally weighted portfolio and 4.09% p.a. with a $t$-statistic of 2.04 in the case of a value-weighted portfolio. This slightly lower significance in the case of the value-weighted portfolio may be partially caused by the fact that the value-weighted portfolios are more concentrated, which leads to more volatile returns.

On the other hand, if we consider the returns of the zero-cost portfolios associated with either central or upper-tail $\Delta$CIQ factors, these portfolios yield premia indistinguishable from zero. This observation holds across weighting and sorting schemes. The value investors place on exposures to the downside and upside idiosyncratic events are clearly asymmetrical, and they clearly prefer to hedge against the former but not the latter. The fact that only the exposures to the lower-tail common movements yield a premium suggests that the $\Delta$CIQ risks are not driven by the effect of the common volatility. If volatility was the main driver of the factors, we would observe symmetrical compensation for the exposures to both the downside and upside factors, which we do not observe here. To illustrate this point in further detail, in Internet Appendix D, we provide evidence indicating that, in an economy that specifically rewards holding assets with exposure to a common volatility factor, the upside and downside factors are priced symmetrically. On the basis of these results, we argue that the typical location-scale model is not consistent with the premia that we observe here.

FIGURE 2

**Performance of the ΔCIQ Portfolios**

The figure depicts performance of a strategy that buys stocks with high exposure to the ΔCIQ factors and sells stocks with low exposure. It plots cumulative log-return obtained from sorting the stocks into decile portfolios with equal-weighting the stocks. The return sample covers period between January 1968 and December 2024. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months and exclude penny stocks with prices of $1 or less. The shaded areas represent NBER recessions. We include the performance of the excess market return as a comparison.




To visually inspect the performance of the portfolios sorted on the basis of the exposure to the ΔCIQ factors, we present in Figure 2 the cumulative log-returns of the equally weighted high-minus-low portfolios. Consistent with the numerical portfolio results, only the portfolio based on the lower-tail ΔCIQ factor has a strong performance during the sample period that is no worse than the performance of the aggregate market as measured by the value-weighted return of all the CRSP firms.

The newly discovered lower-tail ΔCIQ premium raises the question of whether previously reported risks associated with a particular factor model are mirrored. Accordingly, we regress the returns of the high-minus-low portfolios on various sets of factors and report the estimated intercepts—alphas—from this exercise. We summarize the results in Table 3.[16] We report the

[16]The Fama–French factors are obtained from Kenneth French's data library. The intermediary capital risk factors, $q$-factors, mispricing factors, betting-against-beta factor, and liquidity factor are obtained from the authors' publicly

annualized alphas for both the equal- and value-weighted portfolios that are sorted either in quintiles or deciles. We start the investigation by regressing the returns on the market factor (CAPM). We observe that the market factor is not able to explain the abnormal returns across the specifications. We repeat the regressions on the basis of the three-factor model developed by Fama and French (1993) (FF3), the five-factor model developed by Fama and French (2015) (FF5), and its extension, incorporating the momentum factor following Fama and French (2018) (FF6). The alphas remain economically large and generally statistically significant; thus, standard Fama–French models do not fully span the lower-tail $\Delta$CIQ premium.

Next, we extend the four-factor model developed by Carhart (1997) by augmenting it with the $\Delta$CIV idiosyncratic volatility factor found in Herskovic et al. (2016) and the betting-against-beta (BAB) factor used by Frazzini and Pedersen (2014). We then add either the short-term reversal (Rev.) or liquidity factor following Pastor and Stambaugh (2003) (Liquid). Similarly, we observe that the premia associated with these risks do not subsume the abnormal returns of the lower-tail $\Delta$CIQ portfolio.

We also verify that the premium does not mirror the risk premium associated with the benchmark two-factor intermediary pricing model developed by He et al. (2017), which includes the market factor and traded capital factor. We observe that neither of these models captures the complex risk associated with our lower-tail idiosyncratic factor.

Finally, we also regress the portfolio returns on the five-factor model developed by Hou et al. (2020) (Q5) and the four factors of Stambaugh and Yuan (2016), which include two

available data libraries or other maintained public sources where available. We replicated the common idiosyncratic volatility factor following Herskovic et al. (2016).

TABLE 3

## Alphas of the Zero-Cost Lower-Tail ∆CIQ Portfolios

The table reports annualized abnormal returns of zero-cost portfolios obtained from buying high-exposure and selling low-exposure stocks with respect to the lower-tail ∆CIQ factor. Each month, stocks are sorted on the exposure estimated from the last 60 months into decile or quintile portfolios. Returns within a portfolio are either equal- or value-weighted based on the market capitalization at the time of the portfolio formation. Zero-cost portfolio return is calculated as a difference between high-beta and low-beta portfolio. The portfolio is then held for one month and subsequently rebalanced. We report estimated intercepts (alphas) from regressing the portfolio out-of-sample returns on various sets of asset pricing factors: market (CAPM), three factors of Fama and French (1993) (FF3), five factors of Fama and French (2015) (FF5) and its extension with the momentum factor of Fama and French (2018) (FF6), two factors of He et al. (2017) (Intermediary), five factors of Hou, Mo, Xue, and Zhang (2020) (Q5), and four factors of Stambaugh and Yuan (2016) (M4). Moreover, we formulate an ad-hoc model based on four factors of Carhart (1997), common idiosyncratic volatility shocks of Herskovic et al. (2016), and betting-against-beta factor of Frazzini and Pedersen (2014) (FF alt.) and augment it with either short-term reversal factor (Rev.) or liquidity factor of Pastor and Stambaugh (2003) (Liquid.). The corresponding $t$-statistics (in parentheses) are computed using the robust standard errors suggested by Newey and West (1987) with six lags. The return sample covers period between January 1968 and December 2024, except for M4, which is only available up to December 2016, and Intermediary, which covers period between January 1970 and November 2018. Each month, we use all the CRSP stocks for which at least 48 monthly observations are available over the last 60 months and exclude penny stocks with prices of $1 or less.

| | Decile | | Quintile | |
|---|---|---|---|---|
| | EW | VW | EW | VW |
| CAPM | 9.72 | 9.31 | 7.35 | 5.78 |
| | (6.39) | (3.65) | (5.73) | (3.04) |
| FF3 | 8.22 | 6.74 | 6.08 | 3.44 |
| | (5.52) | (2.83) | (5.01) | (2.04) |
| FF5 | 7.59 | 6.71 | 6.14 | 3.57 |
| | (4.87) | (2.56) | (4.91) | (2.05) |
| FF6 | 8.31 | 7.85 | 6.83 | 4.85 |
| | (4.97) | (2.84) | (5.11) | (2.72) |
| FF alt. | 8.51 | 8.39 | 6.75 | 5.21 |
| | (4.47) | (2.91) | (4.43) | (2.72) |
| FF alt. + Rev. | 8.47 | 8.64 | 6.72 | 4.88 |
| | (4.15) | (2.84) | (4.05) | (2.37) |
| FF alt. + Liquid. | 8.28 | 8.17 | 6.57 | 5.07 |
| | (4.36) | (2.77) | (4.32) | (2.65) |
| Intermediary | 8.07 | 9.34 | 6.11 | 5.53 |
| | (5.24) | (3.82) | (4.70) | (3.03) |
| Q5 | 7.18 | 8.06 | 6.09 | 5.58 |
| | (3.59) | (2.70) | (3.55) | (2.53) |
| M4 | 5.94 | 7.61 | 5.52 | 5.18 |
| | (3.17) | (2.85) | (3.79) | (2.73) |

mispricing factors (M4). The results also suggest that these models do not span the abnormal premium associated with the lower-tail ∆CIQ risk.[17]

[17] Although none of the factor models considered here can explain the lower-tail ∆CIQ premium from the portfolio sorting procedure, some stock-level characteristics might. For this reason, we investigate how stock-level lower-tail exposures are related to other stock-specific characteristics and exposures by performing dependent bivariate sorting. This procedure yields single-sorted portfolios that vary in terms of their exposure to the lower-tail ∆CIQ factor but exhibit approximately equal control variable values. The results are summarized in Internet Appendix I in Table A6 and show that none of the variables considered can subsume the discovered premium.

## C. Fama–MacBeth Regressions

In the next step, we perform two-stage Fama and MacBeth (1973) predictive regressions. In contrast to portfolio sorting, this type of asset pricing test conveniently allows for the simultaneous estimation of many risk premia associated with various stock-level characteristics. This means that we can estimate the risk premium associated with the $\Delta\text{CIQ}^j$ factors $j \in \{LT, C, UT\}$ while controlling for other risk measures that have previously been proposed in the literature. Specifically, for each time $t = 1, \ldots, T-1$ using all of the stocks $i = 1, \ldots, N$ available at time $t$ and $t+1$,[18] we cross-sectionally regress all the returns at time $t+1$ on the betas estimated using only the information available up to time $t$. This procedure yields estimates of the risk prices $\lambda_{t+1,j}^{CIQ}$ while controlling for the most widely used measures of risk. More specifically, we use variations of the following cross-sectional regressions:

$$r_{i,t+1} = \alpha_{t+1} + \sum_j \beta_{i,t,j}^{CIQ} \lambda_{t+1,j}^{CIQ} + Z_{i,t}' \lambda_{t+1}^{Z} + e_{i,t+1} \tag{5}$$

where $Z_{i,t}$ is a vector of control variables and $\lambda_{t+1}^{Z}$ is a vector of corresponding risk prices. Using $T-1$ cross-sectional estimates of the risk prices, we compute the average risk price associated with $\lambda_j^{CIQ}$ as follows:

$$\widehat{\lambda}_j^{CIQ} = \frac{1}{T-1} \sum_{t=2}^{T} \widehat{\lambda}_{t,j}^{CIQ}, \quad j = LT, C, UT \tag{6}$$

[18] A stock is identified as available if 48 monthly return observations are available during the last 60-month window up to time $t$ and an observation occurs at time $t+1$.

and report them alongside their $t$-statistics on the basis of robust standard errors following Newey and West (1987) with six lags. Moreover, unlike in the case of portfolio sorting, this setup enables us to jointly evaluate the risk premia of the $\Delta$CIQ factors by including them simultaneously in the regressions.

We summarize the first set of results in Panel A of Table 4, where we report the estimation outcomes of the regressions with general risk measures. First, by considering the settings featuring all three $\Delta$CIQ exposures, we observe that only the lower-tail exposure significantly predicts future returns, with a coefficient of 1.26 ($t$-statistic of 2.78). Adding idiosyncratic volatility, total and idiosyncratic skewness, and the market beta to the regression does not alter the results, and the coefficient for lower-tail exposure is 0.82 ($t$-statistic of 2.68) in the full setting. On the other hand, exposures to the central or upper-tail $\Delta$CIQ factors remain unpriced.

We also investigate whether firm-specific characteristics based on accounting and trading information can capture the lower-tail premium.[19]. In Panel B, we predict the returns using the exposure to the lower-tail $\Delta$CIQ factor and size, book-to-price ratio, net payout yield, turnover, illiquidity, profit, and investment. All these specifications do not affect the significance or magnitude of the effect of $\beta_{LT}^{CIQ}$ on the expected returns, and we observe that the cross-section of stock returns robustly compensates for the exposure to the common idiosyncratic lower-tail events. in For example, in the setting featuring all the characteristics, the estimated price of risk is equal to 1.08 ($t$-statistic of 3.17).

Finally, we focus on nonlinear risk measures that may be correlated with exposure to common idiosyncratic lower-tail movements. We report the results from the regressions in

[19]We construct the variables following Langlois (2020).

## TABLE 4

## Fama–MacBeth Regressions with General Characteristics

The table shows estimated prices of risk and their $t$-statistics from Fama–MacBeth predictive regressions. Each month, we cross-sectionally regress next-month stock returns on current-month estimate of the exposures to the ΔCIQ factors while controlling for various stock and firm characteristics. In Panel A, we control for idiosyncratic volatility (IVOL) and skewness (ISKEW), total skewness (SKEW) and market beta ($\beta^{CAPM}$). In Panel B, we focus on lower-tail ΔCIQ exposure and various firm-specific characteristics constructed as in Langlois (2020). The resulting coefficients are calculated as averages of the monthly estimated coefficients and corresponding $t$-statistics are based on the robust standard errors suggested by Newey and West (1987) with six lags. The return sample covers period between January 1968 and December 2024. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months, and exclude penny stocks with prices of $1 or less. Note the coefficients are multiplied by 100 to ensure the clarity of the presentation.

*Panel A. Risk Characteristics*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $\beta_{LT}^{CIQ}$ | 1.32 | | | 1.26 | 1.15 | 1.25 | 1.26 | 0.81 | 0.82 |
| | (3.38) | | | (2.78) | (2.82) | (2.77) | (2.79) | (2.39) | (2.68) |
| $\beta_{C}^{CIQ}$ | | 3.32 | | 1.16 | -0.06 | 0.93 | 0.94 | 1.60 | 0.27 |
| | | (1.20) | | (0.39) | (-0.02) | (0.31) | (0.32) | (0.56) | (0.11) |
| $\beta_{UT}^{CIQ}$ | | | -0.15 | -0.48 | -0.36 | -0.44 | -0.45 | -0.07 | -0.01 |
| | | | (-0.34) | (-1.09) | (-0.91) | (-1.02) | (-1.04) | (-0.18) | (-0.04) |
| IVOL | | | | | -16.70 | | | | -15.80 |
| | | | | | (-4.51) | | | | (-4.68) |
| SKEW | | | | | | 0.00 | | | -0.14 |
| | | | | | | (-0.13) | | | (-2.15) |
| ISKEW | | | | | | | 0.01 | | 0.14 |
| | | | | | | | (0.32) | | (2.14) |
| $\beta^{CAPM}$ | | | | | | | | -0.19 | -0.14 |
| | | | | | | | | (-1.59) | (-1.14) |
| Intercept | 0.82 | 0.80 | 0.80 | 0.83 | 1.12 | 0.81 | 0.81 | 0.92 | 1.15 |
| | (3.77) | (3.44) | (3.67) | (4.07) | (6.39) | (4.06) | (4.04) | (5.18) | (7.55) |
| $R^2_{adj}$ | 0.66 | 0.41 | 0.43 | 1.50 | 2.62 | 1.65 | 1.64 | 3.14 | 4.23 |
| $\bar{n}$ | 3045 | 3045 | 3045 | 3045 | 3043 | 3043 | 3043 | 3043 | 3043 |
| $T$ | 684 | 684 | 684 | 684 | 684 | 684 | 684 | 684 | 684 |

*Panel B. Firm Characteristics*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\beta_{LT}^{CIQ}$ | 1.34 | 1.30 | 1.23 | 1.20 | 1.32 | 1.32 | 1.19 | 1.08 |
| | (3.48) | (3.47) | (3.48) | (3.12) | (3.30) | (3.36) | (3.14) | (3.17) |
| Size | -0.02 | | | | | | | -0.01 |
| | (-1.91) | | | | | | | (-1.71) |
| Book-to-price | | 0.13 | | | | | | 0.12 |
| | | (2.33) | | | | | | (2.00) |
| Net payout yield | | | 1.16 | | | | | 0.84 |
| | | | (1.59) | | | | | (1.18) |
| Turnover | | | | -0.08 | | | | -0.08 |
| | | | | (-1.48) | | | | (-1.84) |
| Illiquidity | | | | | 1.49 | | | 1.92 |
| | | | | | (1.27) | | | (1.22) |
| Profit | | | | | | 0.45 | | 0.48 |
| | | | | | | (3.75) | | (3.87) |
| Investment | | | | | | | -0.43 | -0.38 |
| | | | | | | | (-6.62) | (-6.38) |
| Intercept | 0.83 | 0.68 | 0.79 | 0.83 | 0.80 | 0.68 | 0.87 | 0.60 |
| | (3.73) | (3.33) | (3.55) | (3.88) | (3.73) | (3.21) | (4.03) | (2.99) |
| $R^2_{adj}$ | 0.99 | 1.45 | 1.25 | 1.52 | 1.34 | 1.19 | 1.06 | 3.26 |
| $\bar{n}$ | 3045 | 2897 | 2900 | 2893 | 2871 | 2887 | 2887 | 2741 |
| $T$ | 684 | 684 | 684 | 684 | 684 | 684 | 684 | 684 |

Table 5. We start by considering coskewness and cokurtosis, following the specifications reported in Harvey and Siddique (2000) and Dittmar (2002), respectively, and control for these measures simultaneously. We show that these measures do not drive out the significance of the lower-tail $\Delta$CIQ betas and neither factor is significant in this setting. Next, we consider the effect of a stock's lagged return and its momentum (cumulative return over the past twelve months while excluding the most recent return). Both control variables are highly significant, but the lower-tail $\Delta$CIQ exposure remains significant, with a slightly diminished coefficient of 0.94 ($t$-statistic of 2.67).

Next, we consider the simultaneous effect of the exposures to three systematic volatility measures on the lower-tail premium: the $\Delta$PCA-SQ factor, the $\Delta$CIV factor of Herskovic et al. (2016), and the $\Delta$VIX factor. We conclude that the lower-tail factor extracts different pricing information, as neither its magnitude nor significance decrease with exposure to volatility. The common distribution must be investigated in further depth if pricing information regarding the common distributional movements is to be identified.

Similarly, we jointly consider exposure to the tail-risk factor reported in Kelly and Jiang (2014) and the downside beta reported in Ang, Chen, and Xing (2006a). The results show that these two measures do not drive out the effect of the lower-tail $\Delta$CIQ betas, which remains significant, similar to the univariate specification.

Finally, we consider the following three measures separately: the hybrid tail covariance risk (HTCR) proposed by Bali, Cakici, and Whitelaw (2014), the multivariate crash risk (MCRASH) reported in Chabi-Yo, Huggenberger, and Weigert (2022), and the predicted systematic skewness (PSS) employed in Langlois (2020). Although the HTCR and MCRASH are

TABLE 5

## Fama–MacBeth Regressions with Nonlinear Risk Measures

The table shows estimated prices of risk and their $t$-statistics from Fama–MacBeth predictive regressions. Each month, we cross-sectionally regress next-month stock returns on current-month estimates of exposure to the lower-tail $\Delta$CIQ factor and other risk measure that captures firm's risk exposure. We control for co-skewness (CSK), co-kurtosis (CKT), previous-month return (STR), cumulative return over $t-11$ to $t-1$ months (MOM), exposure to the $\Delta$PCA-SQ factor ($\beta^{PCA-SQ}$), exposure to the $\Delta$CIV factor ($\beta^{CIV}$), exposure to the $\Delta VIX$ ($\beta^{VIX}$), tail-risk beta ($\beta^{tail}$), downside-risk beta ($\beta^{down}$), hybrid tail covariance risk (HTCR), multivariate crash risk (MCRASH) and predicted co-skewness (PSS). The resulting coefficients are calculated as averages of the monthly estimated coefficients and corresponding $t$-statistics are based on the robust standard errors suggested by Newey and West (1987) with six lags. The return sample covers period between January 1968 and December 2024, with the exception of the case of $\beta^{VIX}$, which begins in February 1990. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months, and exclude penny stocks with prices of $1 or less. Note the coefficients are multiplied by 100 to ensure the clarity of the presentation.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\beta_{LT}^{CIQ}$ | 1.32 | 1.22 | 0.94 | 1.32 | 1.02 | 1.35 | 1.48 | 1.25 |
| | (3.38) | (3.25) | (2.67) | (3.47) | (3.14) | (3.52) | (3.43) | (3.44) |
| CSK | | -0.30 | | | | | | |
| | | (-1.10) | | | | | | |
| CKT | | -0.09 | | | | | | |
| | | (-1.28) | | | | | | |
| STR | | | -3.86 | | | | | |
| | | | (-9.11) | | | | | |
| MOM | | | 0.64 | | | | | |
| | | | (4.28) | | | | | |
| $\beta^{PCA\text{-}SQ}$ | | | | 19.07 | | | | |
| | | | | (0.64) | | | | |
| $\beta^{CIV}$ | | | | -0.23 | | | | |
| | | | | (-2.12) | | | | |
| $\beta^{VIX}$ | | | | -6.25 | | | | |
| | | | | (-1.62) | | | | |
| $\beta^{down}$ | | | | | -0.04 | | | |
| | | | | | (-0.42) | | | |
| $\beta^{tail}$ | | | | | 0.09 | | | |
| | | | | | (1.24) | | | |
| HTCR | | | | | | 1.02 | | |
| | | | | | | (2.75) | | |
| MCRASH | | | | | | | 2.39 | |
| | | | | | | | (2.51) | |
| PSS | | | | | | | | -2.53 |
| | | | | | | | | (-1.48) |
| Intercept | 0.82 | 0.86 | 0.71 | 0.91 | 0.81 | 0.90 | 0.57 | 2.11 |
| | (3.77) | (3.37) | (3.23) | (3.48) | (4.44) | (4.52) | (2.40) | (2.13) |
| $R^2_{adj}$ | 0.66 | 1.87 | 2.64 | 1.34 | 2.48 | 1.56 | 1.43 | 2.09 |
| $\bar{n}$ | 3045 | 3043 | 3042 | 3009 | 3043 | 3044 | 2039 | 2739 |
| $T$ | 684 | 684 | 684 | 418 | 684 | 684 | 684 | 684 |

highly relevant for determining expected returns, these factors do not alter the effect of CIQ($\tau$) risk.

Using portfolio sorts and firm-level cross-sectional regressions, we showed that exposure to common downside idiosyncratic tail risk is significantly priced in the cross-section of stock returns. Assets with higher exposure to lower-tail $\Delta CIQ$ innovations earn higher expected returns, whereas those with higher exposure to upper-tail or central CIQ innovations earn no significant premium. Importantly, this pricing effect is not subsumed by common volatility or other existing measures of aggregate risk, reinforcing the conclusion that downside CIQ captures economically distinct information.

# IV. Economic Origins of the CIQ Risks

We now investigate the economic origins of the lower-tail CIQ premium that we observe in the data. We start by providing time series evidence that points toward an intermediary-based mechanism. Next, we identify firm characteristics associated with exposure to common idiosyncratic tail movements, clarifying which firms are most vulnerable to ΔCIQ risk. By doing so, we provide further evidence in favor of the intermediary-based explanation of the observed premium. Finally, we provide a theoretical mechanism consistent with our empirical findings.

## A. CIQ Risks in the Economy: Time Series Evidence

We link the lower-tail CIQ innovations to proxies for intermediary constraints, uncertainty, and market-wide liquidity. Because constraints bind primarily on the downside, we expect the relationships for lower $\tau$ values to be significant and those for upper-tail values to be insignificant. Motivated by theories in which adverse states amplify downside outcomes, we test whether $\Delta CIQ$ innovations are predictable on the basis of intermediary balance-sheet conditions, aggregate uncertainty, and market-wide liquidity.

We estimate various versions of the following:

$$\Delta CIQ^j_{t+1} = \beta_0 + \beta_1 \Delta CIQ^j_t + \beta_2 ICF_t + \beta_3 VIX^2_t + \beta_4 X_t + \beta_5 (ICF_t \times X_t) + \varepsilon_{t+1}, \quad (7)$$

for $j \in \{LT, UT\}$, where $ICF_t$ is the intermediary capital factor of He et al. (2017), $VIX_t^2$ controls for aggregate uncertainty, and $X_t$ denotes alternative illiquidity measures based on Amihud (2002): changes in average illiquidity $\Delta Avg(ILLQ)_t$, downside and upside illiquidity $\Delta Avg(ILLQ)_t^-$, and $\Delta Avg(ILLQ)_t^+$ (computed within negative- and positive-return stocks, respectively) and the change in illiquidity dispersion $\Delta Var(ILLQ)_t$.[20] The standard errors are from Newey and West (1994).

Panel A of Table 6 shows that the lower-tail factor is strongly mean-reverting ($\beta_1 < 0$) and, more importantly, is systematically related to financial conditions. A deterioration in intermediary capital predicts a decline in $\Delta CIQ^{LT}$ (i.e., worse downside tail conditions), and a higher $VIX^2$ value similarly predicts a decline. Liquidity conditions add incremental predictive content: increases in average illiquidity predict decreases in $\Delta CIQ^{LT}$, and this effect is concentrated in sell-side illiquidity ($\Delta Avg(ILLQ)^-$), which is consistent with an inherently asymmetric mechanism. In addition, increases in illiquidity dispersion predict decreases in $\Delta CIQ^{LT}$, indicating that downside tail conditions worsen when liquidity stress becomes uneven and widespread. The interaction terms imply state dependence: the predictive effect of weak intermediary capital is amplified when average illiquidity or illiquidity dispersion increases.

Panel B reports analogous results for the upper-tail factor. While $\Delta CIQ^{UT}$ is also mean-reverting, its relationship with intermediary capital and illiquidity is markedly weaker and less robust. In particular, the illiquidity variables that strongly predict the lower-tail factor have little explanatory power for the upper-tail factor, and interaction effects are negligible.

Taken together, the results reveal a pronounced asymmetry between downside and upside

[20] Illiquidity for stock $i$ in month $t$ is estimated as $ILLIQ_{i,t} = \frac{1}{D_{i,t}} \sum_{d=1}^{D_{i,t}} \frac{|R_{i,d}|}{\mathrm{V}_{i,d}}$, where $D_{i,t}$ is the number of valid trading days in month $t$, $R_{i,d}$ is the daily return, and $\mathrm{V}_{i,d}$ is daily price times shares traded.

TABLE 6

## Predicting the $\Delta$CIQ Factors

The table reports predictive regressions of $\Delta CIQ$ factors on its lagged value, intermediary capital factor of He et al. (2017) ($ICF$), squared CBOE volatility index ($VIX^2$), and various cross-sectional measures of illiquidity based on Amihud (2002): average illiquidity ($Avg(ILLQ)$), variance of illiquidity ($Var(ILLQ)$), and downside (upside) illiquidity averaged by using only stocks that experienced negative (positive) monthly returns ($Avg(ILLQ)^-$ and $Avg(ILLQ)^+$), respectively). Explanatory variables enter the regressions at time $t$, while the dependent variables are in time $t+1$. The $t$-statistics (in parentheses) are based on standard errors of Newey and West (1994). The $\Delta$CIQ factors and the illiquidity measures cover the period from February 1968 to December 2024, $ICF$ covers the period between February 1970 to December 2018, $VIX^2$ data cover the period between February 1990 and December 2024.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| *Panel A.* *Dependent variable: $\Delta CIQ^{LT}_{t+1}$* | | | | | | | | | |
| $\Delta CIQ_t$ | -0.44 | -0.41 | -0.51 | -0.50 | -0.50 | -0.51 | -0.50 | -0.50 | -0.50 |
| | (-13.20) | (-11.72) | (-19.25) | (-15.61) | (-15.33) | (-14.21) | (-15.17) | (-16.21) | (-15.45) |
| $ICF_t$ | | 0.52 | | 0.67 | 0.63 | 0.69 | 0.66 | 0.65 | 0.67 |
| | | (4.39) | | (7.33) | (6.69) | (6.65) | (7.11) | (6.79) | (7.21) |
| $VIX^2_t$ | | | -1.11 | -0.71 | -0.71 | -0.70 | -0.71 | -0.61 | -0.71 |
| | | | (-5.78) | (-4.66) | (-4.46) | (-4.28) | (-4.68) | (-3.10) | (-4.56) |
| *Cross-sectional liquidity measures* | | | | | | | | | |
| $\Delta Avg(ILLQ)_t$ | | | | | -0.09 | | | -0.12 | |
| | | | | | (-3.05) | | | (-3.50) | |
| $\Delta Avg(ILLQ)^-_t$ | | | | | | -0.06 | | | |
| | | | | | | (-3.06) | | | |
| $\Delta Avg(ILLQ)^+_t$ | | | | | | 0.20 | | | |
| | | | | | | (1.37) | | | |
| $\Delta Var(ILLQ)_t$ | | | | | | | -0.07 | | -0.10 |
| | | | | | | | (-5.83) | | (-10.93) |
| *Interactions with capital factor* | | | | | | | | | |
| $ICF_t \times \Delta Avg(ILLQ)_t$ | | | | | | | | 0.97 | |
| | | | | | | | | (3.62) | |
| $ICF_t \times \Delta Var(ILLQ)_t$ | | | | | | | | | 1.10 |
| | | | | | | | | | (10.25) |
| Intercept | -0.00 | -0.01 | 0.04 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| | (-0.75) | (-0.97) | (3.17) | (1.41) | (1.43) | (1.38) | (1.43) | (1.23) | (1.46) |
| $R^2_{adj}$ | 19.29 | 22.43 | 28.17 | 33.74 | 33.92 | 34.08 | 33.68 | 34.05 | 33.65 |
| $T$ | 683 | 587 | 419 | 347 | 347 | 347 | 347 | 347 | 347 |
| *Panel B.* *Dependent variable: $\Delta CIQ^{UT}_{t+1}$* | | | | | | | | | |
| $\Delta CIQ_t$ | -0.43 | -0.42 | -0.45 | -0.45 | -0.44 | -0.44 | -0.45 | -0.44 | -0.45 |
| | (-14.94) | (-10.67) | (-11.41) | (-9.62) | (-9.53) | (-10.56) | (-9.57) | (-9.57) | (-9.53) |
| $ICF_t$ | | -0.51 | | -0.34 | -0.37 | -0.39 | -0.34 | -0.39 | -0.35 |
| | | (-3.58) | | (-1.69) | (-1.76) | (-2.01) | (-1.69) | (-1.84) | (-1.69) |
| $VIX^2_t$ | | | 0.61 | 0.26 | 0.25 | 0.25 | 0.26 | 0.11 | 0.26 |
| | | | (3.32) | (1.48) | (1.47) | (1.51) | (1.47) | (0.43) | (1.47) |
| *Cross-sectional liquidity measures* | | | | | | | | | |
| $\Delta Avg(ILLQ)_t$ | | | | | -0.06 | | | -0.02 | |
| | | | | | (-0.84) | | | (-0.41) | |
| $\Delta Avg(ILLQ)^-_t$ | | | | | | 0.01 | | | |
| | | | | | | (0.31) | | | |
| $\Delta Avg(ILLQ)^+_t$ | | | | | | -0.30 | | | |
| | | | | | | (-1.44) | | | |
| $\Delta Var(ILLQ)_t$ | | | | | | | -0.03 | | -0.03 |
| | | | | | | | (-1.67) | | (-2.28) |
| *Interactions with capital factor* | | | | | | | | | |
| $ICF_t \times \Delta Avg(ILLQ)_t$ | | | | | | | | -1.45 | |
| | | | | | | | | (-1.25) | |
| $ICF_t \times \Delta Var(ILLQ)_t$ | | | | | | | | | -0.10 |
| | | | | | | | | | (-0.26) |
| Intercept | -0.01 | -0.01 | -0.04 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 |
| | (-1.45) | (-1.26) | (-2.66) | (-1.34) | (-1.32) | (-1.29) | (-1.33) | (-1.09) | (-1.33) |
| $R^2_{adj}$ | 17.68 | 20.92 | 19.78 | 20.21 | 20.11 | 20.19 | 20.00 | 20.35 | 19.76 |
| $T$ | 683 | 587 | 419 | 347 | 347 | 347 | 347 | 347 | 347 |

idiosyncratic dynamics. Innovations in the lower-tail CIQ factor are closely linked to intermediary balance-sheet conditions, aggregate uncertainty, sell-side liquidity stress, and general cross-sectional dispersion of illiquidity. In contrast, upside CIQ innovations exhibit substantially weaker connections to liquidity and intermediary constraints. This asymmetry suggests that the lower-tail CIQ factor relates to endogenous, liquidity-driven tail risk that emerges when intermediary capacity is strained, whereas upside tail realizations are less affected by these financial frictions.

Such asymmetry is consistent with the inherently one-sided nature of balance-sheet constraints. When intermediaries are constrained, selling pressure is amplified into large negative price movements, while there is no comparable mechanism through which buying is forced. As a result, intermediation stress manifests itself primarily through a deterioration of the lower tail of the cross-sectional distribution of idiosyncratic returns rather than through changes in the upper tail.

## B. Cross-Sectional Determinants

Our cross-sectional pricing tests assume that firms differ in their exposure to the lower-tail $\Delta CIQ$ factor. To understand the economic origins of this heterogeneity, we examine which firm characteristics are associated with sensitivity to common idiosyncratic tail movements. This analysis provides an economic interpretation of the $\Delta CIQ$ factors by identifying firm attributes—such as liquidity fragility, financial slack, and vulnerability—that are systematically related to CIQ exposures and, ultimately, to the associated risk premium. Crucially, the results presented further align with the time series properties of CIQ risk, which are related to the intermediary-based interpretation.

We use a subset of firm characteristics from Freyberger, Neuhierl, and Weber (2020) and Kim, Korajczyk, and Neuhierl (2020), which are reported in Internet Appendix L in Table A12, and organize them into the following economically motivated channels suggested by the intermediary-liquidity mechanism: (i) *liquidity fragility and trading*, (ii) *funding and financial slack*, (iii) *risk controls*, and (iv) *fundamental, vulnerability, and investment controls*.[21] Within each channel, we select a parsimonious set of representative variables to limit redundancy among highly correlated characteristics. We estimate monthly Fama–MacBeth regressions of firm-level CIQ betas on the basis of these characteristics after each characteristic is cross-sectionally ranked and scaled to the interval $(-1, +1)$. CIQ betas are similarly standardized, allowing coefficient magnitudes to be compared across specifications and across tails. Reported coefficients and $t$-statistics are obtained from the time series of monthly estimates using the corrections recommended by Newey and West (1994). Owing to data availability, the sample of characteristics spans the period between January 1968 and December 2018.

Panel A of Table 7 shows that exposure to lower-tail CIQ risk is concentrated in firms with fragile trading conditions and limited financial slack. Measures of liquidity fragility—such as low turnover depth (`dto` and `lturnover`), high turnover volatility (`std_turn`), and elevated unexplained volume (`suv`)—are strongly associated with higher CIQ betas. Firms with greater financial flexibility, as measured by cash holdings (`c`), exhibit lower CIQ exposure levels, whereas firms with higher net payouts (`nop`) exhibit higher exposure levels. The market beta (`beta`) has a negative sign, which is consistent with the view that the CIQ exposure level is higher for stocks whose risk is less hedgeable and therefore more costly for intermediaries to warehouse during stress. Firms trading closer to recent price highs (`rel_to_high_price`) also exhibit greater lower-tail CIQ exposure, which is consistent with sharper valuation adjustments

[21] We are grateful to Andreas Neuhierl for sharing the data with us.

when downside risk materializes. Measures of fundamental vulnerability, such as industry-adjusted profit margins (pm_adj), further indicate that CIQ exposure is concentrated among firms more susceptible to funding-driven sell pressure.[22]

The economic interpretation of CIQ further depends on whether these cross-sectional determinants are symmetric across return tails. If CIQ merely reflected a generic order flow or trading activity, similar characteristics would explain both downside and upside exposures. In contrast, theories of constrained intermediation predict strong asymmetries: downside order flow is amplified by forced selling and balance-sheet constraints, whereas upside order flow reflects discretionary trading in favorable states. Consistent with this prediction, the results of the Fama–MacBeth regressions for upper-tail exposures in Panel B of Table 7 show that liquidity fragility and funding vulnerability lose explanatory power in the upper tail but load positively on turnover depth and the market beta.

This sharp contrast across tails reinforces the relationship between lower-tail $\Delta$CIQ and downside liquidity risk arising from constrained intermediation. These patterns support the interpretation of lower-tail CIQ betas capturing exposure to idiosyncratic downside sell pressure amplified by constrained intermediation rather than a generic volatility or value effect. CIQ-sorted portfolios tilt toward firms that are difficult to intermediate in stress states—stocks with fragile liquidity and limited financial slack—providing an economic foundation for the pricing results.

[22] In Table A11, we provide regression results using all the characteristics considered in our investigation.

## TABLE 7

## Cross-Sectional Determinants of CIQ Exposure

This table reports Fama–MacBeth cross-sectional regressions explaining firm-level exposure to innovations in the CIQ factors. The dependent variable is each stock's rolling 60-month beta with respect to either $\Delta CIQ^{LT}$ (Panel A) or $\Delta CIQ^{UT}$ (Panel B). Firm characteristics are ranked cross-sectionally each month and linearly scaled to lie between $-1$ and 1. Regressions are estimated monthly, and reported coefficients are time-series averages. The $t$-statistics based on Newey and West (1994) are reported in parentheses. Firm-level characteristics are from Freyberger et al. (2020) and Kim et al. (2020). The sample covers the period from January 1968 to December 2018.

| | 1<br>Liquidity | 2<br>Funding | 3<br>Risk | 4<br>Vulnerability | 5<br>Full |
|---|---|---|---|---|---|
| *Panel A.*<br>*Dependent variable:* $\beta^{CIQ}_{LT}$ | | | | | |
| *Liquidity fragility* | | | | | |
| `dto` | -0.02 | | | | -0.01 |
| | (-7.94) | | | | (-6.85) |
| `lturnover` | -0.28 | | | | -0.24 |
| | (-11.47) | | | | (-10.27) |
| `std_turn` | 0.10 | | | | 0.10 |
| | (4.84) | | | | (4.84) |
| `suv` | 0.02 | | | | 0.01 |
| | (6.35) | | | | (3.63) |
| *Funding and financial slack* | | | | | |
| `c` | | -0.06 | | | -0.04 |
| | | (-5.01) | | | (-3.70) |
| `nop` | | 0.11 | | | 0.07 |
| | | (3.49) | | | (3.83) |
| *Risk controls* | | | | | |
| `beta` | | | -0.10 | | -0.04 |
| | | | (-13.05) | | (-8.45) |
| `rel_to_high_price` | | | 0.07 | | 0.05 |
| | | | (4.77) | | (4.44) |
| *Vulnerability control* | | | | | |
| `pm_adj` | | | | -0.06 | -0.04 |
| | | | | (-5.39) | (-6.42) |
| Intercept | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 |
| | (-1.07) | (-0.72) | (-1.20) | (-1.11) | (-1.13) |
| $R^2_{adj}$ | 7.21 | 4.65 | 4.54 | 2.35 | 10.85 |
| $\bar{n}$ | 2578 | 2578 | 2578 | 2578 | 2578 |
| $T$ | 612 | 612 | 612 | 612 | 612 |
| *Panel B.*<br>*Dependent variable:* $\beta^{CIQ}_{UT}$ | | | | | |
| *Liquidity fragility* | | | | | |
| `dto` | 0.01 | | | | 0.00 |
| | (2.35) | | | | (1.75) |
| `lturnover` | 0.10 | | | | 0.08 |
| | (5.40) | | | | (4.53) |
| `std_turn` | -0.02 | | | | -0.01 |
| | (-1.06) | | | | (-1.10) |
| `suv` | 0.00 | | | | 0.00 |
| | (-1.19) | | | | (-0.13) |
| *Funding and financial slack* | | | | | |
| `c` | | 0.01 | | | 0.01 |
| | | (0.56) | | | (0.62) |
| `nop` | | -0.06 | | | -0.03 |
| | | (-3.24) | | | (-3.37) |
| *Risk controls* | | | | | |
| `beta` | | | 0.05 | | 0.03 |
| | | | (5.93) | | (5.77) |
| `rel_to_high_price` | | | -0.03 | | -0.01 |
| | | | (-2.20) | | (-1.57) |
| *Vulnerability control* | | | | | |
| `pm_adj` | | | | -0.01 | -0.02 |
| | | | | (-1.11) | (-2.61) |
| Intercept | 0.00 | -0.00 | 0.00 | 0.00 | -0.00 |
| | (0.09) | (-0.03) | (0.52) | (0.04) | (-0.11) |
| $R^2_{adj}$ | 3.54 | 2.52 | 2.64 | 1.44 | 5.87 |
| $\bar{n}$ | 2578 | 2578 | 2578 | 2578 | 2578 |
| $T$ | 612 | 612 | 612 | 612 | 612 |

## C. Theoretical Mechanism

Considering the previous results, we now present an economic mechanism that can rationalize the observed premium and is consistent with the observed empirical patterns. It is based on an intermediary-driven mechanism whereby financial intermediaries provide liquidity by absorbing inventory risk, subject to balance sheet constraints. When intermediary capital deteriorates or aggregate uncertainty increases, these constraints tighten, reducing intermediaries' willingness to absorb sell-side order flow. Consequently, negative idiosyncratic shocks are more likely to propagate into common downside tail realizations across assets, resulting in elevated price impact and transaction costs.

In this environment, the lower-tail CIQ factor captures situations in which the risk-bearing capacity of intermediaries is impaired and sell-side liquidity evaporates across the board. The significant impact of downside illiquidity and its dispersion reflects the uneven distribution of liquidation pressure among firms: when intermediaries are under pressure, heterogeneous sell pressure results in correlated downside tail outcomes. In contrast, positive idiosyncratic shocks do not face comparable balance sheet frictions, meaning that upside CIQ innovations are weakly related to liquidity conditions and do not command a risk premium.

From an asset-pricing perspective, exposure to lower-tail CIQ innovations means exposure to states in which liquidity provision is costly and intermediary capital is scarce. In such states, the stochastic discount factor is high, meaning that assets that perform poorly when the lower-tail CIQ deteriorates require a higher expected return to compensate for this. The core mechanism is a state-dependent intermediary common stress intensity, which amplifies downside idiosyncratic realizations through forced selling but has limited relevance for upside quantiles.

Dynamic intermediary asset pricing models formalize this idea. In He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and He et al. (2017), intermediary net worth or leverage acts as a state variable that drives the equilibrium risk premium. In the systemic-risk literature, measures such as CoVaR and SRISK similarly condense a high-dimensional system into a low-dimensional fragility index (Adrian and Brunnermeier (2016), Acharya, Engle, and Richardson (2017), Giglio et al. (2016)). Within our simple reduced-form theoretical setting, we assume that the lower-tail CIQ factor represents a reduced-form proxy for the shadow cost of intermediaries' risk-bearing capacity. Nevertheless, we do not dismiss the possibility of a more general downside risk explanation of the observed risk premia.

## 1. A Reduced-Form Representation

Let $\varepsilon_{i,t}$ denote the idiosyncratic return from a linear factor model $r_{i,t} = \alpha_i + \beta_i' f_t + \varepsilon_{i,t}$, where $f_t$ collects benchmark factors (e.g., Fama–French factors). We assume a latent systemic state $s_t \in \mathbb{R}$ that summarizes the tightness of intermediaries' balance-sheet constraints. In fragile states (high $s_t$), risk-bearing capacity is scarce: intermediaries absorb less inventory, funding becomes more expensive, and the sell-side price impact increases. These conditions affect firm-level downside risk through tighter credit lines, higher margins and haircuts, stricter lending standards, and weaker secondary-market liquidity. Consequently, firm-specific negative shocks are more likely to result in extreme residual losses, particularly for firms with high leverage, low cash reserves, or illiquid equity.

We model idiosyncratic returns as responses to the liquidity demand absorbed by competitive intermediaries as follows:

$$\varepsilon_{i,t} \;=\; \lambda_t(s_t)\, u_{i,t} + \xi_{i,t}, \tag{8}$$

where $u_{i,t}$ is the idiosyncratic liquidity demand, $\xi_{i,t}$ is the idiosyncratic noise orthogonal to the liquidity demand, and $\lambda_t(s_t) > 0$ is a common price-impact coefficient (the shadow cost of the balance-sheet capacity). We assume that the price impact increases when the constraints are binding, $\partial\lambda_t/\partial s_t > 0$. This channel captures a common change in the scale of residual returns, akin to a volatility-type effect. On its own, however, it does not cause asymmetry between the downside and upside quantiles.

To capture the effects of *asymmetric* forced selling, we introduce a one-sided stress component to liquidity demand, as follows:

$$u_{i,t} \;=\; \tilde{u}_{i,t} \;-\; \ell_i \, J_t(s_t)\, \eta_{i,t}, \tag{9}$$

where $\tilde{u}_{i,t}$ is the baseline demand, $J_t(s_t) \geq 0$ is a common stress intensity, $\eta_{i,t} \geq 0$ is an idiosyncratic nonnegative shock, and $\ell_i \geq 0$ captures liquidation sensitivity or liquidity fragility (e.g., low turnover depth, unstable turnover, unexplained volume, or drawdown sensitivity). We assume that $J_t(s_t)$ increases when the constraints are binding, $\partial J_t/\partial s_t > 0$. This second channel is the primary source of asymmetry between downside and upside common factor structures: when aggregate stress intensifies, stocks with larger $\ell_i$ values experience negative tail realizations induced by forced selling.

The distinction between the two channels is important. If $J_t(s_t) = 0$, aggregate stress affects idiosyncratic returns only through the common scaling term $\lambda_t(s_t)$; thus, the distribution of residual returns is rescaled symmetrically across the upper and lower quantiles, and CIQ loses its distinctive left-tailed interpretation. In contrast, the one-sided component in equation (9) generates differential comovement specifically in the lower tail, which is the feature that the CIQ is designed to capture. Stocks with $\ell_i = 0$ are therefore not exposed to the asymmetric forced-selling channel, although they may still be affected by the common price-impact channel in (8).

Taken together, equations (8) and (9) should be interpreted as a reduced-form data-generating device designed to motivate CIQ rather than as a fully identified structural model of intermediary balance-sheet constraints. The purpose of this setup is to isolate two distinct channels through which the systemic state $s_t$ affects idiosyncratic returns: a common price-impact channel and a one-sided stress channel.

A key implication of the asymmetry assumption is that higher fragility $s_t$ generates a stronger coordinated downward shift in many firms' left-tail idiosyncratic outcomes in comparison to right-tail shifts. We capture this with a parsimonious quantile representation, as follows:

$$\epsilon_{i,t} = b_i(\tau)s_t + \nu_{i,t}(\tau) \tag{10}$$

where $b_i(\tau)$ is more negative for some sufficiently low $\tau = \tau_L$ when the constraints disproportionately amplify the downside outcomes, $b_i(\tau) \approx 0$ for the median and upper tail, and $\nu_{i,t}(\tau)$ satisfies the appropriate quantile restriction. Under this structure, the quantile factor extracted from the data is a reduced-form proxy for the latent state $CIQ_t(\tau) \approx G_\tau(s_t)$ for some monotone mapping $G_\tau(\cdot)$ that is steeper in the lower tail. Because $s_t$ is persistent, the levels of $CIQ_t(\tau)$ contain slow-moving components. Asset prices, however, respond to news about future investment opportunities and risk-bearing capacity. Standard intertemporal asset-pricing logic therefore motivates a focus on innovations,

$\Delta CIQ_t(\tau) \equiv CIQ_t(\tau) - CIQ_{t-1}(\tau) \;\approx\; G'_\tau(s_{t-1})\,\Delta s_t$, so that $\Delta CIQ_t(\tau_L)$ isolates "news" about aggregate downside idiosyncratic conditions rather than slowly evolving tail levels.

## 2. Intermediary Constraints and the Pricing Kernel

In a constrained-intermediary setting,[23] the stochastic discount factor (SDF) permits the following the representation:

$$m_{t+1} = \overline{m}_{t+1} + \psi_t\,\Delta s_{t+1}, \tag{11}$$

where $\psi_t > 0$ in constrained states. When the constraints tighten, $\psi_t$ increases, and the SDF assigns more weight to states with large portfolio losses; see, e.g., Brunnermeier and Pedersen (2009) or He and Krishnamurthy (2013). A standard linearization implies that expected excess returns satisfy the following:

$$\mathbb{E}_t[r_{i,t+1}] = \lambda_M \beta_i^M + \lambda_{s,t}\,\beta_i^s, \tag{12}$$

where $\beta_i^s = \mathrm{Cov}_t(r_{i,t+1}, \Delta s_{t+1})/\mathrm{Var}_t(\Delta s_{t+1})$ is exposure to risk-bearing-capacity news and $\lambda_{s,t}$ is its (time-varying) price of risk.

Quantile factor analysis applied to idiosyncratic returns results in recovery of the dominant common component in lower-tail quantiles. Accordingly, lower-tail CIQ innovations at $\tau_L$ (e.g., $\tau_L = 0.2$) estimate an affine transformation of $\Delta s_t$:

[23] Internet Appendix C formalizes a simple environment in which a cross-section of balance-sheet constraints aggregates into a single state $s_t$. Under standard conditions, the representative constrained intermediary's shadow cost of risk-bearing is proportional to $s_t$, and the lower-tail CIQ factor identifies this state.

**Proposition 1 (Quantile factor representation)** *Under standard large-$N$ conditions for approximate factor models, a lower-tail CIQ innovation at $\tau_L$ consistently estimates an affine transformation of $\Delta s_t$,*

$$\Delta CIQ_t^L \equiv \Delta CIQ_t(\tau_L) = \kappa\, \Delta s_t + \nu_t,$$

*with $\kappa \neq 0$ and an error term $\nu_t$.*

For the proof, see Proposition 2 and the discussion in Internet Appendix C. Empirically, we find that a single CIQ factor accounts for a significant proportion of the variation in realized lower-tail idiosyncratic quantiles and is distinct from volatility-based factors such as CIV or PCA-SQ. The estimated exposure to the $\Delta$CIQ$(\tau_L)$ factor,

$$\beta_i^{CIQ} = \frac{\text{Cov}(r_{i,t+1}, \Delta CIQ_{t+1}^L)}{\text{Var}(\Delta CIQ_{t+1}^L)},$$

therefore corresponds to exposure to innovations in systemic fragility. Assets whose returns are more adversely affected by these innovations must offer higher average returns as compensation, which is consistent with our empirical pricing results from the previous section.

# V. Robustness Checks

In this section, we examine how the lower-tail CIQ premium varies across different specifications. First, we analyze how the CIQ premium varies across a finer grid of quantiles. Second, we examine how altering the data and the model specification behind the abnormal returns affects the results. Finally, we investigate whether CIQ risk influences time variation in the aggregate equity premium by evaluating how CIQ risk innovations map onto market-wide expected return fluctuations.

## A. CIQ Premium across the Distribution

Figure 3 reports estimated risk premia for CIQ factors across quantile levels $\tau \in [0.05, 0.95]$ in increments of 0.05.[24] The pricing pattern varies systematically across quantiles: only lower-tail exposures ($\tau < 0.5$) command a significant premium, with the largest premia concentrated around $\tau \approx 0.2$. These results reinforce the central theme of the paper that downside idiosyncratic tail risk, rather than symmetric dispersion or upside potential, is the primary price dimension of common idiosyncratic risk.

To better understand the economic forces driving the cross-quantile variation in premia, we examine how differences in portfolio returns across quantiles vary over time. Specifically, for two quantile levels $\tau_1 < \tau_2$, we construct the return differential between the corresponding high-minus-low portfolios from equal-weighted decile sorts,

$$r_t^{diff(\tau_1,\tau_2)} \equiv r_t^{CIQ(\tau_1)} - r_t^{CIQ(\tau_2)}.$$

This difference is positive when exposure to more extreme lower tails is compensated for more than exposure to less severe states. We then compute the correlation between these return differentials and the lagged variance risk premium (VP) computed as suggested by Bekaert and Hoerova (2014), which proxies for fluctuations in risk aversion and risk-bearing capacity.

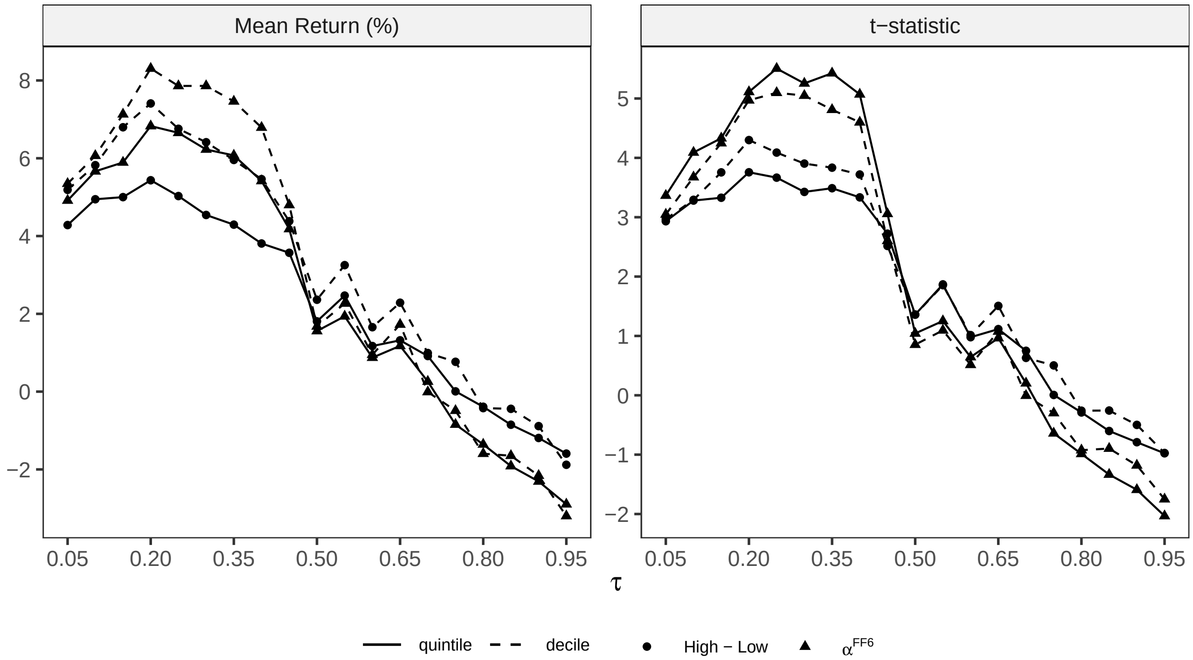
Panel A of Table 8 shows that these correlations are predominantly positive and increase

[24]Full results across specifications are reported in Internet Appendix K.

FIGURE 3

## CIQ Premia across Quantiles

The figure describes premia related to the exposures to the $\Delta$CIQ factors. We report average annual returns of high-minus-low equal-weighted portfolios from decile and quintile sorts and their alphas with respect to the six-factor model of Fama and French (2018). We also report $t$-statistics based on the correction of Newey and West (1994). The data come from the CRSP and cover the period from January 1968 to December 2024. We exclude penny stocks with prices of $1 or less.




as the distance between $\tau_1$ and $\tau_2$ increases. For example, the return differential between portfolios associated with $\tau = 0.05$ and those associated with $\tau = 0.35$ is strongly positively correlated with the lagged VP. This pattern implies that when risk aversion is elevated, compensation for exposure to more extreme downside tail risk increases disproportionately relative to milder tail exposures. In other words, investors require especially high compensation for assets that perform poorly in states associated with severe common downside realizations.

This evidence is consistent with quantile-dependent preference models, in which agents focusing on lower quantiles behave in a more risk-averse manner (de Castro and Galvao, 2019). Importantly, repeating the same analysis using conditional variance instead of the VP in Panel B of Table 8 yields no comparable pattern, indicating that the cross-quantile variation in premia is

TABLE 8

**State-Dependent CIQ Premia**

The table reports forward correlations between differences in CIQ portfolio returns and lagged measures of market risk aversion and volatility. The CIQ return differential is defined as the month-$t$ return difference between two high-minus-low portfolios constructed using exposures to the $\Delta CIQ(\tau)$ factor at quantile levels $\tau_1$ and $\tau_2$, with $\tau_1 < \tau_2$. CIQ portfolios are equal-weighted decile-sorted portfolios based on $\Delta CIQ(\tau)$ betas. Market measures are observed at month $t-1$ and correspond to either the variance risk premium (VP) in Panel A or the conditional variance (CV) in Panel B. The VP and CV data are from Bekaert and Hoerova (2014) and cover the period from February 1990 to January 2022.

| $\tau_1/\tau_2$ | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 |
|---|---|---|---|---|---|---|---|
| *Panel A. VP* | | | | | | | |
| 0.05 | 0.04 | 0.02 | 0.06 | 0.11 | 0.15 | 0.18 | 0.17 |
| 0.10 | | -0.01 | 0.05 | 0.12 | 0.17 | 0.19 | 0.18 |
| 0.15 | | | 0.08 | 0.16 | 0.20 | 0.23 | 0.20 |
| 0.20 | | | | 0.13 | 0.21 | 0.23 | 0.20 |
| 0.25 | | | | | 0.17 | 0.21 | 0.18 |
| 0.30 | | | | | | 0.14 | 0.12 |
| 0.35 | | | | | | | 0.04 |
| *Panel B. CV* | | | | | | | |
| 0.05 | 0.00 | -0.02 | 0.00 | 0.01 | 0.01 | 0.03 | 0.02 |
| 0.10 | | -0.03 | 0.00 | 0.02 | 0.01 | 0.04 | 0.02 |
| 0.15 | | | 0.04 | 0.05 | 0.04 | 0.07 | 0.04 |
| 0.20 | | | | 0.02 | 0.01 | 0.05 | 0.02 |
| 0.25 | | | | | 0.00 | 0.06 | 0.02 |
| 0.30 | | | | | | 0.08 | 0.02 |
| 0.35 | | | | | | | -0.04 |

driven by risk aversion and risk-bearing capacity rather than by fluctuations in aggregate uncertainty.

In addition, Internet Appendix M provides numerous approaches for aggregating exposures across CIQ factors and further evidence of the clear asymmetry between the pricing implications of downside and upside CIQ factors.

## B. Stability Checks

In this section, we investigate the stability of the premium associated with the lower-tail $\Delta$CIQ factor across various alternative data and model specifications. First, in Panel A of Table 9, we further address the concern that the results are driven by volatility. To alleviate the risk that the driving force behind the premium is firm-level time-varying idiosyncratic volatility, we standardize the idiosyncratic returns from equation (1) by their estimate of time-varying volatility using the simple exponentially weighted moving average (EWMA) model and use the standardized returns to estimate the CIQ factors.[25] We present the results of high-minus-low portfolios from the decile and quintile equal-weighted sorts. We observe that the premium remains highly significant, with annual returns of 6.92% ($t$-statistic of 3.88) and 4.68% ($t$-statistic of 3.32) in the cases of the decile and quintile sorts, respectively. Furthermore, the premium is not subsumed by the six-factor model of Fama and French (2018).

Next, to precisely measure the sensitivity of stocks to the lower-tail $\Delta$CIQ factor while controlling for the sensitivity to the changes in common idiosyncratic volatility, we estimate the lower-tail $\Delta$CIQ betas from equation (4) using multiple regression, and we also include the $\Delta$PCA-SQ factor as a control. The results show that this alternative does not affect the premium associated with lower-tail $\Delta$CIQ risks, as the premia remain at 6.75% ($t = 4.26$) and 4.83% ($t = 4.01$) in the cases of the decile and quintile sorts, respectively.

We also include results in which the exposures are estimated with respect to the *levels* of the CIQ factors. The premium remains strong even with respect to this specification, with values of 8.01% ($t = 2.88$) and 5.87% ($t = 2.57$) per annum for the respective sorts. Finally, we report the results using AR(1) lower-tail CIQ factor innovations, which also yield strong performance within this setting.

In Panel B, we report the portfolio results using idiosyncratic returns with respect to the FF5 and FF6 models. While the FF5 specification does not quantitatively alter the results from our baseline FF3 specification, the results using the FF6 model yield a slightly diminished premium. Given that the only difference between the FF5 and FF6 models is the inclusion of the

[25]The EWMA volatility model for random variable $e_t$ is defined as $\sigma^2_{t+1} = \lambda\sigma^2_t + (1-\lambda)e^2_t$, where we opt for the standard value of $\lambda = 0.94$.

TABLE 9

## Lower-Tail △CIQ Premium across Different Specifications

The table contains risk premia associated with exposure to the lower-tail △CIQ factor across various specifications. Premia are obtained as differences between high- and low-exposure decile or quintile equal-weighted portfolio returns. We also report alphas with respect to the six-factor model of Fama and French (2018) (FF6). In Panel A, firstly, we report results based on the lower-tail △CIQ factor that is estimated by reference to idiosyncratic returns that are standardized by their time-varying volatility using the EWMA model. Second, we report results using △CIQ betas that are estimated from multiple regression when controlling for the exposure to the △PCA-SQ factor. Third, we report results using betas estimated on levels and AR(1) innovations of the lower-tail CIQ factor, respectively. In Panel B, we report results from models that use FF5 and FF6 model to compute the idiosyncratic returns. In Panel C, we report results using data that do not exclude stocks based on their price, as well as for dataset that includes stocks that are traded with price above $5. In Panel D, first, we report results from sorting the stocks into portfolios based on the breakpoints obtained from NYSE stocks only. Second, we progressively exclude stocks with market capitalization below certain quantile level of NYSE traded companies. In Panel E, we report separately the premia across two disjoint time periods. Panel G captures returns that are obtained from average multi-period stock returns followed after the formation of the portfolios. The reported $t$-statistics are computed using Newey and West (1987) robust standard errors with six lags. The sample covers period between January 1968 and December 2024. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months, and exclude penny stocks with prices of $1 or less if not stated otherwise.

| | Decile Sorts | | | | Quintile Sorts | | | |
|---|---|---|---|---|---|---|---|---|
| | Premium | $t$-stat | $\alpha^{FF6}$ | $t$-stat | Premium | $t$-stat | $\alpha^{FF6}$ | $t$-stat |
| *Panel A. Volatility Control* | | | | | | | | |
| TV volatility std. | 6.92 | 3.88 | 7.71 | 4.49 | 4.68 | 3.32 | 6.16 | 4.89 |
| PCA-SQ control | 6.75 | 4.26 | 6.39 | 3.97 | 4.83 | 4.01 | 5.03 | 4.20 |
| CIQ level | 8.01 | 2.88 | 9.74 | 4.86 | 5.87 | 2.57 | 7.37 | 4.56 |
| AR(1) shocks | 8.70 | 3.56 | 9.82 | 5.12 | 6.54 | 3.23 | 7.75 | 5.12 |
| *Panel B. Linear Specification* | | | | | | | | |
| FF5 | 6.03 | 3.85 | 7.21 | 4.68 | 4.84 | 3.71 | 6.55 | 5.17 |
| FF6 | 4.38 | 2.84 | 6.08 | 3.77 | 3.73 | 2.93 | 5.65 | 4.32 |
| *Panel C. Stock Price* | | | | | | | | |
| All stocks | 7.76 | 4.44 | 8.64 | 4.99 | 5.79 | 3.96 | 7.20 | 5.15 |
| Price $>$ $5 | 6.71 | 3.97 | 7.30 | 5.27 | 4.64 | 3.34 | 5.89 | 5.37 |
| *Panel D. Firm Size* | | | | | | | | |
| NYSE breakpoints | 6.03 | 3.80 | 7.36 | 5.12 | 4.64 | 3.43 | 6.05 | 5.18 |
| Market cap $> q_{NYSE}$(10%) | 7.34 | 3.61 | 8.21 | 4.64 | 5.21 | 3.18 | 6.40 | 4.77 |
| Market cap $> q_{NYSE}$(20%) | 6.75 | 3.28 | 7.85 | 4.41 | 4.83 | 2.87 | 6.10 | 4.39 |
| Market cap $> q_{NYSE}$(50%) | 4.92 | 2.33 | 6.01 | 3.35 | 4.06 | 2.38 | 4.83 | 3.37 |
| *Panel E. Time Split* | | | | | | | | |
| 01/1968 - 12/1996 | 4.06 | 1.72 | 4.00 | 1.93 | 2.89 | 1.37 | 3.58 | 2.13 |
| 01/1997 - 12/2024 | 10.87 | 4.48 | 9.49 | 3.87 | 8.06 | 4.25 | 7.43 | 3.96 |
| *Panel F. Multi-Period Returns* | | | | | | | | |
| Next 3 months | 5.61 | 3.78 | 6.56 | 4.31 | 3.92 | 3.12 | 5.38 | 4.39 |
| Next 6 months | 5.13 | 3.87 | 6.65 | 4.30 | 3.61 | 3.24 | 5.26 | 4.42 |
| Next 12 months | 4.50 | 4.35 | 5.22 | 4.12 | 3.24 | 3.69 | 3.83 | 3.65 |
| $t+2$ to $t+12$ | 4.30 | 4.05 | 5.22 | 4.20 | 3.04 | 3.39 | 3.90 | 3.76 |

momentum factor in the linear specification, we can conclude that the momentum exposure is partially related to the common lower-tail events. The observation that momentum returns are associated with extreme negative events, so-called momentum crashes, has been well documented (see, e.g., Daniel and Moskowitz (2016), Barroso and Santa-Clara (2015)). This observation sheds further light on the drivers of the momentum risk premium, capturing the risk of lower-tail events in the cross-section of stock returns. Moreover, the alphas associated with the lower-tail $\Delta$CIQ risk remain high and significant, with annualized values of 6.08% ($t = 3.77$) and 5.65% ($t = 4.32$) for the decile and quintile sorts, respectively.

Next, in Panel C, we vary the stocks that we consider when forming the portfolios. First, as we restrict our universe to stocks with prices above $1 in our baseline specification, we report here the results using all the available stocks. We observe that the premia estimated using either decile or quintile sorts remain high and significant. This situation holds if we restrict our universe to stocks with prices above the $5 threshold.

In Panel D, we first investigate how the results change if we sort stocks into portfolios on the basis of New York Stock Exchange breakpoints. We see that the results of both decile- and quintile-based high-minus-low portfolios remain high and significant. These results show that the portfolio results are not driven by microcap stocks.

Next, in the same panel, we restrict our investment universe to stocks with market capitalization above the $p$th percentile of the distribution of stocks traded on the New York Stock Exchange at a given time. We report the results for the stocks with market capitalization greater than the 10%, 20%, and 50% percentiles. We observe a stable premium that, even if we focus on the stocks with capitalization above the NYSE median, yields annual returns of 4.92% ($t = 2.33$) and 4.06% ($t = 2.38$) for decile- and quintile-based premia, respectively.

In Panel E, we report separate results for two disjointed periods. The first period covers the time between January 1968 and December 1996, and the second period covers the time from January 1997 to December 2024. We observe that the premium is substantially lower for the first period—4.06% and 2.89% for the decile and quintile sorts, respectively. As shown in Figure 2, this situation is caused mostly by the period between approximately 1974 and 1980, in which the strategy did not yield any significant premium. On the other hand, since that time, relatively steady growth was observed from using the strategy, with a premium of $10.87\%$ ($t$-statistic of 4.48) for the decile-based high-minus-low strategy and a premium of $8.06\%$ ($t$-statistic of 4.25) for the quintile-based strategy.

A strengthening of the lower-tail premium in the second period is broadly consistent with our intermediary risk interpretation because the late 1990s saw the rise of market-based finance and leveraged dealer intermediation (e.g., expansion of securitization, repo funding, and hedge funds), making asset prices more sensitive to fluctuations in intermediary balance sheets. As a result, shocks such as the long-term capital management collapse or global financial crisis revealed that constrained intermediaries reduce risk-bearing capacity, increasing the required returns on assets exposed to funding and liquidity risk.

Finally, in Panel F, we consider the performance of the lower-tail $\Delta$CIQ-sorted portfolios as captured by the following multiperiod returns. This exercise helps us understand two points. First, we examine whether the investment strategy based on the lower-tail $\Delta$CIQ factor is feasible in terms of turnover by examining the returns of portfolios that are rebalanced with a lower than monthly frequency. Second, we can infer whether the premium associated with the exposure to the lower-tail $\Delta$CIQ factor is a compensation for risk and not just a reversal effect. If risk is the driving force underlying the abnormal returns, then multiperiod returns should remain economically and statistically significant. We proceed as follows. Each month, we form the portfolios using the same method used in the previous case. Instead of saving the next one-month return of the sorted portfolios, we record the three-, six-, and twelve-month returns after the formation period. We observe returns that are consistent with the results obtained using the one-month returns; for example, the high-minus-low portfolio rebalanced every six months yields a 5.13% ($t = 3.87$) decile-based premium and a 3.61% ($t = 3.24$) quintile-based premium on an annual basis. These results suggest that an investor does not have to suffer the high turnover costs associated with the strategy to exploit the associated risk premium.

To mitigate the effect of return reversals, we also extend this analysis by working with one-year returns but excluding returns immediately following the formation period. We report the results in the last row of Panel F. The resulting returns are almost indistinguishable from the returns over the full one-year period, with premia of 4.30% ($t = 4.05$) and 3.04% ($t = 3.39$) for the decile and quintile sorts, respectively.

Furthermore, consistent with the intuition that assets hedging the states in which liquidity provision is costly and intermediary capital is scarce command higher prices and lower expected returns, we examine whether the lower-tail CIQ premium is related to forward-looking measures of risk.[26] Regressions of portfolio returns on expected variance and the variance risk premium show that the CIQ premium is primarily associated with variation in the variance risk premium rather than expected variance. This distinction is important: while expected variance reflects changes in aggregate uncertainty, the variance risk premium is typically interpreted as capturing fluctuations in risk-bearing capacity and liquidity supply. The results therefore reinforce the view that lower-tail CIQ risk is priced because it loads on states in which liquidity provision is costly rather than on periods of elevated uncertainty alone.

[26] See Table A7 in Internet Appendix J.

## C. Time Series Implications for the Equity Premium

Having shown that innovations in the lower-tail ΔCIQ factor reflect periods of strained intermediary balance sheets, elevated aggregate uncertainty, and sell-side liquidity stress, we now examine whether these states also carry information about the aggregate equity premium. If the lower-tail ΔCIQ factor captures fluctuations in economy-wide risk-bearing capacity, it should predict subsequent market returns by identifying periods in which compensation for bearing risk is unusually high.

We investigate this hypothesis by examining the ability of the lower-tail ΔCIQ factor to forecast short-horizon equity returns, measured as the value-weighted excess return of all CRSP firms. We show that the predictive content of the lower-tail ΔCIQ factor is economically meaningful, robust to controls for existing predictors, and persists in an out-of-sample setting.

We begin with in-sample univariate predictive regressions of the following form:

$$r_{m,t+1} = \gamma_0 + \gamma_1 \Delta CIQ_t^j + \epsilon_{t+1}, \quad j = LT, C, UT, \tag{13}$$

where $r_{m,t+1}$ denotes the monthly market excess return. Each month, CIQ factors are estimated using the preceding 60 months of data, and the most recent innovation is used to predict the subsequent market return. Coefficients are scaled to represent the effect of a one-standard-deviation change in the predictor on the annualized market return. Statistical inference is based on Newey–West standard errors with six lags.

The results, reported in Table 10, show that the lower-tail ΔCIQ factor strongly predicts the equity premium. A one-standard-deviation decrease in the factor—corresponding to a deterioration in common idiosyncratic downside conditions—predicts an increase of 5.49

TABLE 10

## Market Return Predictability by ΔCIQ Factors

The table reports results from various specifications of predictive regressions of the value-weighted return of all CRSP firms on the ΔCIQ factors and control variables. We employ increments of the PCA-SQ factor ($\Delta PCA\text{-}SQ$), innovations of the CIV factor of Herskovic et al. (2016) ($\Delta CIV$), tail risk factor of Kelly and Jiang (2014) (TR), lagged market return ($\text{MKT}_{t-1}$), cross-sectional bivariate idiosyncratic volatility of Han and Li (2025) (CBIV), and short-term reversal factor. Coefficients are scaled to capture the effect of one-standard-deviation increase in the factor on the annualized market return in percentage points. The corresponding $t$-statistics reported in parentheses are computed using the Newey and West (1987) robust standard errors using six lags. We report both in-sample (IS) and out-of-sample (OOS) $R^2$s. We also truncate the predictions at zero following Campbell and Thompson (2007) (CT) and report corresponding OOS $R^2$s. The data cover the period from January 1968 to December 2024, with the exception of the CBIV, which ends in December 2022.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Delta CIQ^{LT}$ | -5.49 | | | -5.52 | -6.27 | -6.60 | -5.53 | -4.97 | -4.98 | -5.70 | -5.52 | -7.24 | -6.38 |
| | (-2.26) | | | (-2.22) | (-2.11) | (-2.38) | (-2.24) | (-1.96) | (-1.96) | (-2.12) | (-2.19) | (-2.10) | (-1.80) |
| $\Delta CIQ^{C}$ | | -2.30 | | -3.50 | -3.47 | -3.37 | -3.60 | -3.52 | -3.59 | -3.48 | -3.50 | -3.48 | -3.49 |
| | | (-0.96) | | (-1.32) | (-1.29) | (-1.22) | (-1.35) | (-1.30) | (-1.33) | (-1.30) | (-1.31) | (-1.21) | (-1.21) |
| $\Delta CIQ^{UT}$ | | | 3.75 | 5.98 | 6.84 | 7.09 | 6.22 | 5.96 | 6.08 | 6.07 | 5.98 | 8.30 | 7.89 |
| | | | (1.71) | (2.49) | (2.39) | (2.81) | (2.60) | (2.43) | (2.48) | (2.47) | (2.50) | (2.74) | (2.54) |
| $\Delta PCA\text{-}SQ$ | | | | | -1.51 | | | | | | | -1.53 | -1.35 |
| | | | | | (-0.49) | | | | | | | (-0.48) | (-0.42) |
| $\Delta CIV$ | | | | | | -3.48 | | | | | | -3.80 | -3.60 |
| | | | | | | (-0.99) | | | | | | (-1.10) | (-1.02) |
| TR | | | | | | | 4.83 | | 2.27 | | | 4.92 | 2.61 |
| | | | | | | | (2.34) | | (0.59) | | | (2.36) | (0.66) |
| CBIV | | | | | | | | 4.87 | 2.89 | | | | 2.63 |
| | | | | | | | | (2.21) | (0.70) | | | | (0.62) |
| $\text{MKT}_{t-1}$ | | | | | | | | | | 0.43 | | -0.41 | -0.75 |
| | | | | | | | | | | (0.21) | | (-0.20) | (-0.34) |
| STR | | | | | | | | | | | 0.00 | -0.53 | -0.40 |
| | | | | | | | | | | | (0.00) | (-0.21) | (-0.15) |
| $R^2$ IS | 1.00 | 0.17 | 0.46 | 2.00 | 2.04 | 2.34 | 2.77 | 2.56 | 2.60 | 2.01 | 2.00 | 3.18 | 2.97 |
| $R^2$ OOS | 0.62 | -0.17 | 0.01 | 0.95 | 0.20 | 0.27 | 1.67 | 1.30 | 0.87 | 0.65 | 0.33 | -0.70 | -1.65 |
| $R^2$ OOS CT | 1.28 | -0.06 | 0.29 | 1.12 | 0.19 | 1.03 | 1.59 | 1.31 | 1.15 | 0.81 | 0.85 | 0.32 | -0.34 |

percentage points in the subsequent annualized market return ($t = -2.26$). In contrast, univariate predictive power of the upper-tail ΔCIQ factor is substantially weaker and less stable, with a smaller economic magnitude and lower explanatory power. This asymmetry mirrors the earlier evidence that downside CIQ innovations are closely tied to binding intermediation constraints, whereas upside realizations are not.

From an economic perspective, large negative innovations in the lower-tail ΔCIQ factor are tied to periods in which the intermediary risk-bearing capacity is impaired and sell-side liquidity is scarce. In such states, assets command higher expected returns, leading to an elevated subsequent equity premium. The ability of the lower-tail ΔCIQ factor to predict the market return therefore reflects time variations in the price of aggregate risk driven by fluctuations in liquidity provisions and balance-sheet capacity.

We also evaluate the performance of the predictive regressions in an expanding-window out-of-sample framework. Specifically, we use data up to time $t$ to estimate a prediction model and then forecast a return at time $t+1$ (the first window contains 120 monthly periods to obtain sufficiently reasonable estimates). Then, the window is extended by one observation, the prediction model is re-estimated, and a new forecast is obtained. We repeat this procedure until the entire sample is exhausted. The corresponding $R^2$ value is computed by comparing the conditional forecast and historical mean computed using the available data up to time $t$.[27] Unlike the case of the in-sample $R^2$ value, the OOS $R^2$ value can be negative if the conditional forecasts perform worse than the historical mean forecast.

We observe that the lower-tail ΔCIQ factor delivers positive and economically meaningful out-of-sample $R^2$ values, whereas the predictive performance of the upper-tail factor deteriorates substantially out of sample. Applying the truncation following Campbell and Thompson (2007) further improves the out-of-sample performance for the lower-tail factor.

Next, we examine whether the predictive power of the lower-tail ΔCIQ factor is subsumed by existing predictors. We estimate multivariate regressions of the following form:

$$r_{m,t+1} = \gamma_0 + \sum_j \gamma_{1,j} \Delta CIQ_t^j + \gamma_2 f_t + \epsilon_{t+1}, \tag{14}$$

where $f_t$ includes controls such as innovations in common idiosyncratic volatility (PCA-SQ and

[27] I.e., $1 - \sum_t (r_{m,t+1} - \widehat{r}_{m,t+1|t})^2 / \sum_t (r_{m,t+1} - \bar{r}_{m,t})^2$, where $\widehat{r}_{m,t+1|t}$ is the out-of-sample forecast of the $t+1$ return using data up to time $t$, and $\bar{r}_{m,t}$ is the historical mean of the market return computed up to date $t$.

CIV), the tail risk reported by Kelly and Jiang (2014) (TR), the cross-sectional bivariate idiosyncratic volatility reported by Han and Li (2025) (CBIV), lagged market returns, and short-term reversal (STR). Across all specifications, the coefficient on the lower-tail $\Delta$CIQ factor remains economically large and statistically significant, while the inclusion of volatility-based controls does not diminish its predictive ability. This evidence indicates that the ability of $\Delta$CIQ to aid in predictions is not driven by common volatility or generic tail risk but instead reflects a distinct channel related to fluctuations in aggregate liquidity provision and intermediary constraints. The predictability of the market return therefore complements the cross-sectional pricing evidence and reinforces the interpretation of $\Delta$CIQ as a state variable governing the price of risk in the economy.[28]

# VI. Conclusion

This paper examines the circumstances in which idiosyncratic downside returns become systematically related across firms and the reasons why this comovement is priced. Using a quantile factor model applied to idiosyncratic returns, we identify common idiosyncratic quantile factors that alter the lower, central, and upper regions of the residual return distribution. Our central empirical finding is that exposure to innovations in the lower-tail CIQ factor commands a large premium, whereas exposure to the median and upper-tail CIQ factors is not priced. The downside CIQ premium is robust to standard factor models and a wide range of volatility- and tail-based risk controls, suggesting that it represents a unique source of systematic risk.

[28]In unreported results, we also control for a set of eleven macroeconomic variables investigated by Welch and Goyal (2007), and the results remain identical. These results are available upon request.

This paper makes three contributions. First, it introduces a new return-based measure of systematic risk that explicitly considers the distribution and distinguishes between downside and upside idiosyncratic comovement. Second, it demonstrates that only the downside of common idiosyncratic tail comovement is priced, with premia that survive an extensive set of risk and characteristic controls and that vary systematically across quantiles. Third, it links the priced downside factor to intermediary constraints and liquidity conditions, demonstrating that the same state variable forecasts the aggregate equity premium.

Our findings have implications for both asset pricing and risk management. In terms of asset pricing, they identify a state variable that links distributional shifts in residual returns to equilibrium risk premia, helping to explain why compensation for tail exposure is concentrated in the left tail. For practitioners and regulators, the results highlight that monitoring downside tail comovement in residual returns can provide information about latent fragility in market-making and risk-bearing capacity. Future work could investigate how these tail comovement states interact with market structure, dealer inventories, and the propagation of shocks across asset classes at higher frequencies.

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# Internet Appendix

## A. Quantile Factor Model

We assume a panel of returns of length $T$ and width $N$ after eliminating the common mean factors from the time series regression

(1) $$r_{i,t} = \alpha_i + \beta_i^\top f_t + \epsilon_{i,t},$$

to have a $\tau$-dependent structure-$g_t(\tau)$-in idiosyncratic errors that we coin the common idiosyncratic quantile–CIQ($\tau$)-factors, which satisfies

(2) $$Q_{\epsilon_{i,t}}\left[\tau \middle| g_t(\tau)\right] = \gamma_i^\top(\tau) g_t(\tau),$$

which implies

(3) $$\epsilon_{i,t} = \gamma_i^\top(\tau) g_t(\tau) + u_{i,t}(\tau),$$

where $g_t(\tau)$ is an $r(\tau) \times 1$ vector of random common factors, $\gamma_i(\tau)$ is an $r(\tau) \times 1$ vector of nonrandom factor loadings with $r(\tau) \ll N$, $Q_{\epsilon_{i,t}}\left[\tau \middle| g_t(\tau)\right]$ is a conditional quantile function of $\epsilon_{i,t}$ at $\tau$, and the quantile-dependent idiosyncratic error $u_{i,t}(\tau)$ almost certainly satisfies the quantile restriction $P[u_{i,t}(\tau) < 0 | g_t(\tau)] = \tau$ for all $\tau \in (0, 1)$.

To estimate the common factors that capture the comovement of quantile-specific features of distributions of the idiosyncratic parts of the stock returns, we use quantile factor analysis (QFA) introduced by Chen et al. (2021). In contrast to principal component analysis (PCA), QFA facilitates the capture of hidden factors that may shift more general characteristics, such as moments or quantiles of the distribution of returns other than the mean. The methodology is also suitable for large panels and requires less strict assumptions about the data generation process.

The quantile-dependent factors and their loadings can be estimated as follows:

$$\underset{(\gamma_1,\ldots,\gamma_N,g_1,\ldots,g_T)}{\operatorname{argmin}} \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\rho_\tau\left(\epsilon_{it}-\gamma_i^\top(\tau)g_t(\tau)\right) \tag{4}$$

where $\rho_\tau(u)=(\tau-\mathbf{1}\{u\leq 0\})u$ is the check function while imposing the following normalizations: $\frac{1}{T}\sum_{t=1}^{T} g_t(\tau)g_t(\tau)^\top=\mathbb{I}_r$, and $\frac{1}{N}\sum_{i=1}^{N}\gamma_i(\tau)\gamma_i(\tau)^\top$ is diagonal with nonincreasing diagonal elements.

As discussed in Chen et al. (2021), this estimator is related to the PCA estimator studied in Bai and Ng (2002) and Bai (2003), similar to quantile regression, which is related to classical least squares regression. Unlike the PCA estimator of Bai (2003), this estimator does not yield an analytical closed-form solution. To solve for the stationary points of the objective function, Chen et al. (2021) proposed a computational algorithm called iterative quantile regression. These authors also showed that the estimator possesses the same convergence rate as the PCA estimators for the approximate factor model. We follow their approach when estimating the quantile factors.[29]

## B. Beyond Volatility Factors

To illustrate the discussion and provide the link between volatility and quantiles, let's consider the data generating process to be a typical location-scale model with two unrelated

[29]We employ the authors' MATLAB codes, which are provided on the Econometrica webpage.

factors in the first and second moments. Idiosyncratic returns $\epsilon_{i,t}$ of such model will be zero mean i.i.d. process independent of both factors with cumulative distribution function $F_{\epsilon_{i,t}}$. Further let $Q_{\epsilon_{i,t}}(\tau) = F^{-1}_{\epsilon_{i,t}}(\tau) = \inf\{s : F_{\epsilon_{i,t}}(s) \leq \tau\}$ be a quantile function of $\epsilon_{i,t}$ and assume the median is zero. Then the following model that is typical for finance

$$r_{i,t} = \beta_i f_{1,t} + (\sigma_{i,t}^\top f_{2,t})\epsilon_{i,t}, \tag{5}$$

where $\sigma_{i,t}$ is time-varying volatility of an $i$th stock and $\sigma_{i,t} f_{2,t} > 0$ can be assumed to generate returns. When $f_{1,t}$ and $f_{2,t}$ do not share common elements, then

$$Q_{r_{i,t}}\left[\tau \middle| f_t(\tau)\right] = \beta_i f_{1,t} + \sigma_{i,t}^\top f_{2,t} Q_{\epsilon_{i,t}}(\tau) \tag{6}$$

for $\tau \neq 0.5$ and $Q_{r_{i,t}}\left[\tau \middle| f_t(\tau)\right] = \beta_i f_{1,t}$ for $\tau = 0.5$. Note that here loadings on the factor are the only quantile-dependent objects and structure in the mean and volatility describes well the structure in quantiles. While this is already restrictive example that operates with the assumption on first two moments, even in such case standard PCA will not provide consistent estimates if the distribution of $\epsilon_{i,t}$ is heavy-tailed (Chen et al., 2021).

But what if the data follows more complicated models than the one implied by location-shift models? Consider adding asymmetric dependence such as

$$r_{i,t} = \beta_i f_{1,t} + f_{2,t}\epsilon_{i,t} + f_{3,t}\epsilon_{i,t}^3, \tag{7}$$

where $\epsilon_{i,t}$ is standard normal random variable with cumulative distribution function $\Phi(.)$. The quantiles of the returns will then follow

$$Q_{r_{i,t}}\left[\tau \middle| f_t(\tau)\right] = \beta_i f_{1,t} + \Phi^{-1}(\tau)\left[f_{2,t} + f_{3,t}\Phi^{-1}(\tau)^2\right], \tag{8}$$

for $\tau \neq 0.5$ and we can clearly see that second factor in $f(\tau) = [f_{1,t}, f_{2,t} + f_{3,t}\Phi^{-1}(\tau)^2]^\top$ is quantile dependent.

## C. A Simple Intermediary-Fragility Model

This appendix provides a simple model that formalizes the mechanism in Section 2. The goal is not to offer a fully calibrated structural model, but to show that a continuum of intermediary balance-sheet constraints can aggregate into a one-dimensional fragility state $s_t$ that (i) drives common shifts in firms' idiosyncratic left-tail quantiles and (ii) appears in the stochastic discount factor through innovations $\Delta s_{t+1}$.

### 1. Setup

There is a continuum of competitive intermediaries $j \in [0, 1]$ with equity $E_{j,t}$. Intermediary $j$ chooses a portfolio over a market factor $F_{t+1}$ and a continuum of firm claims with returns $r_{i,t+1}$, subject to an expected shortfall (ES) constraint on portfolio losses. Let $\lambda_{j,t}$ denote the Lagrange multiplier on this constraint.

We define *systemic fragility* as the equity-weighted average multiplier

$$s_t := \int_0^1 \omega_{j,t}\, \lambda_{j,t}\, dj, \tag{9}$$

where $\omega_{j,t}$ are equity weights. Intuitively, $s_t$ is the aggregate shadow cost of risk-bearing in the intermediary sector.

Firm $i$'s idiosyncratic return $\varepsilon_{i,t+1}$ has a conditional quantile representation

$$\epsilon_{i,t} = b_i(\tau)s_t + \nu_{i,t}(\tau), \tag{10}$$

with $b_i(\tau) < 0$ for $\tau \leq \tau_L$, $b_i(\tau) \approx 0$ for $\tau \geq 0.5$, and $\nu_{i,t}(\tau)$ satisfies the appropriate quantile restriction. This captures the idea that tighter constraints shift firms' left-tail quantiles downward through funding, covenant, and liquidity channels, but leave the center and right tail largely unaffected.

## 2. Quantile Factor Representation

**Proposition 2 (Quantile factor representation)** *Under standard large-$N$ conditions for approximate factor models, the first quantile principal component of idiosyncratic returns at $\tau_L$ consistently estimates an affine transformation of $s_t$:*

$$CIQ_t^L = \alpha + \kappa\, s_t + \eta_t,$$

*with $\kappa \neq 0$ and an error term $\eta_t$.*

*Sketch of proof.* Equation (10) defines a one-factor structure in the conditional $\tau_L$-quantiles of idiosyncratic returns. Under the assumptions of Chen et al. (2021) on cross-sectional pervasiveness and bounded idiosyncratic components, the quantile factor analysis estimator recovers the space spanned by $s_t$ up to an affine transformation. Normalization conditions pin down the scale and sign up to a constant.[30]

### 3. Pricing Kernel

Each intermediary's optimality condition with an ES constraint yields an SDF containing a term proportional to its marginal shortfall loss $\ell_{j,t+1}$, in addition to a standard component $m_{t+1}^{\text{std}}$. In equilibrium, all intermediaries share the same pricing kernel, which can be written as

$$m_{t+1} = m_{t+1}^{\text{std}} + \phi_t \, \ell_{t+1}, \tag{11}$$

where $\ell_{t+1}$ is the aggregate marginal shortfall and $\phi_t$ is proportional to the cross-sectional average of $\lambda_{j,t}$, hence to $s_t$ by (9).

Assuming that portfolio shortfalls materialize precisely when the fragility state worsens, we can approximate $\ell_{t+1}$ as a linear function of $\Delta s_{t+1}$:

$$\ell_{t+1} = a_0 + a_1 \, \Delta s_{t+1} + \epsilon_{t+1},$$

with $a_1 > 0$. Substituting into (11) yields

$$m_{t+1} = m_{t+1}^{\text{std}} + \psi_t \, \Delta s_{t+1}, \tag{12}$$

with $\psi_t = \phi_t a_1 > 0$ in constrained states.

[30]See Assumption 1 from Chen et al. (2021) for the exact formulation of the conditions for the consistency of the QFA.

**Proposition 3 (CIQ beta as fragility exposure)** *With $CIQ_t^L = \alpha + \kappa s_t + \eta_t$, the innovation $\Delta CIQ_t^L$ is proportional to $\Delta s_t$ up to noise. The CIQ beta*

$$\beta_i^{CIQ} = \frac{\text{Cov}(r_{i,t+1}, \Delta CIQ_{t+1}^L)}{\text{Var}(\Delta CIQ_{t+1}^L)}$$

*is therefore proportional to the covariance of $r_{i,t+1}$ with $\Delta s_{t+1}$. Expected excess returns satisfy*

$$\mathbb{E}_t[r_{i,t+1}] = \lambda_M \beta_i^M + \lambda_{CIQ,t}\, \beta_i^{CIQ},$$

*with $\lambda_{CIQ,t} > 0$ when constraints are tight.*

*Sketch of proof.* Combine $\Delta CIQ_t^L = \kappa \Delta s_t + \Delta \eta_t$ with (12) and apply the standard linear beta pricing argument. Under mild assumptions on the noise term $\Delta \eta_t$, the projection of $r_{i,t+1}$ on $\Delta CIQ_{t+1}^L$ coincides with its projection on $\Delta s_{t+1}$ up to a constant.

**Proposition 4 (Left-tail asymmetry)** *Because ES constraints depend on losses, the marginal shortfall $\ell_{t+1}$ is insensitive to upside co-movement and depends only on left-tail realizations. Under the quantile representation* (10)*, the resulting price of quantile-factor risk is monotone in $\tau$ for $\tau < 0.5$ and negligible for $\tau \geq 0.5$. In particular, only the lower-tail CIQ factor is priced, while median and upper-tail CIQ factors do not command premia.*

*Sketch of proof.* The ES constraint binds only in states where portfolio returns fall below a left-tail threshold. By construction, $\ell_{t+1}$ is orthogonal to purely central or right-tail shifts. Under (10), $s_t$ affects $q_{i,t}(\tau)$ only for $\tau \leq \tau_L$. Thus, only quantile factors estimated from the left tail proxy for $\Delta s_{t+1}$ and receive a non-zero price of risk in (12).

## D. Simulation Study

We present a simulation exercise to illustrate how the $\Delta$CIQ premia would look like if the driving force behind them were simply common volatility. We simulate stock returns from the following model

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \gamma_i (V_t - \bar{V}) + \gamma_i \lambda^V + e_{i,t} \tag{13}$$

where $V_t$ is the common variance factor, and the variance of the idiosyncratic error follows the factor structure proposed by Ding, Engle, Li, and Zheng (2025)

$$\begin{aligned} & e_{i,t} = \sqrt{V_{i,t}} z_{i,t}, \\ & V_{i,t} = V_t \exp(\mu_i + \sigma_i u_{i,t}) = V_t \tilde{V}_{i,t}, \\ & z_{i,t},\ u_{i,t} \sim i.i.d.\, N(0,1). \end{aligned} \tag{14}$$

Time-series variation of the returns drive two common factors – market factor, $r_{m,t}$, and variance factor $V_t$ with unconditional mean $\bar{V}$. The expected return of a stock is then equal to

$$\mathbb{E}[r_i] = \alpha_i + \beta_i \mathbb{E}[r_m] + \gamma_i \lambda^V. \tag{15}$$

We assume that the market factor follows a simple GARCH(1,1) process of Bollerslev (1986), which we fit on the market return from the empirical analysis. We assume that the log of the variance factor follows a modified HAR model of Corsi (2009)

$$\log V_{t+1} = \theta_0 + \theta_m x_t^m + \theta_y x_t^y + v_{t+1}$$
$$v_{t+1} \sim i.i.d.\, N(0, \sigma_v^2) \tag{16}$$

where $x_t^m$ and $x_t^y$ are the previous month's log-variance and average log-variance over the last 12-month period, respectively. The common variance process is approximated by the cross-sectional average of the squared residuals from the time series regression of stock returns on the market factor. We fit the model from Equation (16) on this time series. When simulating this time series, we initialize the process by randomly selecting 12 consequent observations of the common variance process estimated from the data and using those observations for iterating forward.

We calibrate the simulation setting to match the CRSP data sample we employ in the empirical investigation. We estimate stock-level market beta, $\beta_i$, using time-series regression of stock return on the market return. Exposure to the common variance, $\gamma_i$, is estimated by regressing the stock return on the estimate of the common variance process. Price of risk associated with the variance exposure, $\lambda^V$ is chosen to be equal to $3 \times 10^{-3}$.[31] We estimate stock-level parameters of the idiosyncratic error variance–$\mu_i$, $\sigma_i$–as the sample mean and standard deviation of $\log \tilde{V}_{i,t}$. To approximate the $\tilde{V}_{i,t}$, we use squared residuals from the time-series regression of the stock return on the market return. Then, to simulate these parameters, we

[31] This value corresponds to approximately 6% annual high minus low premium obtained from ten portfolios portfolios sorted on the exposure to the common variance. The choice of this value is not essential for the results that we present here.

approximate their distribution by normal distribution, with the mean equal to the estimates' cross-sectional average and the variance equal to the cross-sectional variance of the estimates.

We simulate a panel of 2,500 stocks with 120 observations. We repeat this simulation 1,000 times. Each time, we simulate stock returns by randomly choosing parameters for the stock-level process from the normal distribution with mean and variance corresponding to their sample counterparts. We remove the common time variation in stock returns by first forming the common linear factor

$$f_t = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}, \quad t = 1, \dots, T \tag{17}$$

and then regressing the returns on this factor

$$r_{i,t} = \alpha_i + \hat{\beta}_i f_t + \hat{e}_{i,t}, \tag{18}$$

which yields the residuals $\hat{e}_{i,t}$. Those residuals are then used to form the common volatility and quantile factors. We construct the volatility factor as the first principal component of those squared residuals. $\Delta$CIQ$(\tau)$ factors are estimated as discussed in Section II. Exposures to those factors are then estimated using univariate time-series regressions of stock returns on the increments of the volatility or quantile factors, respectively.

Similarly, as in the empirical investigation, we sort stocks into decile portfolios based on their estimated exposure to the factors to infer the associated risk premia. We proxy the premia by computing high-minus-low returns of the portfolios. Table A1 reports the average premia for the three $\Delta$CIQ factors that we investigate in the empirical analysis. We observe that the premium is

TABLE A1

## Simulated Risk Premia

The table contains risk average premia computed from high-minus-low returns of decile portfolios sorted on exposure to the CIQ($\tau$) risks. We simulate the returns using common variance factor model proposed by Ding et al. (2025). We simulate panel of 2,500 stocks with 120 monthly observations. We perform the simulation 1,000 times. The $t$-statistics are obtained by dividing the average premium by its standard deviation. We also report proportion of rejections of non-significance of ΔCIQ betas from multivariate cross-sectional regressions of average returns on those betas and market betas.

| $\tau$ | Premium | $t$-stat | Rejections |
|---|---|---|---|
| $\Delta CIQ_{LT}$ | 9.37 | 2.44 | 0.96 |
| $\Delta CIQ_{C}$ | 0.26 | 0.03 | 0.96 |
| $\Delta CIQ_{UT}$ | -9.47 | -2.56 | 0.96 |

positive for the lower-tail quantile factor, negative for the upper-tail factor and close to zero for the central factor. The magnitude of the premia are comparable across lower and upper tail factors and in absolute value approximately equal to 9.4%. The premium associated with the exposure to the ΔPCA-SQ factor is -6.09%. We also compute associated $t$-statistics as a ratio between average premium and its standard deviation across all the simulations. premia for the lower and upper tail factors are significant, unlike the value for the central factor, with $t$-statistics of around 2.6 in absolute value. The $t$-value associated with the ΔPCA-SQ factor is -2.33, so the premium estimated using this approach is also significant. Next, we present the proportion of rejections of non-significance of the ΔCIQ betas at a 5% significance level from multivariate cross-sectional regressions of average returns on those betas and market betas. We can see that the proportions are virtually identical for both lower- and upper-tail betas of around 96%. The ratio for the ΔPCA-SQ betas is 90%.

As we can see from the results, if there was a simple common volatility element present in the returns, which is compensated in the cross-section, the ΔCIQ risk premium would be symmetrical for lower and upper tail quantile factors. Moreover, the exposure to the ΔPCA-SQ factor would be priced in this case, too. Overall, the evidence from the simulation exercise suggests that the $\Delta$CIQ risk premia we observe in the data are not attributable to the common volatility compensation.

## E. Cross-Sectional Quantiles

TABLE A2

**Quantiles of the Cross-Sectional Stock Returns**

The table provides a summary of cross-sectional dispersion of excess stock returns. Each month, we compute cross-sectional quantile of raw or idiosyncratic excess stock returns with respect to market factor (CAPM), three factors of Fama and French (1993), five factors of Fama and French (2015) (FF5) or six factors of Fama and French (2018) (FF6). We report results for 20% (lower-tail) and 80% (upper-tail) quantiles. In Panel A, we report variance ratios between time-series of these quantile series. Rows correspond to the quantile series in the denominator, while columns corresponds to the quantile series in the nominator. In Panel B, we report regression results from regressing FF3 idiosyncratic quantiles on intercept, idiosyncratic volatility and CIQ (either lower- or upper-tail) factor. The data come from the CRSP database and cover the period from January 1968 to December 2024.

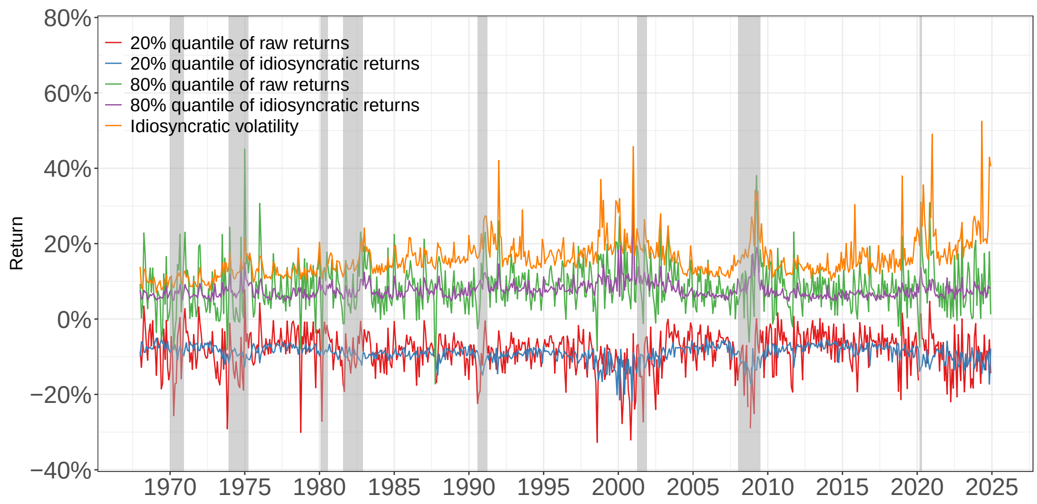
| ***Panel A:*** Variance ratios | Lower-Tail | | | | Upper-Tail | | | |
|---|---|---|---|---|---|---|---|---|
| | CAPM | FF3 | FF5 | FF6 | CAPM | FF3 | FF5 | FF6 |
| Raw | 0.31 | | | | 0.32 | | | |
| CAPM | | 0.46 | | | | 0.37 | | |
| FF3 | | | 0.90 | | | | 0.96 | |
| FF5 | | | | 0.94 | | | | 0.81 |

| ***Panel B:*** Regressions | Lower-Tail | | | Upper-Tail | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Intercept | -0.09 | -0.05 | -0.02 | 0.08 | 0.03 | 0.02 |
| | (-106.88) | (-23.72) | (-4.35) | (88.70) | (15.96) | (5.37) |
| Volatility | | -0.23 | | | 0.26 | |
| | | (-17.83) | | | (20.48) | |
| CIQ factor | | | 0.07 | | | 0.06 |
| | | | (21.40) | | | (20.75) |
| adj. $R^2$ | 0.00 | 0.32 | 0.40 | 0.00 | 0.38 | 0.39 |

FIGURE A1

## Cross-Sectional Dispersion of Stock Returns

The figure shows various measures of cross-sectional stock returns dispersion through time. For each month, we plot 20% and 80% cross-sectional quantiles of either excess or idiosyncratic excess returns. The idiosyncratic returns are computed with respect to the three-factor model of Fama and French (1993). The data come from the CRSP database and cover the period from January 1968 to December 2024. The shaded areas represent NBER recessions.



## F. Risk Measures Definitions

This section provides a brief exposition of the estimation process of each of the control risk measures employed in the main text. For further details regarding the nuances of the related computations, consult the original papers.

We use two sources of data to compute these measures. First, we use either daily or monthly data of stock returns from the CRSP database. Second, we utilize the value-weighted return of the CRSP stocks from Kenneth French's online library to approximate the overall market return.

Variables are estimated using moving windows of various lengths following the procedures proposed in their original papers. In the case of measures estimated from the daily stock returns, we use mostly a moving window of one year. We require at least 200 daily observations during the window to be included. If we estimate a measure based on monthly return data we use a window of at least 60 months and demand at least 36 monthly observations.

The measures are estimated following the definition proposed in the literature. In some cases, we slightly change the requirements regarding the minimal history of stocks to be included in the analysis. This modification aims at the precision of the estimates as well as the broadest possible dataset.

Throughout this section, we use $r_{i,t}$ ($r^e_{i,t}$) to denote a raw (excess) return of an asset $i$ at time $t$. The raw (excess) market return is denoted by $f_t$ ($f^e_t$). Corresponding variables with a bar denote their time-series averages computed in a given window.

## 1. Market Beta

Market beta is estimated using daily data over the previous year for stocks that possess at least 200 observations as

(19) $$\beta_i^{CAPM} = \frac{\sum_t (r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)}{\sum_t (f_t^e - \bar{f}^e)^2}.$$

## 2. Idiosyncratic Volatility

Following Ang et al. (2006b), idiosyncratic volatility is estimated using daily data over the previous month relative to the model of Fama and French (1993)

(20) $$r_{i,t}^e = \alpha_i + \beta_i^{MKT} f_t^e + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + e_{i,t}$$

and taking standard deviation of the estimated residuals, $IVOL_i = \sqrt{var(e_i)}$.

## 3. Total and Idiosyncratic Skewness

Following Langlois (2020), we estimate total skewness as mean of the cubed standardized daily returns $r_{i,t}^e$, and idiosyncratic skewness from the model

(21) $$r_{i,t}^e = \alpha_i + \beta_{1,i} f_t^e + \beta_{2,i} (f_t^e)^2 + e_{i,t}$$

and taking mean of the cubed standardised residuals $e_{i,t}$. We estimate these quantities using one year of data and requiring at least 200 observations.

## 4. Co-skewness

Co-skewness of Harvey and Siddique (2000) is estimated using daily excess returns and is defined as

$$CSK_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)^2}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)^2}\frac{1}{T}\sum_{t=1}^{T}(f_t^e - \bar{f}^e)^2}. \tag{22}$$

Estimation window is set to one year, at least 200 daily observations are required.

## 5. Co-kurtosis

Co-kurtosis of Dittmar (2002) is estimated using daily data and is defined as

$$CKT_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)^3}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)^2}\left(\frac{1}{T}\sum_{t=1}^{T}(f_t^e - \bar{f}^e)^2\right)^{3/2}}. \tag{23}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

## 6. PCA-SQ Betas

Both the factor and the exposures are estimated using 60-month moving window of monthly data similarly as in the case of the ΔCIQ factors. Exposure is estimated from time series regression of regressing excess stock returns on a constant and ΔPCA-SQ factor. We require at least 48 observations during the estimation period.

## 7. CIV Beta

Following Herskovic et al. (2016), CIV beta is estimated by regressing monthly excess stock returns on a constant, increments of the CIV factor and increments of the monthly market variance

$$r^e_{i,t} = \alpha_i + \beta_i^{CIV} \Delta CIV_t + \beta_i^{MV} \Delta MV_t + e_{i,t}. \tag{24}$$

We use 60-month rolling window and require at least 48 observations.

## 8. VIX Beta

VIX beta is estimated following Ang et al. (2006a) using daily data over the previous month by regressing stock returns on a constant, market factor and increments of the CBOE volatility index as

$$r^e_{i,t} = \alpha_i + \beta_i^{MKT} f^e_t + \beta_i^{VIX} \Delta VIX_t + e_{i,t}. \tag{25}$$

We require at least 17 observations during the estimation month.

## 9. Downside Beta

Downside beta of Ang et al. (2006a) is estimated using daily data and is defined as

$$\beta_i^{down} = \frac{\sum\limits_{f^e_t < \bar{f}^e} (r^e_{i,t} - \bar{r}^e_i)(f^e_t - \bar{f}^e)}{\sum\limits_{f^e_t < \bar{f}^e} (f^e_t - \bar{f}^e)^2}. \tag{26}$$

Estimation window is set to one year, at least 200 daily observations are required.

## 10. Tail Risk Beta

Tail risk beta of Kelly and Jiang (2014) is estimated using monthly return data using 120-month window with requirement of at least 36 monthly observations. Following the original setting, we require stocks to have price higher than \$5. Beta is computed by means of least-square estimator from the predictive regression of the form

$$r_{i,t+1} = \mu_i + \beta_i^{tail}\lambda_t + \epsilon_{t+1,i} \tag{27}$$

where the tail risk factor is obtained as

$$\lambda_t = \frac{1}{K_t}\sum_{k=1}^{K_t} \ln\frac{e_{k,t}}{u_t} \tag{28}$$

where $e_{k,t}$ is the $k$th daily idiosyncratic return that falls below an extreme value threshold $u_t$ during month $t$, and $K_t$ is the total number of such exceedences within month $t$. Idiosyncratic return is computed relative to the three-factor model of Fama and French (1993), and the threshold value is taken to be 5% quantile from the monthly cross-section of daily returns.

## 11. Hybrid Tail Covariance Risk

Hybrid tail covariance risk of Bali et al. (2014) is estimated using daily data using 6-month window with at least 80 daily observations as

$$HTCR_i = \sum_{r_{i,t} < h_i} (r_{i,t} - h_i)(f_t - h_f) \tag{29}$$

where $h_i$ and $h_f$ are the 10% empirical quantiles of stock and market return, respectively.

## 12. Multivariate Crash Risk

Multivariate crash risk of Chabi-Yo et al. (2022) is estimated using daily data with 1-year window and minimum of 200 nonzero observations in the following steps. First, for each stock separately, using stock and $N$ factor returns, we estimate $N+1$ GARCH(1,1) models of Bollerslev (1986) to obtain a series of conditional distribution functions $F_{i,t}(h) = \mathbb{P}_{t-1}[r_{i,t} \leq h]$ and use it to compute probability integral transforms as $\hat{u}_{i,t} = F_{i,t}(r_{i,t})$. Second, we estimate MCRASH as

$$\text{MCRASH}_{i,t} = \frac{\sum_t \mathbb{I}(\{\hat{u}_{1,t} \leq p\}) \cdot \mathbb{I}(\cup_{j=2}^{N+1} \{\hat{u}_{j,t} \leq p\})}{\sum_t \mathbb{I}(\cup_{j=2}^{N+1} \{\hat{u}_{j,t} \leq p\})} \tag{30}$$

where $\mathbb{I}$ denotes the indicator function and $p$ is set to 0.05. We follow the baseline specification of Chabi-Yo et al. (2022) and use the five factors of Fama and French (2015), momentum factor of Carhart (1997) and betting-against-beta factor of Frazzini and Pedersen (2014).

## 13. Predicted Systematic Co-skewness

Predicted systematic co-skewness of Langlois (2020) is based on

(31) $$Cos_{i,t} = \mathbb{C}ov_{t-1}\big(r_{i,t}, f_t^2\big),$$

then, each month we run the panel regression using all available stocks and history of data

(32) $$F\big(Cos_{i,k-12\to k-1}\big) = \kappa + F\big(Y_{i,k-24\to k-13}\big)\theta + F\big(X_{i,k-13}\big)\phi + \epsilon_{i,k-12\to k-1}$$

where $Cos_{i,k-12\to k-1}$ is the co-skewness from Equation (31) computed using daily returns from month $k-12$ to month $k-1$, $Y_{i,k-24\to k-13}$ are risk measures (volatility, market beta, etc.) estimated using daily data from month $k-24$ to month $k-13$, and $X_{i,k-13}$ are characteristics (size, book-to-price, etc.) observed at the end of month $k-13$. The function $F(x_{i,t}) = \frac{Rank(x_{i,t})}{N_t+1}$ transforms the original variable into its normalised rank in the cross-section of variable $x_t$, which posses $N_t$ observations.

The predicted systematic co-skewness for each stock is then obtained using the estimated coefficients of $\hat{\kappa}, \hat{\theta}, \hat{\phi}$ as

(33) $$F\big(\widehat{Cos_{i,t\to t+11}}\big) = \hat{\kappa} + F\big(Y_{i,t-12\to t-1}\big)\hat{\theta} + F\big(X_{i,t-1}\big)\hat{\phi}.$$

The choice of risk measures and characteristics employed in the prediction of systematic skewness follows closely Langlois (2020).

# G. Summary of the CIQ Factors

TABLE A3

**Summary Statistics of the $\triangle$CIQ$(\tau)$ Factors**

The table provides summary of the estimated $\Delta$CIQ$(\tau)$ factors. In Panel A, we report descriptive statistics of the $\Delta$CIQ$(\tau)$ factors including their means, standard deviations, skewness, kurtosis and autocorrelation coefficients of order between one and three. In Panel B, we report correlations between $\Delta$CIQ$(\tau)$ factors. The data cover the period from January 1968 to December 2024.

| | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Panel A:*** Descriptive statistics | | | | | | | | | | | | | | | | | | | |
| Mean $\times 10^3$ | -5.83 | -5.17 | -3.45 | -3.21 | -4.02 | -4.07 | -6.18 | -9.23 | -35.29 | -41.48 | -78.70 | -52.38 | -16.06 | -9.55 | -7.92 | -8.34 | -5.93 | -6.25 | 0.57 |
| St. Dev. | 0.15 | 0.16 | 0.17 | 0.19 | 0.22 | 0.26 | 0.33 | 0.49 | 1.27 | 1.46 | 1.46 | 1.32 | 0.57 | 0.36 | 0.28 | 0.23 | 0.20 | 0.18 | 0.17 |
| Skewness | -0.10 | -0.06 | 0.06 | 0.06 | 0.22 | 0.15 | 0.25 | 0.27 | -0.27 | -0.35 | -0.30 | -0.06 | 0.19 | 0.19 | 0.03 | -0.05 | -0.19 | -0.01 | 0.26 |
| Kurtosis | 4.42 | 4.21 | 4.57 | 5.09 | 5.80 | 5.69 | 6.06 | 6.09 | 8.09 | 6.41 | 6.55 | 5.46 | 6.90 | 7.20 | 6.63 | 6.86 | 6.78 | 6.18 | 5.57 |
| AR(1) | -0.36 | -0.40 | -0.41 | -0.44 | -0.43 | -0.46 | -0.46 | -0.46 | -0.29 | -0.27 | -0.27 | -0.33 | -0.47 | -0.46 | -0.44 | -0.42 | -0.40 | -0.37 | -0.33 |
| AR(2) | -0.14 | -0.08 | -0.05 | -0.02 | -0.00 | 0.03 | 0.03 | 0.05 | 0.07 | 0.07 | 0.02 | -0.01 | 0.01 | -0.00 | -0.05 | -0.07 | -0.10 | -0.13 | -0.13 |
| AR(3) | 0.17 | 0.12 | 0.08 | 0.03 | 0.00 | -0.02 | -0.02 | -0.04 | -0.07 | -0.02 | -0.00 | -0.01 | -0.00 | 0.02 | 0.06 | 0.07 | 0.09 | 0.09 | 0.08 |
| ***Panel B:*** Correlations | | | | | | | | | | | | | | | | | | | |
| 0.05 | 1.00 | 0.93 | 0.89 | 0.85 | 0.79 | 0.72 | 0.65 | 0.54 | 0.19 | 0.07 | 0.05 | 0.11 | 0.15 | 0.05 | -0.06 | -0.15 | -0.24 | -0.33 | -0.39 |
| 0.10 | | 1.00 | 0.97 | 0.93 | 0.89 | 0.84 | 0.78 | 0.67 | 0.28 | 0.14 | 0.11 | 0.18 | 0.25 | 0.16 | 0.03 | -0.06 | -0.17 | -0.26 | -0.34 |
| 0.15 | | | 1.00 | 0.98 | 0.95 | 0.91 | 0.85 | 0.76 | 0.35 | 0.19 | 0.16 | 0.26 | 0.35 | 0.26 | 0.13 | 0.03 | -0.08 | -0.19 | -0.27 |
| 0.20 | | | | 1.00 | 0.98 | 0.95 | 0.91 | 0.82 | 0.40 | 0.22 | 0.20 | 0.32 | 0.44 | 0.36 | 0.23 | 0.13 | 0.01 | -0.10 | -0.20 |
| 0.25 | | | | | 1.00 | 0.98 | 0.95 | 0.87 | 0.45 | 0.26 | 0.23 | 0.36 | 0.51 | 0.43 | 0.31 | 0.21 | 0.09 | -0.03 | -0.14 |
| 0.30 | | | | | | 1.00 | 0.98 | 0.92 | 0.51 | 0.31 | 0.28 | 0.41 | 0.58 | 0.52 | 0.40 | 0.31 | 0.19 | 0.07 | -0.07 |
| 0.35 | | | | | | | 1.00 | 0.96 | 0.58 | 0.38 | 0.34 | 0.47 | 0.66 | 0.61 | 0.50 | 0.41 | 0.29 | 0.16 | 0.02 |
| 0.40 | | | | | | | | 1.00 | 0.69 | 0.47 | 0.43 | 0.55 | 0.74 | 0.69 | 0.60 | 0.52 | 0.41 | 0.29 | 0.14 |
| 0.45 | | | | | | | | | 1.00 | 0.77 | 0.72 | 0.71 | 0.67 | 0.61 | 0.56 | 0.51 | 0.45 | 0.36 | 0.26 |
| 0.50 | | | | | | | | | | 1.00 | 0.90 | 0.73 | 0.53 | 0.47 | 0.44 | 0.41 | 0.38 | 0.31 | 0.24 |
| 0.55 | | | | | | | | | | | 1.00 | 0.80 | 0.52 | 0.47 | 0.44 | 0.41 | 0.38 | 0.32 | 0.26 |
| 0.60 | | | | | | | | | | | | 1.00 | 0.69 | 0.61 | 0.57 | 0.53 | 0.48 | 0.40 | 0.29 |
| 0.65 | | | | | | | | | | | | | 1.00 | 0.96 | 0.92 | 0.87 | 0.80 | 0.69 | 0.54 |
| 0.70 | | | | | | | | | | | | | | 1.00 | 0.97 | 0.93 | 0.87 | 0.77 | 0.62 |
| 0.75 | | | | | | | | | | | | | | | 1.00 | 0.97 | 0.93 | 0.86 | 0.72 |
| 0.80 | | | | | | | | | | | | | | | | 1.00 | 0.97 | 0.91 | 0.78 |
| 0.85 | | | | | | | | | | | | | | | | | 1.00 | 0.96 | 0.85 |
| 0.90 | | | | | | | | | | | | | | | | | | 1.00 | 0.91 |
| 0.95 | | | | | | | | | | | | | | | | | | | 1.00 |

## H. Cross-Sectional Summary Statistics

TABLE A4

### Summary Statistics of the $\Delta$CIQ$(\tau)$ Betas

The table provides time-series averages of monthly cross-sectional statistics of the estimated $\Delta$CIQ$(\tau)$ betas. Betas are estimated by regressing stock's monthly excess returns on $\Delta$CIQ using a rolling window of 60 months. We keep only betas that are estimated with at least 48 observations. The return sample covers period between January 1968 and December 2024. We exclude penny stocks with prices of $1 or less.

| $\tau$ | Mean | St. Dev. | Skewness | Kurtosis | Min. | 5% | 25 % | Median | 75% | 95% | Max. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | -0.06 | 0.15 | -0.63 | 19.42 | -1.13 | -0.29 | -0.13 | -0.05 | 0.03 | 0.17 | 0.90 |
| 0.10 | -0.06 | 0.14 | -0.89 | 22.86 | -1.14 | -0.29 | -0.14 | -0.06 | 0.02 | 0.15 | 0.81 |
| 0.15 | -0.06 | 0.13 | -1.02 | 25.23 | -1.11 | -0.28 | -0.13 | -0.06 | 0.01 | 0.13 | 0.74 |
| 0.20 | -0.06 | 0.12 | -1.11 | 26.65 | -1.03 | -0.26 | -0.12 | -0.06 | 0.01 | 0.11 | 0.65 |
| 0.25 | -0.06 | 0.11 | -1.25 | 28.52 | -0.94 | -0.22 | -0.11 | -0.05 | 0.00 | 0.10 | 0.55 |
| 0.30 | -0.05 | 0.09 | -1.31 | 29.78 | -0.81 | -0.19 | -0.09 | -0.04 | 0.00 | 0.08 | 0.47 |
| 0.35 | -0.03 | 0.07 | -1.29 | 28.67 | -0.61 | -0.15 | -0.07 | -0.03 | 0.00 | 0.07 | 0.36 |
| 0.40 | -0.02 | 0.05 | -1.22 | 28.34 | -0.42 | -0.09 | -0.04 | -0.02 | 0.01 | 0.05 | 0.25 |
| 0.45 | -0.00 | 0.02 | -0.85 | 28.53 | -0.17 | -0.03 | -0.01 | -0.00 | 0.01 | 0.03 | 0.12 |
| 0.50 | -0.00 | 0.02 | -0.81 | 23.15 | -0.14 | -0.03 | -0.01 | -0.00 | 0.01 | 0.02 | 0.09 |
| 0.55 | -0.00 | 0.02 | -0.93 | 25.86 | -0.14 | -0.03 | -0.01 | -0.00 | 0.01 | 0.02 | 0.09 |
| 0.60 | -0.00 | 0.02 | -1.35 | 32.91 | -0.18 | -0.03 | -0.01 | -0.00 | 0.01 | 0.02 | 0.10 |
| 0.65 | -0.00 | 0.04 | -1.59 | 40.01 | -0.42 | -0.07 | -0.02 | 0.00 | 0.02 | 0.06 | 0.25 |
| 0.70 | 0.01 | 0.06 | -1.31 | 33.67 | -0.61 | -0.09 | -0.02 | 0.01 | 0.04 | 0.10 | 0.39 |
| 0.75 | 0.02 | 0.08 | -1.24 | 35.05 | -0.76 | -0.10 | -0.02 | 0.02 | 0.07 | 0.15 | 0.51 |
| 0.80 | 0.04 | 0.10 | -1.02 | 31.07 | -0.86 | -0.10 | -0.01 | 0.04 | 0.09 | 0.19 | 0.61 |
| 0.85 | 0.06 | 0.11 | -0.79 | 26.85 | -0.90 | -0.10 | 0.00 | 0.06 | 0.12 | 0.23 | 0.72 |
| 0.90 | 0.08 | 0.12 | -0.59 | 25.00 | -0.93 | -0.09 | 0.01 | 0.08 | 0.15 | 0.28 | 0.83 |
| 0.95 | 0.11 | 0.13 | -0.28 | 21.59 | -0.90 | -0.08 | 0.03 | 0.10 | 0.17 | 0.32 | 0.92 |

# TABLE A5

## Summary Statistics of Stock-Level Variables

The table provides time-series averages of monthly cross-sectional statistics of stock-level variables used in our analysis. We also report time-series averages of cross-sectional correlations between ΔCIQ betas and these characteristics. The sample covers period between January 1968 and December 2024.

| | $\beta^{CIQ}_{LT}$ | $\beta^{CIQ}_{C}$ | $C\beta^{CIQ}_{UT}$ | Mean | St. Dev. | Skewness | Kurtosis | Min. | 5% | 25 % | Median | 75% | 95% | Max. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Panel A:*** Firm Characteristics | | | | | | | | | | | | | | |
| Size | 0.00 | 0.00 | -0.02 | 3725.16 | 17615.94 | 13.95 | 285.74 | 1.67 | 16.42 | 93.11 | 405.45 | 1683.60 | 14345.46 | 445702.01 |
| Book-to-price | 0.08 | 0.01 | -0.03 | 0.88 | 1.11 | 11.36 | 362.68 | 0.00 | 0.15 | 0.42 | 0.71 | 1.09 | 2.06 | 33.31 |
| Net payout yield | 0.06 | 0.01 | -0.02 | 0.00 | 0.42 | -10.12 | 805.13 | -18.16 | -0.11 | -0.00 | 0.01 | 0.04 | 0.10 | 2.94 |
| Turnover | -0.12 | -0.02 | 0.03 | 1.21 | 6.09 | 13.46 | 426.80 | 0.00 | 0.08 | 0.33 | 0.68 | 1.25 | 3.22 | 259.46 |
| Illiquidity | 0.04 | -0.00 | -0.01 | 0.03 | 0.22 | 18.28 | 564.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.13 | 7.17 |
| Investment | -0.05 | -0.01 | -0.00 | 0.15 | 0.62 | 15.29 | 492.33 | -0.81 | -0.17 | -0.00 | 0.07 | 0.18 | 0.61 | 21.70 |
| Profit | -0.01 | 0.01 | 0.01 | 0.32 | 0.29 | 0.06 | 33.36 | -2.25 | 0.02 | 0.12 | 0.29 | 0.46 | 0.81 | 2.53 |
| ***Panel B:*** Risk Characteristics | | | | | | | | | | | | | | |
| $\beta^{CAPM}$ | -0.24 | -0.03 | 0.11 | 0.85 | 0.55 | 0.45 | 4.53 | -1.36 | 0.05 | 0.47 | 0.80 | 1.18 | 1.83 | 3.50 |
| SKEW | 0.00 | -0.00 | -0.01 | 0.48 | 1.30 | 2.12 | 22.50 | -8.30 | -0.98 | -0.03 | 0.34 | 0.81 | 2.36 | 13.02 |
| ISKEW | 0.00 | -0.00 | -0.01 | 0.52 | 1.40 | 1.73 | 19.52 | -8.78 | -1.17 | -0.04 | 0.38 | 0.91 | 2.58 | 13.04 |
| IVOL | -0.06 | -0.03 | 0.03 | 0.02 | 0.02 | 4.40 | 71.37 | 0.00 | 0.01 | 0.01 | 0.02 | 0.03 | 0.05 | 0.30 |
| ***Panel C:*** Non-Linear Risk Measures | | | | | | | | | | | | | | |
| CKT | -0.14 | 0.01 | 0.09 | 1.78 | 1.11 | 0.06 | 2.82 | -2.20 | 0.04 | 0.98 | 1.76 | 2.54 | 3.64 | 5.23 |
| CSK | 0.01 | 0.01 | 0.02 | -0.10 | 0.16 | 0.09 | 3.44 | -0.73 | -0.36 | -0.21 | -0.10 | 0.01 | 0.16 | 0.59 |
| MOM | -0.01 | 0.00 | -0.01 | 0.15 | 0.51 | 3.69 | 54.30 | -0.85 | -0.42 | -0.12 | 0.08 | 0.31 | 0.93 | 8.34 |
| STR | 0.00 | -0.00 | -0.01 | 0.01 | 0.13 | 2.55 | 45.69 | -0.56 | -0.16 | -0.05 | 0.00 | 0.07 | 0.21 | 1.78 |
| HTCR | -0.05 | 0.01 | 0.03 | -0.09 | 0.21 | -2.56 | 19.69 | -2.12 | -0.46 | -0.17 | -0.05 | 0.04 | 0.14 | 0.56 |
| MCRASH | -0.05 | -0.01 | 0.02 | 0.08 | 0.03 | 0.22 | 2.91 | 0.00 | 0.03 | 0.06 | 0.08 | 0.10 | 0.14 | 0.20 |
| $\beta^{CIV}$ | -0.36 | -0.08 | 0.24 | -0.05 | 0.48 | -0.19 | 14.42 | -4.12 | -0.82 | -0.29 | -0.03 | 0.21 | 0.66 | 3.40 |
| $\beta^{PCA-SQ}$ | -0.38 | 0.04 | 0.47 | 0.00 | 0.00 | -1.44 | 109.84 | -0.03 | -0.00 | -0.00 | 0.00 | 0.00 | 0.00 | 0.02 |
| $\beta^{VIX}$ | -0.01 | -0.00 | -0.00 | 0.00 | 0.01 | 0.18 | 54.68 | -0.13 | -0.02 | -0.01 | 0.00 | 0.01 | 0.02 | 0.14 |
| $\beta^{down}$ | -0.18 | -0.03 | 0.08 | 0.92 | 0.70 | 0.21 | 7.42 | -3.20 | -0.08 | 0.47 | 0.86 | 1.31 | 2.12 | 4.74 |
| $\beta^{tail}$ | -0.01 | 0.08 | 0.14 | 0.13 | 0.60 | -0.20 | 15.00 | -4.99 | -0.74 | -0.15 | 0.11 | 0.40 | 1.06 | 3.98 |

## I. Bivariate Portfolio Sorts

# TABLE A6

## Dependent Bivariate Portfolio Sorts

The table reports annualized out-of-sample excess returns of portfolios double-sorted on the exposure to the lower-tail $\Delta$CIQ factor and a control variable. In Panel A, the portfolios are constructed by first sorting the stocks into deciles based on a control variable and then within each portfolio we sort stocks into deciles based on the exposure to the lower-tail $\Delta$CIQ factor. Final portfolio returns are calculated by averaging the returns across the control deciles for every decile of the $\beta_{LT}^{CIQ}$. This procedure yields spread in the exposure to the lower-tail $\Delta$CIQ factor, while holding control variable approximately constant across portfolios. In Panel B, we repeat the procedure by sorting stocks into quintiles. The portfolios are formed every month and returns within portfolios are equally weighted. We also report returns of the high minus low portfolios, annualized alphas with respect to the six-factor model of Fama and French (2018) and their $t$-statistics using the robust standard errors suggested by Newey and West (1987) with six lags. The return sample covers period between January 1968 and December 2024, with the exception of the case of $\beta^{VIX}$, which begins in February 1990. Each month, we use all the CRSP stocks for which at least 48 monthly observations are available over the last 60 months and exclude penny stocks with prices of $1 or less.

| | $\beta^{CAPM}$ | Size | B/P | MOM | STR | IVOL | ISKEW | CSK | CKT | $\beta^{down}$ | $\beta^{PCA\text{-}SQ}$ | $\beta^{CIV}$ | $\beta^{tail}$ | $\beta^{VIX}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Panel A:*** Decile Sorts | | | | | | | | | | | | | | |
| Low | 6.82 | 4.85 | 5.99 | 5.51 | 5.66 | 6.71 | 4.54 | 4.64 | 5.27 | 5.89 | 6.00 | 6.34 | 5.55 | 6.48 |
| 2 | 8.59 | 7.53 | 7.54 | 8.11 | 8.94 | 7.77 | 8.09 | 8.23 | 7.53 | 8.20 | 8.14 | 9.13 | 7.78 | 9.32 |
| 3 | 9.38 | 9.66 | 9.86 | 8.80 | 9.44 | 8.65 | 8.87 | 8.98 | 9.43 | 9.09 | 8.88 | 9.00 | 9.06 | 9.97 |
| 4 | 9.21 | 9.37 | 9.19 | 9.57 | 9.52 | 9.40 | 9.57 | 9.58 | 9.92 | 9.30 | 9.84 | 9.68 | 9.41 | 10.97 |
| 5 | 9.40 | 9.88 | 9.37 | 9.55 | 10.38 | 9.34 | 10.14 | 9.60 | 9.20 | 9.64 | 9.24 | 9.68 | 9.63 | 10.44 |
| 6 | 9.66 | 10.19 | 9.89 | 10.24 | 9.59 | 9.61 | 10.54 | 10.46 | 10.06 | 9.21 | 10.29 | 10.30 | 10.67 | 10.98 |
| 7 | 9.88 | 10.75 | 10.14 | 10.65 | 10.63 | 10.97 | 10.03 | 9.97 | 10.47 | 10.42 | 10.05 | 10.46 | 10.61 | 12.06 |
| 8 | 10.48 | 11.16 | 10.56 | 9.88 | 9.87 | 10.21 | 11.13 | 10.34 | 10.86 | 10.58 | 10.47 | 10.41 | 9.98 | 12.37 |
| 9 | 11.22 | 11.60 | 11.31 | 11.35 | 11.19 | 11.16 | 11.34 | 11.88 | 11.56 | 11.41 | 11.09 | 10.40 | 11.72 | 13.77 |
| High | 11.69 | 10.62 | 11.74 | 11.87 | 10.31 | 11.76 | 11.35 | 11.83 | 11.24 | 11.76 | 11.56 | 11.02 | 11.14 | 14.34 |
| High - Low | 4.87 | 5.76 | 5.75 | 6.36 | 4.65 | 5.05 | 6.81 | 7.19 | 5.97 | 5.87 | 5.55 | 4.68 | 5.60 | 7.85 |
| $t$-statistic | (3.64) | (3.45) | (3.77) | (4.29) | (3.02) | (3.79) | (4.02) | (4.34) | (3.86) | (4.25) | (3.91) | (3.19) | (3.63) | (3.99) |
| $\alpha^{FF6}$ | 6.11 | 6.52 | 7.36 | 7.70 | 5.24 | 5.99 | 7.64 | 8.07 | 6.75 | 7.23 | 6.06 | 6.76 | 6.93 | 7.20 |
| $t$-statistic | (4.20) | (4.39) | (4.73) | (5.71) | (3.73) | (5.07) | (4.84) | (5.04) | (4.66) | (4.84) | (4.17) | (4.87) | (4.74) | (3.74) |
| ***Panel B:*** Quintile Sorts | | | | | | | | | | | | | | |
| Low | 7.60 | 6.09 | 6.79 | 6.66 | 7.10 | 7.22 | 6.28 | 6.31 | 6.33 | 6.87 | 7.17 | 7.60 | 6.57 | 7.86 |
| 2 | 9.17 | 9.71 | 9.55 | 9.46 | 9.54 | 9.08 | 9.20 | 9.34 | 9.49 | 9.33 | 8.75 | 9.28 | 9.11 | 10.29 |
| 3 | 9.59 | 9.92 | 9.55 | 9.70 | 9.96 | 9.39 | 10.12 | 10.13 | 9.79 | 9.56 | 10.23 | 9.85 | 10.29 | 11.05 |
| 4 | 10.31 | 10.83 | 10.46 | 10.33 | 10.40 | 10.62 | 10.72 | 10.18 | 10.75 | 10.35 | 10.27 | 10.60 | 10.42 | 11.92 |
| High | 11.49 | 11.23 | 11.46 | 11.60 | 10.76 | 11.46 | 11.46 | 11.78 | 11.38 | 11.63 | 11.35 | 10.87 | 11.37 | 14.20 |
| High - Low | 3.88 | 5.14 | 4.68 | 4.93 | 3.66 | 4.24 | 5.18 | 5.47 | 5.05 | 4.75 | 4.18 | 3.27 | 4.80 | 6.34 |
| $t$-statistic | (3.46) | (3.78) | (3.60) | (3.96) | (2.79) | (3.77) | (3.73) | (3.94) | (3.76) | (4.11) | (3.79) | (2.74) | (3.72) | (4.08) |
| $\alpha^{FF6}$ | 5.12 | 6.23 | 6.26 | 6.41 | 4.69 | 5.24 | 6.47 | 6.83 | 6.20 | 6.02 | 4.70 | 5.12 | 6.28 | 6.26 |
| $t$-statistic | (4.31) | (5.19) | (4.82) | (5.67) | (3.94) | (5.19) | (5.16) | (5.17) | (4.95) | (5.01) | (4.24) | (4.87) | (5.27) | (4.08) |

## J. Predictability of the CIQ Premium

TABLE A7

### Predicting the Lower-Tail △CIQ Premium

The table reports predictive regressions of the high-minus-low equal-weighted portfolio returns from decile sorts on the exposures to the $\Delta CIQ^{LT}$ factor. We regress the portfolio's one-month-ahead returns on the lagged market return ($Mkt$), on the CBOE variance factor ($VIX^2$) and on its two components: the conditional variance of stock returns (CV) and the equity variance premium (VP) based on the procedure of Bekaert and Hoerova (2014). Coefficients are scaled to capture the effect of one-standard-deviation increase in the factor on the annualized portfolio return in percentage points. The $t$-statistics (in parentheses) are based on standard errors of Newey and West (1994). The data cover the period between February 1990 and January 2022.

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $VIX^2$ | 6.10 | | | | | |
| | (1.90) | | | | | |
| $VP$ | | 7.28 | | | 8.61 | 7.81 |
| | | (2.27) | | | (2.71) | (2.34) |
| $CV$ | | | 4.30 | | -1.86 | 1.23 |
| | | | (1.37) | | (-0.84) | (0.43) |
| $Mkt$ | | | | 2.74 | | 5.36 |
| | | | | (1.14) | | (2.23) |
| $R^2$ IS | 2.22 | 3.16 | 1.10 | 0.45 | 3.26 | 4.58 |
| $R^2$ OOS | 0.90 | 2.44 | -0.77 | -2.45 | -0.10 | -0.83 |
| $R^2$ OOS CT | 1.49 | 2.44 | 0.42 | -0.97 | 2.34 | 2.00 |

## K. CIQ Premia

TABLE A8

**Decile Portfolios Sorted on the Exposure to the $\Delta$CIQ$(\tau)$**

The table contains annualized out-of-sample excess returns of ten portfolios sorted on the exposure to the $\Delta$CIQ($\tau$) factors. We also report returns of the high minus low (H - L) portfolios, their $t$-statistics, and annualized alphas with respect to the six-factor model of Fama and French (2018). All $t$-statistics are based on the correction of Newey and West (1994). The data cover the period from January 1968 to December 2024. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months, and exclude penny stocks with prices of $1 or less.

| $\tau$ | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | H - L | $t$-stat | $\alpha$ | $t$-stat |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | *Equal-weighted* | | | | | | | |
| 0.05 | 5.68 | 8.17 | 9.25 | 9.51 | 9.93 | 9.55 | 10.08 | 10.95 | 11.55 | 10.87 | 5.19 | 2.97 | 5.35 | 3.05 |
| 0.10 | 5.14 | 7.91 | 8.76 | 9.59 | 9.50 | 10.28 | 10.68 | 10.72 | 11.98 | 10.96 | 5.82 | 3.29 | 6.07 | 3.68 |
| 0.15 | 4.78 | 8.02 | 9.13 | 9.79 | 10.01 | 9.69 | 10.60 | 10.72 | 11.22 | 11.58 | 6.80 | 3.76 | 7.13 | 4.25 |
| 0.20 | 4.36 | 7.94 | 9.47 | 9.63 | 9.46 | 10.36 | 10.41 | 10.74 | 11.39 | 11.77 | 7.41 | 4.30 | 8.31 | 4.97 |
| 0.25 | 4.78 | 7.76 | 9.26 | 9.93 | 10.20 | 10.15 | 10.41 | 10.45 | 11.06 | 11.54 | 6.76 | 4.09 | 7.86 | 5.10 |
| 0.30 | 4.86 | 7.98 | 9.14 | 9.89 | 10.03 | 10.51 | 10.35 | 10.87 | 10.64 | 11.27 | 6.41 | 3.90 | 7.86 | 5.05 |
| 0.35 | 4.93 | 8.37 | 9.63 | 9.65 | 9.74 | 10.47 | 10.46 | 10.40 | 11.00 | 10.89 | 5.96 | 3.83 | 7.46 | 4.81 |
| 0.40 | 5.06 | 8.57 | 9.49 | 9.56 | 9.73 | 10.28 | 10.77 | 10.84 | 10.72 | 10.53 | 5.46 | 3.72 | 6.79 | 4.60 |
| 0.45 | 5.80 | 7.95 | 10.31 | 9.34 | 10.00 | 10.28 | 10.49 | 10.46 | 10.71 | 10.19 | 4.38 | 2.52 | 4.80 | 2.60 |
| 0.50 | 6.90 | 9.05 | 10.35 | 9.05 | 10.23 | 9.85 | 10.31 | 10.24 | 10.30 | 9.26 | 2.36 | 1.36 | 1.68 | 0.85 |
| 0.55 | 6.36 | 8.85 | 10.03 | 9.27 | 10.18 | 9.99 | 10.43 | 10.28 | 10.53 | 9.61 | 3.25 | 1.86 | 2.27 | 1.09 |
| 0.60 | 7.16 | 9.26 | 10.92 | 9.61 | 10.32 | 10.00 | 9.93 | 9.58 | 9.95 | 8.82 | 1.66 | 1.01 | 0.95 | 0.51 |
| 0.65 | 6.84 | 9.89 | 9.69 | 10.31 | 9.77 | 9.62 | 9.97 | 10.08 | 10.24 | 9.13 | 2.29 | 1.51 | 1.73 | 1.08 |
| 0.70 | 7.80 | 9.37 | 10.39 | 10.26 | 9.49 | 9.72 | 10.05 | 9.45 | 10.21 | 8.79 | 0.99 | 0.63 | -0.01 | -0.01 |
| 0.75 | 8.26 | 9.79 | 10.35 | 9.78 | 9.81 | 9.88 | 9.81 | 9.78 | 9.04 | 9.03 | 0.77 | 0.50 | -0.49 | -0.30 |
| 0.80 | 8.86 | 10.09 | 10.46 | 9.62 | 9.73 | 9.70 | 9.90 | 9.00 | 9.74 | 8.43 | -0.42 | -0.26 | -1.60 | -0.92 |
| 0.85 | 9.07 | 10.28 | 9.79 | 10.03 | 9.67 | 9.59 | 9.77 | 9.69 | 9.02 | 8.63 | -0.44 | -0.26 | -1.65 | -0.90 |
| 0.90 | 9.31 | 10.25 | 9.88 | 9.97 | 9.33 | 10.22 | 10.04 | 9.34 | 8.76 | 8.43 | -0.89 | -0.50 | -2.16 | -1.18 |
| 0.95 | 9.65 | 10.44 | 10.28 | 9.67 | 10.03 | 9.36 | 10.41 | 8.78 | 9.14 | 7.77 | -1.88 | -0.97 | -3.20 | -1.75 |
| | | | | | | | *Value-weighted* | | | | | | | |
| 0.05 | 6.39 | 5.95 | 6.45 | 7.62 | 6.64 | 7.58 | 7.52 | 9.51 | 9.96 | 8.75 | 2.36 | 0.96 | 0.70 | 0.28 |
| 0.10 | 5.16 | 6.12 | 5.98 | 7.81 | 6.80 | 7.95 | 7.83 | 8.66 | 10.35 | 8.96 | 3.80 | 1.51 | 3.02 | 1.22 |
| 0.15 | 4.41 | 6.59 | 7.11 | 6.79 | 7.47 | 7.75 | 8.84 | 7.49 | 10.26 | 9.76 | 5.35 | 2.05 | 5.21 | 2.03 |
| 0.20 | 3.35 | 7.29 | 6.65 | 6.56 | 7.17 | 8.11 | 7.98 | 8.24 | 9.81 | 10.73 | 7.37 | 2.73 | 7.86 | 2.84 |
| 0.25 | 4.40 | 6.68 | 6.80 | 6.88 | 8.15 | 7.84 | 7.90 | 7.30 | 9.47 | 10.35 | 5.95 | 2.27 | 6.57 | 2.43 |
| 0.30 | 4.05 | 7.65 | 6.98 | 7.36 | 7.71 | 8.07 | 7.51 | 7.06 | 8.73 | 10.23 | 6.18 | 2.27 | 6.69 | 2.37 |
| 0.35 | 5.18 | 7.16 | 7.45 | 7.36 | 7.22 | 8.19 | 7.37 | 6.77 | 8.42 | 9.86 | 4.68 | 1.77 | 5.83 | 2.18 |
| 0.40 | 4.81 | 7.57 | 6.50 | 8.16 | 7.56 | 7.95 | 6.73 | 7.37 | 7.47 | 9.99 | 5.18 | 2.15 | 6.96 | 2.83 |
| 0.45 | 4.43 | 5.41 | 7.10 | 6.92 | 7.76 | 8.16 | 8.26 | 7.29 | 7.58 | 10.13 | 5.70 | 2.44 | 6.55 | 2.71 |
| 0.50 | 5.02 | 5.18 | 6.63 | 6.58 | 7.51 | 8.43 | 8.15 | 7.73 | 6.49 | 9.38 | 4.36 | 1.81 | 3.99 | 1.56 |
| 0.55 | 5.03 | 4.72 | 6.90 | 6.21 | 7.85 | 7.60 | 8.31 | 8.53 | 6.86 | 9.06 | 4.03 | 1.60 | 3.46 | 1.22 |
| 0.60 | 4.61 | 6.73 | 7.90 | 6.50 | 7.43 | 7.47 | 8.07 | 7.50 | 6.94 | 7.90 | 3.29 | 1.40 | 2.45 | 1.02 |
| 0.65 | 4.62 | 7.98 | 7.98 | 8.94 | 7.32 | 7.17 | 6.40 | 7.22 | 6.92 | 7.32 | 2.70 | 1.22 | 2.30 | 0.95 |
| 0.70 | 5.64 | 7.48 | 8.84 | 8.98 | 7.55 | 7.34 | 6.93 | 6.89 | 6.49 | 7.08 | 1.44 | 0.59 | 0.87 | 0.36 |
| 0.75 | 4.86 | 8.91 | 7.78 | 8.74 | 7.98 | 7.58 | 6.48 | 6.63 | 5.38 | 6.69 | 1.83 | 0.76 | 0.90 | 0.37 |
| 0.80 | 7.23 | 9.11 | 8.15 | 7.07 | 8.11 | 7.84 | 6.81 | 6.37 | 5.87 | 6.33 | -0.90 | -0.36 | 0.03 | 0.01 |
| 0.85 | 6.91 | 8.92 | 7.71 | 6.76 | 8.25 | 7.22 | 7.36 | 7.01 | 6.01 | 5.63 | -1.27 | -0.51 | 0.08 | 0.03 |
| 0.90 | 6.74 | 8.84 | 7.44 | 6.75 | 7.98 | 7.80 | 6.82 | 6.69 | 6.79 | 5.49 | -1.24 | -0.50 | 0.03 | 0.01 |
| 0.95 | 5.99 | 8.62 | 7.72 | 8.08 | 7.15 | 7.39 | 7.68 | 6.16 | 6.03 | 5.14 | -0.85 | -0.31 | 0.09 | 0.04 |

TABLE A9

## Quintile Portfolios Sorted on the Exposure to the $\Delta$CIQ$(\tau)$

The table contains annualized out-of-sample excess returns of five portfolios sorted on the exposure to the $\Delta$CIQ($\tau$) factors. We also report returns of the high minus low (H - L) portfolios, their $t$-statistics, and annualized alphas with respect to the six-factor model of Fama and French (2018). All $t$-statistics are based on the correction of Newey and West (1994). The data cover the period from January 1968 to December 2024. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months, and exclude penny stocks with prices of $1 or less.

| $\tau$ | Low | 2 | 3 | 4 | High | H - L | $t$-stat | $\alpha$ | $t$-stat |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *Equal-weighted* | | | | | |
| 0.05 | 6.93 | 9.38 | 9.74 | 10.51 | 11.21 | 4.28 | 2.93 | 4.91 | 3.36 |
| 0.10 | 6.53 | 9.17 | 9.89 | 10.70 | 11.47 | 4.95 | 3.28 | 5.66 | 4.09 |
| 0.15 | 6.40 | 9.46 | 9.85 | 10.66 | 11.40 | 5.00 | 3.33 | 5.89 | 4.33 |
| 0.20 | 6.15 | 9.55 | 9.91 | 10.57 | 11.58 | 5.43 | 3.76 | 6.83 | 5.11 |
| 0.25 | 6.27 | 9.59 | 10.18 | 10.43 | 11.30 | 5.03 | 3.67 | 6.65 | 5.50 |
| 0.30 | 6.42 | 9.51 | 10.27 | 10.61 | 10.96 | 4.54 | 3.43 | 6.23 | 5.25 |
| 0.35 | 6.65 | 9.64 | 10.11 | 10.43 | 10.94 | 4.29 | 3.49 | 6.07 | 5.43 |
| 0.40 | 6.81 | 9.52 | 10.01 | 10.80 | 10.62 | 3.81 | 3.33 | 5.42 | 5.07 |
| 0.45 | 6.88 | 9.82 | 10.14 | 10.48 | 10.45 | 3.57 | 2.72 | 4.18 | 3.06 |
| 0.50 | 7.97 | 9.70 | 10.04 | 10.27 | 9.78 | 1.81 | 1.36 | 1.56 | 1.04 |
| 0.55 | 7.60 | 9.65 | 10.09 | 10.36 | 10.07 | 2.47 | 1.87 | 1.94 | 1.25 |
| 0.60 | 8.21 | 10.26 | 10.16 | 9.76 | 9.38 | 1.17 | 0.98 | 0.87 | 0.64 |
| 0.65 | 8.36 | 10.00 | 9.70 | 10.02 | 9.68 | 1.32 | 1.12 | 1.18 | 0.96 |
| 0.70 | 8.59 | 10.32 | 9.61 | 9.75 | 9.50 | 0.92 | 0.75 | 0.26 | 0.20 |
| 0.75 | 9.03 | 10.07 | 9.85 | 9.79 | 9.03 | 0.01 | 0.01 | -0.85 | -0.64 |
| 0.80 | 9.48 | 10.03 | 9.72 | 9.45 | 9.09 | -0.39 | -0.29 | -1.36 | -0.99 |
| 0.85 | 9.67 | 9.91 | 9.63 | 9.73 | 8.82 | -0.85 | -0.60 | -1.91 | -1.33 |
| 0.90 | 9.78 | 9.93 | 9.77 | 9.69 | 8.59 | -1.19 | -0.79 | -2.31 | -1.59 |
| 0.95 | 10.05 | 9.98 | 9.69 | 9.60 | 8.45 | -1.59 | -0.98 | -2.90 | -2.03 |
| | | | | *Value-weighted* | | | | | |
| 0.05 | 6.03 | 7.01 | 6.93 | 8.38 | 9.46 | 3.42 | 1.65 | 2.43 | 1.28 |
| 0.10 | 5.77 | 6.96 | 7.40 | 8.11 | 9.94 | 4.17 | 2.04 | 3.75 | 1.96 |
| 0.15 | 5.91 | 6.91 | 7.62 | 8.21 | 10.02 | 4.12 | 2.00 | 4.28 | 2.31 |
| 0.20 | 5.90 | 6.61 | 7.69 | 7.98 | 9.96 | 4.06 | 2.02 | 4.82 | 2.71 |
| 0.25 | 6.01 | 6.92 | 8.02 | 7.32 | 9.61 | 3.60 | 1.84 | 4.49 | 2.51 |
| 0.30 | 6.37 | 6.99 | 8.02 | 7.11 | 9.13 | 2.77 | 1.45 | 3.72 | 2.08 |
| 0.35 | 6.48 | 7.28 | 7.63 | 6.95 | 8.87 | 2.38 | 1.27 | 3.84 | 2.19 |
| 0.40 | 6.52 | 7.29 | 7.55 | 6.98 | 8.24 | 1.72 | 0.95 | 3.25 | 1.83 |
| 0.45 | 4.92 | 6.88 | 7.86 | 7.64 | 8.82 | 3.90 | 2.31 | 4.96 | 2.72 |
| 0.50 | 5.06 | 6.55 | 7.89 | 8.04 | 7.57 | 2.51 | 1.46 | 2.25 | 1.13 |
| 0.55 | 4.73 | 6.46 | 7.65 | 8.40 | 7.62 | 2.89 | 1.70 | 2.75 | 1.40 |
| 0.60 | 6.07 | 6.99 | 7.43 | 7.66 | 7.23 | 1.16 | 0.70 | 0.74 | 0.40 |
| 0.65 | 6.80 | 8.40 | 7.13 | 6.61 | 7.21 | 0.42 | 0.25 | 0.66 | 0.36 |
| 0.70 | 6.97 | 8.95 | 7.31 | 6.85 | 6.70 | -0.27 | -0.16 | -0.64 | -0.36 |
| 0.75 | 7.78 | 8.44 | 7.61 | 6.53 | 6.04 | -1.74 | -1.04 | -2.24 | -1.24 |
| 0.80 | 8.34 | 7.75 | 7.74 | 6.61 | 6.34 | -2.00 | -1.12 | -2.15 | -1.13 |
| 0.85 | 8.53 | 7.31 | 7.63 | 7.21 | 6.07 | -2.47 | -1.31 | -2.29 | -1.20 |
| 0.90 | 8.24 | 6.88 | 8.02 | 6.76 | 6.51 | -1.73 | -0.88 | -1.11 | -0.59 |
| 0.95 | 7.83 | 8.15 | 7.30 | 6.98 | 5.82 | -2.01 | -0.94 | -1.46 | -0.76 |

## TABLE A10

# Correlations between the $\Delta$CIQ$(\tau)$ Premia

The table provides time-series correlations between the $\Delta$CIQ$(\tau)$ premia. We use the high-minus-low equal-weighted portfolios from decile sorts based on the exposures to the $\Delta$CIQ factors and report the correlations for range of values of $\tau$. The data cover the period from January 1968 to December 2024.

| | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | 1.00 | 0.93 | 0.90 | 0.87 | 0.82 | 0.77 | 0.69 | 0.56 | 0.25 | 0.15 | 0.16 | 0.14 | 0.04 | -0.16 | -0.32 | -0.46 | -0.55 | -0.60 | -0.66 |
| 0.10 | | 1.00 | 0.96 | 0.93 | 0.90 | 0.85 | 0.78 | 0.67 | 0.35 | 0.22 | 0.24 | 0.23 | 0.14 | -0.06 | -0.23 | -0.37 | -0.47 | -0.54 | -0.62 |
| 0.15 | | | 1.00 | 0.96 | 0.93 | 0.89 | 0.84 | 0.72 | 0.41 | 0.28 | 0.30 | 0.26 | 0.18 | -0.02 | -0.19 | -0.33 | -0.43 | -0.51 | -0.61 |
| 0.20 | | | | 1.00 | 0.97 | 0.94 | 0.88 | 0.79 | 0.44 | 0.30 | 0.32 | 0.32 | 0.26 | 0.06 | -0.12 | -0.27 | -0.38 | -0.47 | -0.58 |
| 0.25 | | | | | 1.00 | 0.97 | 0.93 | 0.84 | 0.49 | 0.33 | 0.36 | 0.36 | 0.34 | 0.13 | -0.04 | -0.20 | -0.31 | -0.41 | -0.53 |
| 0.30 | | | | | | 1.00 | 0.96 | 0.89 | 0.53 | 0.36 | 0.39 | 0.43 | 0.42 | 0.23 | 0.05 | -0.11 | -0.23 | -0.34 | -0.47 |
| 0.35 | | | | | | | 1.00 | 0.94 | 0.59 | 0.41 | 0.44 | 0.47 | 0.50 | 0.33 | 0.17 | 0.02 | -0.10 | -0.21 | -0.35 |
| 0.40 | | | | | | | | 1.00 | 0.71 | 0.51 | 0.53 | 0.57 | 0.60 | 0.46 | 0.32 | 0.18 | 0.07 | -0.04 | -0.19 |
| 0.45 | | | | | | | | | 1.00 | 0.77 | 0.77 | 0.69 | 0.58 | 0.47 | 0.42 | 0.34 | 0.28 | 0.20 | 0.09 |
| 0.50 | | | | | | | | | | 1.00 | 0.91 | 0.72 | 0.55 | 0.48 | 0.43 | 0.36 | 0.33 | 0.26 | 0.16 |
| 0.55 | | | | | | | | | | | 1.00 | 0.75 | 0.59 | 0.51 | 0.45 | 0.39 | 0.35 | 0.28 | 0.16 |
| 0.60 | | | | | | | | | | | | 1.00 | 0.75 | 0.65 | 0.56 | 0.49 | 0.42 | 0.33 | 0.17 |
| 0.65 | | | | | | | | | | | | | 1.00 | 0.92 | 0.82 | 0.73 | 0.62 | 0.50 | 0.32 |
| 0.70 | | | | | | | | | | | | | | 1.00 | 0.94 | 0.87 | 0.78 | 0.68 | 0.51 |
| 0.75 | | | | | | | | | | | | | | | 1.00 | 0.95 | 0.89 | 0.81 | 0.68 |
| 0.80 | | | | | | | | | | | | | | | | 1.00 | 0.96 | 0.90 | 0.79 |
| 0.85 | | | | | | | | | | | | | | | | | 1.00 | 0.95 | 0.86 |
| 0.90 | | | | | | | | | | | | | | | | | | 1.00 | 0.92 |
| 0.95 | | | | | | | | | | | | | | | | | | | 1.00 |

## L. $\triangle$CIQ Betas and Firm Characteristics

TABLE A11

### Cross-Sectional Determinants of Lower-Tail CIQ Exposure—Full Results

This table reports Fama–MacBeth cross-sectional regressions explaining firm-level exposure to innovations in the lower-tail CIQ factor. The dependent variable is each stock's rolling 60-month beta with respect to $\Delta CIQ^{LT}$. Firm characteristics are ranked cross-sectionally each month and linearly scaled to lie between $-1$ and 1. Regressions are estimated monthly, and reported coefficients are time-series averages. The $t$-statistics based on Newey and West (1994) are reported in parentheses. Firm-level characteristics are from Freyberger et al. (2020) and Kim et al. (2020). The sample covers the period from January 1968 to December 2018.

| | (1) Liquidity | (2) Funding | (3) Risk | (4) Fundamental | (5) Full |
|---|---|---|---|---|---|
| *Liquidity fragility* | | | | | |
| spread_mean | -0.03 | | | | |
| | (-1.17) | | | | |
| dto | -0.02 | | | | -0.01 |
| | (-7.50) | | | | (-5.66) |
| lturnover | -0.22 | | | | -0.21 |
| | (-11.08) | | | | (-10.64) |
| std_turn | 0.06 | | | | 0.09 |
| | (4.57) | | | | (7.00) |
| std_volume | 0.02 | | | | |
| | (0.58) | | | | |
| suv | 0.02 | | | | 0.01 |
| | (7.36) | | | | (3.77) |
| *Funding and financial slack* | | | | | |
| debt2p | | 0.00 | | | |
| | | (-0.31) | | | |
| roc | | -0.07 | | | -0.03 |
| | | (-5.49) | | | (-4.81) |
| c | | -0.03 | | | -0.03 |
| | | (-2.92) | | | (-3.17) |
| c2d | | 0.00 | | | |
| | | (0.13) | | | |
| free_cf | | 0.01 | | | |
| | | (0.67) | | | |
| nop | | 0.09 | | | 0.06 |
| | | (2.95) | | | (3.65) |
| d_so | | -0.01 | | | |
| | | (-0.28) | | | |
| *Risk controls* | | | | | |
| beta | | | -0.06 | | -0.03 |
| | | | (-11.23) | | (-7.53) |
| total_vol | | | -0.22 | | -0.08 |
| | | | (-8.72) | | (-4.64) |
| idio_vol | | | 0.15 | | 0.05 |
| | | | (4.97) | | (2.27) |
| ret_max | | | 0.01 | | |
| | | | (1.38) | | |
| rel_to_high_price | | | 0.05 | | 0.03 |
| | | | (6.28) | | (4.80) |
| cum_return_1_0 | | | 0.00 | | |
| | | | (-1.34) | | |
| *Fundamental controls* | | | | | |
| lme | | | | -0.02 | |
| | | | | (-0.43) | |
| beme | | | | 0.07 | |
| | | | | (1.22) | |
| e2p | | | | 0.05 | 0.01 |
| | | | | (3.94) | (0.74) |
| q | | | | -0.02 | |
| | | | | (-0.58) | |
| prof | | | | 0.00 | |
| | | | | (0.09) | |
| roa | | | | 0.03 | |
| | | | | (1.88) | |
| pm_adj | | | | -0.06 | -0.04 |
| | | | | (-9.06) | (-7.77) |
| investment | | | | -0.06 | -0.02 |
| | | | | (-5.28) | (-2.76) |
| noa | | | | 0.01 | |
| | | | | (1.79) | |
| Intercept | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 |
| | (-1.37) | (-0.56) | (-0.72) | (-0.59) | (-1.10) |
| $R^2_{adj}$ | 9.47 | 6.88 | 8.90 | 9.17 | 12.86 |
| $\bar{n}$ | 2578 | 2578 | 2578 | 2578 | 2578 |
| $T$ | 612 | 612 | 612 | 612 | 612 |

# TABLE A12

## Firm Characteristics

The table provides a list of firm characteristics of Freyberger et al. (2020) and Kim et al. (2020). We employ them to explain the firm-level exposures to the $\Delta$CIQ factors.

| | | |
|---|---|---|
| **Liquidity Fragility and Trading** | | |
| (1) | spread_mean | Average daily bid-ask spread |
| (2) | dto | De-trended Turnover - market Turnover |
| (2) | lturnover | Last month's volume to shares outstanding |
| (4) | std_turn | Standard deviation of daily turnover |
| (5) | std_volume | Standard deviation of daily volume |
| (6) | suv | Standard unexplained volume |
| **Funding and Financial Slack** | | |
| (7) | debt2p | Total debt to Size |
| (8) | roc | Size + long-term debt - total assets to cash |
| (9) | c | Cash to AT |
| (10) | c2d | Cash flow to total liabilities |
| (11) | free_cf | Free cash flow to BE |
| (12) | nop | Net payouts to Size |
| (13) | d_so | Log change in split-adjusted shares outstanding |
| **Risk Controls** | | |
| (14) | beta | CAPM beta using daily returns |
| (15) | total_vol | Standard deviation of daily returns |
| (16) | idio_vol | Idio vol of Fama-French 3 factor model |
| (17) | ret_max | Maximum daily return |
| (18) | rel_to_high_price | Price to 52 week high price |
| (19) | cum_return_1_0 | Return 1 month before prediction |
| **Fundamental Controls** | | |
| (20) | lme | Price times shares outstanding |
| (21) | beme | Book to market ratio |
| (22) | e2p | Income before extraordinary items to Size |
| (23) | q | Tobin's Q |
| (24) | prof | Gross profitability over BE |
| (25) | roa | Return on assets |
| (26) | pm_adj | Profit margin - mean PM in Fama-French 48 industry |
| (27) | investment | % change in AT |
| (28) | noa | Net-operating assets over lagged AT |

## M. Beyond ∆CIQ Beta

In this section, we extend the ∆CIQ betas to further demonstrate the importance of considering downside- and upside-specific risks and their heterogeneous implications. In particular, this approach has two objectives. First, we specifically capture additional information beyond the median dependence from the downside and upside parts of the distribution and define *the relative* ∆CIQ betas as follows:

$$\beta_{i,j}^{CIQ,rel} := \beta_{i,j}^{CIQ} - \beta_{i,C}^{CIQ}, \quad j = LT, UT \tag{34}$$

The results of the portfolio sorts on the basis of relative betas are summarized in Table A13. These results are similar to the ∆CIQ results presented above. The high-minus-low portfolio sorted on the lower-tail relative betas yields an annual excess return of 7.59% ($t = 4.27$) with a six-factor $\alpha = 8.49$ ($t = 4.90$) for the equal-weighted portfolio. In the case of the value-weighted portfolio, we obtain an annual return of 6.04% ($t = 2.22$) and $\alpha = 6.34$ ($t = 2.33$). Similarly, as in the previous section, we also present the results for the quintile sorts in Panel B and report significant abnormal returns and alphas. Moreover, the results suggest that the incremental upside exposure does not result in any premium with zero-cost portfolio returns being statistically indistinguishable from zero.

Second, to show robustness with respect to the quantile level, $\tau$, which we choose to compute the lower- and upper-tail exposures and to provide a way to aggregate the information from the downside and upside parts, we define two compressed measures. To summarize the dependence across the entire lower or upper part of the factor structure, we define the

## TABLE A13

## Portfolios Sorted on Alternative Specifications of the $\triangle$CIQ Exposures

The table reports the annualized out-of-sample excess returns of portfolios sorted on exposure to the lower-tail and upper-tail $\Delta$CIQ($\tau$) factors. Relative betas are computed by subtracting central $\Delta$CIQ exposure from either lower- and upper-tail exposures. Combination betas are obtained as average rank of either lower- and upper-tai $\Delta$CIQ exposures. Relative combination exposures are obtained by first computing relative exposures and then by averaging across ranks of either lower- or upper-tail $\Delta$ exposures. We also report the returns of zero-cost portfolios obtained by buying a high-exposure portfolio and selling a low-exposure portfolio (High - Low). The corresponding $t$-statistics (in parentheses) are computed using the robust standard errors suggested by Newey and West (1987) with six lags. Stocks are either sorted into ten portfolios in Panel A or into five portfolios in Panel B, with portfolio returns obtained by either equally weighting stock returns or value weighting by their market capitalization. The portfolios are formed each month based on the sensitivity to the $\Delta$CIQ($\tau$) factors estimated using time-series regression over the previous 60 months. The return sample covers period between January 1968 and December 2024. Each month, we use all the CRSP stocks for which at least 48 monthly observations are available over the last 60 months and exclude penny stocks with prices of $1 or less.

| | Relative | | | | Combination | | | | Rel. Comb. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Lower-Tail | | Upper-Tail | | Lower-Tail | | Upper-Tail | | Lower-Tail | | Upper-Tail | |
| | EW | VW | EW | VW | EW | VW | EW | VW | EW | VW | EW | VW |
| ***Panel A:*** Decile Sorts | | | | | | | | | | | | |
| Low | 4.40 | 4.63 | 9.00 | 5.83 | 4.90 | 3.77 | 8.60 | 5.36 | 5.17 | 5.18 | 9.46 | 7.36 |
| 2 | 8.05 | 6.45 | 10.71 | 9.92 | 7.61 | 7.66 | 9.56 | 8.25 | 7.90 | 6.88 | 10.11 | 9.01 |
| 3 | 9.63 | 6.69 | 10.31 | 7.51 | 9.27 | 6.69 | 10.02 | 6.58 | 8.86 | 6.95 | 9.57 | 8.43 |
| 4 | 9.30 | 7.44 | 9.33 | 8.05 | 9.82 | 6.59 | 10.14 | 8.47 | 9.66 | 6.90 | 9.63 | 7.09 |
| 5 | 9.40 | 7.18 | 9.37 | 8.45 | 9.92 | 7.44 | 9.80 | 8.16 | 10.12 | 8.72 | 9.62 | 7.51 |
| 6 | 10.29 | 7.82 | 10.04 | 6.48 | 10.53 | 8.79 | 9.91 | 7.96 | 10.38 | 7.33 | 9.87 | 7.63 |
| 7 | 10.50 | 8.07 | 9.59 | 7.33 | 9.76 | 7.55 | 9.63 | 6.88 | 10.21 | 8.35 | 9.17 | 7.14 |
| 8 | 10.69 | 7.99 | 9.32 | 5.98 | 10.81 | 7.76 | 9.53 | 5.93 | 10.38 | 7.17 | 9.99 | 6.93 |
| 9 | 11.27 | 10.09 | 9.34 | 6.04 | 11.53 | 9.44 | 9.24 | 6.36 | 11.32 | 9.41 | 8.95 | 5.46 |
| High | 11.99 | 10.68 | 8.51 | 6.03 | 11.38 | 10.80 | 9.09 | 7.01 | 11.54 | 10.82 | 9.16 | 6.55 |
| High - Low | 7.59 | 6.04 | -0.49 | 0.20 | 6.48 | 7.04 | 0.49 | 1.65 | 6.36 | 5.64 | -0.30 | -0.81 |
| $t$-statistic | (4.27) | (2.22) | (-0.29) | (0.08) | (3.83) | (2.73) | (0.29) | (0.74) | (3.65) | (2.23) | (-0.20) | (-0.33) |
| $\alpha^{FF6}$ | 8.49 | 6.34 | -1.48 | 1.20 | 7.53 | 7.17 | -0.66 | 2.49 | 7.56 | 5.63 | -1.50 | -0.54 |
| $t$-statistic | (4.90) | (2.33) | (-0.87) | (0.48) | (4.76) | (2.85) | (-0.35) | (1.04) | (4.52) | (2.29) | (-0.92) | (-0.20) |
| ***Panel B:*** Quintile Sorts | | | | | | | | | | | | |
| Low | 6.23 | 5.77 | 9.85 | 8.63 | 6.26 | 6.22 | 9.08 | 7.30 | 6.54 | 6.32 | 9.78 | 8.37 |
| 2 | 9.47 | 7.10 | 9.82 | 7.94 | 9.54 | 6.62 | 10.08 | 7.59 | 9.26 | 6.90 | 9.60 | 7.72 |
| 3 | 9.85 | 7.56 | 9.71 | 7.36 | 10.22 | 8.11 | 9.85 | 8.06 | 10.25 | 7.94 | 9.74 | 7.55 |
| 4 | 10.60 | 7.96 | 9.46 | 6.73 | 10.28 | 7.57 | 9.58 | 6.53 | 10.29 | 7.56 | 9.58 | 6.91 |
| High | 11.63 | 10.09 | 8.92 | 6.18 | 11.46 | 9.66 | 9.17 | 6.65 | 11.43 | 9.63 | 9.06 | 5.74 |
| High - Low | 5.40 | 4.33 | -0.93 | -2.45 | 5.20 | 3.44 | 0.09 | -0.65 | 4.89 | 3.32 | -0.73 | -2.63 |
| $t$-statistic | (3.71) | (2.09) | (-0.68) | (-1.36) | (3.77) | (1.83) | (0.07) | (-0.39) | (3.45) | (1.71) | (-0.59) | (-1.57) |
| $\alpha^{FF6}$ | 6.67 | 4.59 | -1.93 | -2.74 | 6.85 | 4.27 | -0.92 | -0.85 | 6.45 | 4.02 | -1.69 | -2.61 |
| $t$-statistic | (5.03) | (2.55) | (-1.42) | (-1.39) | (5.43) | (2.54) | (-0.65) | (-0.47) | (4.85) | (2.38) | (-1.36) | (-1.46) |

*combination* lower-tail and upper-tail $\Delta$CIQ betas as follows:

(35)
$$\beta_{i,LT}^{CIQ,comb} := \sum_{\tau \in \tau_{LT}} F\left(\beta_{i,\tau}^{CIQ}\right), \quad \tau_{LT} = \{0.05, 0.10, \ldots, 0.45\}$$
$$\beta_{i,UT}^{CIQ,comb} := \sum_{\tau \in \tau_{UT}} F\left(\beta_{i,\tau}^{CIQ}\right), \quad \tau_{LT} = \{0.55, 0.60, \ldots, 0.95\}$$

where $F\left(\beta_{i,\tau}^{CIQ}\right) = \frac{Rank(\beta_{i,\tau}^{CIQ})}{N_t+1}$ and $\beta_{i,\tau}^{CIQ}$ is estimated using the $\Delta$CIQ factor for a given quantile level $\tau$, where we use a set of $\tau$s between 0.05 and 0.45 with 0.05 increments for the lower tail and a set of $\tau$s between 0.55 and 0.95 with same increments for the upper tail. We obtain the combination lower- and upper-tail ΔCIQ betas as the average cross-sectional ranks of the ΔCIQ betas for the lower- and upper-tail $\tau$s, respectively. The results of the portfolio sorts on the basis of those betas are also summarized in Table A13. We observe that the long-short portfolios sorted on the basis of the combination lower-tail ΔCIQ betas provide significant excess annual returns of 6.48% ($t = 3.83$) and 7.04% ($t = 2.73$) using decile sorting and equal- and value-weighted schemes, respectively. On the other hand, an investment strategy based on the combination upper-tail betas does not yield significant abnormal returns when either weighting approach is used.

Finally, to summarize the relative betas through the whole downside or upside parts of the joint structure, we introduce the *relative combination* betas as follows:

$$\begin{aligned} \beta_{i,LT}^{CIQ,rel-comb} &:= \sum_{\tau \in \tau_{LT}} F\big(\beta_{i,\tau}^{CIQ,rel}\big), \quad \tau_{LT} = \{0.05, 0.10, \ldots, 0.45\} \\ \beta_{i,UT}^{CIQ,rel-comb} &:= \sum_{\tau \in \tau_{UT}} F\big(\beta_{i,\tau}^{CIQ,rel}\big), \quad \tau_{LT} = \{0.55, 0.60, \ldots, 0.95\} \end{aligned} \tag{36}$$

which are obtained as a mean cross-sectional rank of the relative betas associated with the exposure to the lower- or upper-tail ΔCIQ($\tau$) factors, respectively. We compute the relative betas with respect to the same quantiles as in the case of the combination betas. The associated returns are also summarized in Table A13. Similarly, as in the case of the relative betas, the lower-tail relative combination betas provide an investment strategy with significant abnormal returns of 6.36% ($t = 3.65$) and 5.64% ($t = 2.23$) on an annual basis using the equal- or value-weighted returns, respectively. The returns of the portfolios based on the relative combination upper-tail betas remain nonsignificant.